LISA

Laser Interferometer Space Antenna

Fundamental Limit in Frequency Stabilization of Lasers

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Abstract

- Frequency stabilization of lasers with rigid cavity
  - Widely used in many fields
    - Including LISA
  - Stability achieved with rigid cavity
    - Unbeatable and unidentified noise
- Thermal fluctuation (Brownian noise) of rigid cavity
  - We point out this as a fundamental limitation.
    - Rigorous evaluation with experiments and calculations
    - Agreement with world-highest level stabilization results

New insights for precision measurement communities

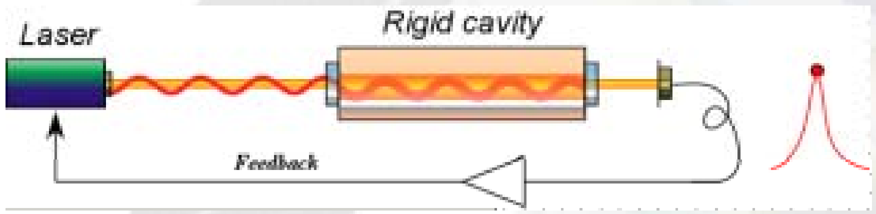
Contents

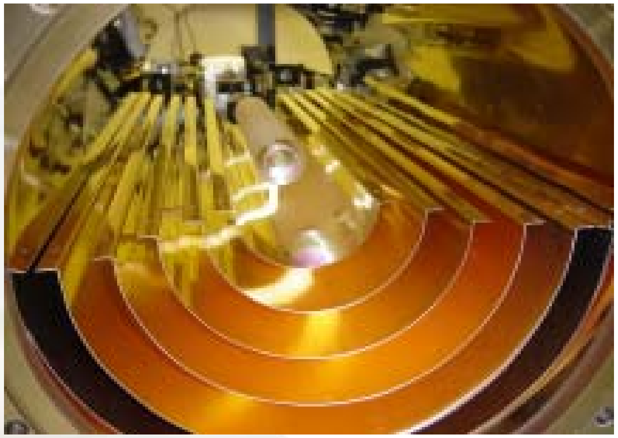
1. Frequency stability of laser

- Why frequency stabilization?
  - Wide range of application
    - Optical frequency standards
    - High-resolution spectroscopies
    - Fundamental physics tests
      - Ex.) Michelson-Morley type experiment: basis of Special/General Relativity
    - Interferometric measurements
      - -Ex.) Gravitational wave detection using laser interferometer: LISA, LIGO…
  - Used as wavelength reference
    - Laser frequency assumed to be fixed to one frequency
    - In reality, it fluctuates! : Needs to be stabilized.
      - LISA requirement: 30Hz/rtHz

Methods of frequency stabilization

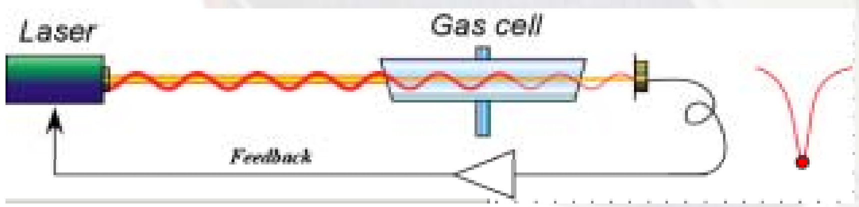
- Two major frequency references
  - Rigid cavity (Basic design for LISA)
    - Made of low CTE material
    - Thermal shield to minimize length variation
    - Laser controlled to be stored within the cavity

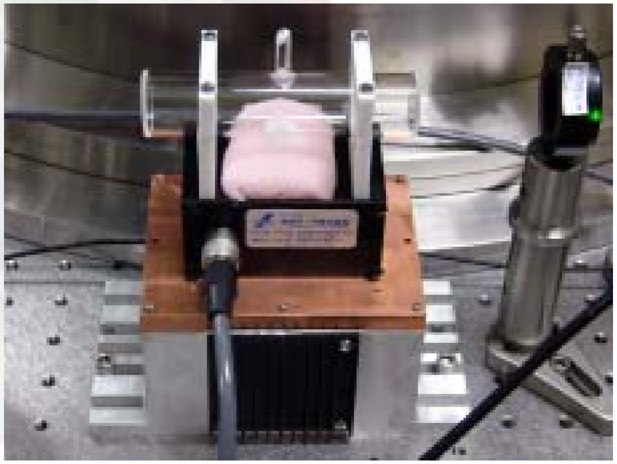




ULE rigid cavity with thermal shields

  - Atomic absorption line (Basic design for TPF)
    - Cooling to avoid Doppler broadening
    - Use of hyperfine structure
    - Laser controlled to be absorbed by one line





Iodine cell with cooling system

Stability of rigid cavity

- Possible noise sources
  - Non-fundamental noise source
    - Length change due to temperature variation
    - Length change due to vibration (seismic noise)
    - Length change due to mirror heating --- coupled to laser intensity noise
    - Coupling from RF amplitude noise
    - Pointing noise coupled to misalignment
    - Circuit noise
    - Etc…
  - Fundamental noise source
    - ***Thermal noise as a result of statistical physics***
      - Hasn't been evaluated from 1970's!!

Fluctuation-Dissipation Theorem

- Calculation of thermal noise spectrum *G(f)*
  - Based on **FDT** (Fluctuation-Dissipation Theorem)

$$G(f) = -\frac{4k_B T}{\omega}\,\mathrm{Im}[H(\omega)]$$

H(f): transfer function

  - Useful form

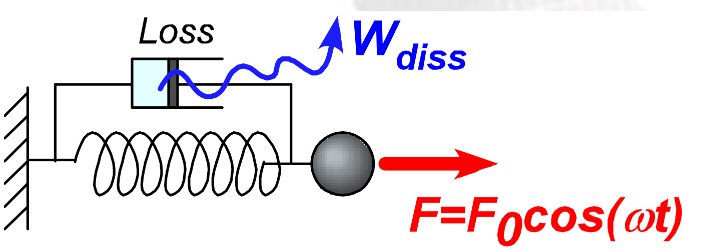
$$G(f) = \frac{4k_B T}{\pi^2 f^2}\,\frac{W_{diss}(f)}{F^2}$$

W_{diss}: dissipated energy under cyclic force

F: force amplitude

ε: strain energy under cyclic force

$$W_{diss}(f) = \int \varepsilon(\vec{r})\,dV \,/\, Q$$



Stored strain energy

Calculation

Quality factor

Experiment

2. Experiment

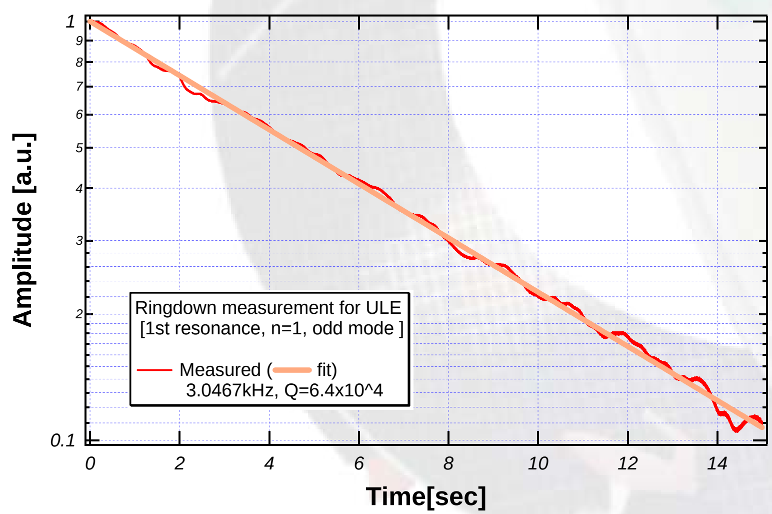
- Basic information in estimating thermal noise level
  - ***Mechanical quality factor (Q)***
    - Measured by "ring-down" method
      - Vibration decay measured by Michelson interferometer
      - Sample supported by thin wires in vacuum to reduce external loss
  - Samples
    - Low CTE materials
    - Imitating rigid cavity

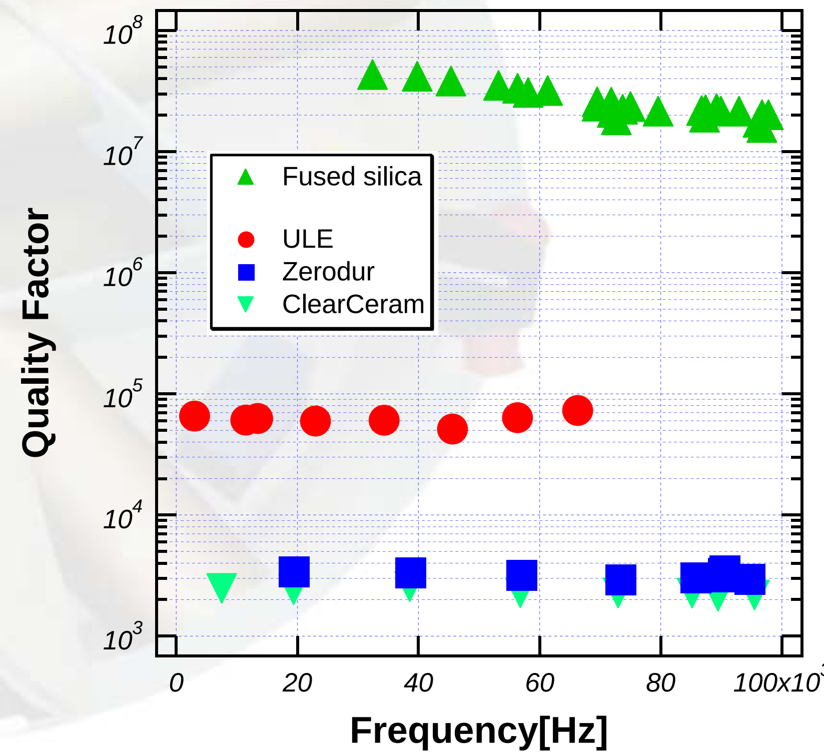
ULE sample

Measurement system

Experimental result

- Fairly low quality factor measured
  - ULE : Q~61000, Zerodure: Q~3100, etc.
    - Usually high Q materials used when thermal noise matters
      - Ex. Ground based gravitational-wave detector: mirror made of silica: $Q>10^7$
  - Constant Q measured





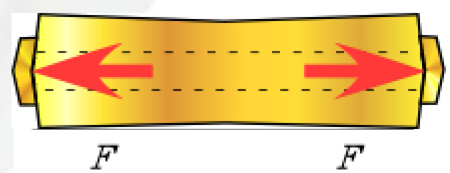
3. Calculation

- Calculation of dissipated energy
  - Calculation of strain energy under cyclic force
    - Done by solving Equation Of Motion (EQM) of the system
  - Numerical approach adopted
    - Finite Element Method (FEM)
    - Procedure
      - 1) Prepare rigid cavity mechanical model
      - 2) Apply cyclic force to the observing (beam-illuminating) points
      - 3) Calculate strain energy within the system based on EQM

$$W_{diss}(f) = \int \varepsilon(\vec{r})\,dV \,/\, Q$$

Stored strain energy — Calculation

Quality factor — Experiment



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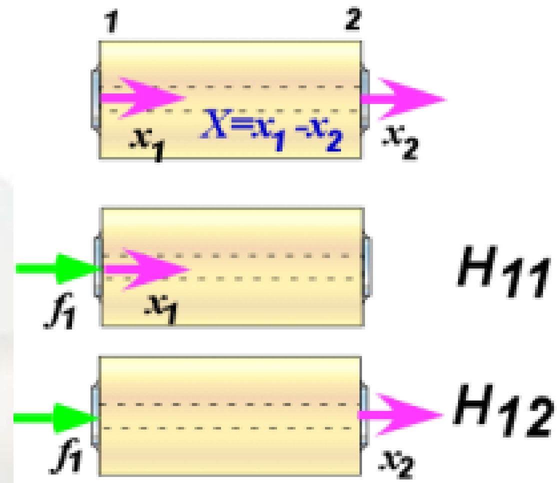
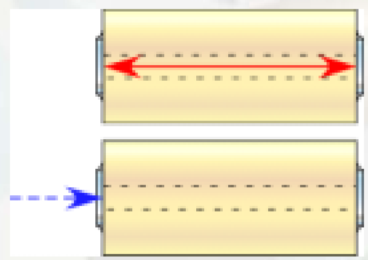
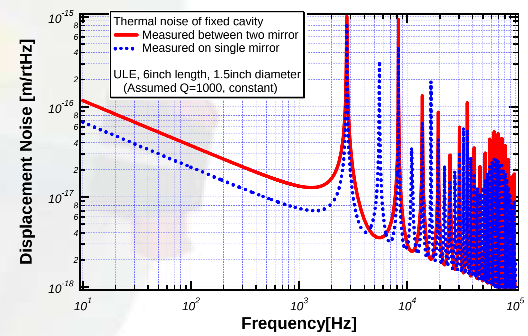
Correlated Thermal Noise



- Calculation of correlated thermal noise
  - Individual calculation but with correlation term
    - $G_X=G_{11}+G_{12}-2G_{12}$
  - Apply two forces simultaneously to measured points (areas)
    - These two give us equivalent results



- Solving equation of motion
  - Numerical approach adopted
    - FEM to get W_{diss}
    - Can be applied for any:
      - Frequency, shape, loss distribution/frequency dependence, weighing…
      - (This method itself should be published somewhere.)
  - In the following
    - Thermal noise in rigid cavity solved
    - Problem includes:
      - Finite sized mass, material combination (loss distribution), Gaussian beam, coating loss, correlated (generalized coordinate) etc…



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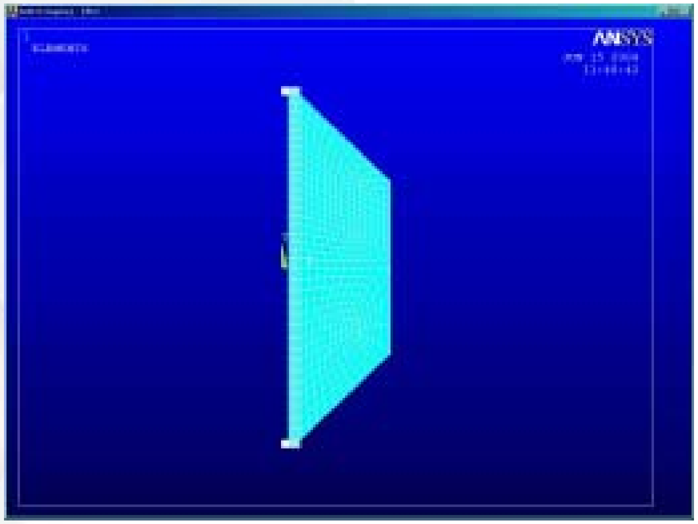
4. Result

- World highest stability achieved by NIST/VIRGO
  - Calculation assumptions
    - Spacer
      - Material: ULE (Q=60000)
      - Length: 15cm (tapered), diameter: 24cm
    - Mirror (optical contacted)
      - Material: ULE
      - Diameter: 1inch, thickness: 5mm
      - Beam radius: 240um on both mirrors
    - Coating
      - Thickness: 2um, phi(1/Q)=$4x10^{-4}$
  - FEM model
    - ANSYS
    - Semi-3D model (2-D axisymmetric)



Half of the cross section model of the cavity

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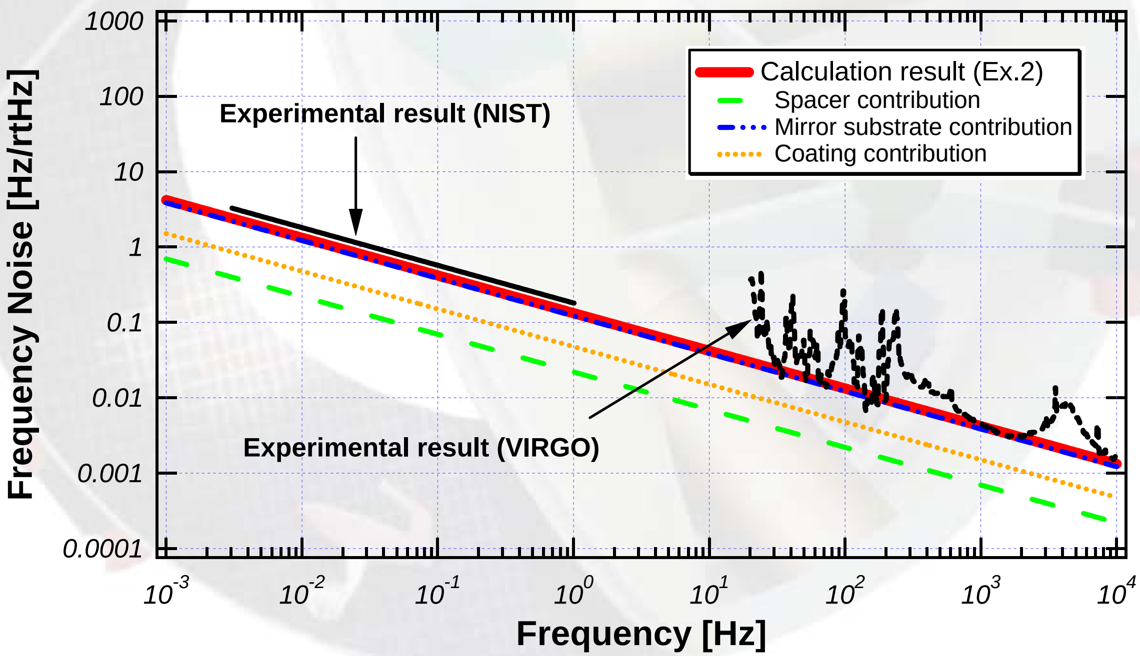
Comparison with experiment

- Agreed pretty well with the measurement
  - ~1Hz/rtHz@0.01Hz level (563nm wavelength)
    - We cannot neglect thermal noise (Brownian motion) anymore!
    - Use of low loss mirrors, larger beam diameter, cooling etc
      - Expected to renew world highest frequency stability



6.Summary

- Frequency stabilization of laser
  - Wide-range of demands and applications in physics and engineering
    - Use of rigid cavity
      - Basic design for LISA
      - Any length fluctuation of cavity limits frequency stability
- Fundamental limit in frequency stabilization with rigid cavity
  - Thermal noise as a result of statistical physics
    - We evaluated the noise level with the FDT.
      - Experiment: Q measurement of cavity materials
      - Calculation: Numerical analysis of strain energy
    - Importance of thermal noise pointed out
      - Agreement with world-highest level stabilization results
      - See PRL **93** (2004) 250602 for details.