

OPTIMIZED COATINGS

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- Current mirror design: *quarter-wavelength* (QWL) alternating SiO_2 and Ta_2O_5 layers.
- Yields *largest reflectance* among all stacked-doublet designs for any *fixed* no. of layers (or equivalently, *smallest* no. of layers at any *fixed reflectance*).
- Coating (structural) noise *dominates thermal-noise budget* in key spectral range.
- QWL coating does *not* yield minimum noise for a prescribed reflectivity, hence *not optimal*.

Coating Design Optimization: Status & Work Plan (2005-2006)

- **Status/Directions**

Genetic optimization (running)

The choice, for highest design flexibility and insight;

Stacked-doublet (completed)

Most obvious generalization of stacked quarter wavelength;

Regular non-periodic (just started)

getting closer to the “perfect mirror”;

- On top of this: *new materials* (e.g., JMM TiO₂-doped Tantalum)

INFN – COAT (2006 - PI Innocenzo M. Pinto)

Goal: prototyping four GA-optimized mirrors to be tested at CALTECH TNI.

Time-span: 1 year (2006).

Participants: TWG (algorithm and code),
CALTECH LIGO-Lab (substrates & TNI),
LMA Lyon, FR (prototyping; bare costs).

Partnerships: ILIAS-Strega (S. Rowan/J. Hough),
TAMA (Tsubono K.), VIRGO (F. Vettrano).

Requested budget: 50 KEU (60 K\$).

- **Nice Features**

Multiple, *heterogeneous* mixed continuous/discrete constraints;
Multi-objective and/or best tradeoff optimization;
Robust.

Available options include:

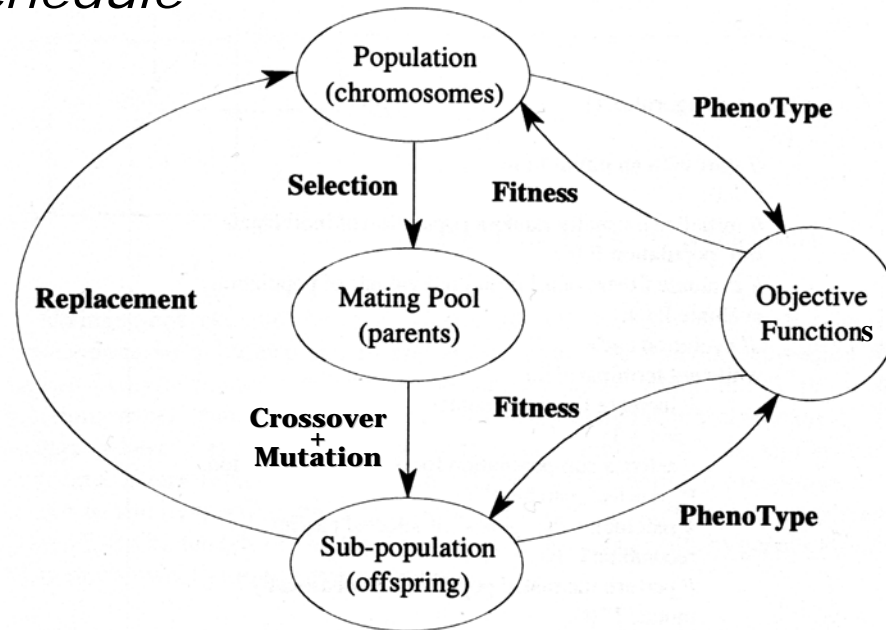
-structural/rheology-related constraints;
-multiple-wavelength operation;
-several (> 2) materials, etc....

Educated ignorance attitude (almost *no a-priori* assumption on structure of sought solution - will shed light on it !);

Effective & well established (e.g. microwave antenna and filter design)...

Status: PIKAIA-based Code-kernel developed.

- Problem unknowns \equiv *genes*;
- Point in search space \equiv *chromosome*;
- Set of points in search space \equiv *population*;
- Evolve *random* initial population according to an *evolutionary schedule*



Most obvious generalization of current stacked- $\lambda/4$ -design.

- Coating reflectivity is a monotonic (increasing) function of Bloch characteristic exponent (BCE) of transmission matrix of basic doublet (true for any *truncated-periodic*);
- Coating noise is closely modeled by a simple (linear) law:

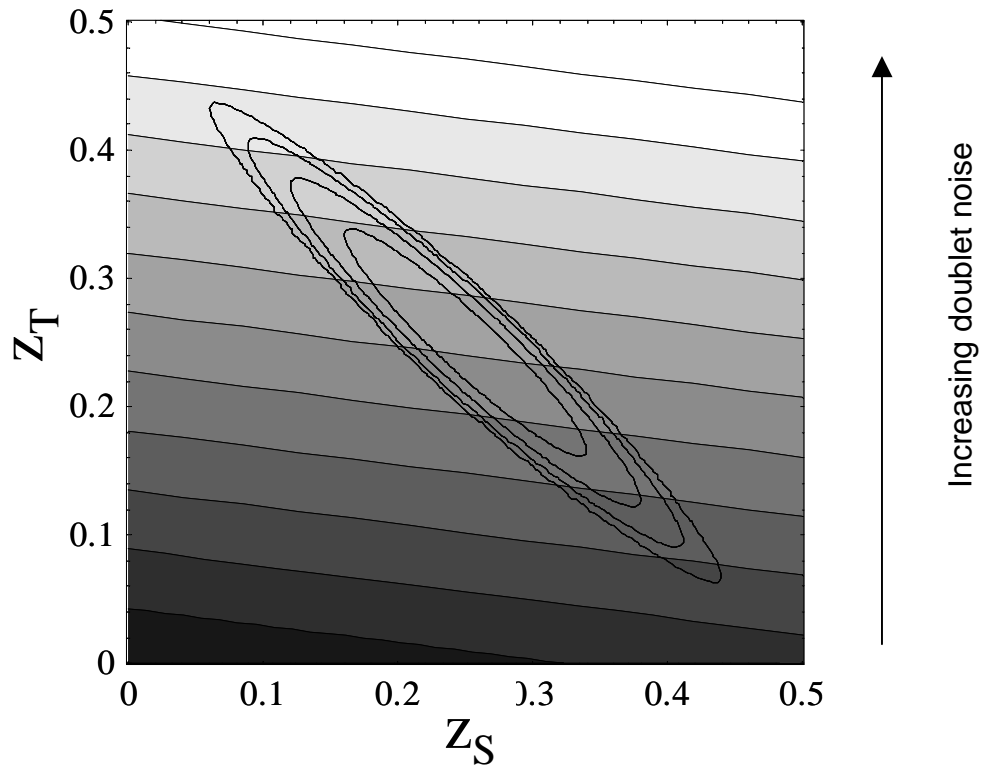
$$\nu = C(\Delta_{\text{Tantala}} + \gamma^{-1} \Delta_{\text{Silica}})$$

total (physical) thicknesses

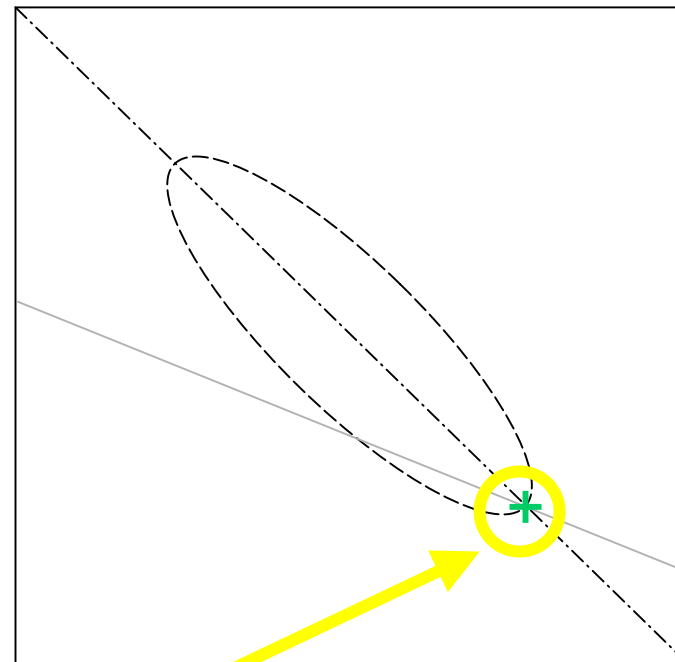
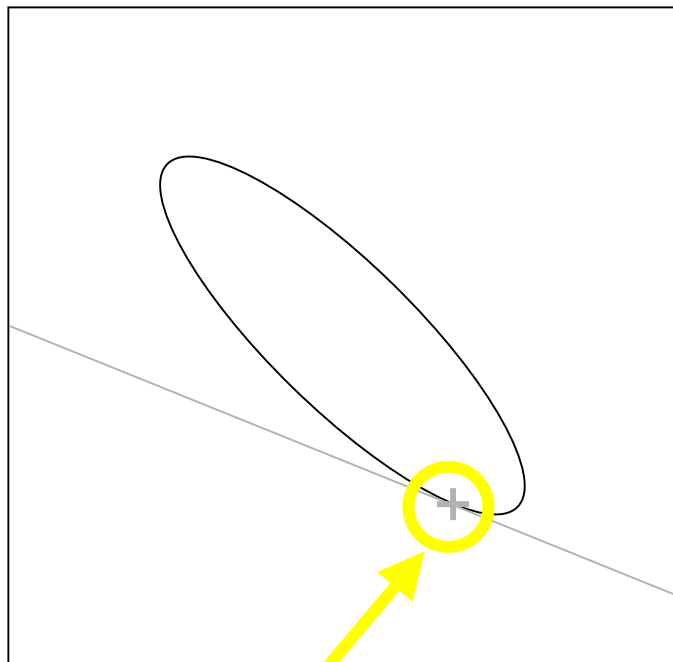
γ related to Young moduli, Poisson ratios & loss angles of both substrate & coating materials.
In view of present measurement uncertainties γ can be anything between 10 and 30.

Stacked Doublet Optimization

z_S = optical length SiO_2 layer
 z_T = optical length Ta_2O_5 layer
 } In units of λ_0

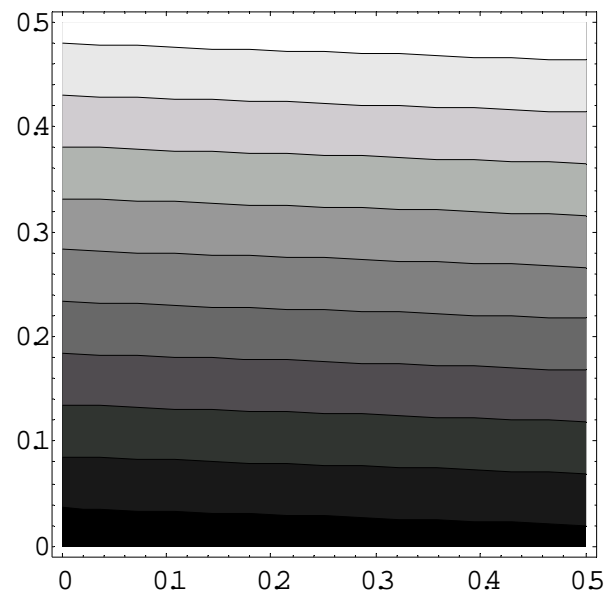
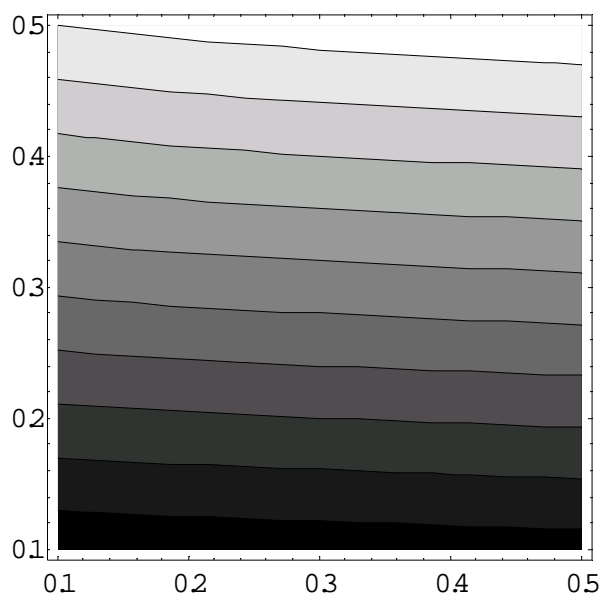


Stacked Doublet Optimization: Approximation # 1



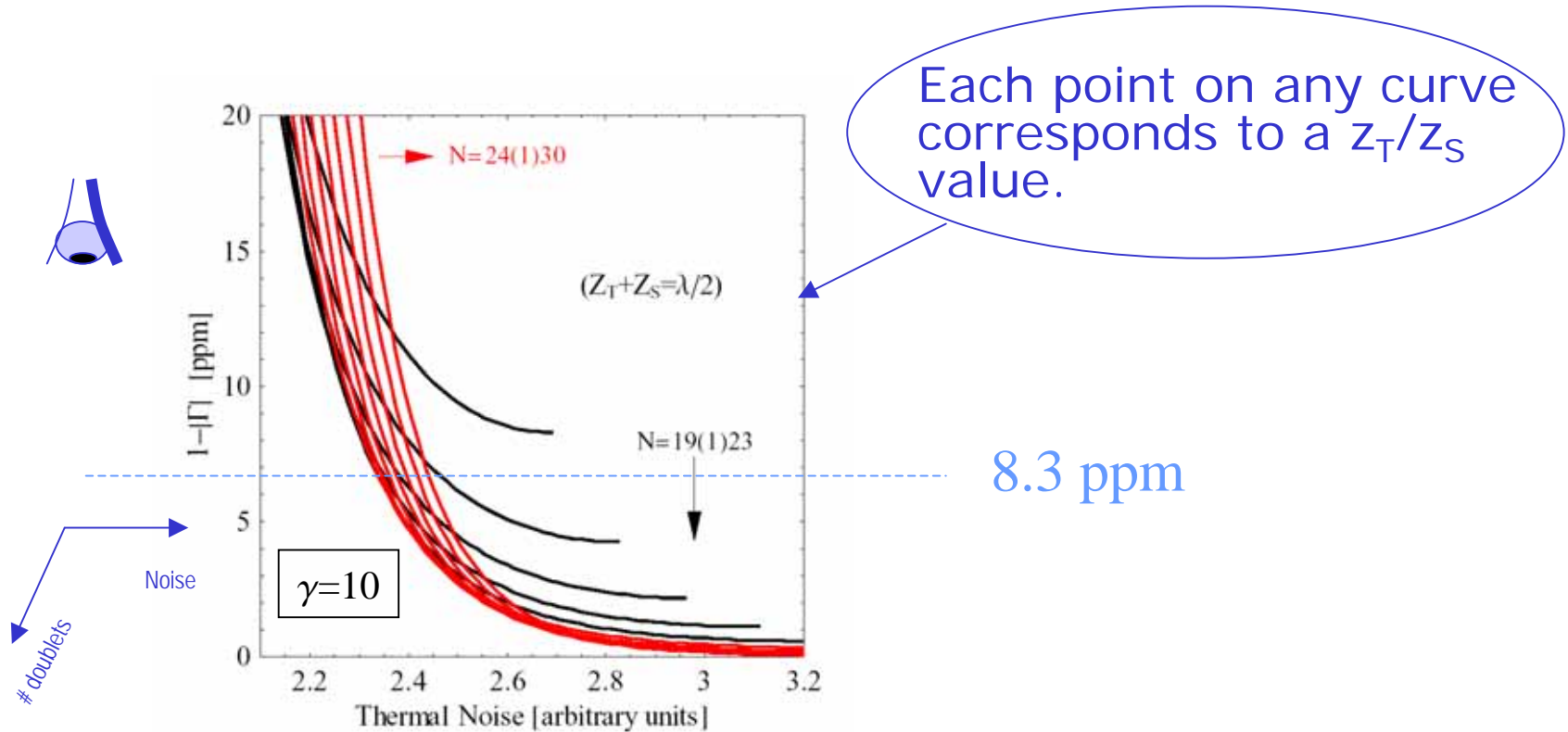
BCE contour lines *very thin*: **no** sensible difference between exact and approximate ($z_T+z_S=1/2$) optimization

Stacked Doublet Optimization: Approximation # 2

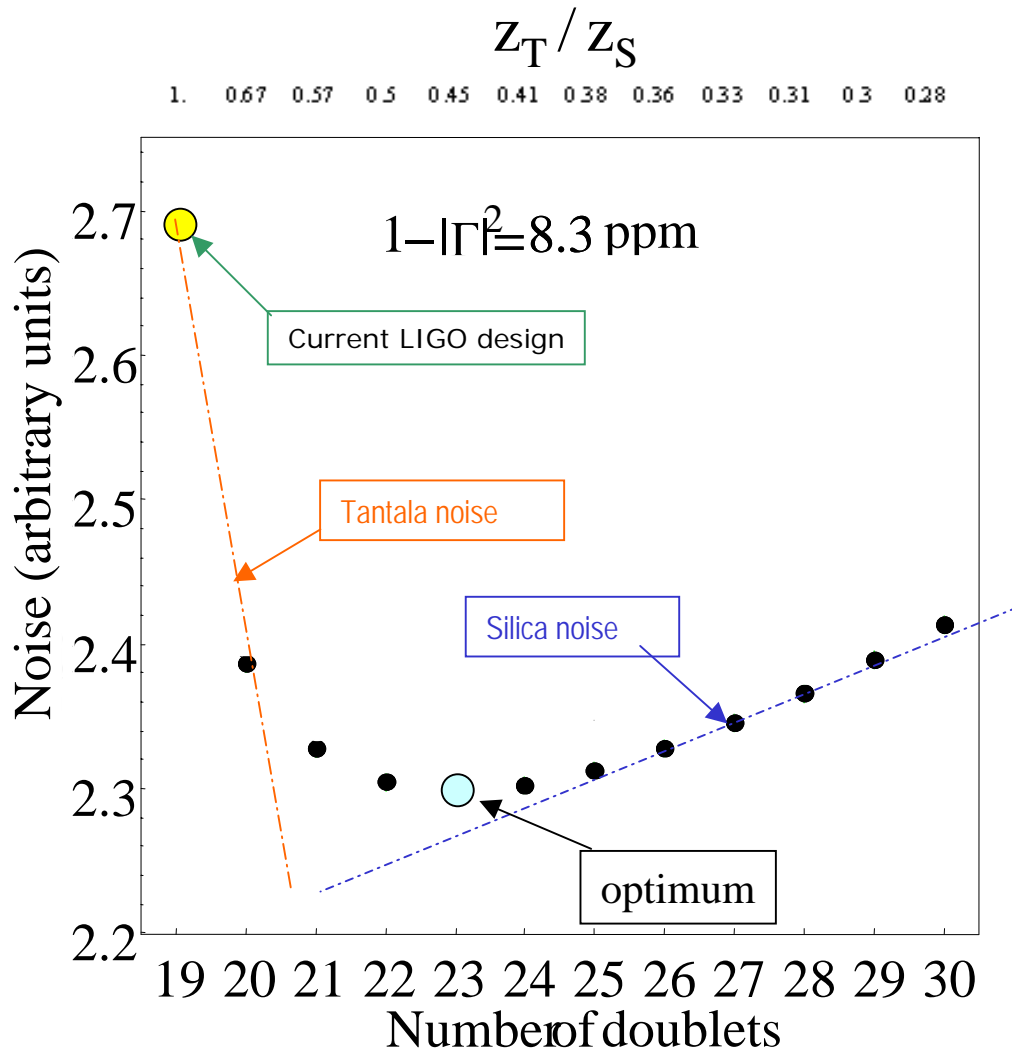


...both absorbed by large by uncertainty in γ

- Assign number N of doublets;
- Assign noise upper-bound noise for *whole* coating;
- Compute corresponding upper-bound for single doublet;
- Determine z_s and z_T so as to maximize BCE;
under the above noise constraint;
- Compute terminated N -doublet reflection coefficient.

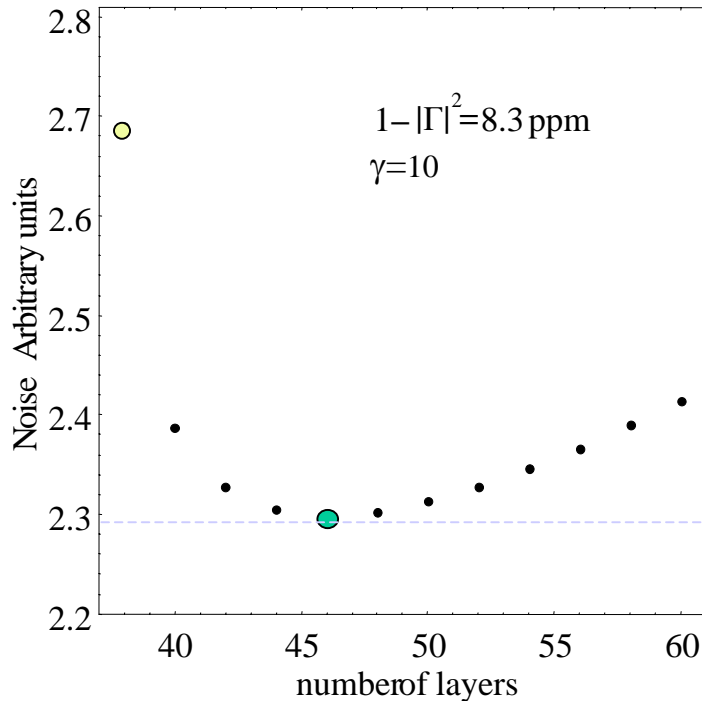


Status: Transmissivity vs. Noise Tradeoff Curves Drawn



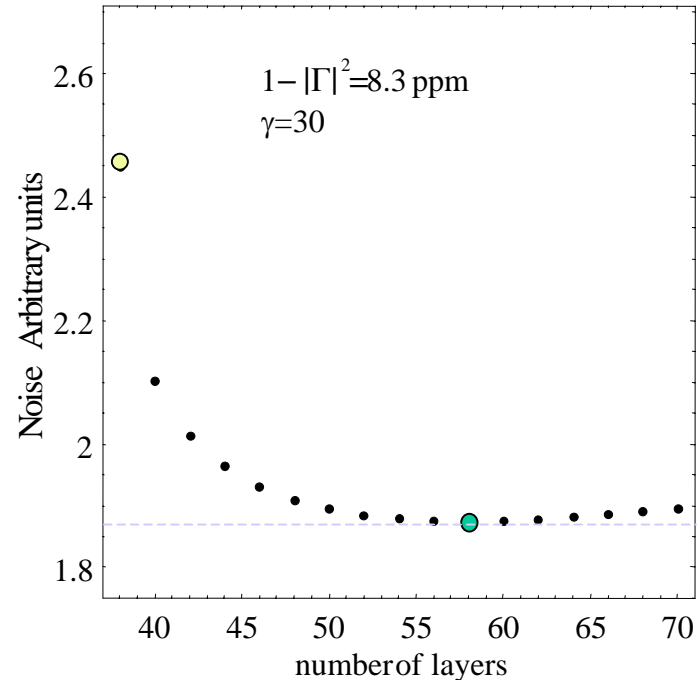
Quarter Wavelength (yellow bullets) vs. Optimized (grey bullets)
Stacked Doublet Design. Transmissivity 8.3 ppm.
Different $\text{SiO}_2/\text{Ta}_2\text{O}_5$ loss ratios.

LIGO G-050363-00-R



~14% noise reduction.
 N_d raised from 19 to 22-24
(absolute optimum at 23)

$\Delta[\text{SiO}_2]= 249.583 \text{ nm}$
 $\Delta[\text{Ta}_2\text{O}_5]= 80.8843 \text{ nm}$

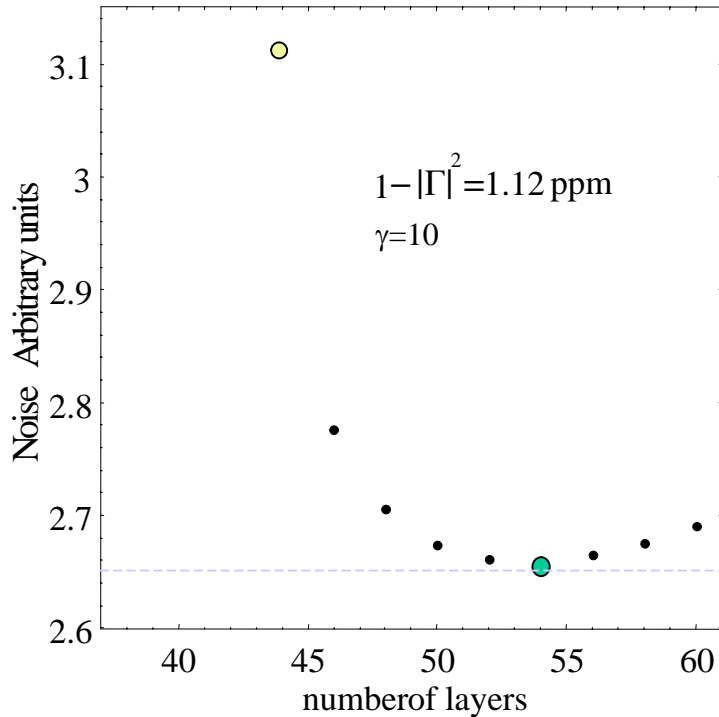


~24% noise reduction.
 N_d raised from 19 to ≥ 25
(absolute optimum at 29)

$\Delta[\text{SiO}_2]= 280.216 \text{ nm}$
 $\Delta[\text{Ta}_2\text{O}_5]= 59.3664 \text{ nm}$

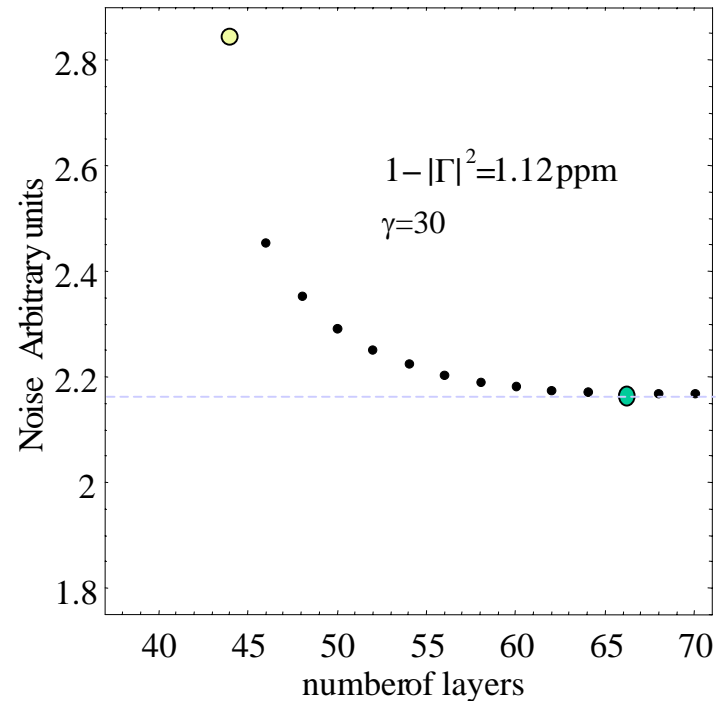
Quarter Wavelength (yellow bullets) vs. Optimized (grey bullets)
Stacked Doublet Design. Transmissivity 1.12 ppm.
Different $\text{SiO}_2/\text{Ta}_2\text{O}_5$ loss ratios.

LIGO G-050363-00-R



~14% noise reduction.
 N_d raised from 22 to 25-28
(absolute optimum at 27)

$\Delta[\text{SiO}_2]= 251.871 \text{ nm}$
 $\Delta[\text{Ta}_2\text{O}_5]= 60.5726 \text{ nm}$



~24% noise reduction.
 N_d raised from 22 to ≥ 28
(absolute optimum at 33)

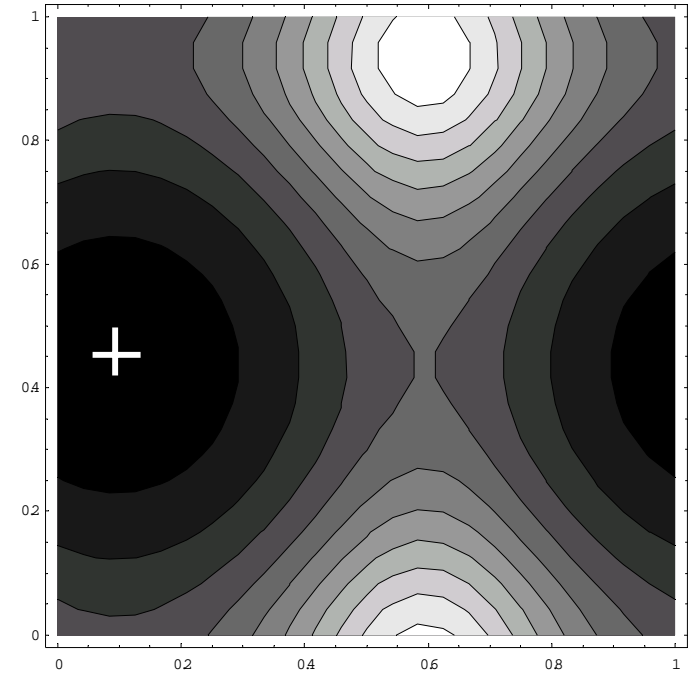
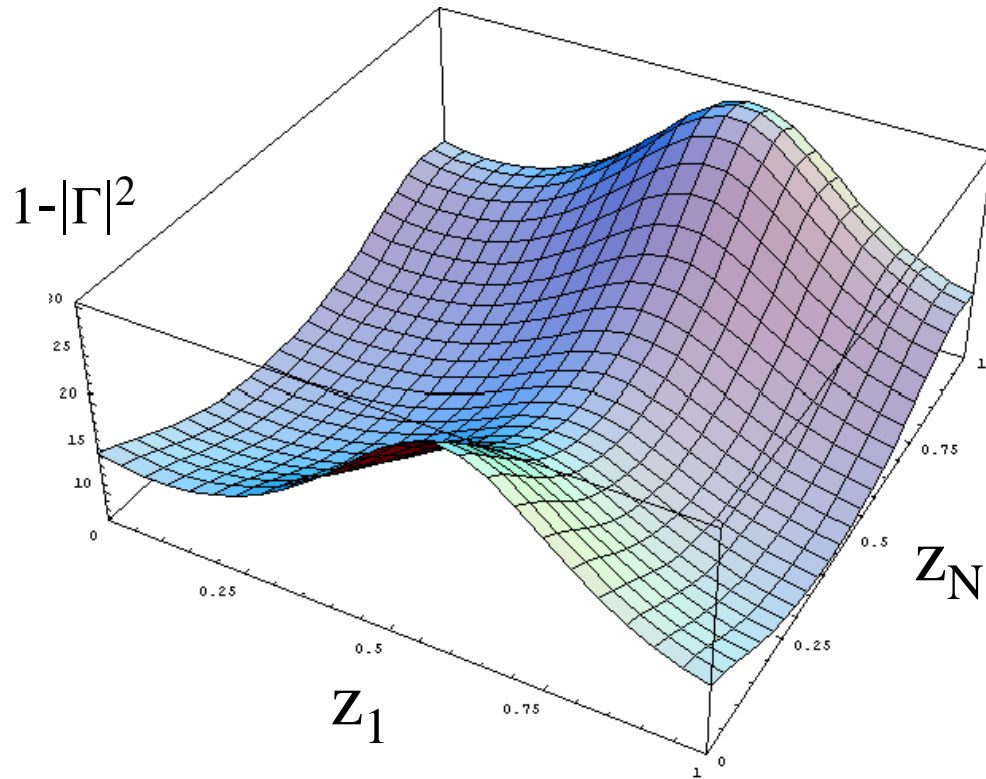
$\Delta[\text{SiO}_2]= 278.465 \text{ nm}$
 $\Delta[\text{Ta}_2\text{O}_5]= 60.5726 \text{ nm}$

vs. nearest-neighbour quarter-wavelengths (QWL)

	QWL-1	Genetic	QWL-2
N (cap included)	36	44	28
$1- \Gamma ^2$ ppm	16.20	14.91	235.46
L(Ta ₂ O ₅) nm	2359.43	1815.61	1835.11
L(SiO ₂) nm	3479.98	5217.4	2747.35
L _{tot} nm	5839.41	7033.01	4582.46

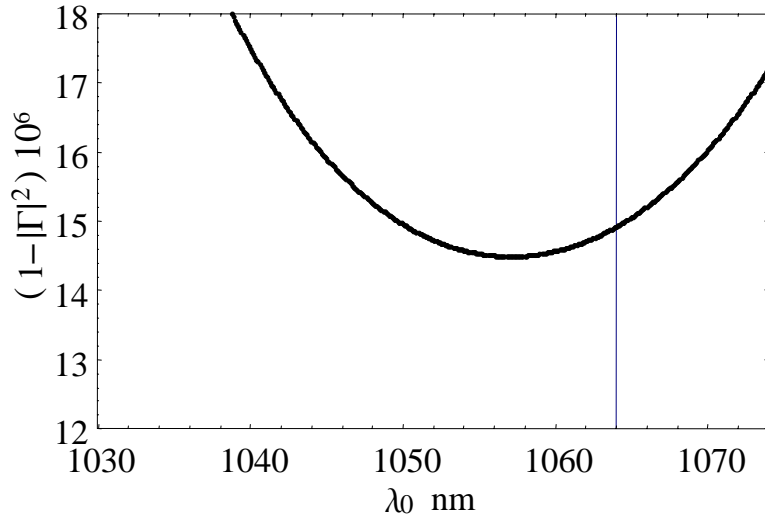
Stacked Doublets: Lesson from GA: Tweak End Layers to Improve Reflectivity !

LIGO G-050363-00-R



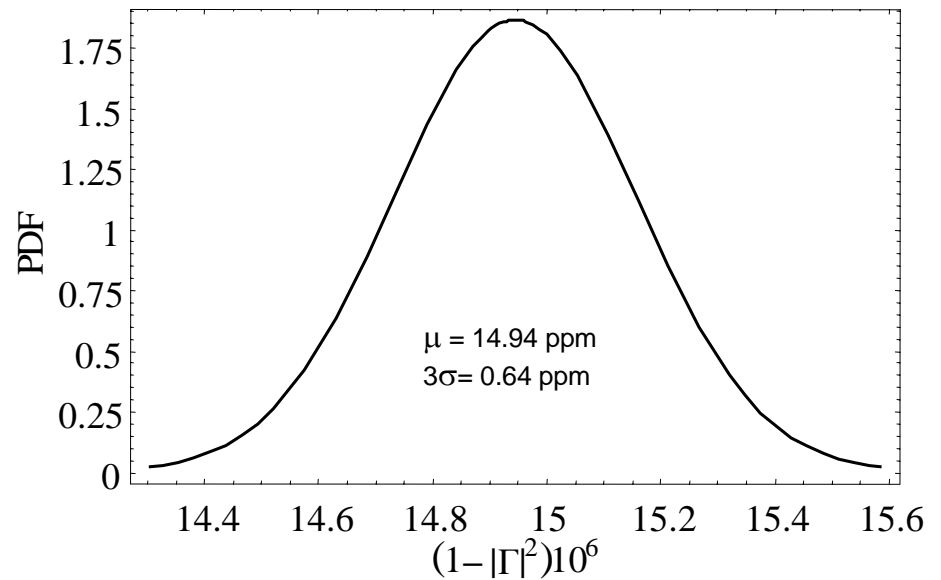
($z_1 = 0.0943$, $z_N = 0.0437$, in units of $\lambda/2$)

reflectance increased by $\sim 10\%$, noise increased by $\sim 1\%$



Distribution of $1-|\Gamma|^2$.
 Random uniform errors,
 $|\delta\ell| \leq 1\text{nm}$ 10^4 trials.

Mirror frequency response
 (normal incidence).



Two main sub-classes: { Fractal (e.g., Cantor);
Substitutional (e.g. Fibonacci);

Goal: large bandwidths (in frequency *and* wavenumber)
[e.g., [Optics Lett. 23 \(1998\) 1573](#)];

Background: applications in antenna array synthesis
[e.g., [IEEE Trans. AP-53 \(2005\) 635](#)]

Status: just started