The Laser Interferometer Gravitational-Wave Observatory

http://www.ligo.caltech.edu

General Relativity…

Astrophysics…

Einstein…

A new window…

Supported by the United States National Science Foundation

The Search For Periodic Gravitational Waves

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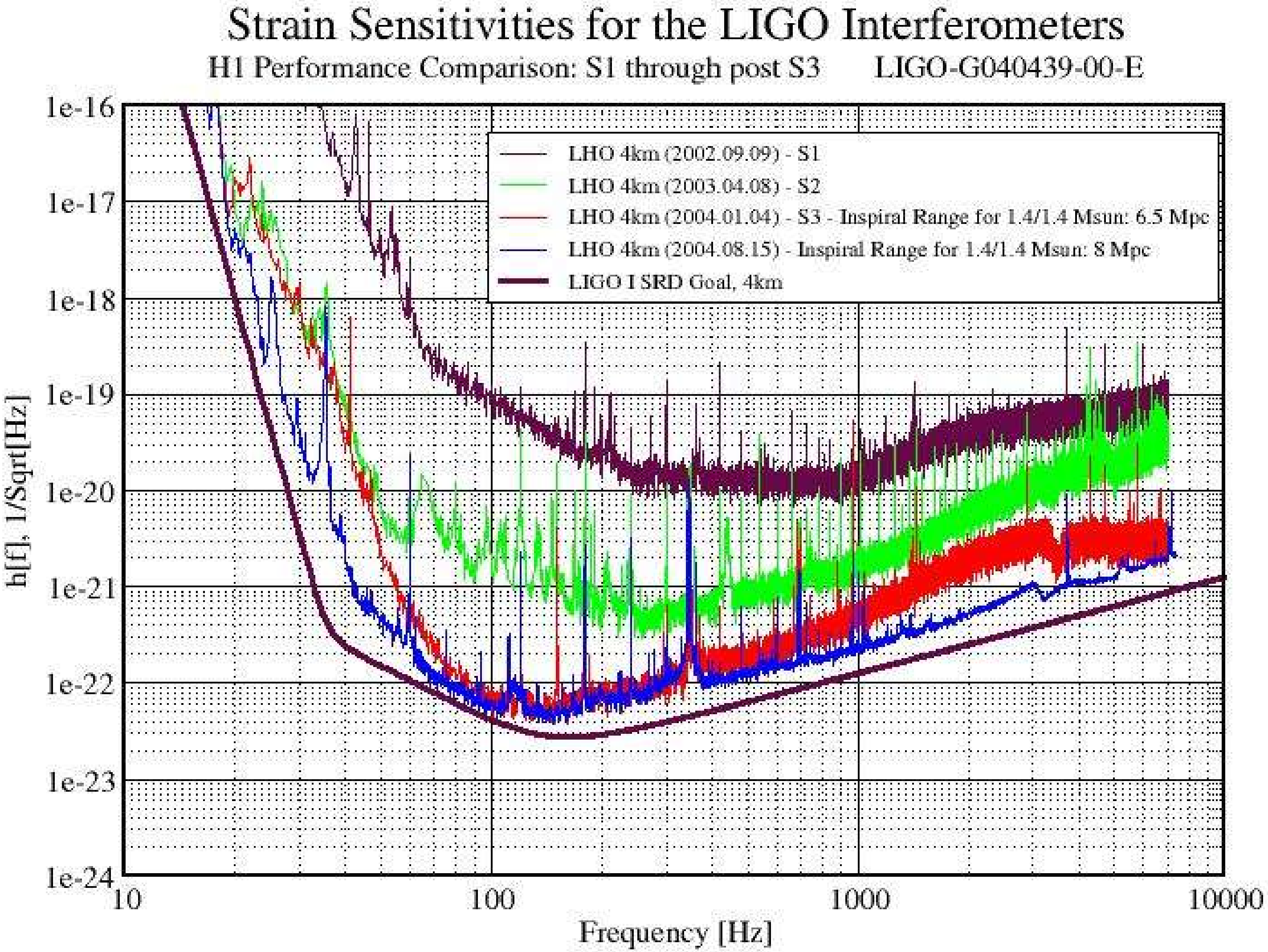
on behalf of the LIGO Science Collaboration

LIGO-G050191-00-W

Noise Amp. Spectral Density: $[S_n(f)]^{1/2}$

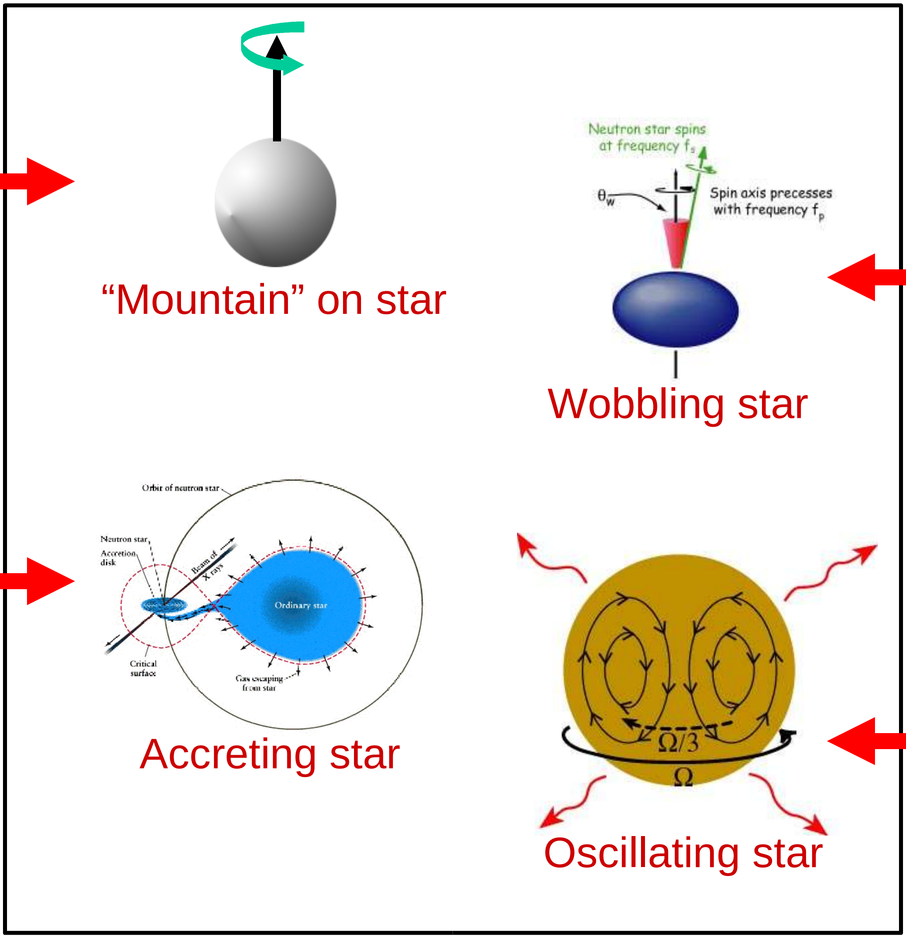


Continuous Periodic Gravitational-Wave Sources

$$= \frac{I_1 - I_2}{I}$$

Mountain mass & height: εMR^2

Low-mass x-ray binary: balance GW torque with accretion torque τ.



“Mountain” on star

Wobbling star

Accreting star

Oscillating star

Free precession wobble angle θ

Unstable vibrations with amplitude A

The back of the envelope please…

Approximate form of the Quadrupole Formula

$$h = \Delta L/L \approx (G/c^4)(\ddot{Q}/r); \quad Q \sim MR^2$$

$$\ddot{Q} \sim (MV^2)_{asym.}; \quad h \sim 10^{-49} erg^{-1} (MR^2 f_{GW}^2)_{asym.}$$

- Triaxial ellipsoid: $h \sim (G/c^4)\, \varepsilon MR^2 4 f_{rot}^{\,2}/r \sim 10^{-26}$ (for ellipticity $\varepsilon \sim 10^{-6}$, $f_{rot} = 200$ Hz, $M = 1.4\, M_{\odot}$ $R = 10^6$ cm, $r = 1$ kpc $= 3\times 10^{21}$ cm)

- Precession: $h \sim (G/c^4) \sin(2\theta)\, \varepsilon\, MR^2 f_{rot}^{\,2}/r \sim 10^{-27}$ (for ellipticity $\varepsilon \sim 10^{-6}$, $r = 1$ kpc, wobble angle $\theta = \pi/4$, etc…)

- LMXB Sco-X1: $h \sim 10^{-26}$ (balance GW torque with accretion torque)

- R-modes: $h \sim (G/c^4)\, MA^2R^2 (16/9)\, f_{rot}^{\,2}/r \sim 10^{-26}$ (for saturation amplitude $A \sim 10^{-3}$, $r = 1$ kpc, etc…)

- Pulsar Glitch $h \sim (G/c^4)\, MA^2R^2 f_{rot}^{\,2}/r \sim 10^{-32}$ (for glitch amplitude $A \sim 10^{-6}$, $r = 1$ kpc, etc…)

LSC Period/CW Search Group
Search Techniques

- False alarm & false dismissal rates determine SNR of detectable signal.

- Coherent matched filtering tracks phase.

- Incoherent power averaging tracks frequency only.

- Optimal search needs 10^{23} templates per 1 Hz for 1 yr of data; Hierarchical

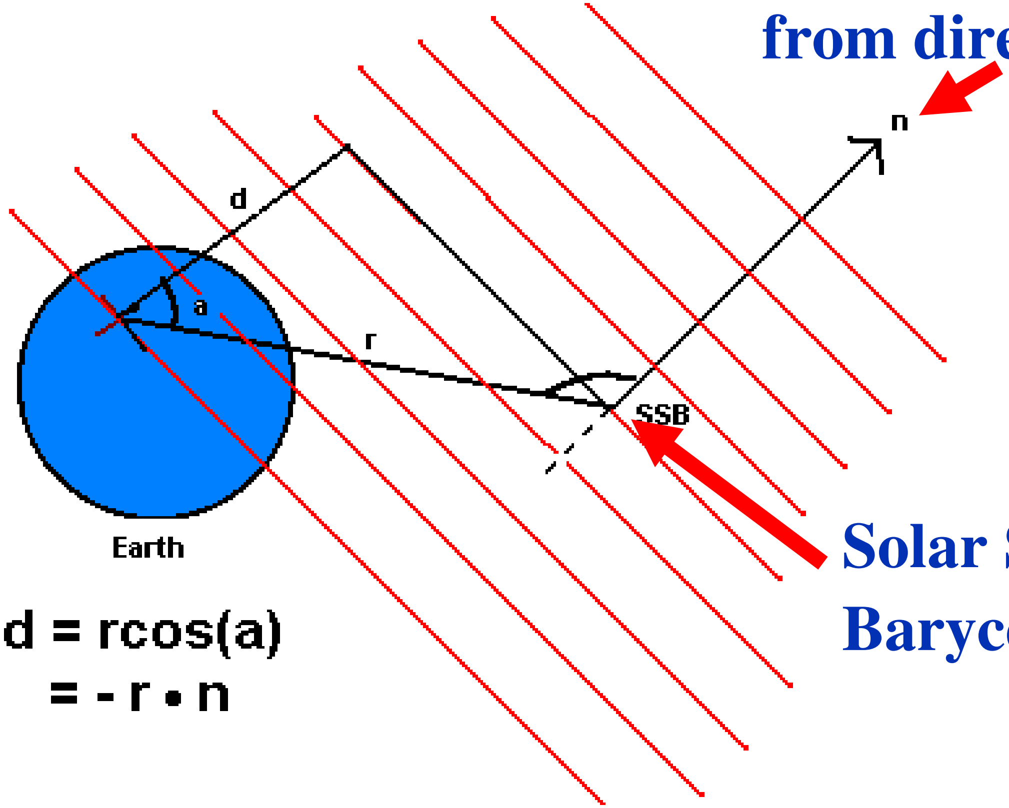
$$SNR = h_c \sqrt{T_{obs}} / \sqrt{S_n}$$

$$\langle h_c \rangle_{coh} = 3 \times 10^{-26} \frac{\sqrt{S_n}}{10^{-23}\, Hz^{-1/2}} \sqrt{\frac{10^7\, s}{T_{obs}}}$$

$$\langle h_c \rangle_{incoh} = 2 \times 10^{-25} \frac{\sqrt{S_n}}{10^{-23}\, Hz^{-1/2}} \left(\frac{1800\, s}{T_{coh}} \frac{10^7\, s}{T_{obs}} \right)^{1/4}$$

(For a 1% false alarm rate, 10% false dismissal rate, an average sky position and source orientation, the effective SNR needed for detection is 11.4 and 8.5$(T_{obs}/T_{coh})^{1/4}$ for coherent and incoherent searches respectively.)

Phase Modulation



GW approaching from direction n

Solar System Barycenter

$d = r\cos(a)$
$= -\mathbf{r} \bullet \mathbf{n}$

T = arrival time at SSB

t = arrival time at detector

$$T = t + \frac{r(t)}{c} \cdot \hat{n}$$

Relativistic corrections are included in the actual code.

Phase and Frequency Modulation

Phase at SSB:

$$= \qquad {}_{0} + \sum_{s=0} \frac{f_{s}}{(s+1)!}\left(T-T_{0}\right)^{s+1}$$

Phase at detector:

$$= \qquad {}_{0} + \sum_{s=0} \frac{f_{s}}{(s+1)!}\left(t+\frac{r(t)}{c}\cdot\hat{n}-T_{0}\right)^{s+1}$$

Frequency at detector:

$$f(t)=\left(1+\frac{v(t)}{c}\cdot\hat{n}\right)\left[f_{0}+\sum_{s=1}\frac{f_{s}}{s!}\left(t+\frac{r(t)}{c}\cdot\hat{n}-T_{0}\right)^{s}\right]$$

df/dt :

$$\dot{f}(t)=\left(\frac{a(t)}{c}\cdot\hat{n}\right)\left[f_{0}+\sum_{s=1}\frac{f_{s}}{s!}\left(t+\frac{r(t)}{c}\cdot\hat{n}-T\right)^{s}{}_{0}\right]+\ldots$$

Amplitude Modulation

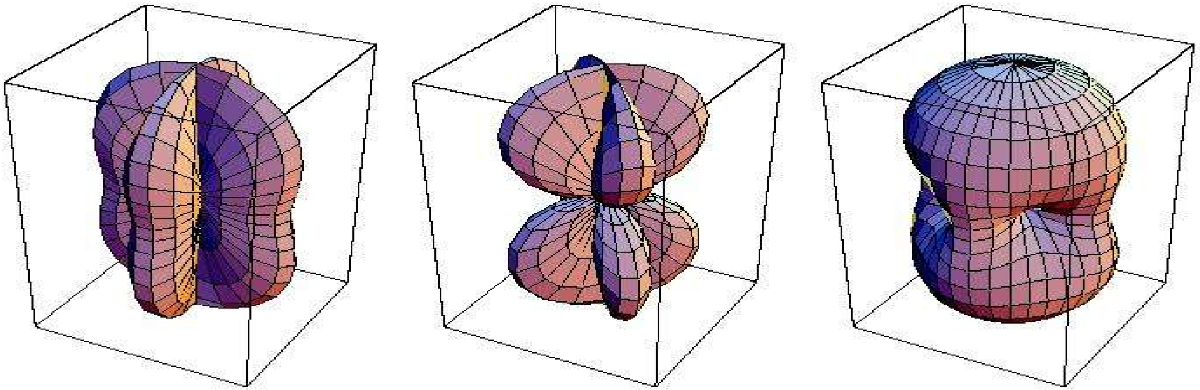


Figure 9. Antenna response function for an interferometric gravitational wave detector. The interferometer is placed at the center of the surrounding box with Michelson arms oriented along the horizontal axes. The distance from a point of the plot surface to the center of the box is a measure for the gravitational wave sensitivity in this direction. The plot to the left is for + polarization, the middle one for × polarization and the right one for unpolarized waves.

Figure: D. Sigg LIGO-P980007-00-D

Polarization Angle ψ

Beam Pattern Response Functions: $F_{+}(t;\quad)$ $F_{¿}(t;\quad)$

Coherent Matched Filtering

$$X_b = \sum_{=0}^{M-1} \sum_{j=0}^{N-1} x_j F_+ (0) e^{-i} \; {}^{jb} \qquad x_j = \frac{1}{N} \sum_{k=0}^{N-1} X_k^{SFT} e^{2} \; {}^{ijk/N}$$

$$\bar{X}_{+b} \simeq \sum_{=0}^{M-1} F_+ (0) e^{-i} \; {}^{0}\sum_k \frac{X_k^{SFT} \sin 2 \quad {}_b -i(1-\cos 2 \quad {}_b)}{\sqrt{S_k} \quad 2 \quad (k- \quad {}_b)}$$

$$F = \frac{4}{M} \frac{\langle F_{\text{¿}}^2 \rangle |\bar{X}_+|^2 + \langle F_+^2 \rangle |\bar{X}_{\text{¿}}|^2 - 2 \langle F_+ F_{\text{¿}} \rangle \Re e \left(\bar{X}_+ \bar{X}_{\text{¿}}^{\text{¿}} \right)}{\langle F_+^2 \rangle \langle F_{\text{¿}}^2 \rangle - \langle F_+ F_{\text{¿}} \rangle^2}$$

Jaranowski, Krolak, & Schutz gr-qc/9804014; Schutz & Papa gr-qc/9905018; Williams and Schutz gr-qc/9912029; Berukoff and Papa LAL Documentation

Time Domain Coherent Search Using Bayesian Analysis

Bayes Theorem: $P(A)P(B|A)=P(B)P(A|B)$

Likelihood: $P(x|h)=\frac{1}{\sqrt{2}\;{}_1}e^{\frac{-(x_1-h_1)^2}{2\;{}_1^2}}\frac{1}{\sqrt{2}\;{}_2}e^{\frac{-(x_2-h_2)^2}{2\;{}_2^2}}\ldots$

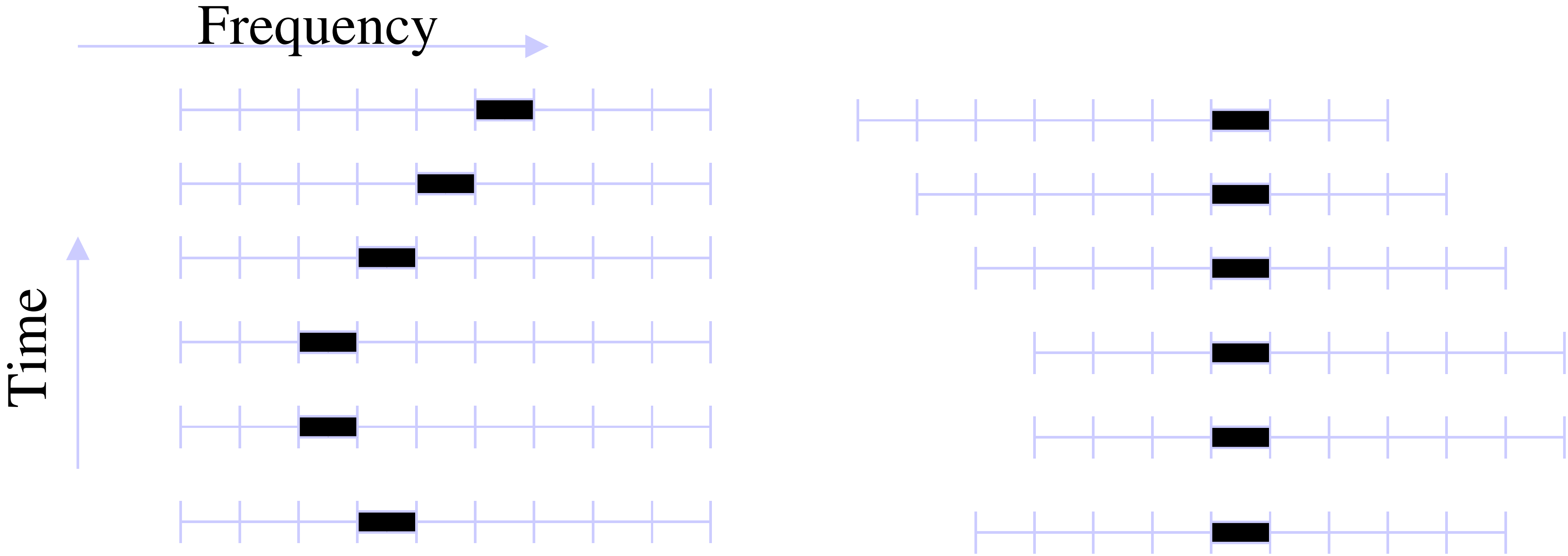
Posterior Probability: $P(h|x)=\frac{P(h)}{P(x)}P(x|h)$

Confidence Interval: $C=\int_0^{h_c}P(h|x)dx$

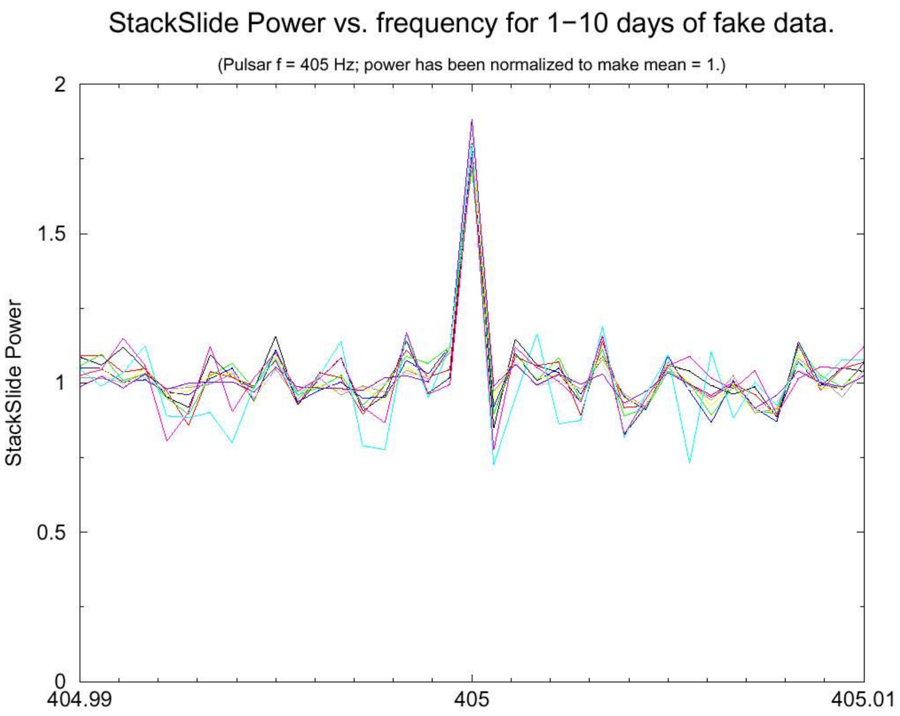
Prior

Incoherent Power Averaging

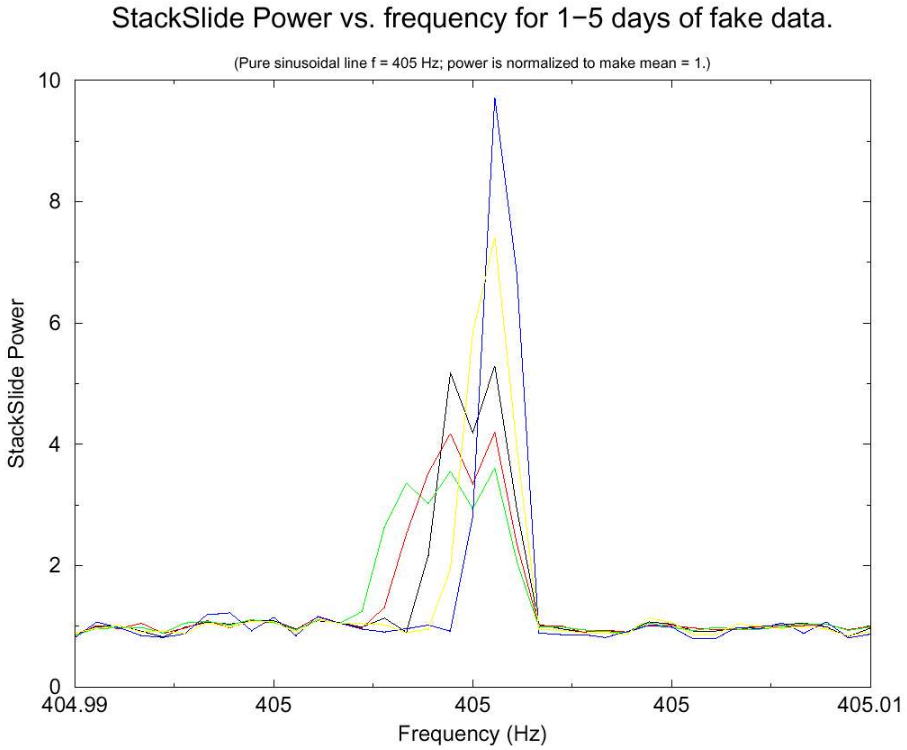
- Break up data into M segments; FFT each segment.
- Track the frequency.
- StackSlide: add the power weighted by the noise inverse.
- Hough: add 1 or 0 if power is above/below a cutoff.
- PowerFlux: add power using weights that maximize SNR



Fake Pulsar vs. Fake Instrument Line


StackSlide Power vs. frequency for 1−10 days of fake data.

(Pulsar f = 405 Hz; power has been normalized to make mean = 1.)

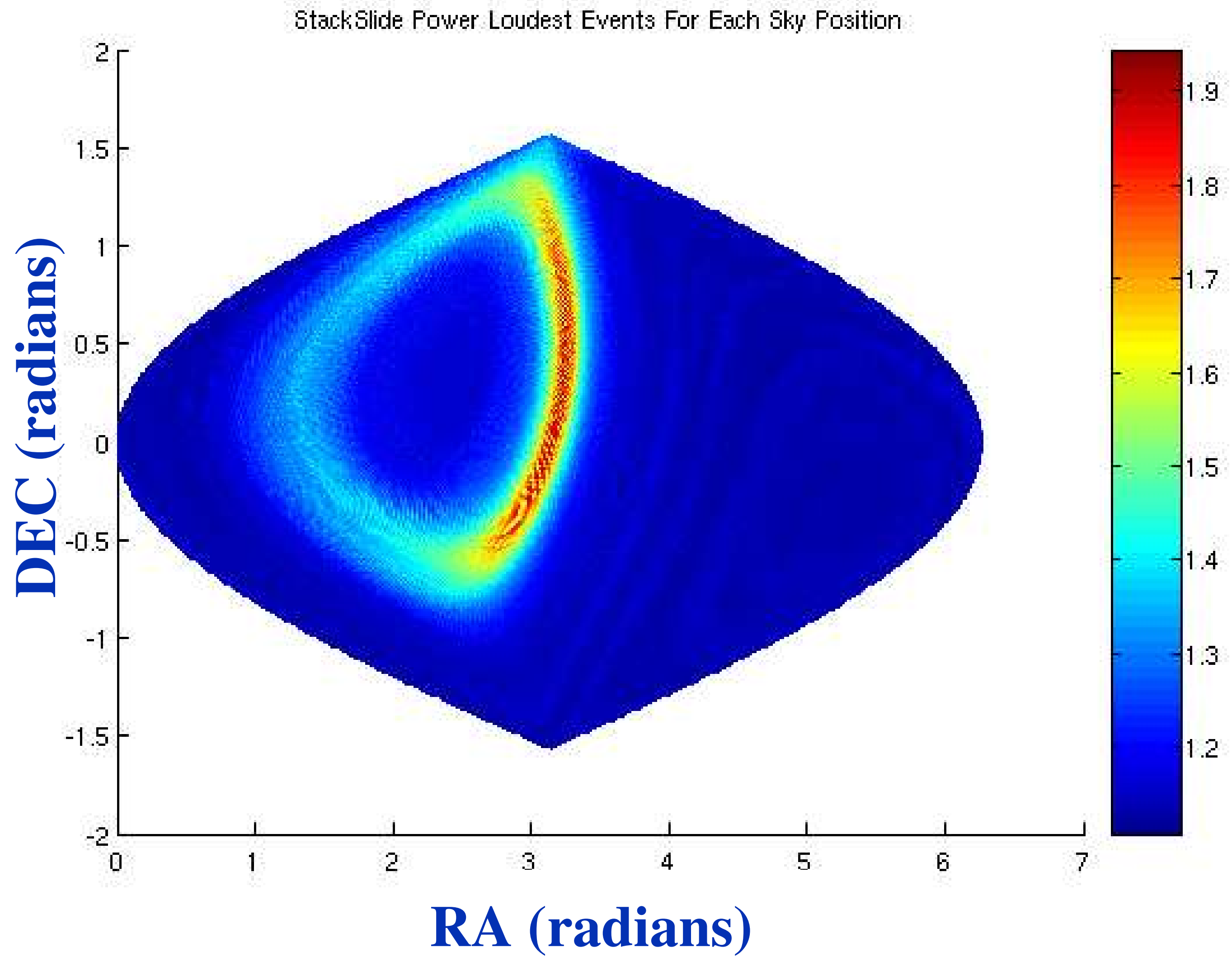

StackSlide Power vs. frequency for 1−5 days of fake data.

(Pure sinusoidal line f = 405 Hz; power is normalized to make mean = 1.)

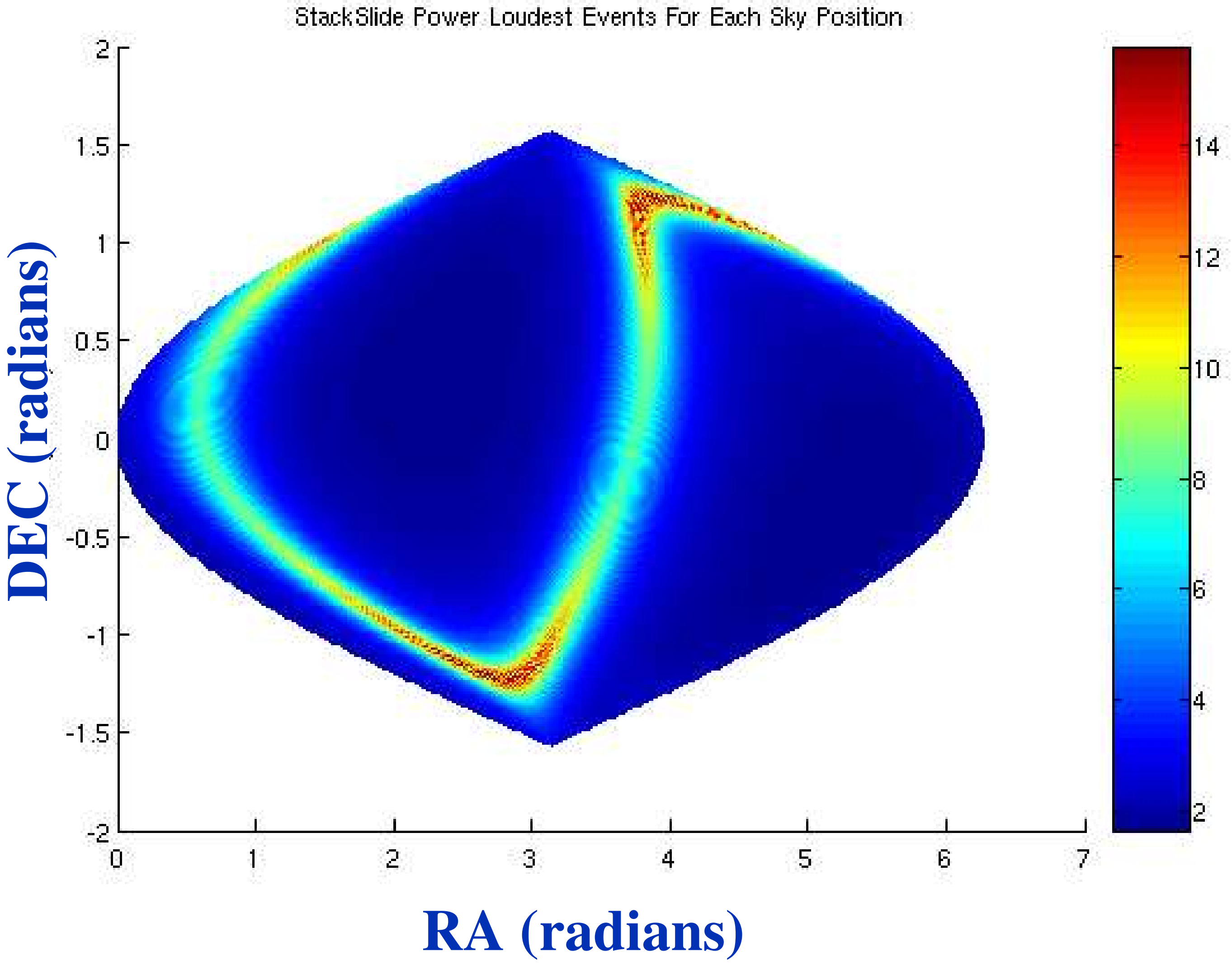
Power averaging reduces noise variance. Pulsar SNR increases with time.

Frequency demodulation broadens instrument line. SNR decreases with time.

All Sky Loudest Events
1000 SFTs, Fake Pulsar & Noise:



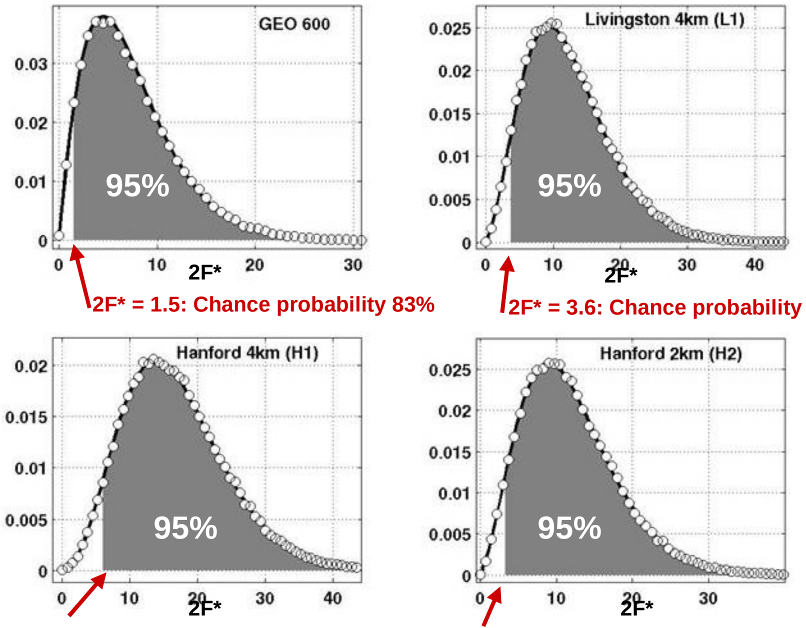
All Sky Loudest Events
1000 SFTs, Fake Inst. Line & Noise:



S1 Results: gr-qc/0308050; Phys.Rev. D69 (2004) 082004

Best h_o UL = 1.4×10^{-22}. Best ellipticity UL = 2.9×10^{-4} ($I = 10^{45}$ gcm²).

$h_0 < 1.9\text{E-}21$ **$h_0 < 2.7\text{E-}22$** **$h_0 < 2.2\text{E-}21$** **$h_0 < 1.4\text{E-}22$**

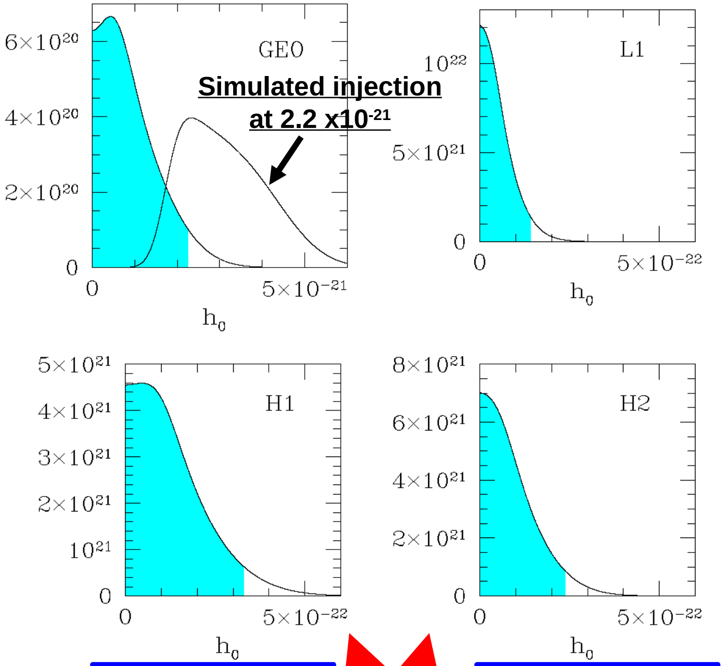


2F* = 1.5: Chance probability 83%

2F* = 3.6: Chance probability 46%

2F* = 6.0: chance probability 20%

2F* = 3.4: chance probability 49%



$h_0 < 4.0\text{E-}22$ **$h_0 < 5.4\text{E-}22$** **$h_0 < 3.3\text{E-}22$** **$h_0 < 2.4\text{E-}22$**

Frequentist Distributions of Log Maximum Likelihood Estimator 2F

Previous results for PSR J1939+2134: **$h_o < 10^{-20}$** (Glasgow, Hough et al., 1983), **$h_o < 1.5 \times 10^{-17}$** (Caltech, Hereld, 1983).

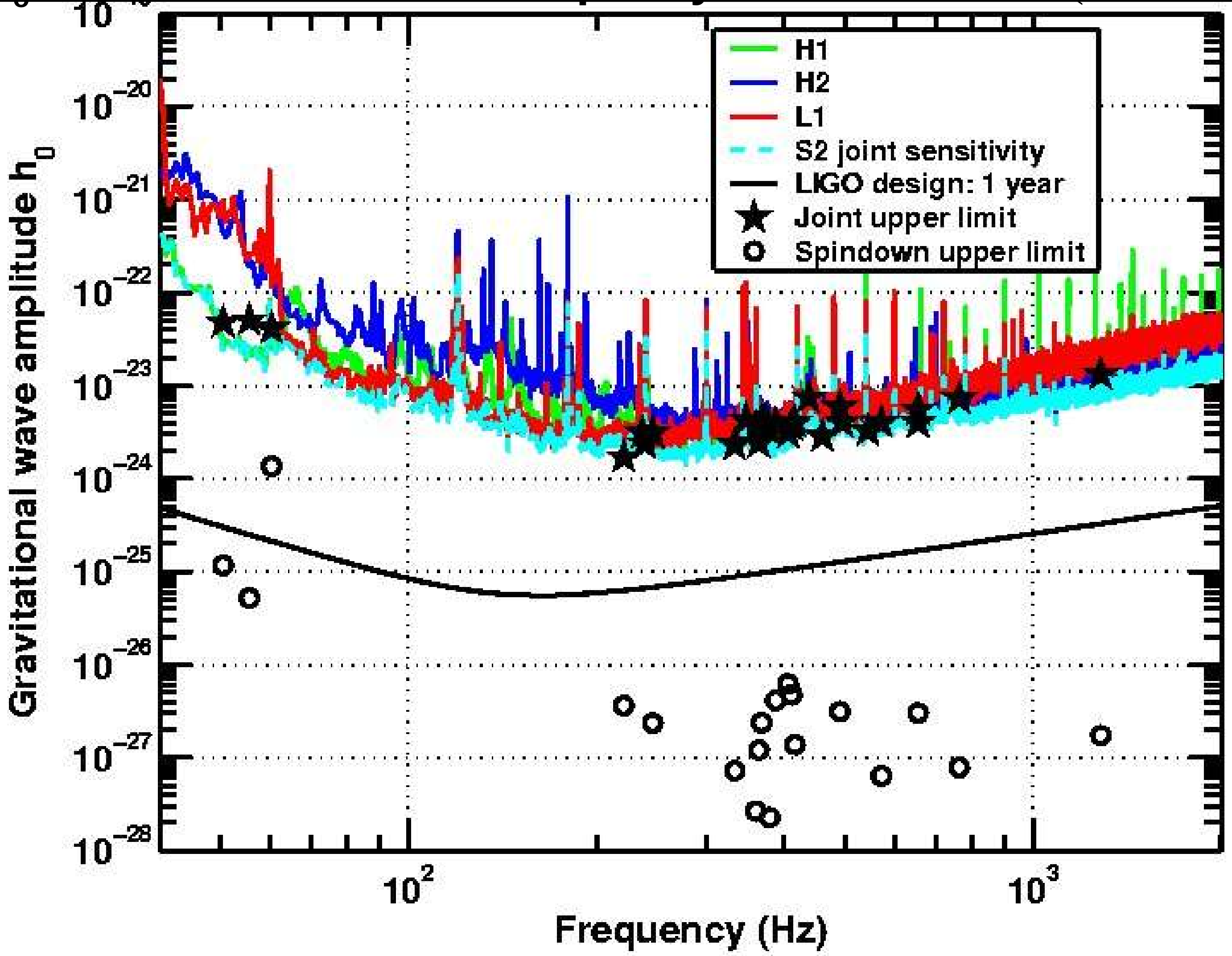
LIGO-G050191-00-W

Bayesian Distributions of Posterior PDF for h_0 for uniform priors

S2 Time Domain Search Results

gr-qc/0410007; accepted PRL

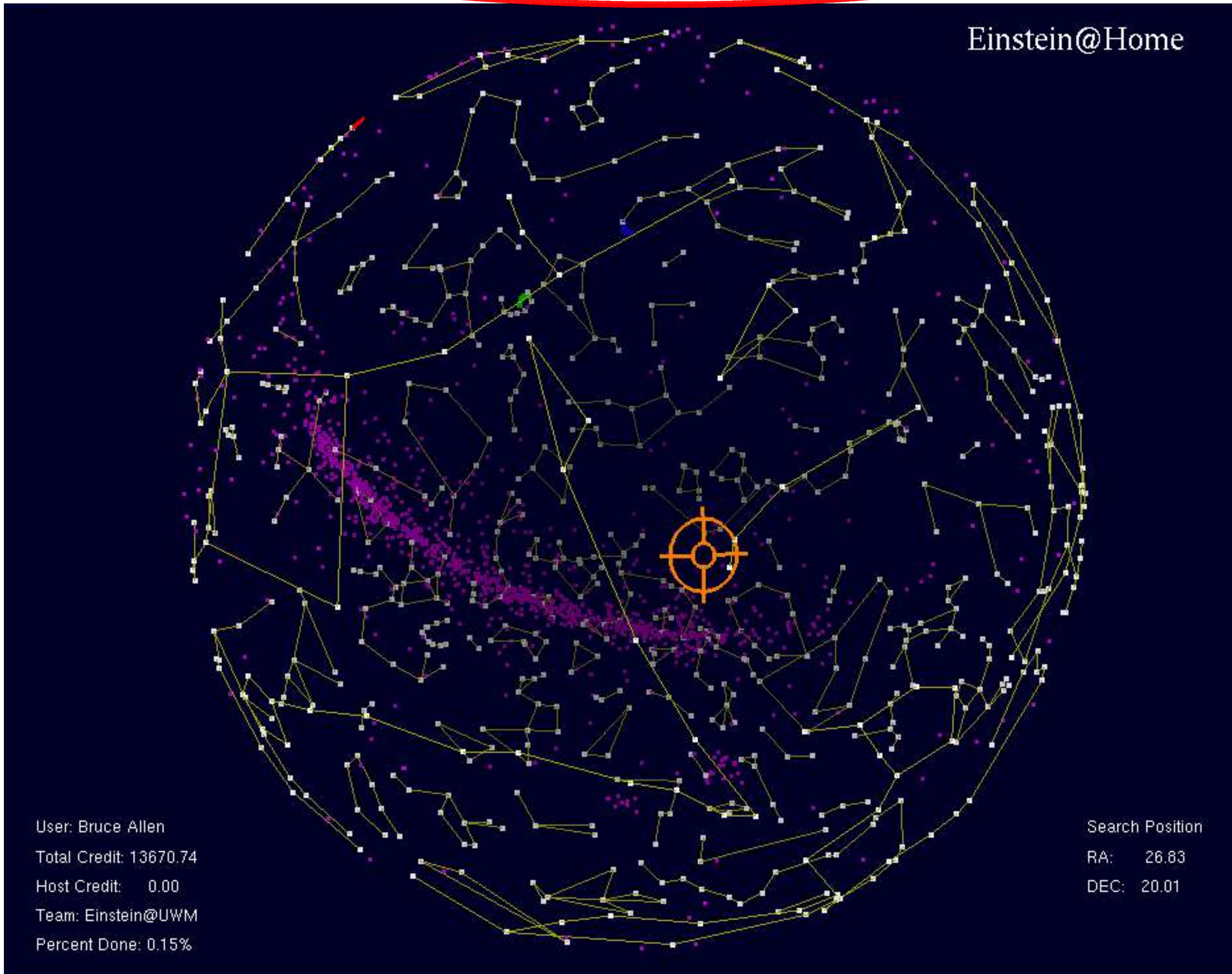
Best h_o UL = 1.7×10^{-24}. Best ellipticity UL = 4.5×10^{-6} ($I = 10^{45}$ gcm^2)



Many searches are underway. You can join via Einstein@home:

- Like SETI@home, but for LIGO/GEO data
- Goal: pulsar searches using ~1 million clients. Support for Windows, Mac OSX, Linux clients
- From our own clusters we can get thousands of CPUs. From Einstein@home hope to many times more computing power at low cost

http://einstein.phys.uwm.edu/



User: Bruce Allen
Total Credit: 13670.74
Host Credit: 0.00
Team: Einstein@UWM
Percent Done: 0.15%

Search Position
RA: 26.83
DEC: 20.01

Summary

- Initial LIGO will search for highly deformed stars in the solar neighborhood of the galaxy.
- Advanced LIGO will search for “average” deformed stars throughout the galaxy.
- Coincidence, hierarchical searches with follow-up targeted searches, and signal recycling are several ways to improve sensitivity beyond back-of-the envelope estimates presented in this talk.