



LIGO I simulation using FFT

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- A few news from LIGO I commissioning
 - thermal compensation system
 - phase camera
- LIGO I mirror phase map
- FFT tools
- Thermal lensing
- Beam splitter curvature
- Interpretation of results using modal model



Phase camera and thermal compensation system

- Phase camera

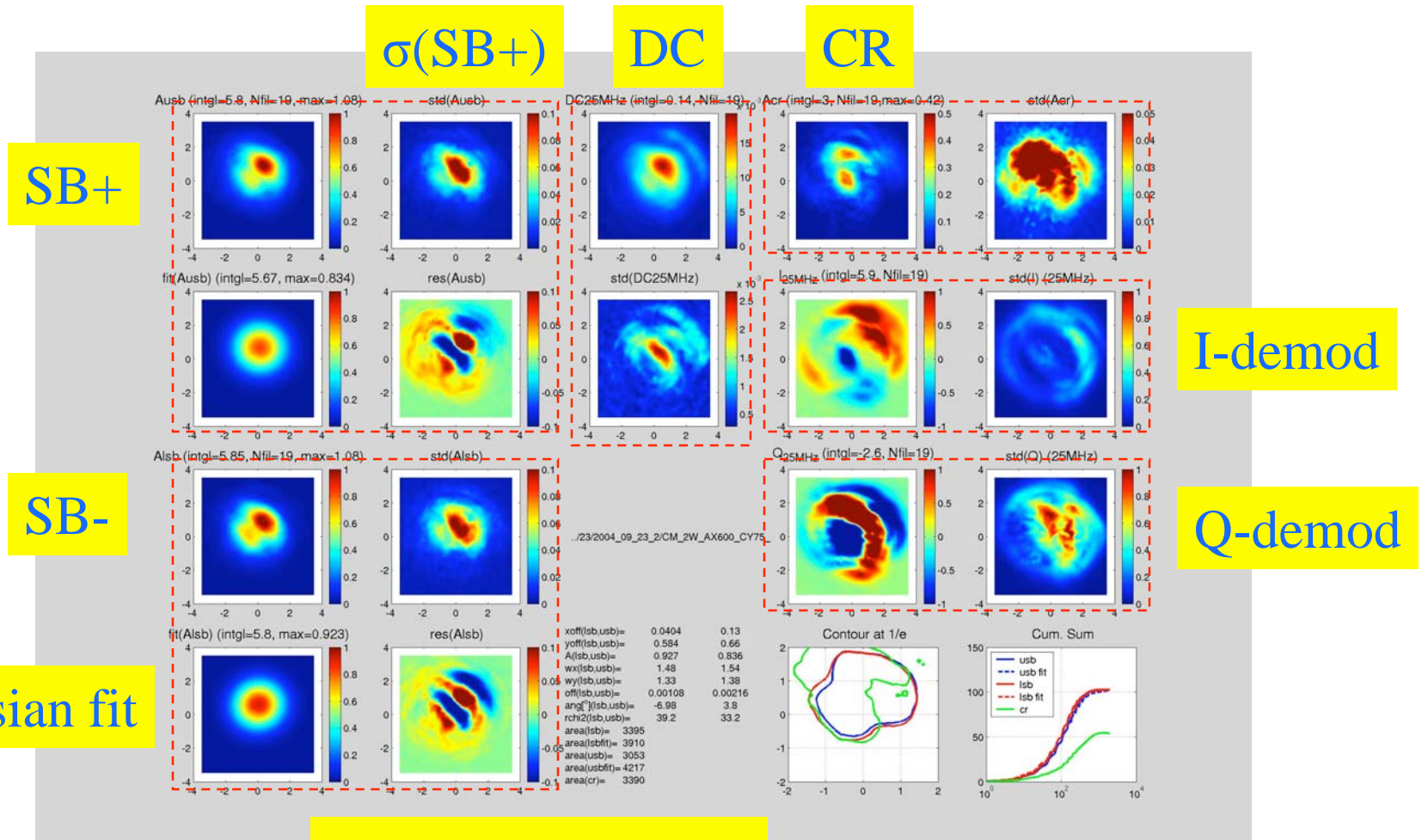
- » E_{IFO} : Field from interferometer
 - SB-, CR, SB+ with modulation frequency of 25MHz
- » E_{laser} : Field from laser frequency shifted by 75MHz
- » Demodulate $E_{\text{IFO}} + E_{\text{laser}}$ by 50, 75 and 100Mhz to measure SB+, CR and SB- separately

- Thermal compensation system (TCS)

- » CO2 laser to heat ITMs
 - Central heating : enhance NdYAG heating effect
 - Annular heating : suppress NdYAG heating effect
- » Somehow, differential heating, inline ITM cooler than offline ITM, preferred
- » It seems SB imbalance is related



Phase camera image at dark port 2W Input, TCS : AX600-CY75



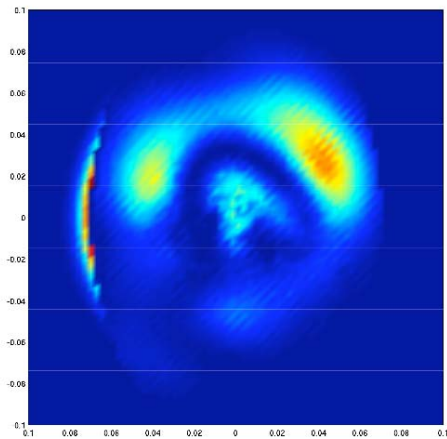


FFT tools

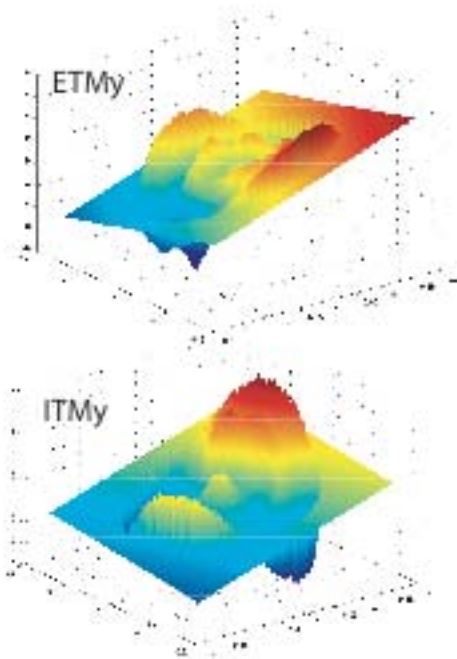
- Calculation of static fields in Core Optics system
 - » Orsay -> MIT
 - » Core optics phase map
 - » Thermal lensing effect
 - » Beam splitter curvature
- Propagation with magnification
 - » Virgo Physics Book, Volume 2 “OPTICS and related TOPICS”, 3.1.7
 - » FFT pixel size can be scaled - 25 cm mirrors to mm detector
 - » Fields can be propagated through telescopes to actual detectors
- FFT lock vs LSC lock
 - » FFT lock uses only CR, LSC lock uses CR and SBs
 - » Lock FFT by itself -> Lock using ASQ,REFL,POB
 - » Arm lengths change by 10^{-12} m, Michelson lengths by 10^{-9} m
 - » Quantitative results affected, most of qualitative results OK

Effect of mirror aberration

Dark port CR

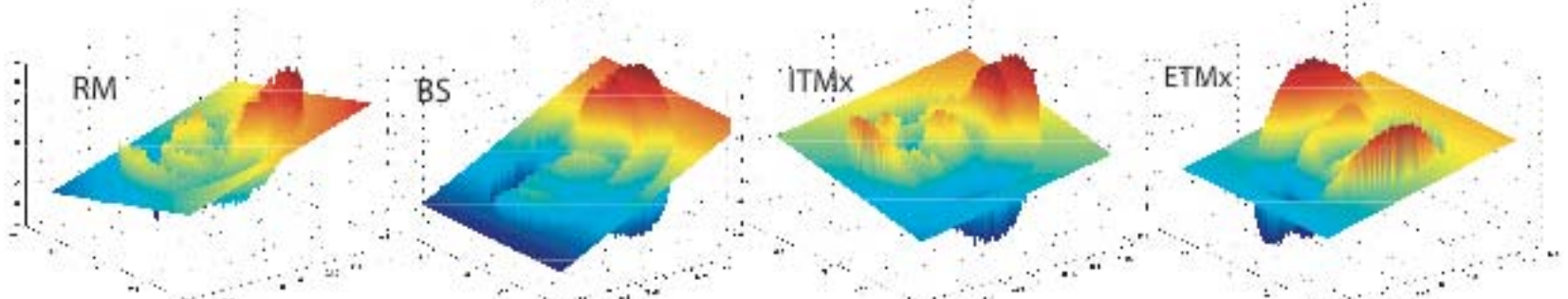


$P(\text{CR})=3.2\text{kW} \rightarrow 2.5\text{kW}$



Contrast defect

Mode matched, identical arms	$5.5e-7$
+ as-built arms	$6.8e-5$
+ BS curvature	$1.2e-4$
+ Mirror phase maps	$2.3e-4$
+ Differential heating	$2.5e-4$



G040

$P(\text{CR})=46\text{W} \rightarrow 36\text{W}$

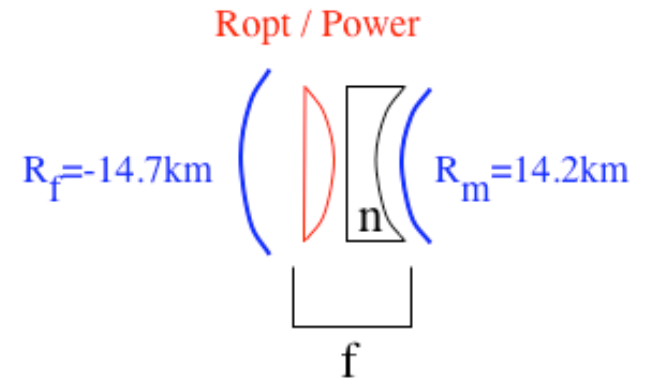
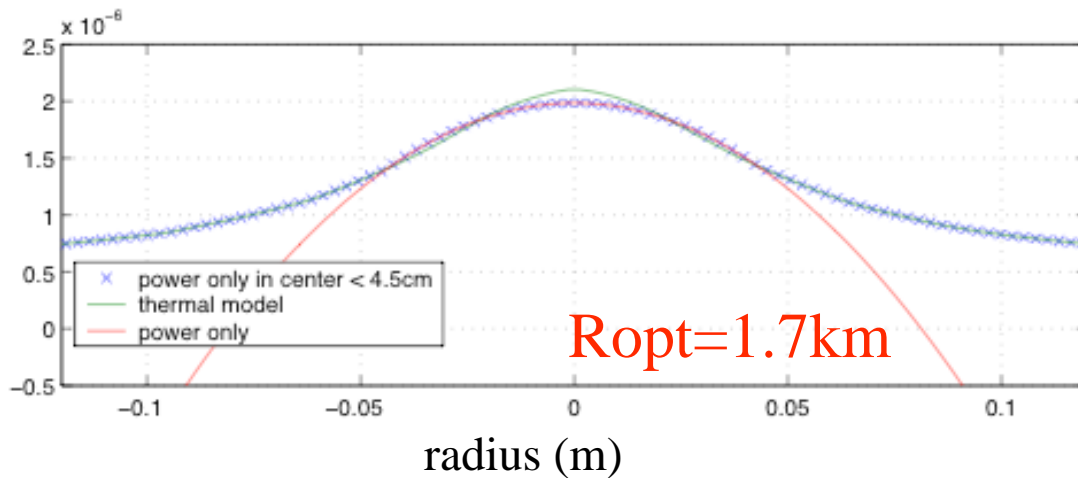
$P(\text{CR})=3.3\text{kW} \rightarrow 2.6\text{kW}$



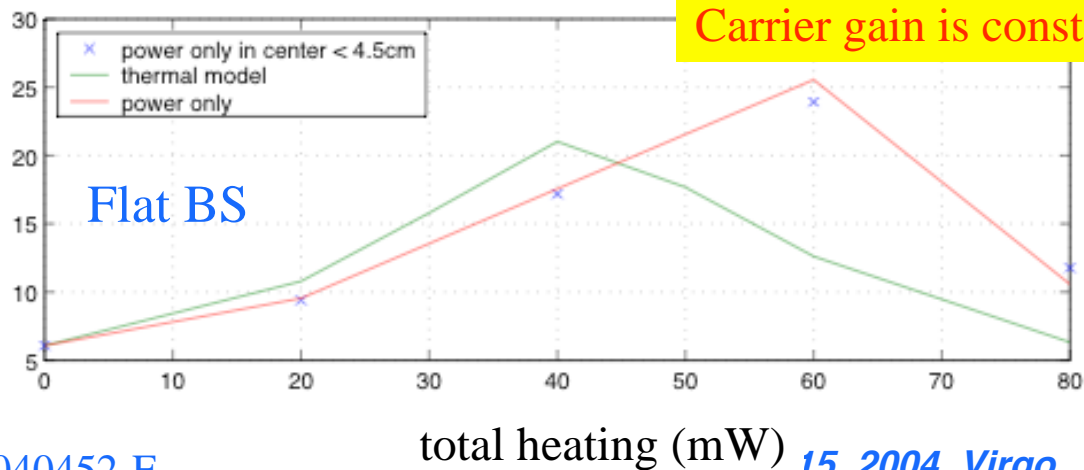
Thermal lensing in FFT

- P. Willems calculated based on MIT model -

Optical thickness @ 1w



Sideband recycling gain



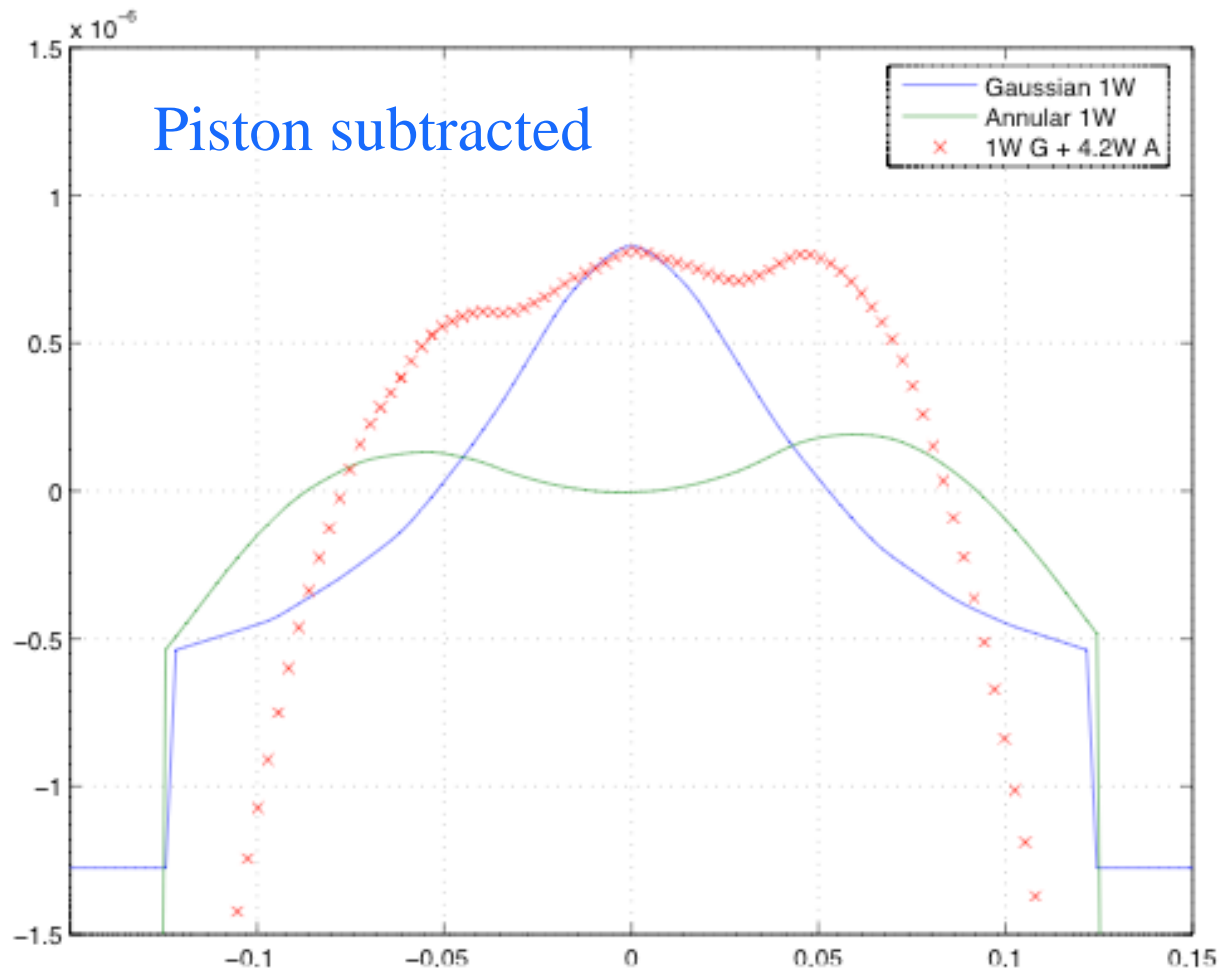
$$\frac{1}{f} = -\frac{n-1}{R_m} + \frac{1}{R_{opt}} Power$$

$$\frac{1}{R_f(HR)} = \frac{1}{R_f(AR)} - \frac{1}{f}$$

Power = 58mW

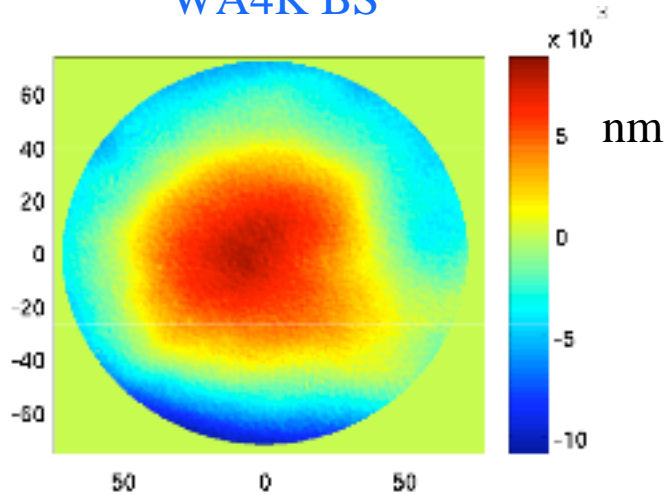
Gaussian and Annular

Optical thickness (10^{-6}m)

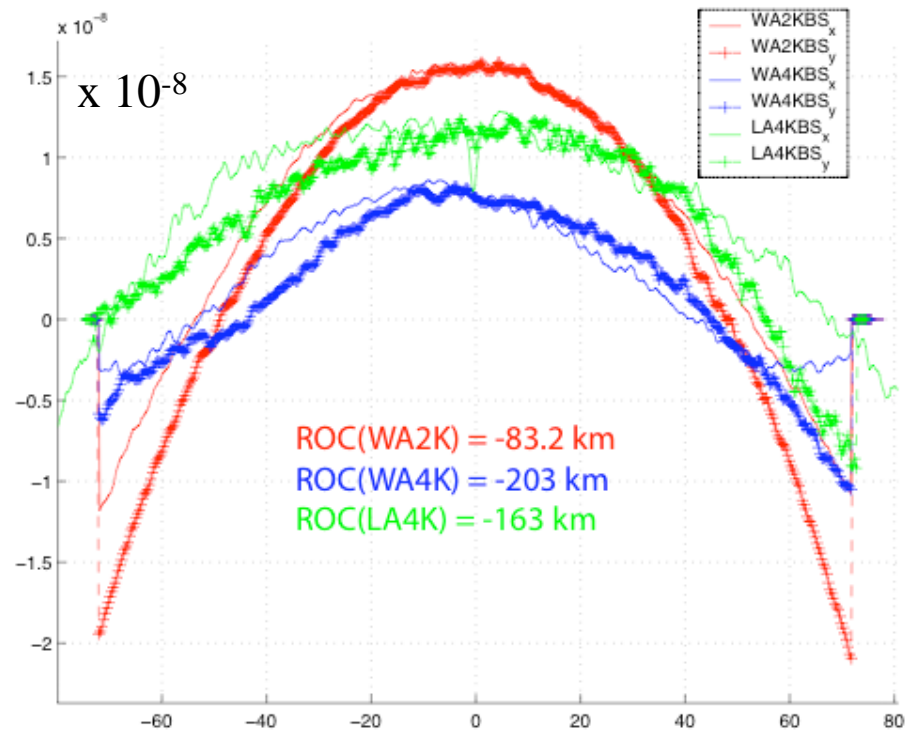
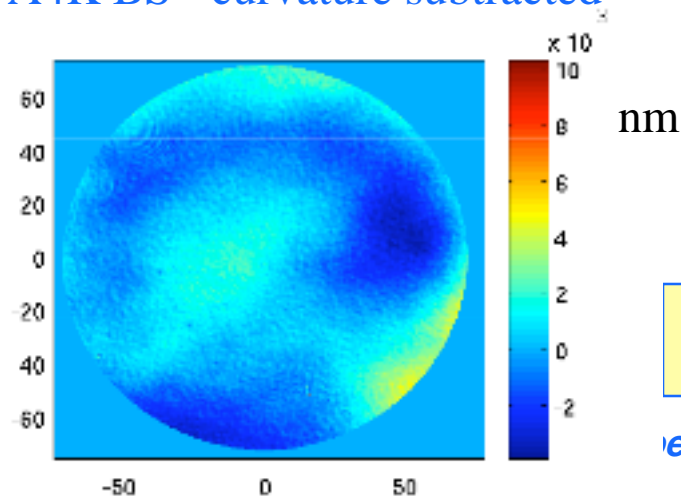


Beam splitter phase map

WA4K BS



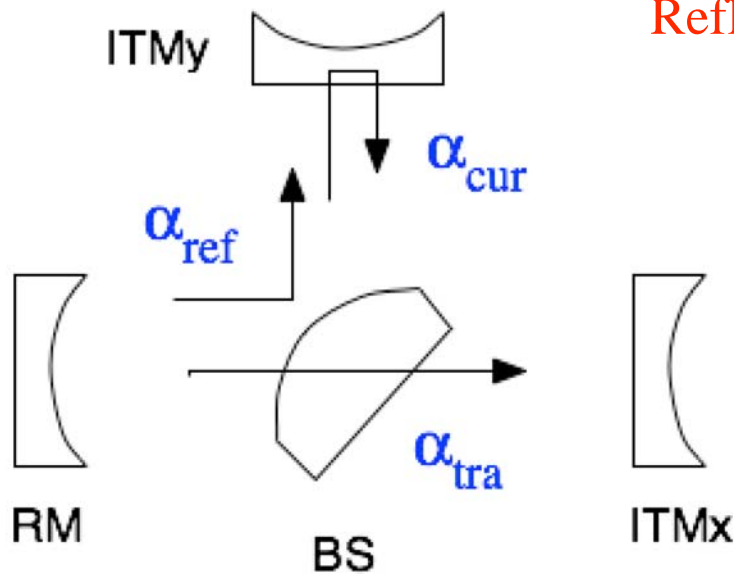
WA4K BS - curvature subtracted



concave ROC > 200km, convex ROC > 720km

Mode disturbance in PRM

- BS and ITM curvature and BS lens -



Reflection and transmission change field curvature

$$\alpha_{cur} = \frac{z}{z_0} \left(1 - \frac{R_f(z)}{R_{ITM}} \right) \quad : 0.23(cold) \sim 0(hot)$$

$$\alpha_{ref}(x/y) = -\frac{z^2 + z_0^2}{z_0 \cdot R_{BS} \sqrt{2}^{\pm 1}} \quad : 0.027$$

$$\alpha_{tra} = -\frac{n-1}{2} \alpha_{ref} \quad : -0.005$$

$$R_{BS} = -200\text{km}$$

$$R_{ITM} = -14\text{km}$$

$$z_0(\text{Rayleigh range}) = 3.6\text{km}$$

$$z(\text{distance to waist}) = -1\text{km}$$

$$TEM00(out) = \frac{1}{\sqrt{(1+i\alpha_x)(1+i\alpha_y)}} TEM00(in)$$



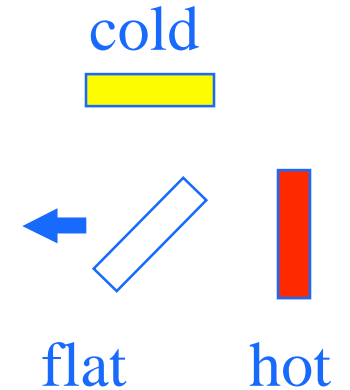
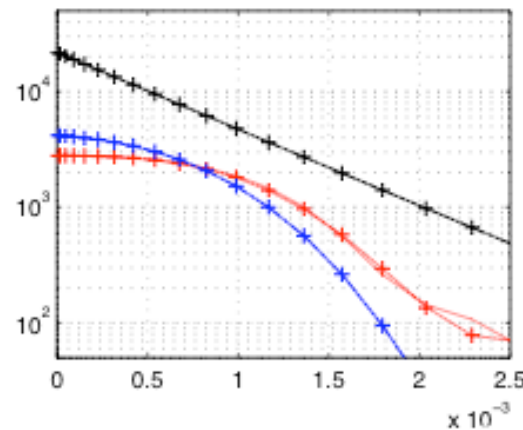
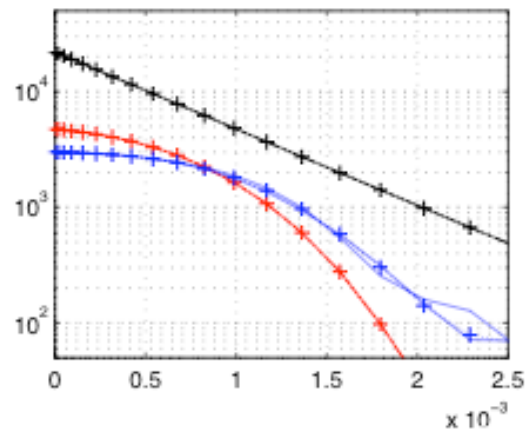
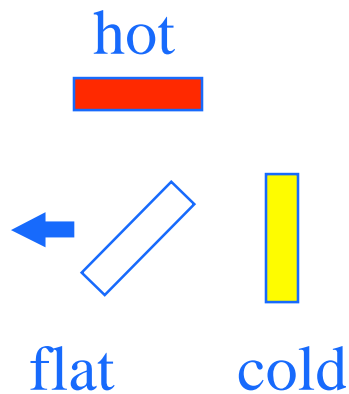
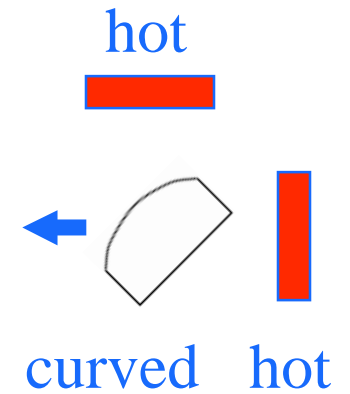
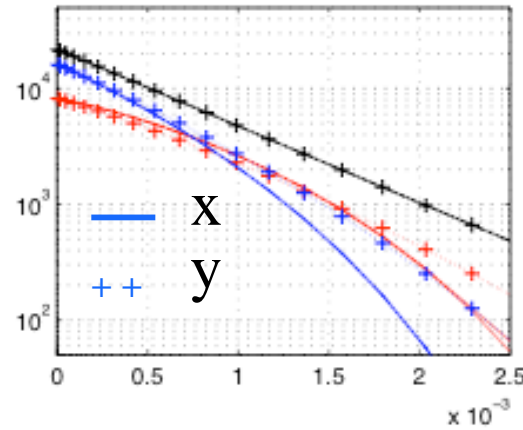
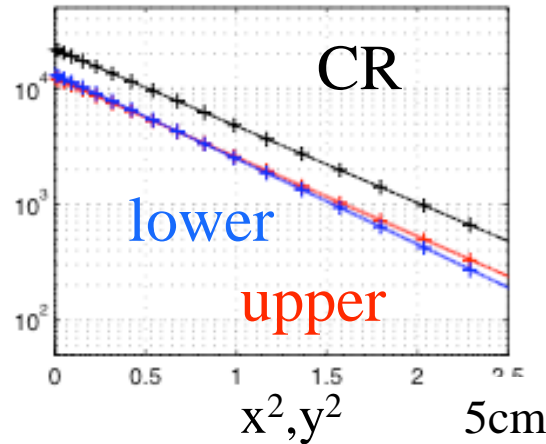
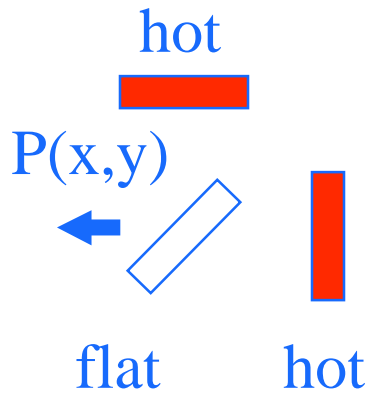
ITM differential heating and beam splitter curvature

Power only
thermal

- Linear line : gaussian
- Blue vs red : sideband imbalance
- --- vs + + + : astigmatism

Power on Symmetric port

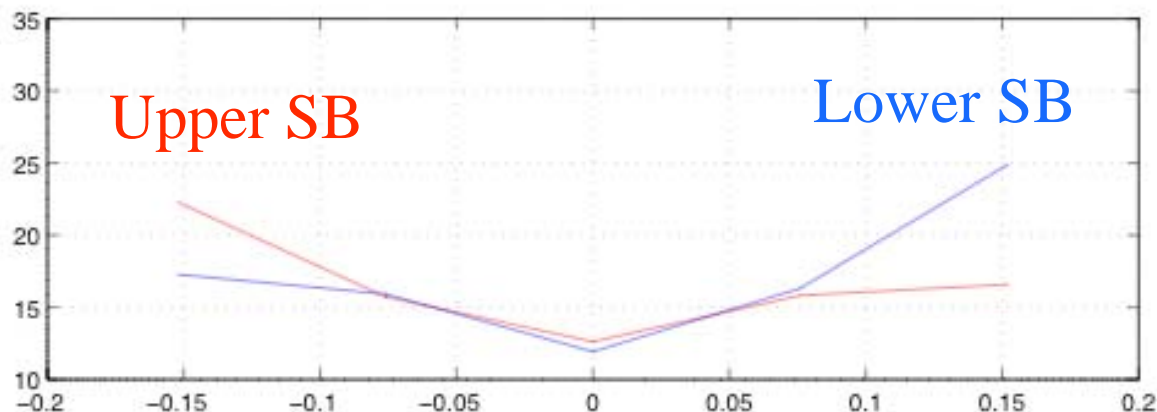
$P(x,0)$ vs x^2 , $P(0,y)$ vs y^2





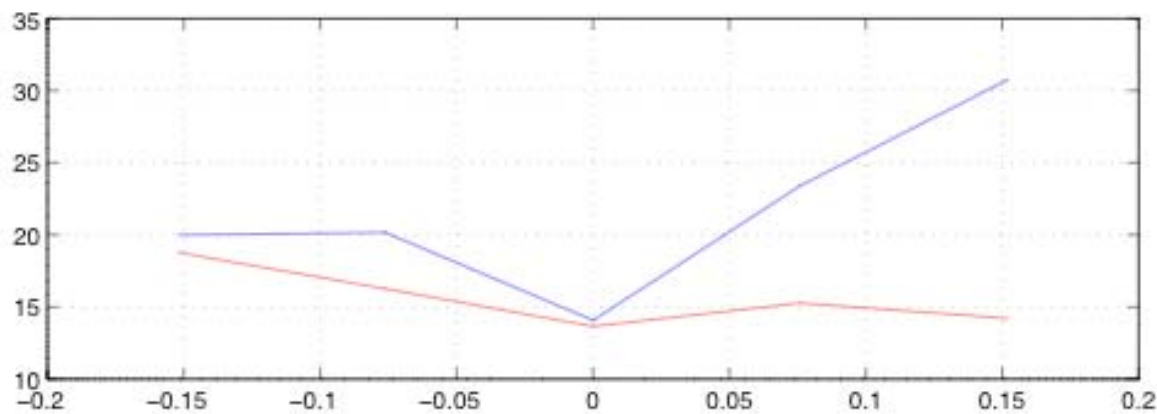
SB gain vs differential heating

Recycling gain



ITMx annular heating

ITMy annular heating

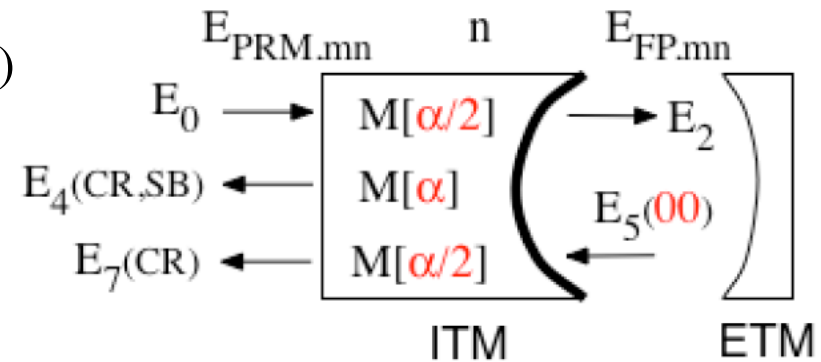




Reflection by a locked arm

- CR~00, SB~00+02/20 -

$$\alpha = \frac{z}{z_0} \left(1 - \frac{R_{Field}}{R_{ITM}}\right) \approx \frac{z}{z_0} \left(n_{substrate} - \frac{R_{ITM}}{R_{RM}} - \frac{R_{ITM}}{R_{thermal}}\right)$$



$$E_{SB} = \frac{1}{1+i\alpha} E_{00} - \frac{i\alpha / \sqrt{2}}{(1+i\alpha)^3} (E_{02} + E_{20}) + O(\alpha^2)$$

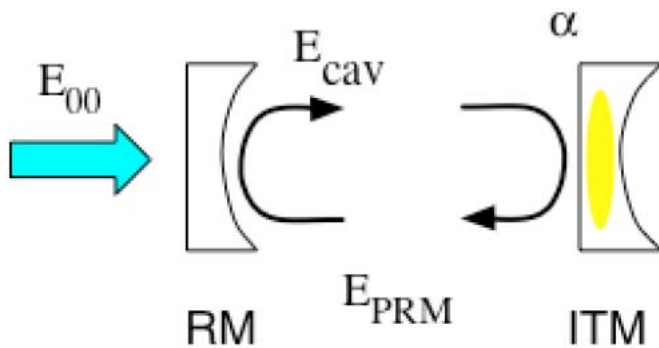
$$E_{CR} = \frac{1}{1+i\alpha} E_{00} - \frac{i\alpha / \sqrt{2}}{(1+i\alpha)^3} (E_{02} + E_{20}) + O(\alpha^2) \Leftarrow E_4$$

$$-2 \frac{1}{1+i\alpha/2} \left(\frac{1}{1+i\alpha/2} E_{00} - \frac{i\alpha/2 / \sqrt{2}}{(1+i\alpha/2)^3} (E_{02} + E_{20}) + O(\alpha^2) \right) \Leftarrow E_7$$

$$= -\frac{1}{1+i\alpha} E_{00} + O(\alpha^2)$$

Fields in mode mismatched FP

$$E_{cav} = \frac{t_{RM} \cdot E_{in}}{(1-R)(1+C_0 \cdot \alpha^2)} (E_{PRM,00} - i \cdot \alpha \cdot C_2 \cdot (E_{PRM,02} + E_{PRM,20})) + O(\alpha^3)$$



$$R = R_0 \cdot \text{Exp}[i\phi_{CR,00} + i\phi], \quad R_0 = r_{RM} \cdot r_{ITM}$$

$$\phi_{CR,00} = -2k_{CR}L + 2\eta - \arctan(\alpha)$$

$$\phi_{mix} = -\frac{1}{2} \cot(2\eta) \cdot \alpha^2$$

$$\phi = -2k_{SB}L + \phi_{mix}$$

$$C_0 = \frac{(1 - i \cdot \cot(2\eta)) \cdot R}{2(1 - \text{Exp}(i4\eta)R)}$$

$$C_2 = \frac{\text{Exp}(i \cdot 2\eta)}{\sqrt{2}(1 - \text{Exp}(i \cdot 4\eta)R)}$$

$$SBPower(k_{SB}) =$$

$$F(k_{SB}L + f_1(\eta) + f_2(\alpha))$$

$$SBPower(k_{SB}) \neq SBPower(-k_{SB})$$

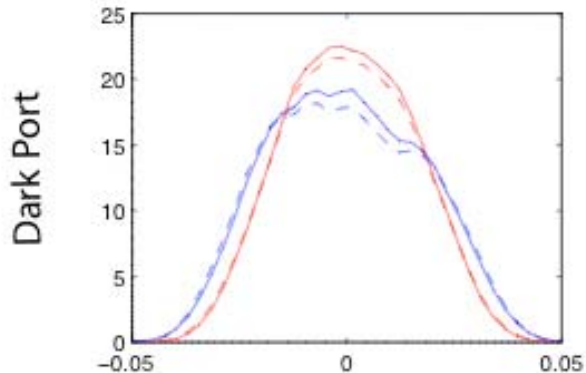


FFT vs LSC lock

$n(\text{ITM}_x) - n(\text{ITM}_y)$

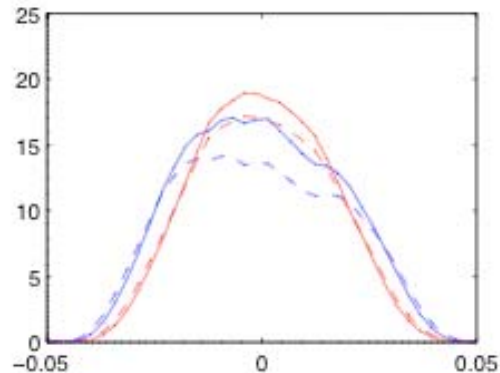
0.96-0.96

Symmetric Heating



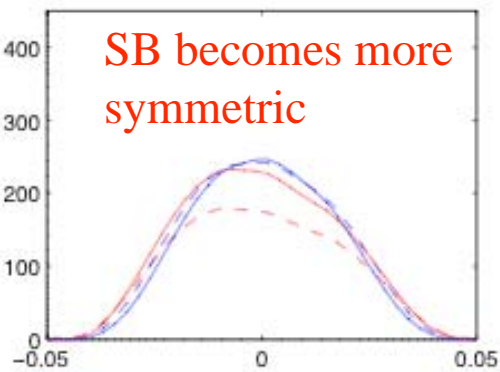
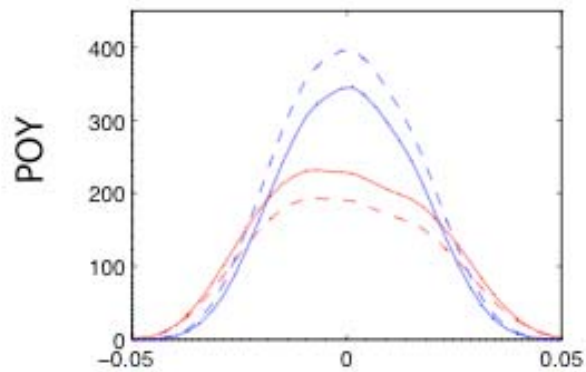
1.10-0.96

Differential Heating
ITMx cooler than ITMy



— lower SB — upper SB

- - - - FFT lock — LSC lock



symmetric

differential

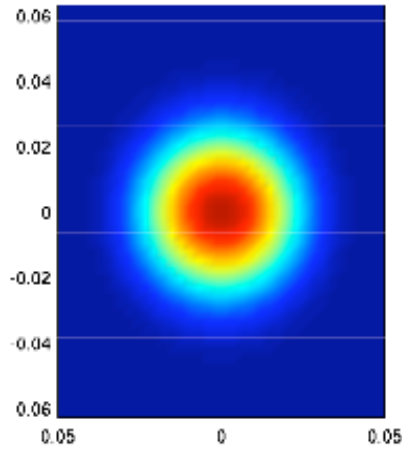
	FFT	LSC
θ_{CR}	0.3	-1.9
θ_{SB+}	-0.6	-2.3
θ_{SB-}	7.2	5.1
Spob	-0.57i	-0.57i
θ_{CR}	0.2	-8
θ_{SB+}	4.9	-1.2
θ_{SB-}	11.8	5.1
Spob	-0.48i	-0.50i



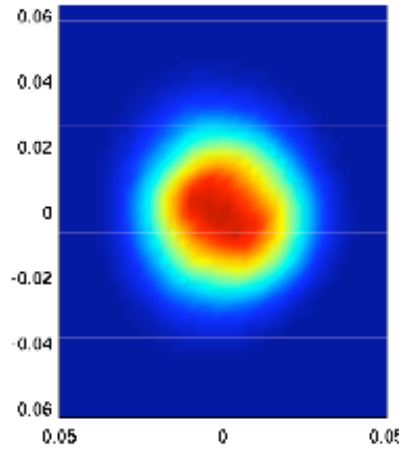
Dark Port sideband profile by FFT

- after LSC lock -

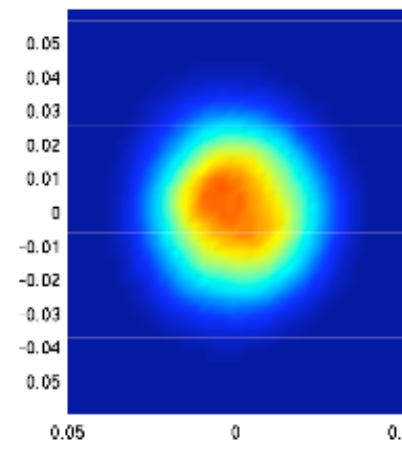
upper SB



No phase map
Symmetric heating



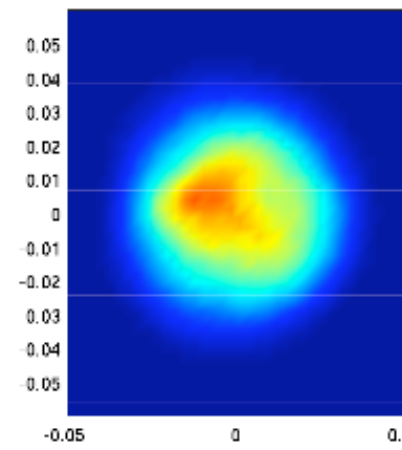
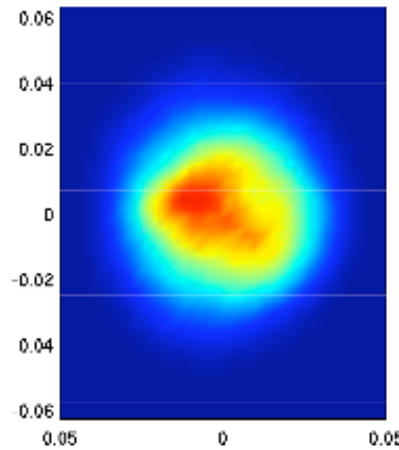
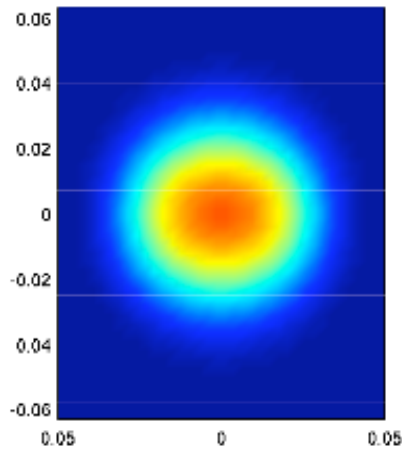
With phase map
Symmetric heating



With phase map
Differential heating

200k BS
curvature

lower SB



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