Parametric Instability: A Few Remarks

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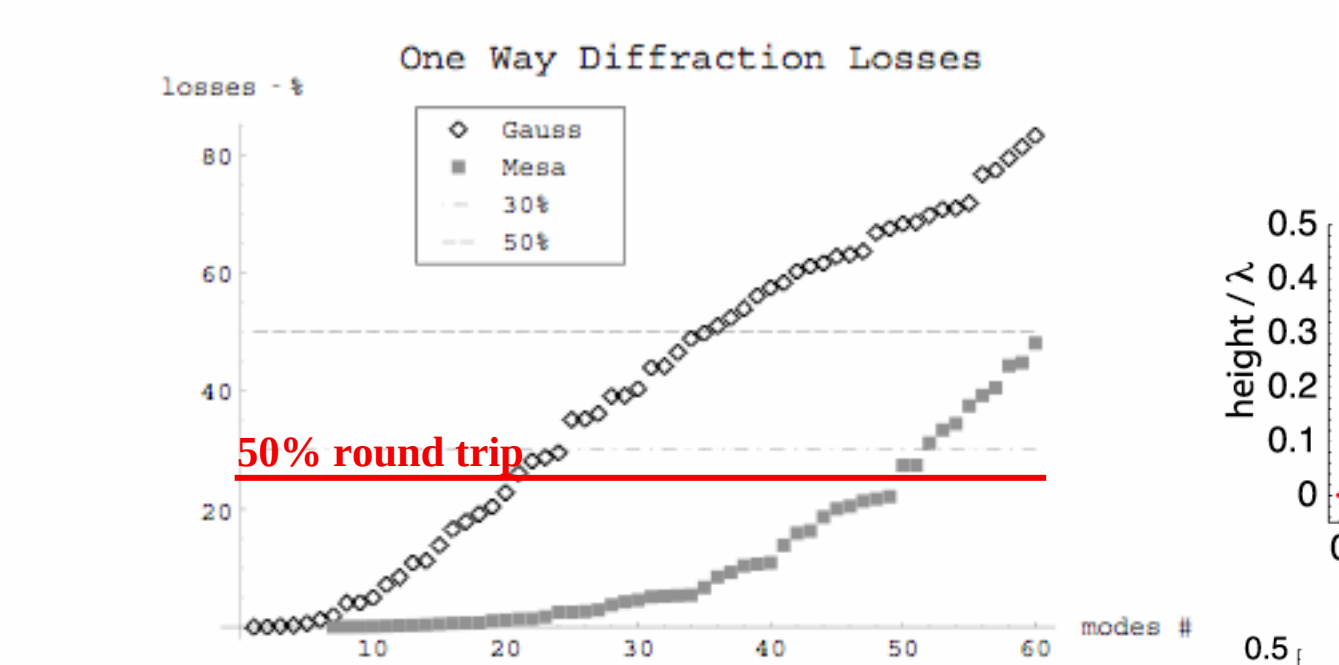
Laguerre-Gauss Stokes & Antistokes Modes: Diffraction Losses: ΔE/E in one round trip

[calculations by Pavlin Savov, solving the cavity's eigenequation by finite difference methods]

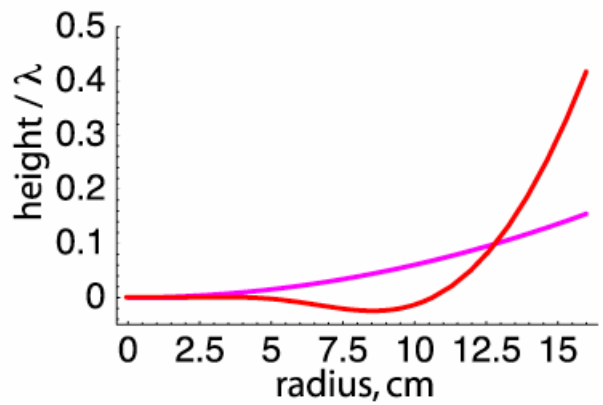
- ΔE/E < 0.01 : 3 modes [Blair's assumed 500Hz line width]
  - n (radial), m (angular) = 0,1 0,2 1,0
- ΔE/E < 0.05: 6 modes
  - Above, plus 0,3 0,4 1,1
- ΔE/E < 0.20: 11 modes
  - Above, plus 0,5 0,6 1,2 1,3 2,0
- ΔE/E < 0.50: 23 modes
  - Above, + 0,7 0,8 0,9 0,10 1,4 1,5 1,6 2,1 2,2 2,3 3,0 3,1

Diffraction Losses for Mexican Hat Mirrors [Pavlin Savov]

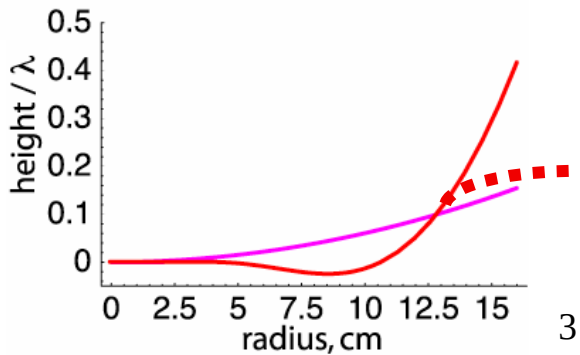
- Mexican Hat Mirrors (Mesa Beams) have ~ 2 or 3 times as many modes at fixed diffraction loss:



Sharply rising MH shape holds light in



- **Reshape mirror to:**
  - **Push parasitic modes outward AND**
  - **Flatten TEM00 mode ?**



Rewrite Formula for Instability Strength

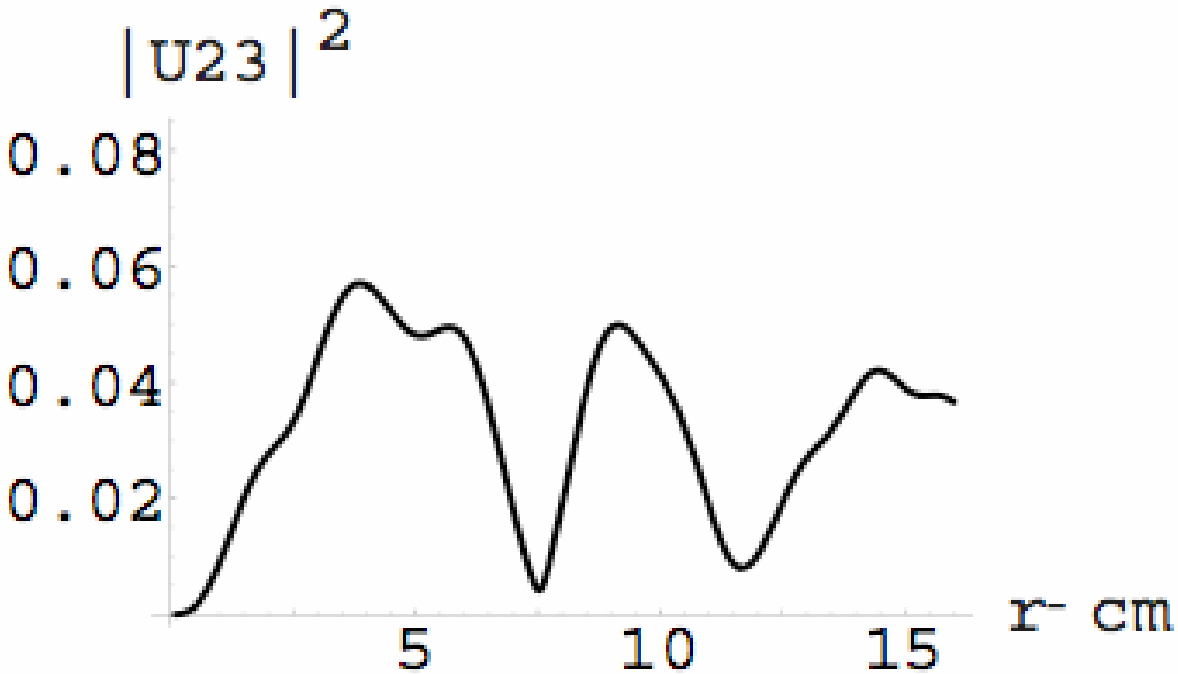
- R = (rate of growth of mechanical vibrations) / (damping rate)
- Q of Stokes & Antistokes modes: change from $Q_1 = \omega_o \tau / \pi$ to Q_{rt} = 1/(fractional energy loss in round trip)

$$R = \frac{4 P_{circ} \omega_o Q_m Q_{rt}}{m c^2 \omega_m^2} \frac{\Lambda}{1+\Delta\omega^2/\delta^2}$$

$$= Q_{rt} \frac{Q_m}{2 \times 10^6} \frac{\Lambda}{1+\Delta\omega^2/\delta^2}$$

- Low - Q_{rt} modes must be considered.

High-Loss Mode: n,m = 2,3

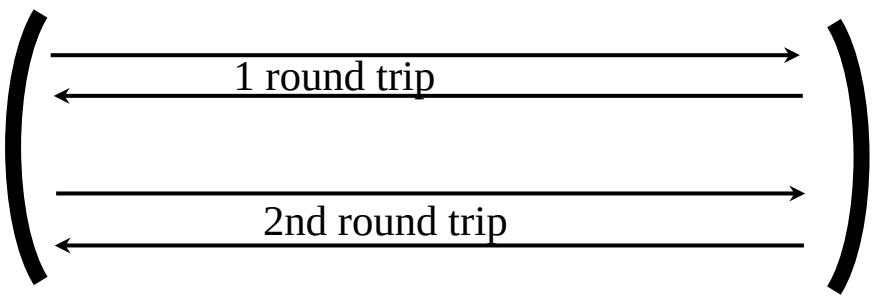


Not a mode
in the usual sense

Must rethink the theory

One Way to Rethink the Theory

- Check whether a chosen mirror vibration mode couples to any high-Q optical modes.
  - If so - that will dominate
- If not, consider low-Q modes (high-loss modes)
  - Consider all simultaneously by propagating light, with paraxial propagator from vibrating mirror, back and forth in cavity several times



Compute oscillatory force of light on mirror

- Are the (lossy) eigenmodes a complete set of functions for evolving the light leaving the vibrating mirror?
  - Vyatchanin suspects “yes”, and so a variant of previous theory still works
  - Thorne is not so sure…