Constraint likelihood analysis with a network of GW detectors

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- **Likelihood analysis**
- **Two detector paradox**
- **Network sensitivity to GW polarizations**
- **Constraint likelihood**
- **Results & Summary**

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Likelihood analysis for bursts

- **Outlined at**
  - **Flanagan, Hughes, PRD57 4577 (1998)**
  - **Mohanty et al, CQG 21 S1831 (2004)**
- **Likelihood for Gaussian noise with variance $\sigma^2{}_k$ and GW waveform u: $x_k[i]$ – detector output, F_k – antenna patterns**

$$L(u) = \sum_i \sum_k \frac{1}{2\sigma_k^2}\left[\left(x_k[i] - \xi_k[i]\right)^2 - x_k^2[i]\right]$$

detector response - $\xi_k[i] = u[i]\tilde{A}_k + \tilde{u}[i]A_k$

$$u = h_+ + ih_x$$

$$\tilde{u} = h_+ - ih_x \qquad A_k = \frac{F_{+k} + iF_{\times k}}{2}, \qquad \tilde{A}_k = \frac{F_{+k} - iF_{\times k}}{2}$$

- **For unknown GW signal treat every sample of u as an independent variable→ find *u[i]* from *variation of L***

Solution for GW waveforms

- **Given by two linear equations:**

$$\begin{bmatrix} \mathrm{Re}(X) \\ \mathrm{Im}(X) \end{bmatrix} = \begin{bmatrix} g_r + \mathrm{Re}(g_c) & \mathrm{Im}(g_c) \\ \mathrm{Im}(g_c) & g_r - \mathrm{Re}(g_c) \end{bmatrix} \begin{bmatrix} h_+ \\ h_\times \end{bmatrix}$$

- **Network response matrix M_R**
- **Network data vector** $X = \sum_k \frac{x_k A_k}{\sigma_k^2}$ Sum over detectors
- **Network antenna patterns** $g_c = \sum_k \frac{A_k^2}{\sigma_k^2}, \quad g_r = \sum_k \frac{A_k \tilde{A}_k}{\sigma_k^2},$
- **Solution:** $u_s = \frac{g_r X - g_c \tilde{X}}{g_r^2 - g_c \tilde{g}_c}$ **-- estimator of GW waveform**

detection statistic is obtained from the likelihood functional by substituting u with the solution

Two detector paradox (Johnston, Mohanty)

- **aligned detectors (unit noise variance for simplicity):**

$$L = \sum_i \left(x_1[i] + x_2[i]\right)\xi[i] - \xi^2[i] \Rightarrow \quad \xi = \frac{x_1 + x_2}{2}$$

$$L_A = \tfrac{1}{4}\left(\langle x_1 x_1 \rangle + \langle x_2 x_2 \rangle + 2\langle x_1 x_2 \rangle\right)$$

power — cross-correlation

  - **If separated → L_A has directional sensitivity (circle on the sky)**

- **misaligned detectors (even infinitesimally):**
  - **solution for GW waveform:** $\xi_1 = x_1, \quad \xi_2 = x_2$

$$L_M = \tfrac{1}{2}\left(\langle x_1 x_1 \rangle + \langle x_2 x_2 \rangle\right)$$

No directional sensitivity for two misaligned detectors!

Dominant polarization wave frame

- **such a wave frame where the network antenna pattern g_c is positive and real**

$$M_R = \begin{bmatrix} g_r + |g_c| & 0 \\ 0 & g_r - |g_c| \end{bmatrix} = g \begin{bmatrix} 1 & 0 \\ 0 & \varepsilon \end{bmatrix}$$

  - **g – network sensitivity factor**
  - **ε – network alignment factor**

- **diagonal M_R → independent statistics for h_1 & h_2 -- GW polarizations in the DP frame**

$$L_{MLR}(h_1, h_2) = L_1(h_1) + L_2(h_2)$$

- **total signal** $SNR = 2g\left(\langle h_1^2 \rangle + \varepsilon \langle h_2^2 \rangle\right) \approx 2L_{MLR}$

$\langle h_1^2 \rangle, \langle h_2^2 \rangle$ **-sum-square energies of GW polarizations**

- **Only first component is detected if** $\varepsilon = 0$

Network alignment factor

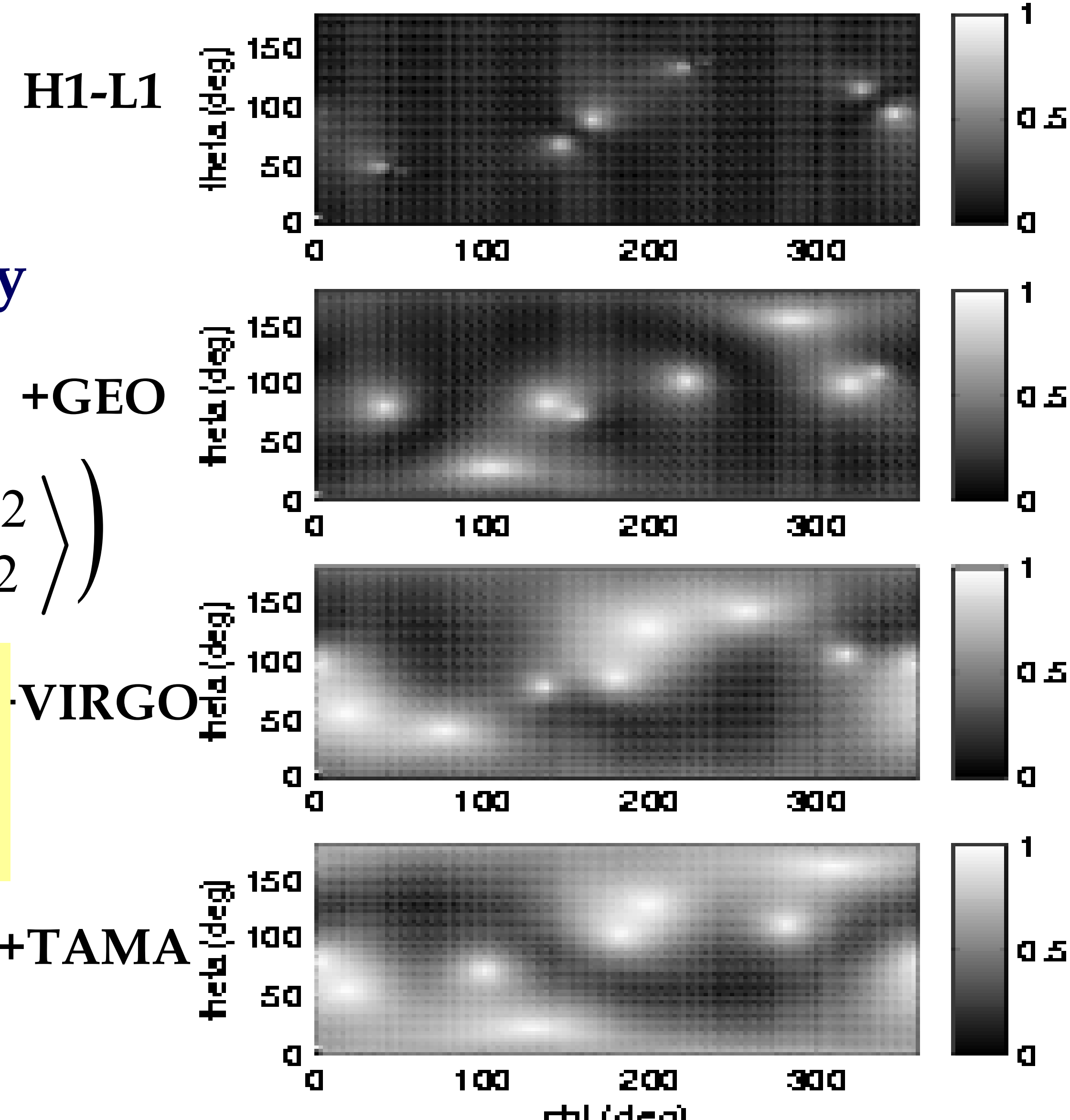
$$\varepsilon = \frac{g_r - |g_c|}{g_r + |g_c|}$$

shows relative sensitivity to two GW components

$$SNR \propto \left(\langle h_1^2 \rangle + \varepsilon \langle h_2^2 \rangle\right)$$

to be detected with the same SNR h_2 should be $1/\varepsilon$ times stronger then h_1

For aligned network ε=0



Network sensitivity factor

$$g = g_r + |g_c|$$

$$SNR = 2g\left(\left\langle h_1^2 \right\rangle + \varepsilon \left\langle h_2^2 \right\rangle\right)$$

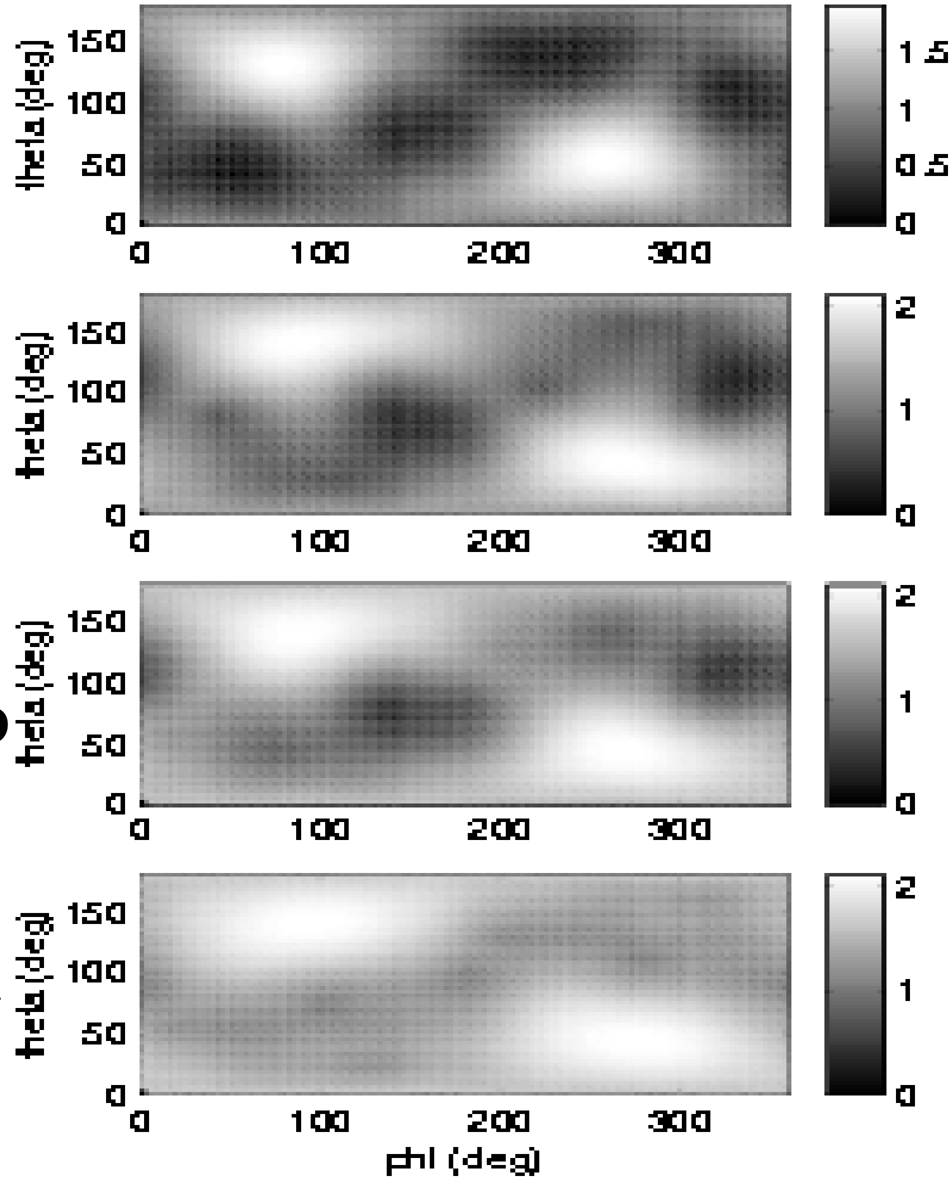
need several detectors for more uniform sky coverage

H1-L1

+GEO

+VIRGO

+TAMA



constraint

- **In average for a population of bursts expect** $\left\langle h_1^2 \right\rangle \approx \left\langle h_2^2 \right\rangle$
- **Hard constraint → ignore the second component**

$$L_{hard} = L_1(h_1) + \cancel{L_2(h_2)}$$

- **Soft constraint → weight the second component**

$$L_{soft} = L_1(h_1) + \varepsilon\, L_2(h_2)$$

  - **remove un-physical solutions produced by noise**
  - **sacrifice small fraction of GW signals but**
  - **enhance detection efficiency for the rest of sources**

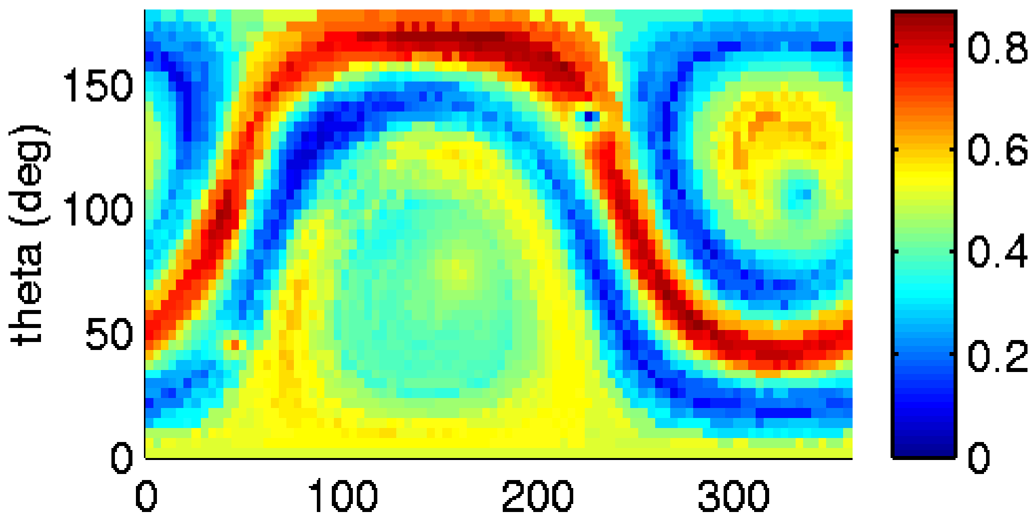
another approach:
penalized likelihood (Mohanty et al, LIGO-G050361-00-Z)

Two detector sky maps

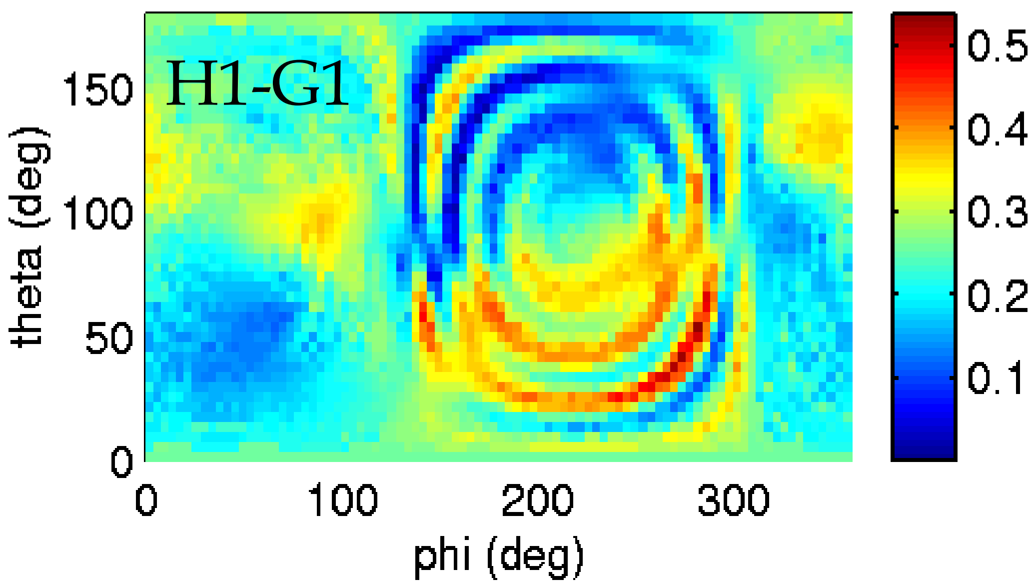
Simulation
- **Equal detector sensitivity**
- **white, G-noise**
- **injections:**

-Lazarus BH-BH
-2-polarizations

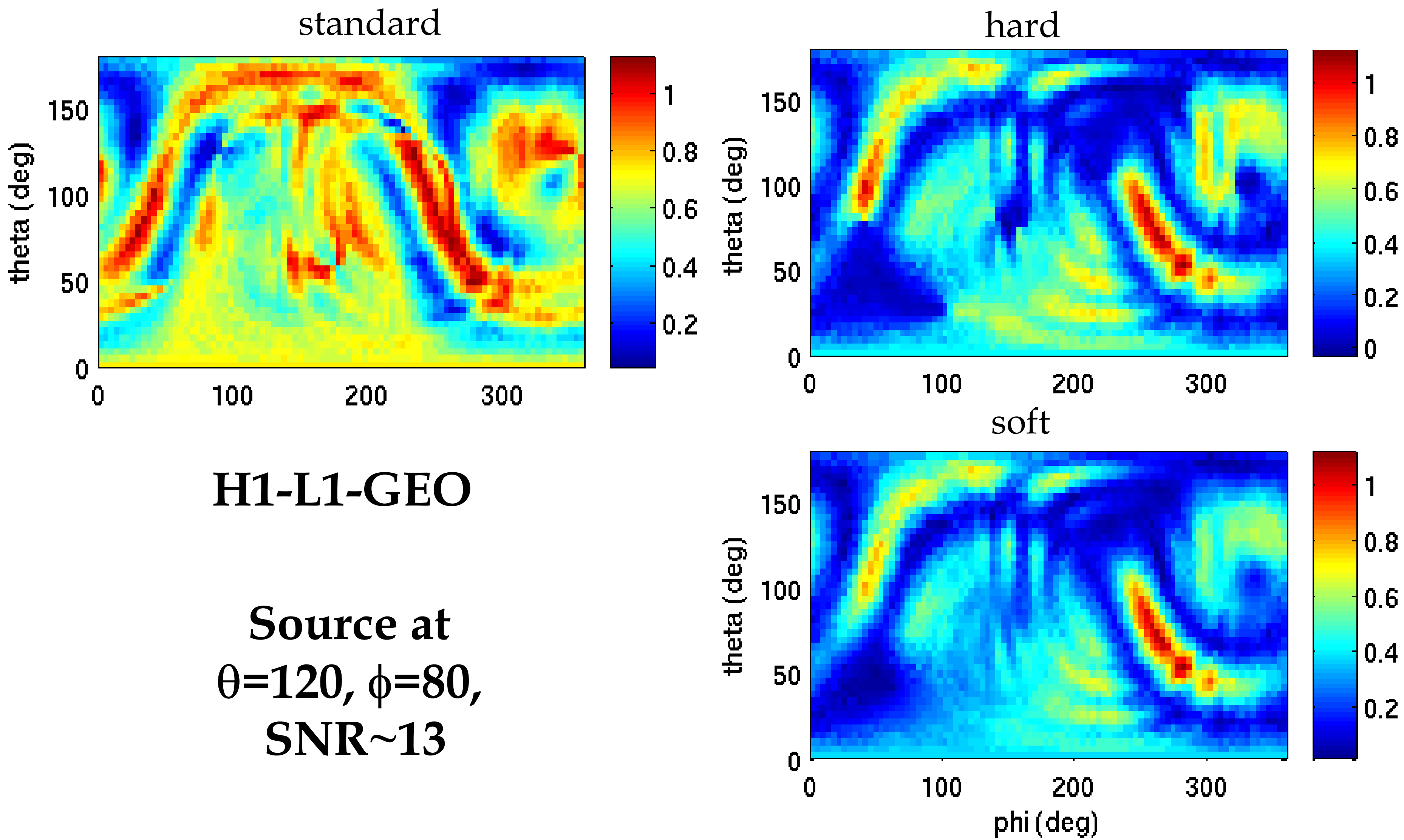
Source at θ=120, φ=80



Constraint likelihood gives directional information in the case of two detectors



Comparison of different methods

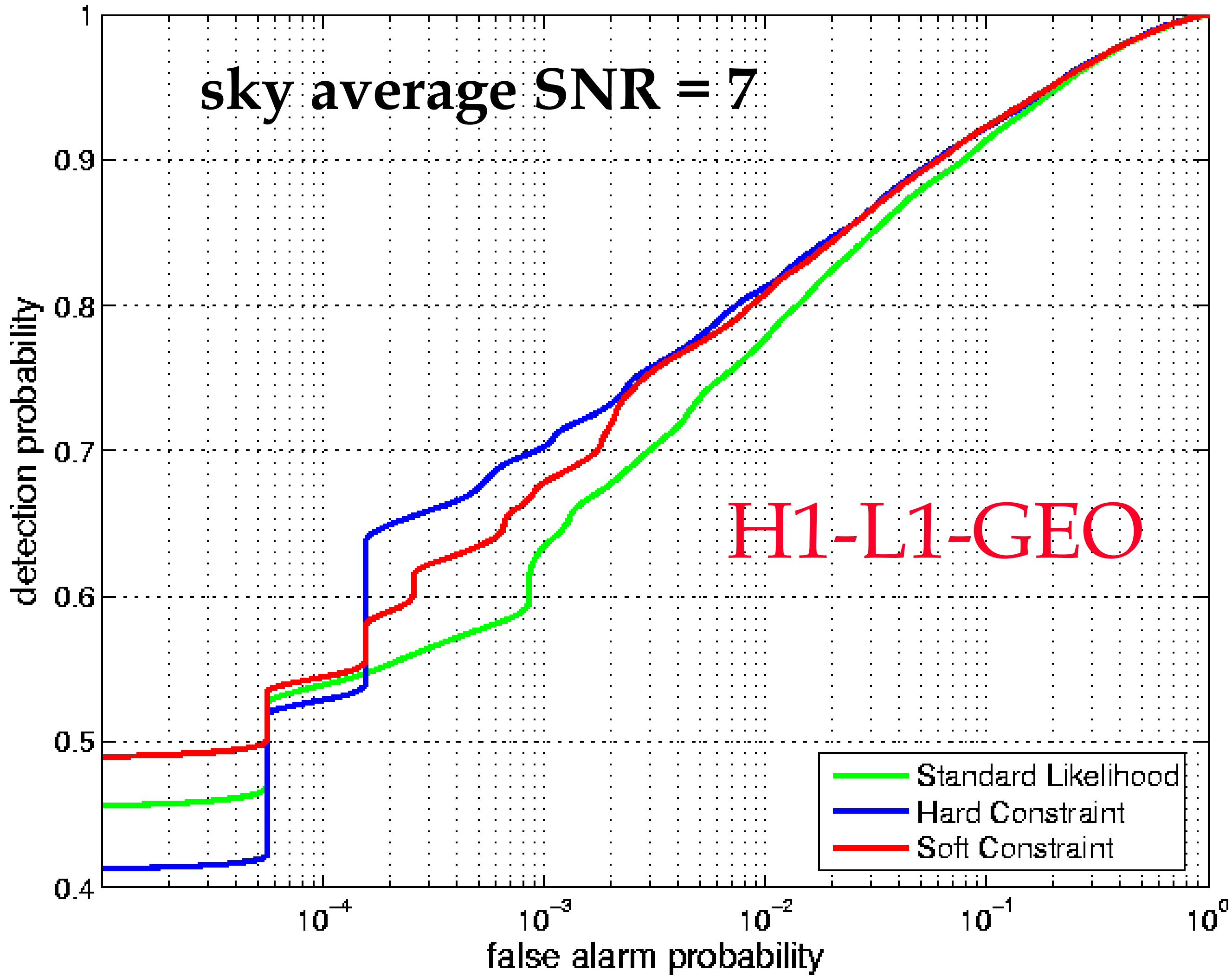


H1-L1-GEO

Source at
θ=120, ϕ=80,
SNR~13

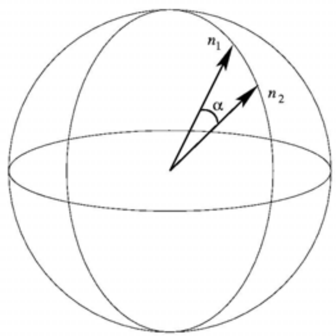
ROC

sources uniformly distributed over the sky

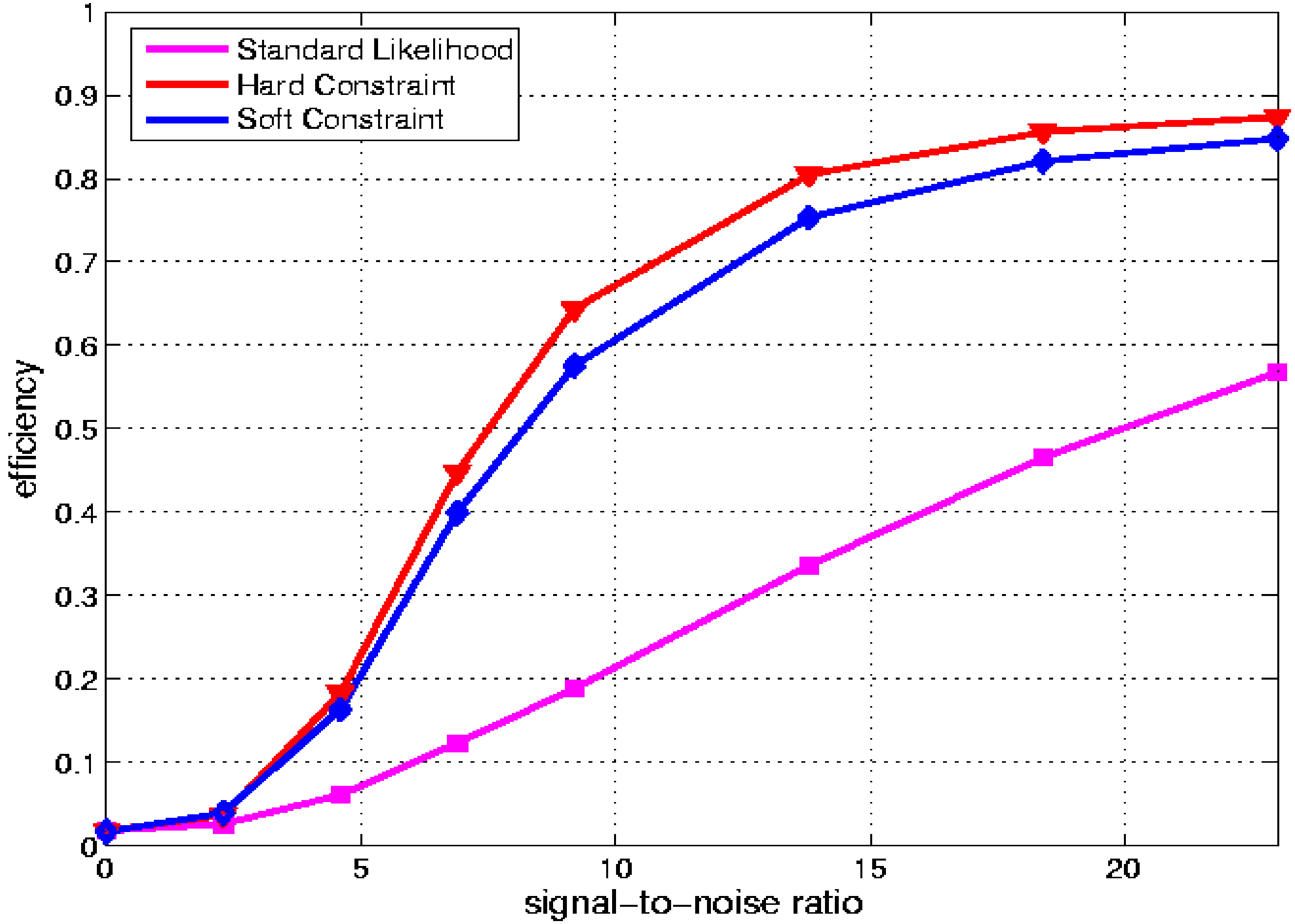


Angular resolution for different methods

fraction of events in the cone
|detected-injected|<16 degrees



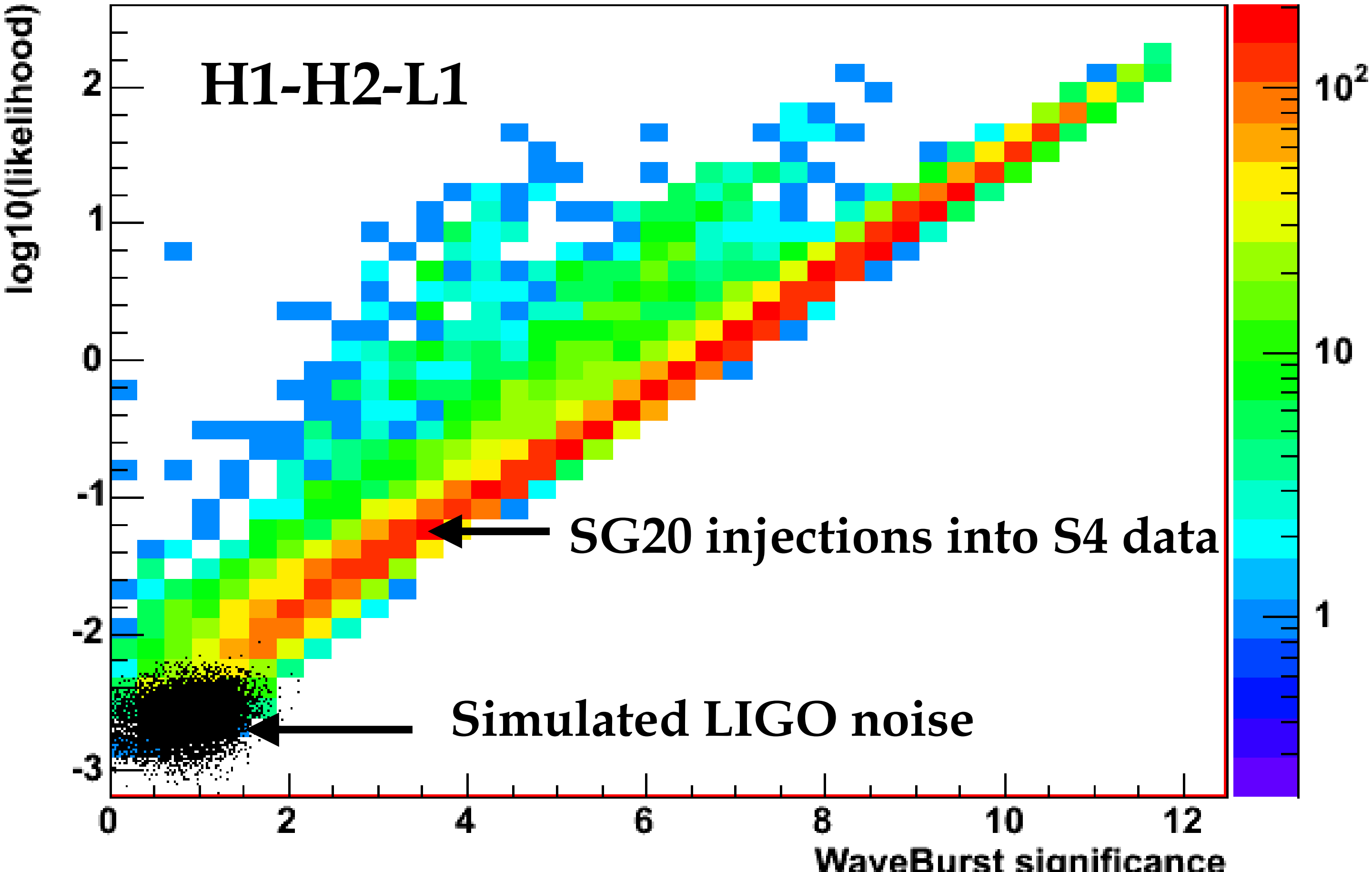
H1
L1
GEO



WaveBurst.net

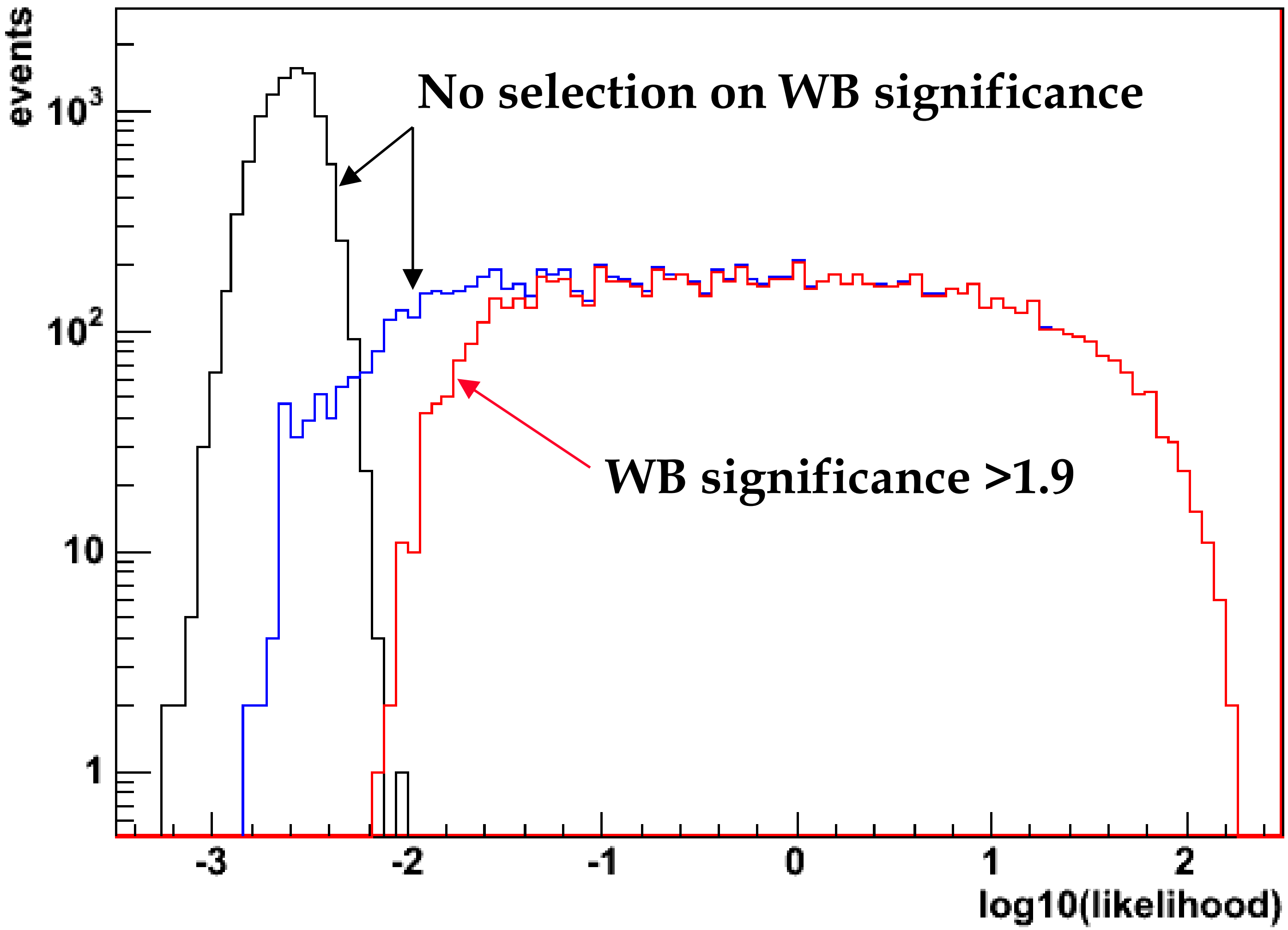
- WAT/DMT implementation (klimenko)
- First run on S4 data with SG20 injections (Yakushin)

Likelihood vs WaveBurst significance



Likelihood

- Likelihood distribution of WaveBurst triggers for
  - Simulated LIGO noise (black)
  - SG20 injections into S4 data (blue, red)



Summary

- Coherent network method for GW bursts searches
  - Based on the likelihood analysis
  - Obtain max likelihood ratio statistics by constrained variation of likelihood functional
  - detection statistic uses both power and cross-correlation terms
  - give information about source location, waveform reconstruction
  - any number of detectors, arbitrary alignment

- Plans
  - Finish debugging of hierarchical search WaveBurst.net
  - Study waveform and coordinate reconstruction on simulations
  - Compare with r-statistics for H1-H2-L1 (useful cross-check)
  - Analyze S 4 LIGO-GEO data → get ready for S5
  - Implement non-hierarchical search (cpu consuming)