Notes about Noise in Gravitational Wave Antennae Created by Cosmic Rays

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We discussed three mechanical effects initiated by cosmic rays which may limit the sensitivity of gravitational wave antennae. Unsolved problems are formulated and several recommendations for the antennae designs are presented.

I. INTRODUCTION

Since 1969 several groups of researchers have presented results of the analysis and estimates of possible contribution of cosmic rays to the noise floor of gravitational wave antennae (both bar antennae and antennae based on free masses, e.g. [1, 2]). The currently achieved sensitivities in Initial LIGO project (see [3, 4]) and the planned sensitivity in the next stage (Advanced LIGO which is planned to operate within a few years from now) are respectively $h \simeq 10^{-21}$ and $h \simeq 10^{-22}$ for the amplitude of the perturbation of the metric (at the mean frequency $f \simeq 100$ Hz in the bandwidth $\simeq 100$ Hz). Independently of Advanced LIGO several groups of researches which belong to the LSC (LIGO Scientific Collaboration) are continuing the analysis of new topologies and designs of the antennae which will permit them to reach a sensitivity even better than in Advanced LIGO (see e.g. [4, 6]).

Apart from this activity during last two decades there is substantial progress in the collection of data and in the resolution in the measurements of cosmic ray showers (cascades) features (see e.g. [7] - [9]). Thus it is reasonable to revise the contribution of cosmic rays to the noise in gravitational wave antennae.

In the present note we limit ourselves to only three possible mechanical "actions" on the rest masses (mirrors):

1. Direct transfer of mechanical momentum from the cascade to the LIGO mirror.
2. Distortion of the mirror's surface due to the heating by the cascade and subsequent thermal expansion — thermoelastic effect.

3. Fluctuating component of the Coulomb force between electrically charged mirror and grounded metal elements located near the mirror's surface.

II. DIRECT TRANSFER OF CASCADE MECHANICAL MOMENTUM TO LIGO ANTENNA MIRROR

In Advanced LIGO it is planned to use nearly cylindrical mirrors in the main Fabry-Perot (FP) optical resonator (mirror's diameter – $2R \simeq 35$ cm, height – $H \simeq 20$ cm, total mass of fused silica (SiO_2) mirror $M = 40$ kg). The mirrors will be suspended on SiO_2 fibers in a horizontal vacuum tube. The axis of the mirrors will be parallel to the tube. The distance between mirrors will be $L \simeq 4$ km. Two mirrors have to respond to the gradient of acceleration generated by a gravitational wave propagating in a perpendicular direction to the FP axis.

TABLE I: The parameters if high energy cascades we used for estimates. $\mathcal{E}$ is cascade energy, J_μ, J_h, J_e are the fluxes of cascades produced by muons, hadrons and by soft component, consequently, at the sea level; $N_{e,max}$ is a number of electrons in the cascade maximum; $\Delta\mathcal{E}$ is energy lost by cascade in the 20 cm of SiO_2; N_{ev} is the expected number per year of events with energy losses higher than $\Delta\mathcal{E}$.

$\mathcal{E}$, TeV	0.5	1	2	Ref
J_μ 1/cm^{2}s	1.8×10^{-9}	2.8×10^{-10}	4.3×10^{-11}	[7, 8, 12, 13]
J_h 1/cm^{2}s	2.5×10^{-9}	4.0×10^{-10}	7.2×10^{-11}	[9]
J_e 1/cm^{2}s	3×10^{-10}	8×10^{-11}	1.7×10^{-11}	[10]
$N_{e,max}$	1000	2000	4000	
$\Delta\mathcal{E}$, GeV	60	120	230	
N_{ev} 1/year	~ 110	20	$3 \div 4$	

It is reasonable to expect that a certain part of all cascades (showers) generated by very high energy muons of cosmic rays will pass through the mirror along axes which have a small angle with the FP resonator axis.

Thus one may expect that not a very small fraction of a cascade with energy $\mathcal{E}$ will be lost in the "travel" through 20 cm of fused silica mirror. This fraction $\Delta\mathcal{E}$ will give a momentum $\Delta P = \Delta\mathcal{E}/c$ to the mirror along its axis (c is speed of light). Correspondingly this mirror position will change during time τ by value

$$\Delta x \simeq \frac{\Delta\mathcal{E}\tau}{Mc}, \quad \tau \simeq 0.01 \text{ s}$$

The narrow cascades in the direction close to vertical can be produced by so-called unaccompanied hadrons [9] and electron-photon (soft) component of cosmic rays [10, 11]. High energy ($\mathcal{E} > 0.5\mathrm{TeV}$) cosmic ray muons are the origin of electromagnetic and nuclear cascades. These particles are born in decay of $\pi^{\pm}$- and $K^{\pm}$ -mesons and charmed particles generated in hadronic showers. The showers are initiated by interactions of primaries with the nuclei of air at high altitudes in the atmosphere. The energy spectra of all particles (and all cascades generated by them) have power law shape $F(\mathcal{E}_c) \sim \mathcal{E}_c^{-\gamma}$, γ is the power index, $\gamma = 2.73 \pm 0.05$ for J_{μ} [8]; $\gamma = 2.5 \pm 0.1$ for J_h [9]; and $\gamma = 2.2 \pm 0.2$ for J_e [10]. For the estimation of the parameters of cascades it is possible to use well elaborated model of this type of cascades [11].

In Table I we present numerical estimates for "an appropriate candidate" for the discussed process. For several energies $\mathcal{E}$ of cascade one can find from the literature the fluxes of cascades produced by muons ($J_{\mu\ \mathrm{und}}$) underground [7, 8], hadrons (J_h) [9] and by the soft component (J_e) [10] at sea level and calculate the mean number $N_{e,\,max}$ of electrons in the cascade maximum. The flux of cascades produced by muons at sea level (J_{μ}) was estimated from experimental data obtained underground ($J_{\mu\ \mathrm{und}}$)[7] taking into account the calculations of energy spectrum and angular distribution of muons at sea level [12, 13]. The energy lost $\Delta\mathcal{E}$ of a cascade was obtained by integration of energy losses of electrons in the mirror taking into account the energy spectrum of electrons. The expected number N_{ev} of events per year with energy losses equal or higher than $\Delta\mathcal{E}$ was calculated by formula $N_{ev} = [J_{\mu} \times S_1 + (J_h + J_e) \times S_2] \times T$, ($S_1, S_2$ are the areas of the mirror perpendicular and parallel to the axis, consequently, T is one year $T \approx 3 \times 10^7$ s). These estimates were made under the assumption that the cascade is coming into the mirror being well developed (more than 6 interactions of the fastest electrons).

The values of $\Delta\mathcal{E}$, presented in Table I permits us to estimate the mirror's displacement $\Delta x \simeq (0.8 \div 3) \times 10^{-18}$ cm. This numerical value of Δx is less than the amplitude of the sensitivity planned in Advanced LIGO: $\Delta L \simeq hL/2 \simeq 2 \times 10^{-17}$ cm.

Evidently one may expect a much more frequent rate of events: e.g. a cascade with initial energy $\mathcal{E} = 1$ GeV will fly through the same mirrors several times per second. But because the value of $\Delta\mathcal{E}$ in this case will be approximately 3 orders smaller the net effect will be negligible compared to the planned sensitivity of Advanced LIGO.

III. DISTORTION OF MIRROR'S SURFACE DUE TO THERMOELASTIC EFFECT AND HEATING BY THE CASCADE

The lost in the mirror's bulk energy of the cascade is likely to be distributed into two parts. The first one is the rise of free energy of the solid (creation of new dislocations, of new clusters, etc) and the second part produces direct heating of a narrow channel. There is no rigorous analysis in the quantum theory of solids which permits us to get a reliable value of the ratio of these two parts. But it is very likely that the second one (the heating) will dominate.

The energy $\Delta\mathcal{E}$ is converted into heat over a length $\mathsf{H} = 20$ cm (thickness of mirror). This heat will be produced in the trace of cascade in the volume $\pi \mathsf{R}_c^2 \mathsf{H}$ where $\mathsf{R}_c \simeq 1 \div 7$ cm is the radius of cascade trace. Assuming that the volume $\simeq \mathsf{R}_c^3$ on the mirror's surface can freely expand we obtain an estimate of the height $\Delta\mathsf{H}$ of the " hill" with footprint $\simeq \mathsf{R}_c^2$ on the surface due to thermal expansion:

$$\Delta \mathsf{H} \simeq \frac{\mathsf{R}_c}{\mathsf{H}} \times \frac{\Delta\mathcal{E}}{\rho C_V \mathsf{R}_c^3} \times \mathsf{R}_c \alpha \tag{3.1}$$

Here $\rho \simeq 2.3\ \mathrm{g/cm^3}$ is the density, $C_V \simeq 7{\times}10^6\ \mathrm{erg/cm^3K}$ is the heat capacity and $\alpha \simeq 5.5{\times}10^{-7}\ \mathrm{K^{-1}}$ is the thermal expansion coefficient of fused silica. The height $\Delta\mathsf{H}_{\mathrm{av}}$ averaged over laser beam spot with radius $r \simeq 10$ cm is approximately equal to

$$\Delta \mathsf{H}_{\mathrm{av}} \simeq \Delta \mathsf{H} \times \frac{\mathsf{R}_c^2}{r^2} \simeq \begin{cases} 2 \times 10^{-18}\,\mathrm{cm} & \text{if } \Delta\mathcal{E} = 60\,\mathrm{GeV}, \text{ if } \mathsf{R}_c = 1\ \mathrm{cm} \\ 4 \times 10^{-17}\,\mathrm{cm} & \text{if } \Delta\mathcal{E} = 230\,\mathrm{GeV}, \text{ if } \mathsf{R}_c = 7\ \mathrm{cm} \end{cases} \tag{3.2}$$

The displacement $\Delta\mathsf{H}_{\mathrm{av}}$ of the surface considered above is produced by cascade developing mainly perpendicular to the surface of the mirror. However, there are cascades having traces approximately parallel to the surface. Such a "parallel" event produces greater contribution to the fluctuational displacement of the mirror's surface:

$$\Delta \mathsf{H}_{\mathrm{av,parall}} \simeq \frac{\Delta\mathcal{E}}{\rho C_V \mathsf{R}_c^2 2r} \times \mathsf{R}_c \alpha \times \frac{\mathsf{R}_c}{r} \simeq \begin{cases} 2 \times 10^{-17}\,\mathrm{cm} & \text{if } \Delta\mathcal{E} = 60\,\mathrm{GeV} \\ 8 \times 10^{-17}\,\mathrm{cm} & \text{if } \Delta\mathcal{E} = 230\,\mathrm{GeV} \end{cases} \tag{3.3}$$

However, such events are more rare than "perpendicular" ones by a factor of about $\mathsf{R}_c^2/r^2 \simeq 0.01 \div 0.5$.

IV. FLUCTUATING COULOMB FORCE

In 1995 R.Weiss pinpointed the potential danger from electrical charge accumulated on the mirror's surface [14]. Direct measurements of the values of electrical charge density σ on models of mirrors were performed independently by several groups [15–17]. In these measurements it was demonstrated that the values of σ on models of mirrors (fabricated from fused silica) was from 10^6 to 10^7 electrons per cm^2 and in several cases even higher.

Recently V.P. Mitrofanov and his colleagues [16, 17] have measured the values of σ in the same vacuum chamber in which record high values quality factors of pendulum and violin modes ($Q > 10^8$) were demonstrated. These measurements were performed during several months and a slow, long lasting drift of σ was observed. The high values of surface density of charge obtained means that the electrostatic potential of the mirror may exceed 100 V.

V.P. Mitrofanov and his colleagues [16] have discovered a monotonic rise of negative charging — $d\sigma/dt \simeq 10^5$ electrons per cm^2 per month. This effect can be qualitatively explained by the model of transition effect occuring during transition of cascade particles, soft component and gamma-quanta of natural radioactivity through iron "envelope" to fused silica mirror (i.e. vacuum chamber).

In the cosmic rays of low energy there are more electrons than positrons. Compton effect, photoelectric effect and the process of ionization are the sources of the excess. In electromagnetic cascades the number of particles with low energy is much larger than the number of particles with high energy. The number of particles having energy less than 0.05 of critical energy in the matter ($\mathcal{E}_{\mathrm{cr,Fe}} = 20,7\mathrm{MeV}$ for iron, $\mathcal{E}_{\mathrm{cr,Al}} = 40\mathrm{MeV}$ for aluminium, $\mathcal{E}_{\mathrm{cr,SiO_2}} = 47,3\mathrm{MeV}$ for $\mathrm{SiO_2}$) is equal to $\simeq 20\%$, and the number of particles having energy less than 0.02 of critical energy in the material is equal to $\simeq 10\%$ of number of particles in the cascade maximum. The cosmic rays are very sensitive even to the thin layers of matter. If a cascade developed in the heavy material comes to the material consisting of lighter atoms, it brings to the light material more electons than takes away. This is explained by the fact that the number N_e of electrons produced in material is proportional to fraction $N_e \sim \mathcal{E}/\mathcal{E}_{\mathrm{cr}}$, where $\mathcal{E}$ is the cascade energy.

As example we consider a cascade in the maximum of its development coming from iron to fused silica. In this case cascade theory gives the formulas for the number of low energy electrons produced in iron and in fused silica:

$$N_{e,\mathrm{Fe}}(\mathcal{E} < 1\mathrm{MeV}) = 0.2 \times \frac{0.2}{\sqrt{\ln(\mathcal{E}/\mathcal{E}_{\mathrm{cr,Fe}})}} \times \frac{\mathcal{E}}{\mathcal{E}_{\mathrm{cr,Fe}}}, \quad (1\,\mathrm{MeV} = 0.05\,\mathcal{E}_{\mathrm{cr,Fe}}), \tag{4.1}$$

$$N_{e,\mathrm{SiO_2}}(\mathcal{E} < 1\mathrm{MeV}) = 0.1 \times \frac{0.3}{\sqrt{\ln(\mathcal{E}/\mathcal{E}_{\mathrm{cr,SiO_2}})}} \times \frac{\mathcal{E}}{\mathcal{E}_{\mathrm{cr,SiO_2}}}, \quad (1\,\mathrm{MeV} = 0.021\,\mathcal{E}_{\mathrm{cr,SiO_2}}). \tag{4.2}$$

So we see that number of low energy electrons produced in iron and coming to the mirror is about 3 times larger than number produced in fused silica and outgoing from it:

$$\frac{N_{e,Fe}}{N_{e,SiO_2}} \approx \frac{0.4\,\mathcal{E}_{cr,SiO_2}}{0.3\,\mathcal{E}_{cr,Fe}} \approx 3$$

This electron excess will stay near the surface of the mirror and will give an additional charge to it. The estimates of the number of electrons with energy less than 1 MeV coming from iron to fused silica are given in the Table II. This effect can qualitatively explain the monotonic rise of *negative* charge observed in [16, 17]. However, for quantitive explanation the detailed analysis has to be performed.

TABLE II: The mean number $N(E < 1$ MeV) of electrons with energy less than 1 MeV coming from iron to fused silica.

$\mathcal{E}$, TeV	0.5	1	2
$N(E < 1\mathrm{MeV})$	450	900	1700

The initial design of the "entourage" of the suspended mirror includes several parts which are planned to be made of metal. These parts include a "cradle" situated under mirror (this " cradle" has to catch the mirror if one or all fibers break). Other parts are called "stoppers" which have to limit large horizontal swings of the mirror in case of an earthquake. These parts have to be grounded. Thus due to electrical charging of the mirror it is very reasonable to expect that a d.c. Coulomb force may act on the mirror. If a grounded metal part has a flat surface S that is close to a part of the mirror's surface, then

$$F_{dc} \simeq 2\pi S\sigma^2 \simeq 1.5\times 10^{-2}\,\mathrm{dyn}, \quad \text{if } S = 10^2\,\mathrm{cm}^2, \quad \sigma = 10^7\,\mathrm{e/cm}^2$$

This numerical estimate of the d.c. force has to be taken into account in the design of feedback actuators which have to maintain the distance between antenna mirrors with accuracy better that $\lambda/\mathcal{F} \simeq 10^{-9}$ cm ($\mathcal{F}$ is the finesse, λ is the optical wave length). More important is another effect: The a.c. component of the Coulomb force may mimic the force F_{grav} which antenna has to register:

$$F_{grav} \simeq \frac{hLM\omega_{grav}^2}{2} \simeq 3\times 10^{-7}\,\mathrm{dyn} \tag{4.3}$$

A comparison of the values presented above indicates that a relative fluctuation $\Delta\sigma/\sigma \simeq 10^{-5}$ of surface charge density will inevitably produce a " step" of F_{ac} approximately equal to the amplitude F_{grav} which is the goal of Advanced LIGO. Note that $N(\mathcal{E} < 1$ MeV) electrons outgoing from iron to fused silica presented in Table II comes to the square about $\pi R_c^2 \simeq 100$ cm^2 (R_c is the radius of cascade) . Then one can estimate the relative fluctuations of charge density $\Delta\sigma/\sigma$ caused by a single cascade with energy $\mathcal{E} = 2$ TeV:

$$\frac{\Delta\sigma}{\sigma} \simeq 10^{-6} \div 2 \times 10^{-5}$$

We see that it is enough to produce an a.c. component of Coulomb force larger than F_{grav} if relatively large surface of grounded metal plate will be located near the mirror.

V. CONCLUSION

It is evident that the first two effects, being not very strong ones for Advanced LIGO, may be relatively easily vetoed by requiring coincidence between detectors (if al least two antennae are operating). On the other hand, the veto can not be considered as an absolute "remedy" for low values of signal to noise ratio. It is worth noting that in the next stage after Advanced LIGO these two effects will make serious contributions to the level of the noise floor.

This conclusion may not be automatically extended to the third effect. First of all because the negative charging may be high when the mirrors "spend" a long time in the vacuum (an year or longer) without scheduled removal of the accumulated electrical charge. The second "reason" is the design of all metal parts of the mirror's "entourage". One recommendation for this design is evident: it is necessary to use small square areas of all metal element which are near the mirror's surface and to place these elements as far as possible.

There are two evident recommendations for the consequent measurement and analysis:

1. To measure bursts of electrons which appears on the mirror's surface with resolution better then 10^2 e/cm^2 and time shorter than 10^{-2} sec.
2. To analyze the possibility to cover the mirror's surface over the coating with a transparent few nanometers thick layer whith substantial conductivity to reduce the d.c. component of electrical charge.

In all three effects considered above the mechanical action on the mirror produces a step like displacement (either of the mirror's center of mass or of its surface). This type of response is similar to the one predicted for shape of gravitational wave bursts created in the process of supernova explosion predicted by V.Imshennik [18]. The predicted rate of these bursts is approximately two orders higher than the rate of neutron star merger events.

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[1] B. L. Beron and R. Hofstadter, Rhys. Rev. Lett. **23** (4), 184 (1969).

[2] J. Chiang and P. Michelson, Nuclear Instruments and Methods in Physics Research **A311**, 603 (1991).

[3] B. Barish and R. Weiss, Physics Today, **52** (10), 44 (1999)

[4] V.B. Braginsky, Physics "Uspechi", **43** (7), 691 (2000).

[5] V.B. Braginsky, M.L.Gorodetsky F.Ya.Khalili and K.S.Thorne , Phys. Rev. **D61**, 4002 (2000).

[6] F.Ya. Khalili, Phys. Lett. **A298**, 308 (2002).

[7] R.I. Enikeev, G.T. Zatsepin, E.V. Korolkova, V.A. Kudryavtsev, A.S. Malguin, O.G. Ryazhskaya, F.F. Khalchukov, Sov. J. Nucl. Phys., **47** (4), 665 ~~1044~~ (1988).

[8] O.G. Ryazhskaya, Il Nuovo Cimento, **19c** (5), 655, (1996).

[9] M. Müller, T. Antoni, W.D. Apel *et al* for KASCADE collaboration, proc. of 28th ICRC, Tsukuba, vol. 1, 101, (2003)

[10] S. Hayakawa, Cosmic Ray Physics, p I, Edited by R.E. Marskak, New York, J. Willey and sons, (1969)

[11] S.Z. Belenkiy, Cascade Processes in Cosmic Rays, M.-L., (1948), in Russian

[12] O.G. Ryazhskaya, L.V. Volkova, Nuclear Physics **B143**, 527 (2005).

[13] L.V. Volkova, G.T. Zatsepin, L.A. Kuzmichev, Sov. J. Nucl. Phys., **29**, 645 (1979), Preprint FIAN, N72 (1969)

[14] R. Weiss, LIGO document T960137-00E. Available in http://admdbsrv.ligo.caltech.edu/dcc/.

[15] M.J. Mortonson, C.C.Vassiliov, D.J.Ottaway, D.H.Shoemakerand G.M.Harry, Rev. Sci. Inst. **74** (11) 4840 (2003).

[16] V.P. Mitrofanov, L.G.Prokhorov and K.V.Tokmakov, Phys. Lett. **A300**, 370 (2002).

[17] V.P. Mitrofanov, L.G.Prokhorov, K.V.Tokmakov, P.Willems, Class.Quantum Grav. **21** (5), 1083 (2004).

[18] V.S. Imshennik, A. Popov, Astron. Lett. **24**, 206 (1998).