To the practical design of the optical lever intracavity topology of gravitational-wave detectors

S.L.Danilishin, F.Ya.Khalili

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Why we need intracavity topologies?

"Practical" version of the optical lever

The local meter

Conclusion

Suppose that we have managed to overcome the SQL.

What are the next limitations?

Pumping power

$$\frac{L^2 S_h}{4} \times S_{\mathrm{B.A.}} = \frac{\hbar^2}{4} \,,$$

$$S_{\mathrm{B.A.}} = \frac{8\hbar\omega_p W}{\zeta^2 c L} \frac{\gamma}{\gamma^2 + \Omega^2} \,,$$

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$$\boxed{\xi^2 \equiv \frac{S_h}{S_h^{\mathrm{SQL}}} = \frac{\zeta^2}{2}\,\frac{W_{\mathrm{SQL}}}{W} \quad (\text{optimization: } \gamma = \Omega)}\,,$$

where

$$S_h^{\mathrm{SQL}} = \frac{4\hbar}{M\Omega^2 L^2}\,, \quad W_{\mathrm{SQL}} = \frac{McL\Omega^3}{8\omega_o}\,, \quad \zeta = e^{-R}\,.$$

Optical losses

$$\xi_{\text{loss}}^2 = \sqrt{\zeta^2 \frac{\gamma_{\text{loss}}}{\gamma_{\text{load}} \approx \gamma}}\,,$$

the half-bandwidth $\gamma = \gamma_{\text{load}} + \gamma_{\text{loss}}\,.$

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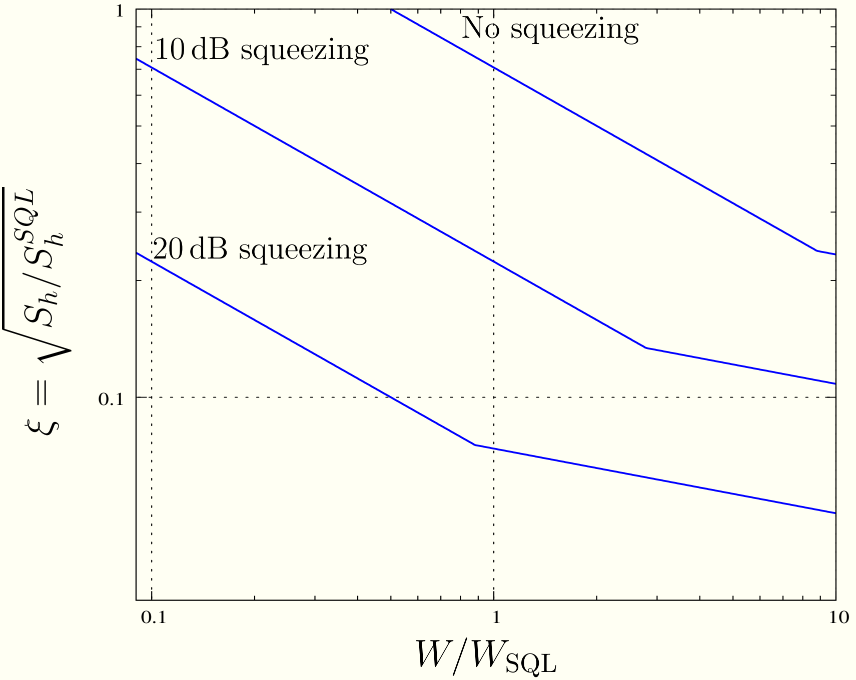
Smaller ξs require $\gamma > \Omega$, hence:

$$\xi_{\rm sum} = \sqrt{\frac{3}{2}}\,\zeta^{2/3}\left(\frac{\gamma_{\rm loss}}{2\Omega}\,\frac{W_{\rm SQL}}{W}\right)^{1/6} \approx 0.35\,\zeta^{2/3}\left(\frac{W_{\rm SQL}}{W}\right)^{1/6}.$$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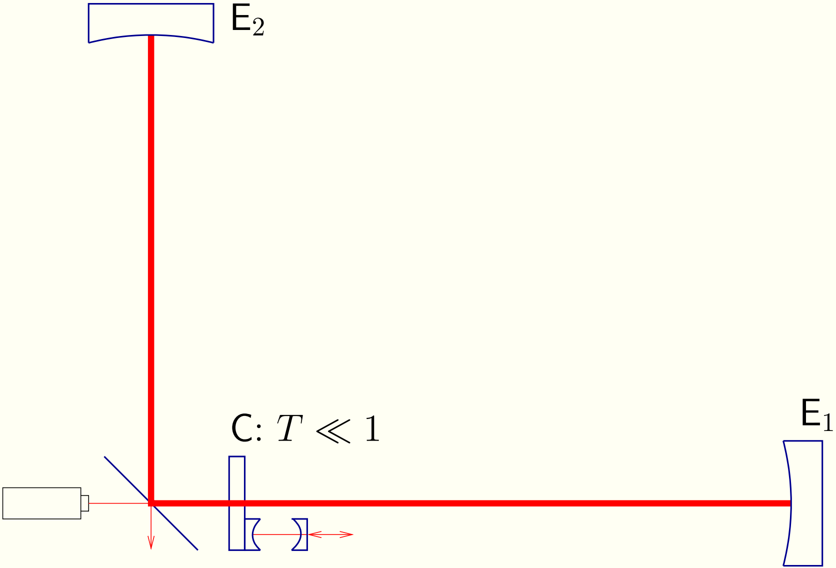
V.B.Braginsky, F.Ya.Khalili, Phys.Lett.A **218**, 167 (1996).
V.B.Braginsky, M.L.Gorodetsky, F.Ya.Khalili, Phys.Lett.A **232**, 340 (1997).

Optical bars



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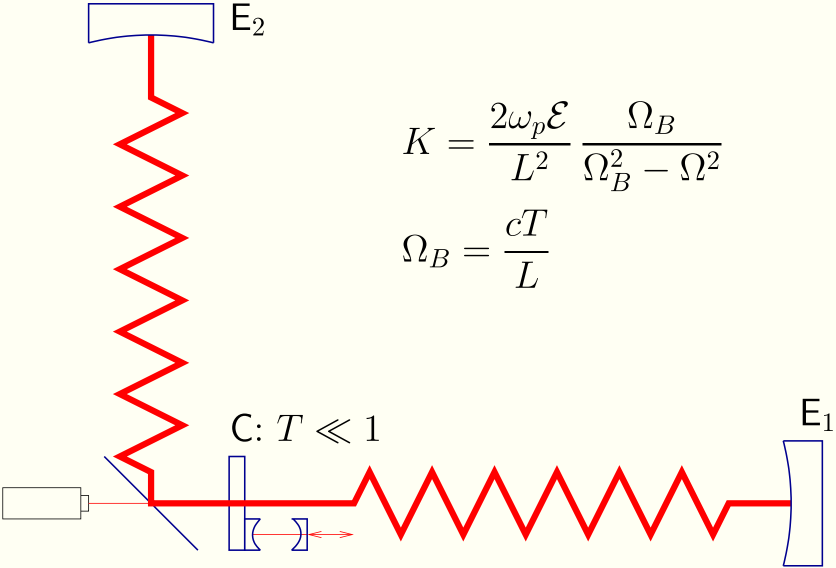
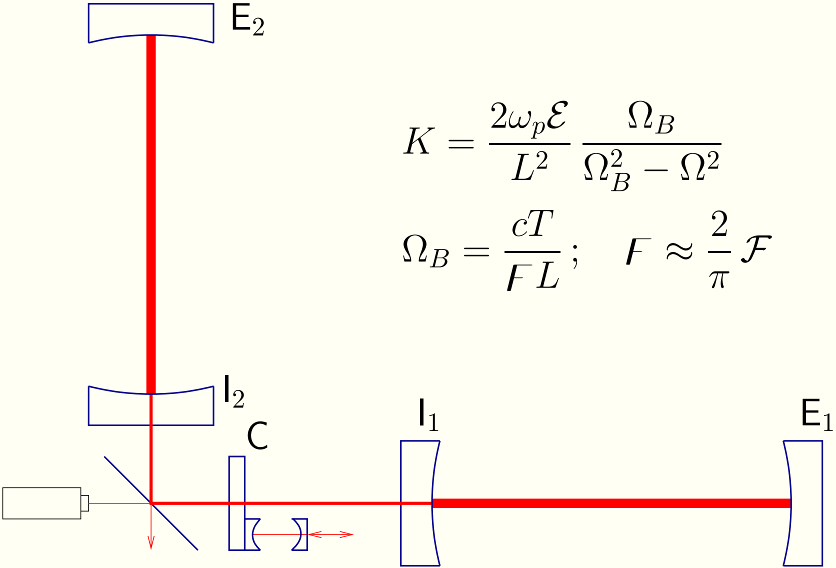


$$K = \frac{2\omega_p\mathcal{E}}{L^2}\,\frac{\Omega_B}{\Omega_B^2 - \Omega^2}$$

$$\Omega_B = \frac{cT}{L}$$

Optical lever



$$K = \frac{2\omega_p \mathcal{E}}{L^2} \frac{\Omega_B}{\Omega_B^2 - \Omega^2}$$

$$\Omega_B = \frac{cT}{F L}\,; \quad F \approx \frac{2}{\pi}\mathcal{F}$$

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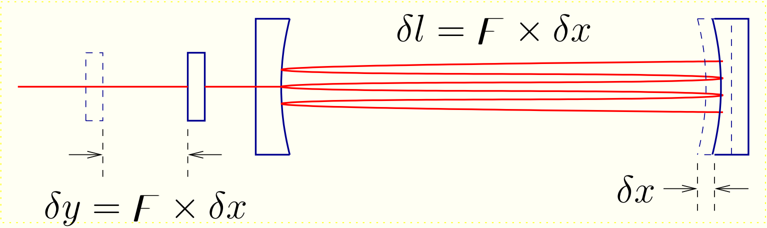
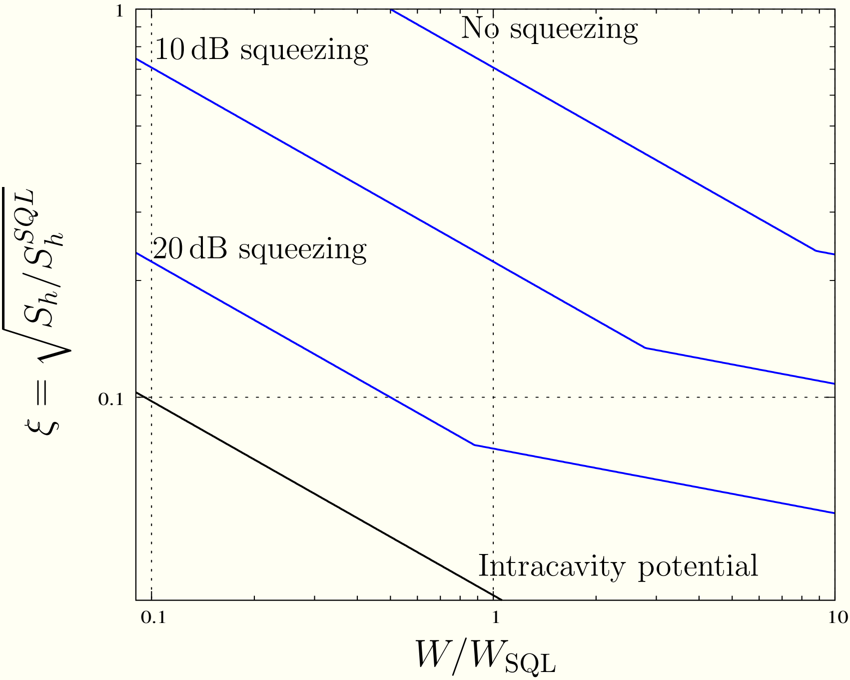


$$\frac{\delta y}{\delta x} = \mathcal{F}\,, \qquad \frac{F_{\mathrm{pond}\,y}}{F_{\mathrm{pond}\,x}} = \frac{1}{\mathcal{F}}$$

(like an ordinary mechanical lever).



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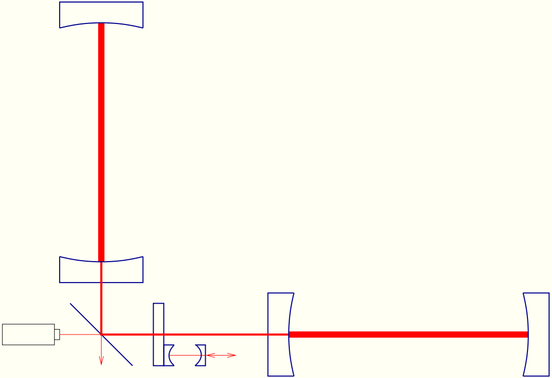
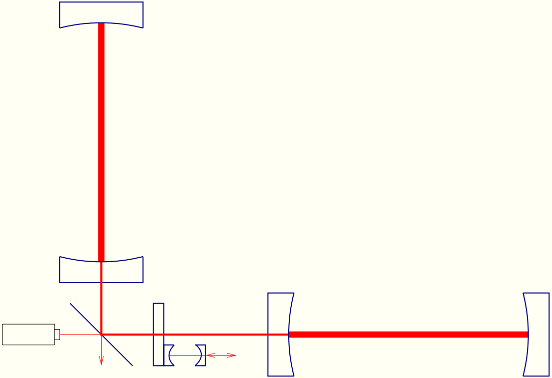
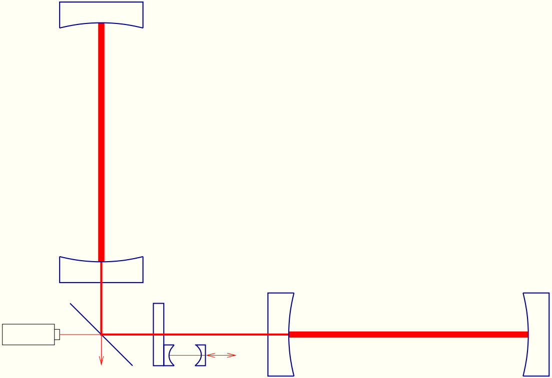




The main disadvantage: non-symmetry $\Rightarrow$ vulnerability to the pumping power fluctuations.



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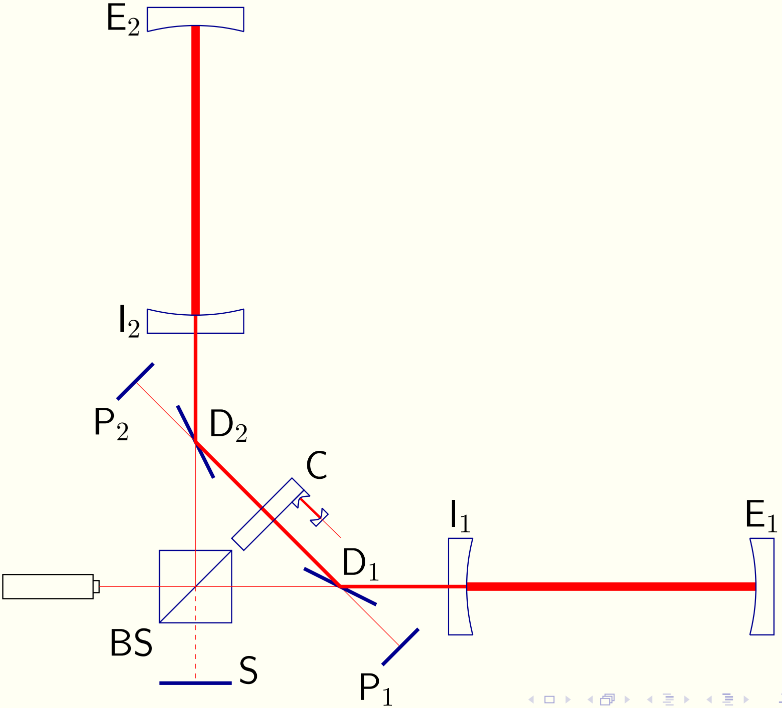
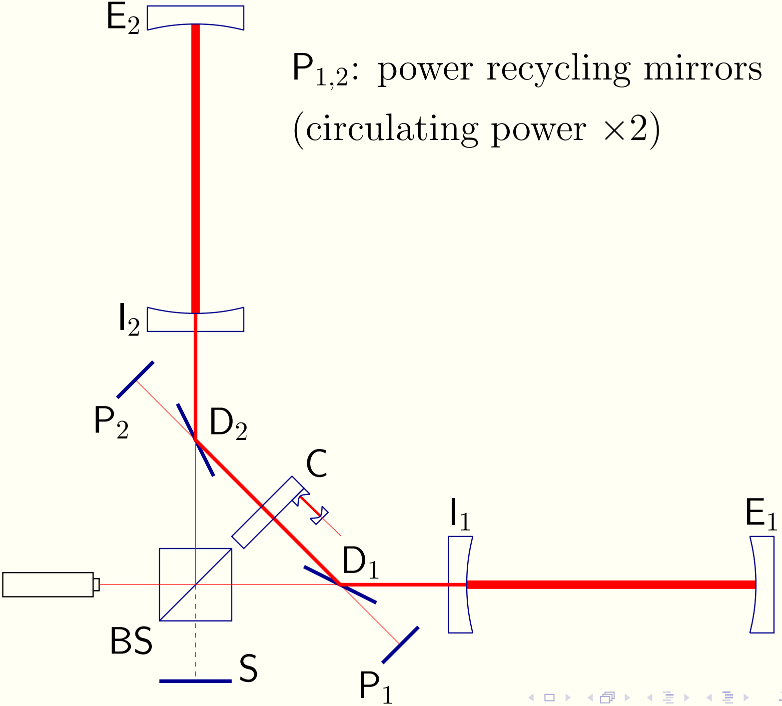
Additional disadvantages:

only half of the pumping power enters inside; input port $\Rightarrow$ additional “hole” for noise.



E2
I2
P2
D2
C
I1
E1
D1
BS
S
P1

$P_{1,2}$: power recycling mirrors
(circulating power $\times 2$)

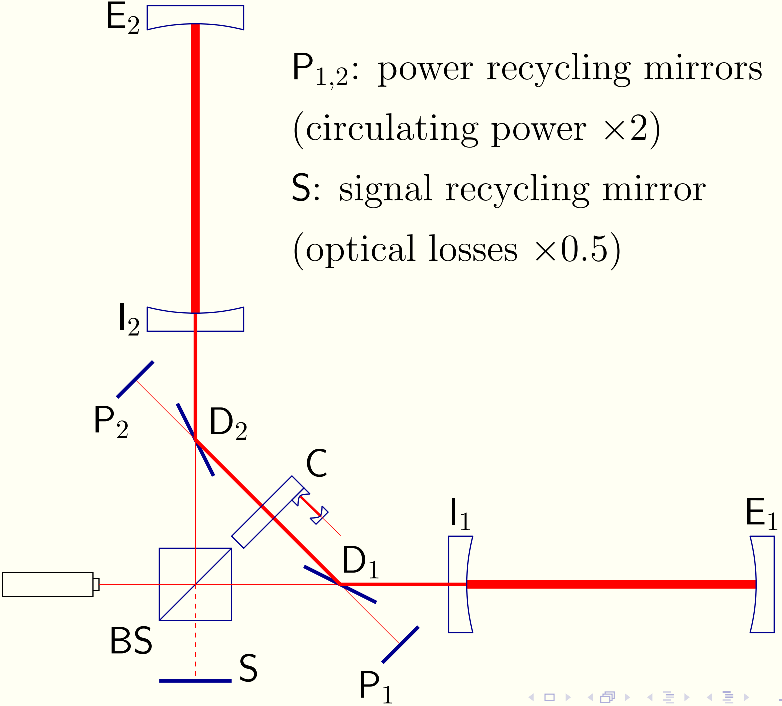


$P_{1,2}$: power recycling mirrors

(circulating power $\times 2$)

S: signal recycling mirror

(optical losses $\times 0.5$)



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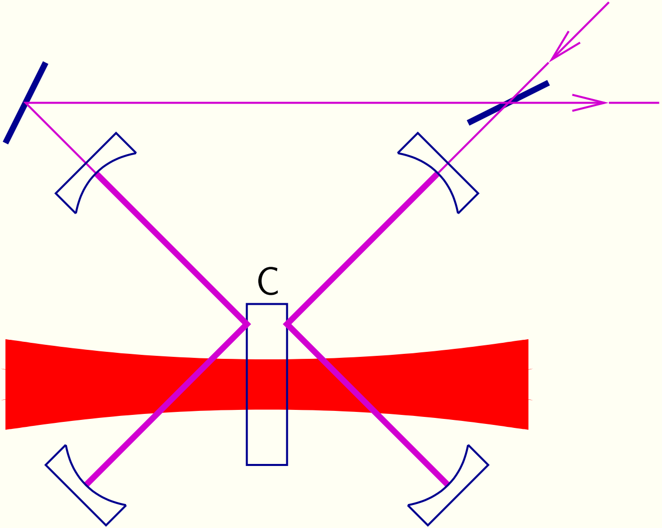
- ???

Possible design of the optical local meter



Thomas Corbitt *et al*, "Ponderomotive Squeezing",

LIGO Document G040147-00 (2004)

Parameters used for the estimates

Local mirror mass	$M_{\mathsf{C}} = 1\,\mathrm{g}$
Circulating power	$w = 10\,\mathrm{kW}$
Optical losses	$A^2_{\mathrm{local}} = 5 \times 10^{-6}$

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Spectral variation measurement

Small-scale version of the KLMTV topology.

Rather gedanken than realistic scheme, but provides a good comparison point for other schemes.

H.J.Kimble *et al*, Physical Review D **65**, 022002 (2002)

Spectral variation measurement

$$\xi^2_{\text{meter}} \approx \frac{1}{2\mathsf{F}^2}\,\frac{M}{M_{\mathsf{C}}}\,\frac{w_{\text{SQL}}}{w}\,,$$

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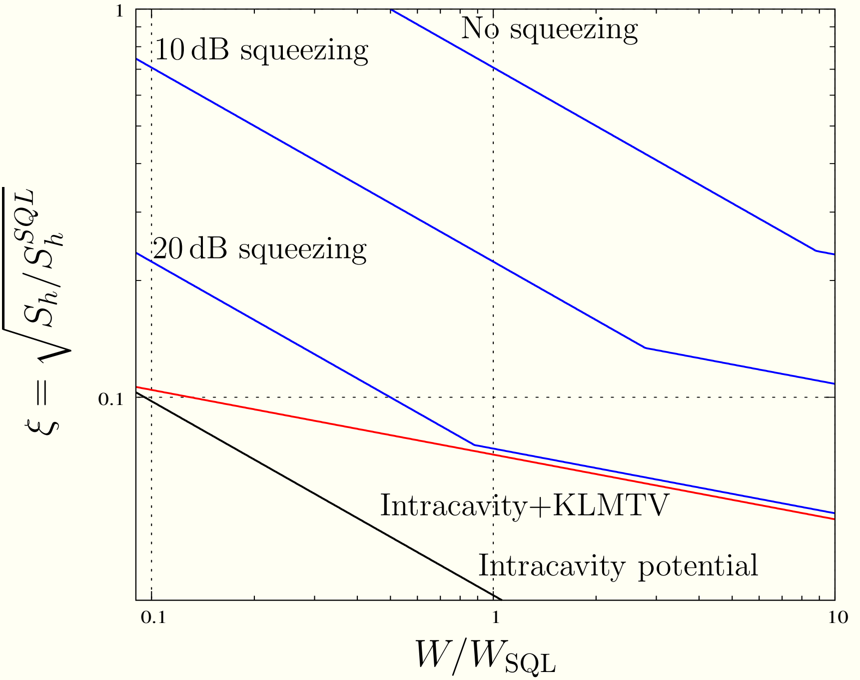
$$\Rightarrow \quad \boxed{\xi^2_{\rm meter\,loss} = \frac{\xi_0^2}{2}\left(\frac{W_{\rm SQL}}{W}\right)^{1/3}}\,,$$

$$\text{where} \quad \xi_0^2 = \frac{3}{2}\left(\frac{M_{\sf C}c^2A^2_{\rm local}\Omega^2}{32\omega_o w}\right)^{1/3} \approx 0.1\,.$$



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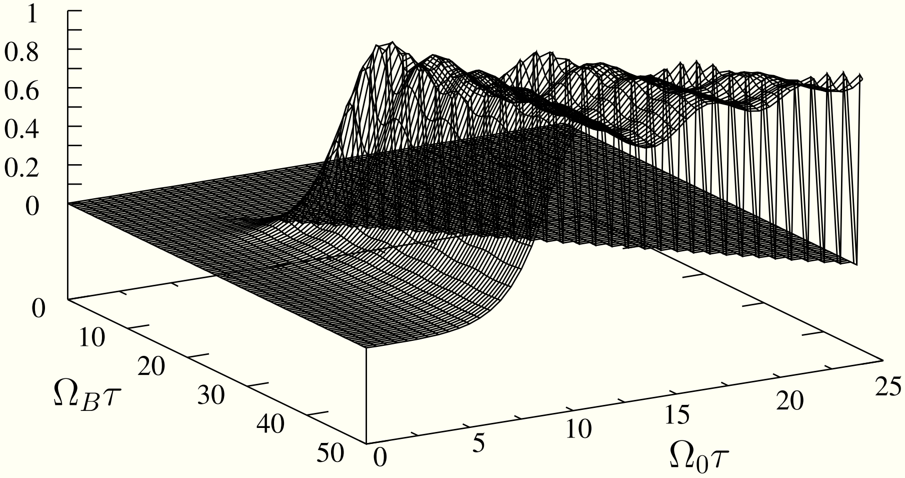
Discrete sampling variation measurement

$$\xi^2_{\mathrm{DSVM}} = \frac{720}{\pi^4}\,\frac{1}{\mathcal{G}(\Omega_B\tau,\Omega_0\tau)}\,\frac{1}{\mathsf{F}^2}\,\frac{M}{M_{\mathrm{C}}}\,\frac{w_{\mathrm{SQL}}}{w}\,,$$

$$W \gtrsim 60\mathsf{F}^2\frac{M_{\mathsf{C}}}{M}\,W_{\mathrm{SQL}}$$

(Yanbei Chen has the explanation for these weird numeric factors)

Plot of $\mathcal{G}(\Omega_B\tau, \Omega_0\tau)$



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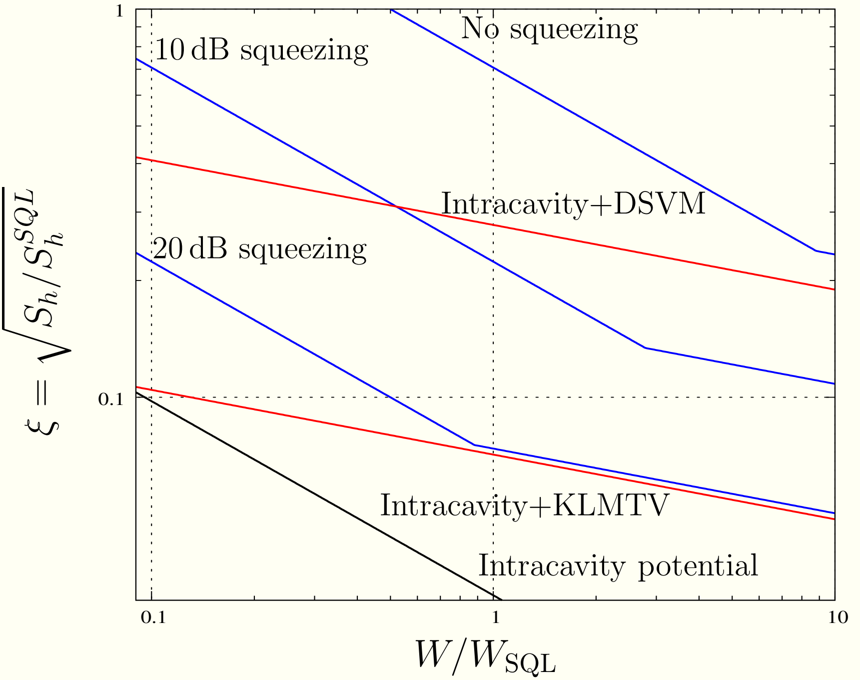
$$\Rightarrow \boxed{\xi^2_{\text{meter loss}} \approx \xi^2_0 \left(\frac{720 \times 60}{\pi^4} \frac{W_{\text{SQL}}}{W} \right)^{1/3}},$$

$$\text{instead of } \xi^2_{\text{meter loss}} = \frac{\xi^2_0}{2} \left(\frac{W_{\text{SQL}}}{W} \right)^{1/3}.$$



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3. However, intracavity topologies promise better sensitivity, especially if the pumping power $W < W_{\text{SQL}}$.
4. Unfortunately, none of the mechanical QND schemes known today can fully realize potential sensitivity of intracavity topologies.