Effect of wire stiffness on pitch modes and correction thereof

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LIGO SUS Design Philosophy/Methodology

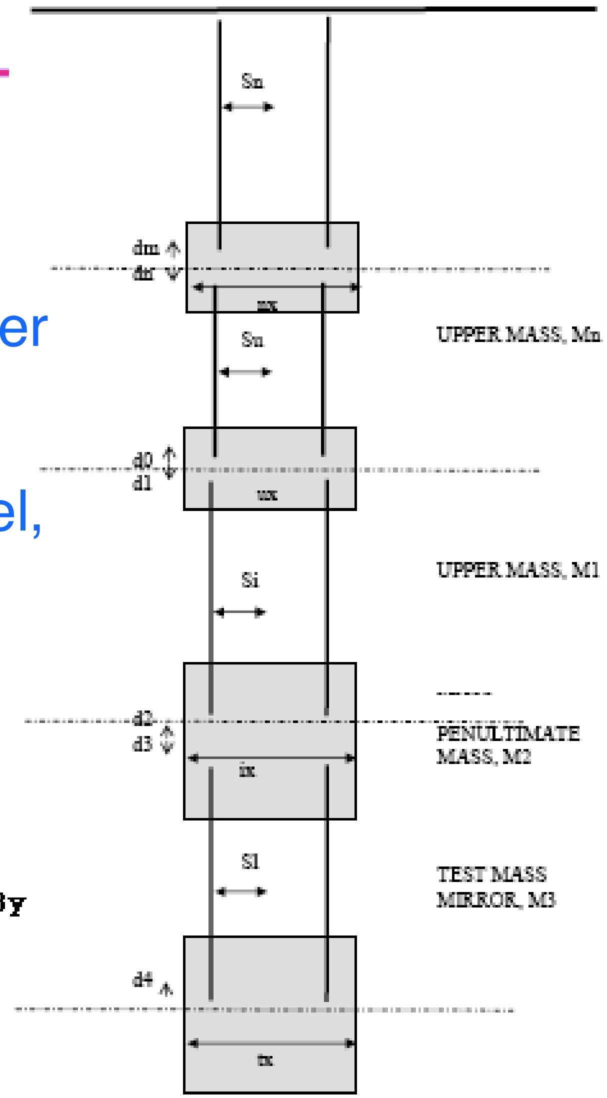
- Quad/triple pendulum has 24/18 rigid body normal modes
  - » Want all frequencies as low as possible
    - – All but highest frequency vertical and roll modes in range ≈0.4 to ≈4 Hz.
  - » Want anti-node at top mass for good local control
- Initial design uses mostly Matlab software by Calum Torrie et al. with improved stiffness matrix elements by Mark Barton
  - » Wire stiffness is neglected
  - » Inaccurate for fundamental pitch mode

Fundamental Pitch Mode

- Only 2 top wires instead of 4 => sn=0
- Fundamental pitch mode has masses moving in phase, ganged together by lower pairs of wires
- For ideally flexible wire as in Matlab model, frequency is set by gravitational restoring torque (weighted sum of d's) and sum of MOIs:

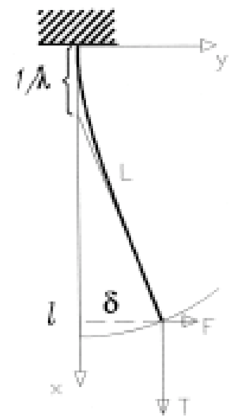
```
kk = (
     (mn+m1+m2+m3) g dm
     +(m1+m2+m3) g (dn+d0)
     +(m2+m3) g (d1+d2)        II = Iny+I1y+I2y+I3y
     +m3 g (d3+d4)
)

              ff = Sqrt[kk/II]/2/Pi
```



Wire Stiffness - Simple Pendulum Case

G. Cagnoli et al., *Physics Letters A 272 (2000): 39 – 45*



- The wire can be modeled as an elastic beam under tension (x=longitudinal, y=displacement, T=tension, Y=Young's modulus, I=moment of area, rho=density): $YIy'''' - Ty'' = \rho \frac{\partial^2 y}{\partial t^2}$
- At low frequency, $y(x) = \frac{F}{T\lambda}[e^{-\lambda x} + \lambda x - 1]$ $\quad \lambda = \sqrt{T/Y_0 I}$
- Pendulum frequency increases due to two effects
  - » Wire is effectively shorter by 0.5/lambda => extra gravitational restoring force
  - » Wire is stiff => elastic restoring force equivalent to another 0.5/ lambda of length change

Old Application To Quad

- For the quad pendulum, the d's from the Matlab model were "corrected" by amounts of 1/lambda
- But 1/lambda is typically rather larger than the raw d's:

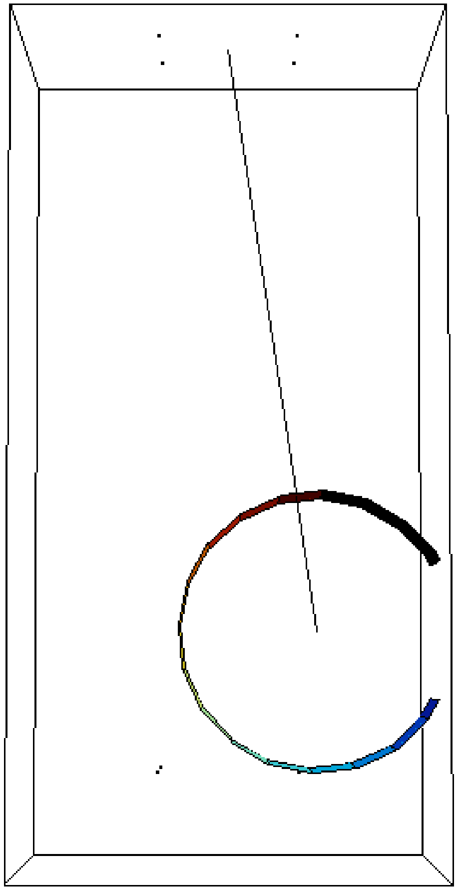
{4.97277,3.26511,2.88802,0.951584}

1/lambda in mm
for ETM prototype, T040214-01
cf. all d's = 1 mm

- If the wire stiffness effect is only slightly less than assumed, the pendulum will be unstable!
- As-built pendulum was unstable(!) - is this the problem?

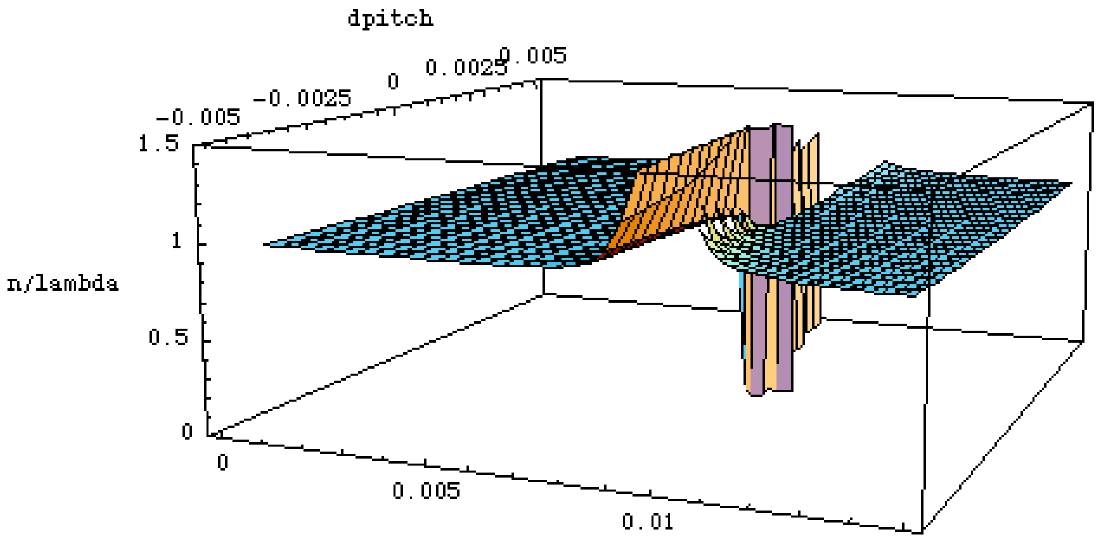
Simple Pendulum With Pitch DOF

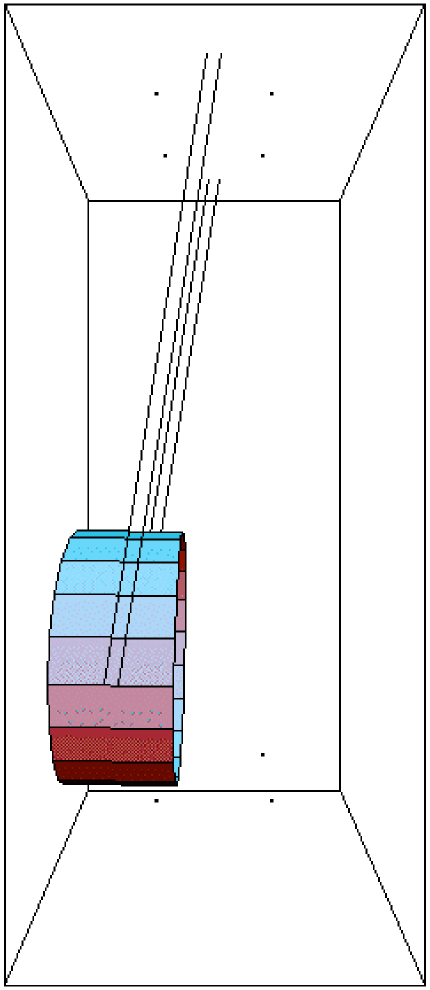
- Does a pitch mode need the same correction as a pendulum mode?
- A simple pendulum model was created to check.
- Yes, the pitch flexure correction is exactly 1/lambda.



Four-Wire Simple Pendulum

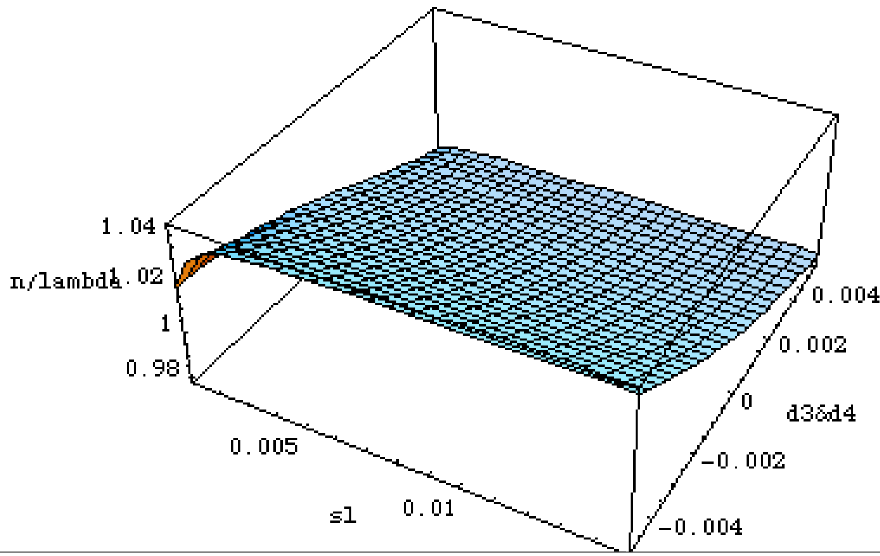
- How about a four-wire pendulum?
- The pitch frequency depends mostly on the wire elasticity, not gravity.
- Yet, the pitch flexure correction is still 1/lambda except where the pitch and pendulum frequencies are nearly equal.

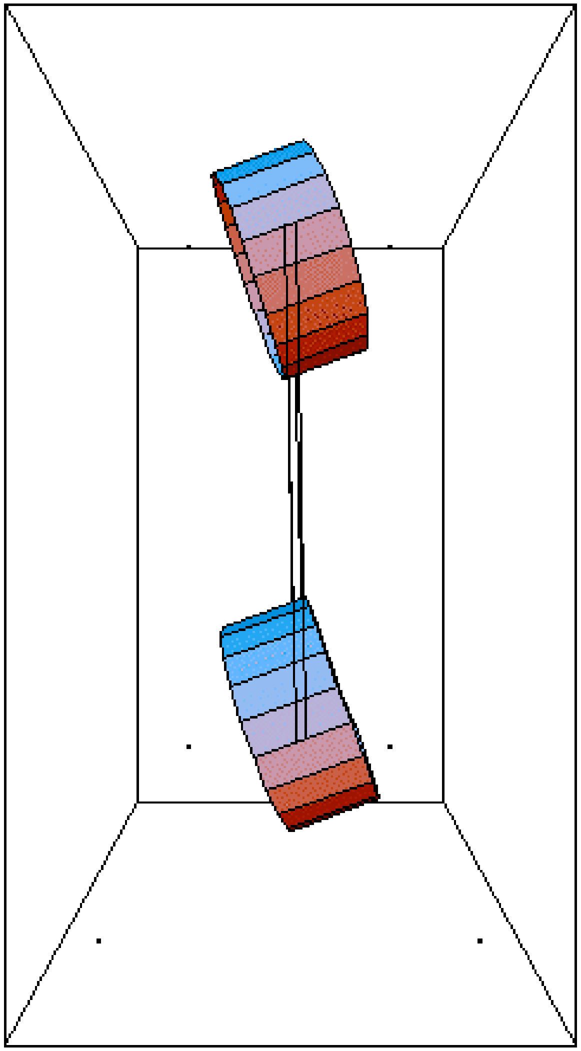




Four-Wire Two-Mass Pendulum

- How about two masses both free to pitch (the upper one hinged)?
- Yes, the flexure correction is still almost exactly 1/lambda (if applied at both top and bottom of each wire).



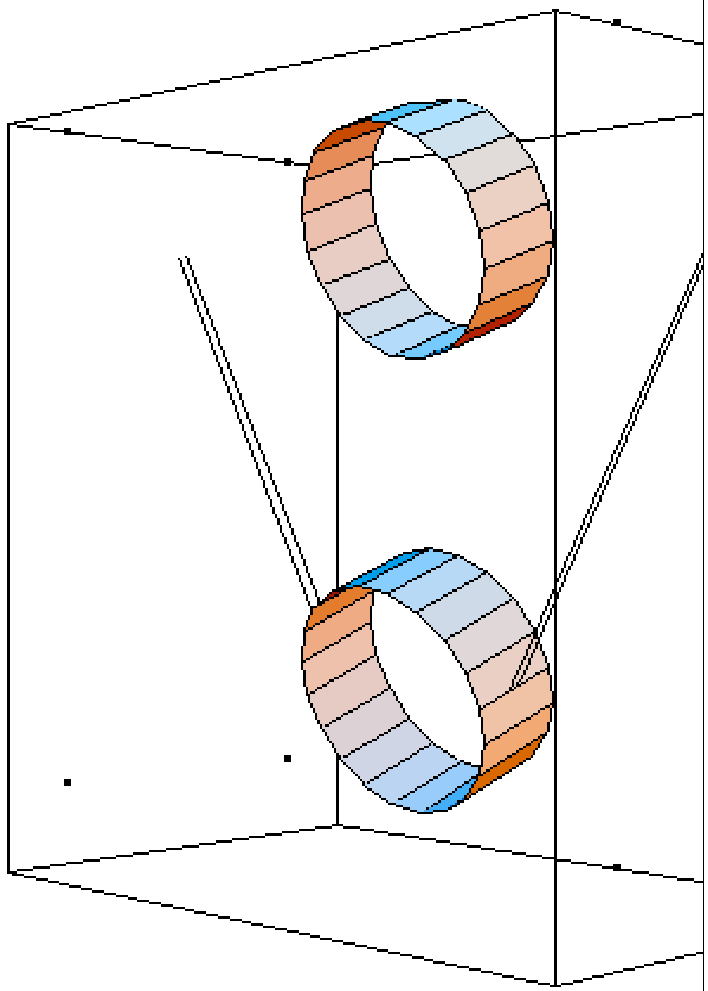


Diagonal Wires

- How about wires further apart in transverse at the top than at the bottom?
- Computation is too slow for a plot to be feasible, but spot checks suggest flexure correction is

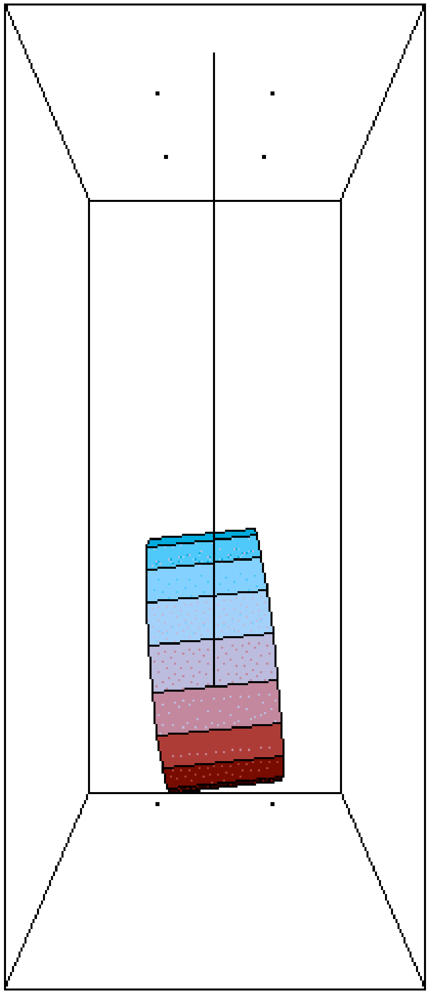
  Cos[theta]/lambda

  if lambda is still calculated in terms of the tension in the wire.
- This is actually a factor of Cos[theta]^(3/2) smaller than what was used for the quad because the vertical load was used instead of the tension.



Experimental Check

- A two-wire LIGO-I style pendulum was constructed in the lab to check.
- Expected frequency with flexible wire: 0.325i Hz (unstable)
- Expected frequency with stiff wire: 0.359 Hz
- Measured frequency: 0.422 Hz
- Fair agreement.



LIGO

Check With Quad Model

- Used parameters from ETM Quad Prototype Design (T040214-01) in Mark Barton's Mathematica quad model.
- Corrected d's using new flexure theory.
- Mathematica "Stage2" results (i.e., with wire stiffness)agree very closely with Matlab results (with neither wires stiffness nor correction thereof).

N	f	type	
1	0.379244	pitch3	pitch2
2	0.443433	pitch3	pitch2
3	0.672038	z3	z2
4	0.997526	pitch0	x2
5	1.27192	pitch0	pitch1
6	1.6833	pitch0	pitch2
7	2.00385	x0	x1
8	2.52876	z0	z1
9	2.95722	pitch1	
10	3.42948	x1	x0
11	4.22801	z1	z0
12	8.83338	z2	z3

longpitch1: [0.3797 0.4408 0.9898 1.2736]

longpitch2: [1.6838 1.9753 2.9580 3.3722]

vertical: [0.6602 2.4795 4.1289 8.8326]

Conclusion

- We think we now have a good understanding of how to allow for wire stiffness.
- The old corrections were not quite right.
- The new corrections are not sufficiently different to explain the instability we're seeing.
- More head-scratching required.