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Equivalence relation between non spherical optical cavities and application to advanced G.W. interferometers.

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This research has logically built on a lot of previous investigations:

D'Ambrosio, O'Shaughnessy, Strigin, Thorne, Vyatchanin (MH-mirror)

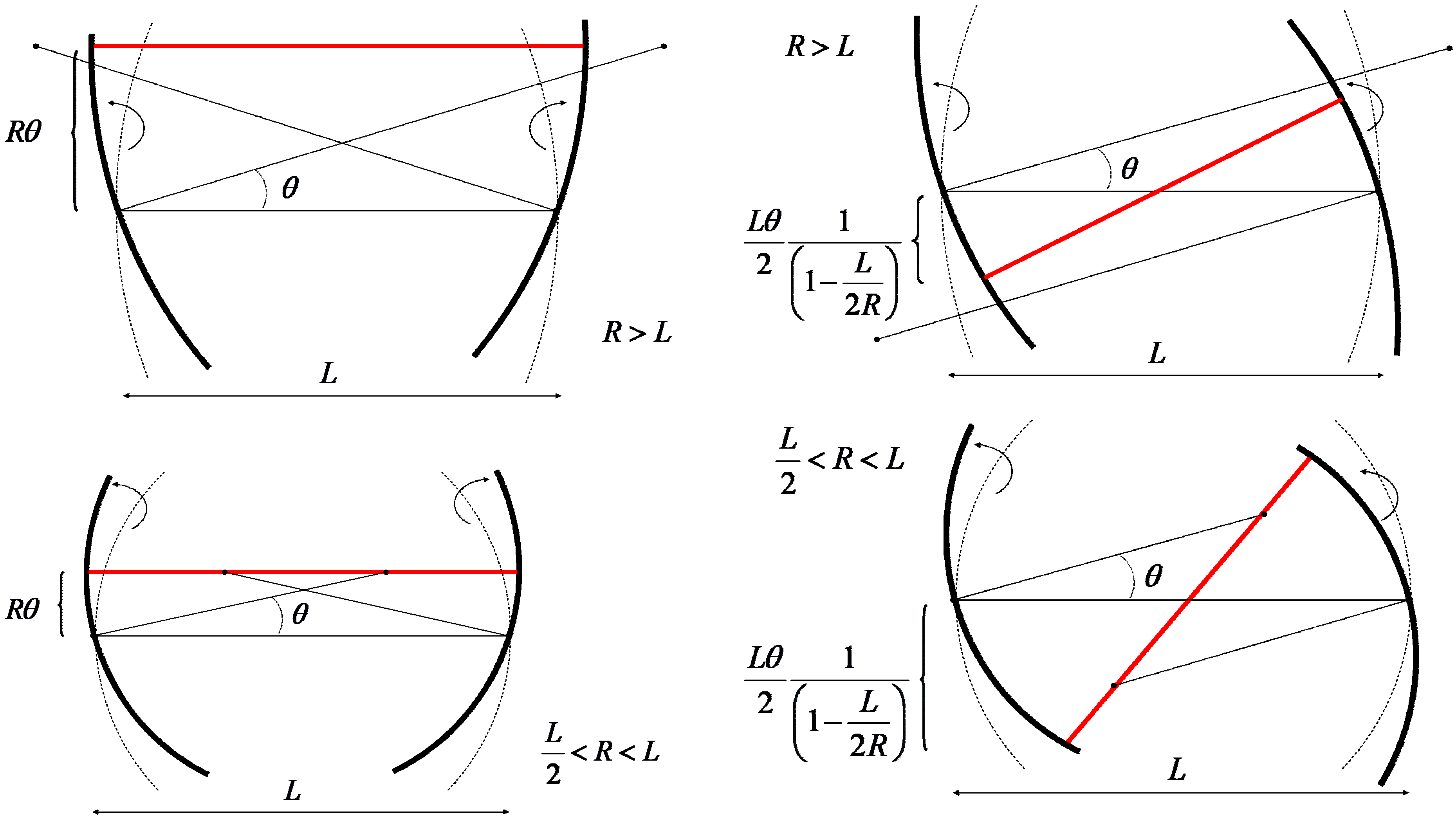
D.Sigg and J.Sidles (radiation pressure coupled to mirror misalignment)

There are also many current investigations on non-gaussian beams:

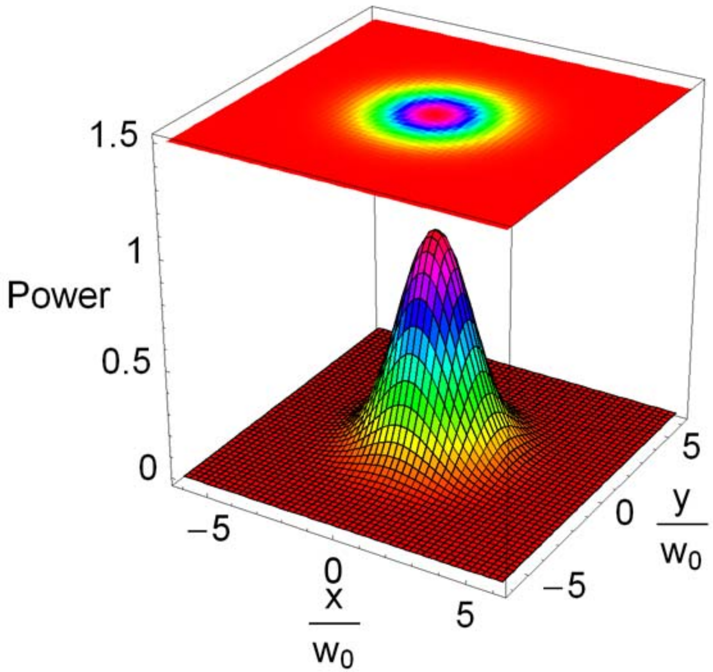
Bagini,Belanger,Gori,Lachance,Li,Lu,Luo,Palma,Pare,Siegman,Tovar

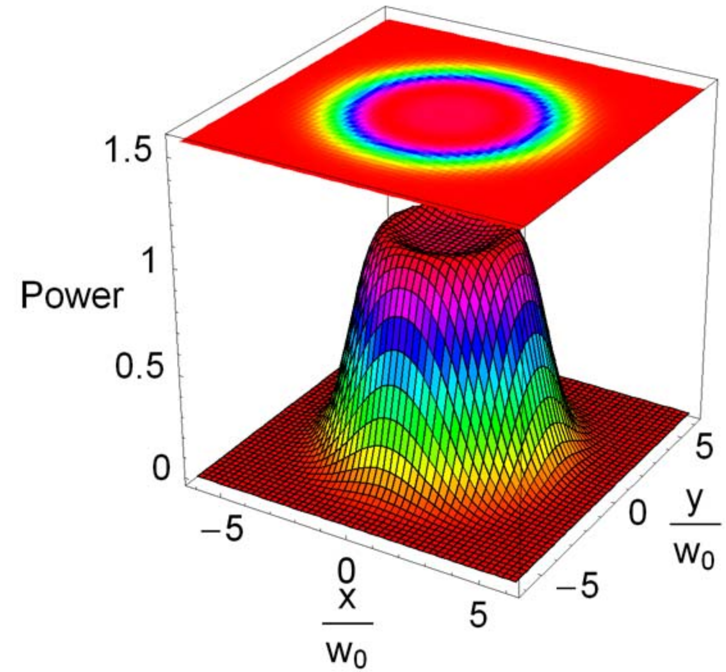
Geometrical interpretation of alignment instability for spherical mirrors

Daniel Sigg had pointed this out at the Fifth Amaldi Conference in Italy (July 2003)



Small-scale experiment: demonstrating the practicability of Flat-Top beams, under any critical aspect and showing the robustness of the corresponding set of cavity modes. (Agresti, D'Ambrosio,Desalvo,Mantovani,Simoni,Willems)





$$w_0 = \sqrt{\frac{L}{k}}$$

Motivated by the important issues illustrated above: Equivalence Relation Proof A

Juri Agresti & Erika D'Ambrosio Generalization to any kind of mirror surface

Lossless specular cavities:

$$\gamma\, u(\vec{r}) = \int_{\substack{Mirror\\ Surface}} K(\vec{r},\vec{r}')\, u(\vec{r}')\, d\vec{r}'$$

$K(\vec{r},\vec{r}')$ propagator from surface to surface

$u(\vec{r})$ light distribution on *both* mirrors

γ eigenvalue for *one-way* trip

$$K_{flat}(\vec{r},\vec{r}') = \frac{ik}{2L\pi} Exp\left[-ikL + ikh(r) - \frac{ik}{2L}\left|\vec{r}-\vec{r}'\right|^2 + ikh(r')\right] \quad K_{conc}(\vec{r},\vec{r}') = \frac{ik}{2L\pi} Exp\left[-ikL - ikh(r) + \frac{ik}{2L}\left|\vec{r}+\vec{r}'\right|^2 - ikh(r')\right]$$

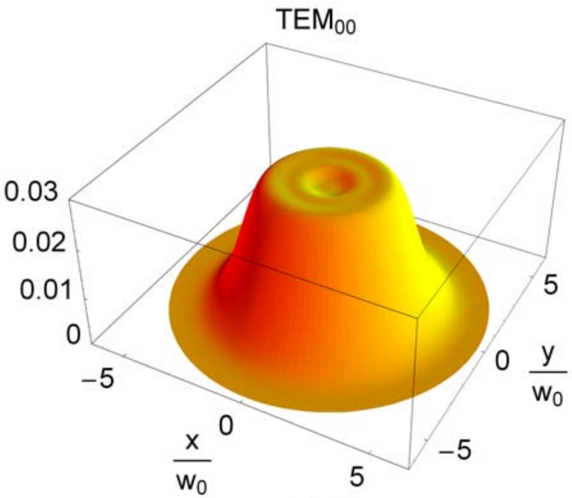
$h(r)$ deviation from perfectly flat surface $\quad$ $-h(r)$ deviation from concentric surface $\quad R = \frac{L}{2}$

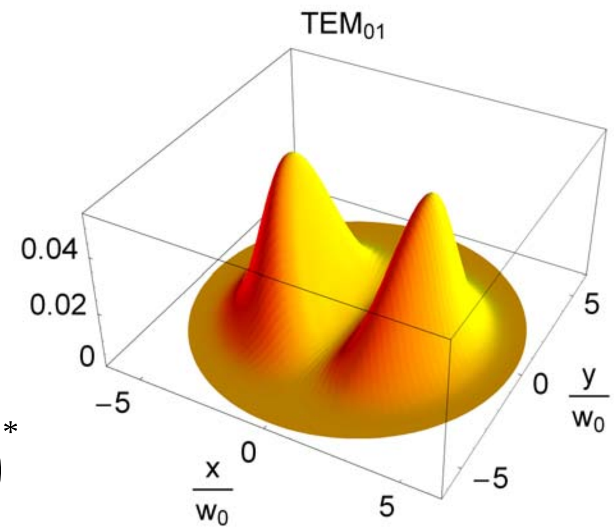
The eigenfunctions are real, being the mirrors constant phase surfaces

The eigenfunctions are the same ➡ automatic mapping

Classification according to quantum numbers by **Equivalence Relation Proof A**

Juri Agresti & Erika D'Ambrosio eigenvalue relation unambiguously identified

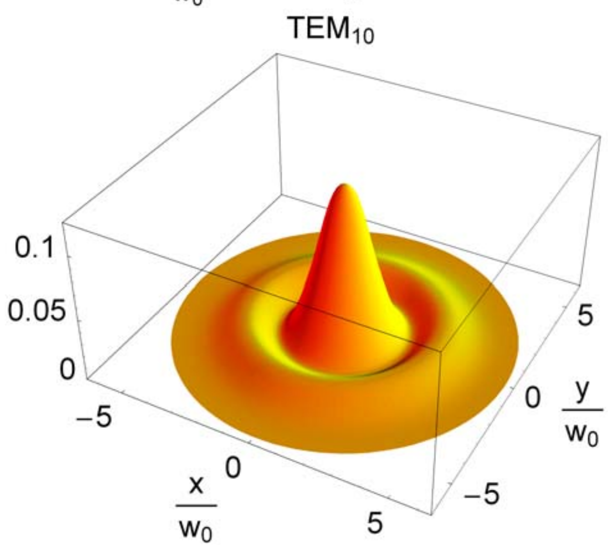


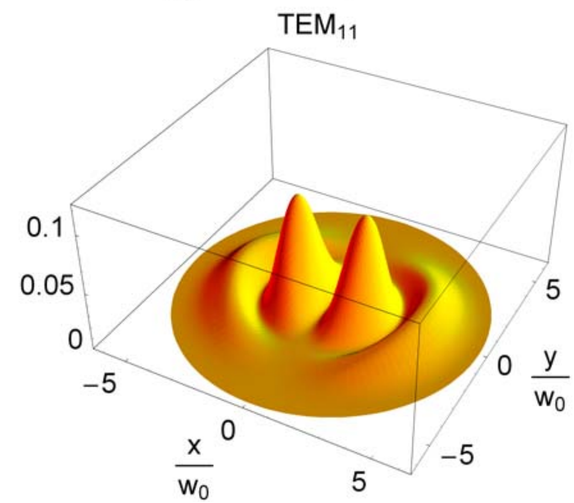


$$e^{ikL}\gamma_{lm}^{conc} = (-1)^{m+1} e^{-ikL} \left(\gamma_{lm}^{flat}\right)^{*}$$

For any l and m : $\left|u_{lm}\right|^2$

Identical loss : $1-\left|\gamma_{lm}\right|^2$





Application to Advanced LIGO of Equivalence Relation Proof A
Juri Agresti & Erika D'Ambrosio : *alignment instability*

For equivalent cavities: $$\frac{T_{conc}}{T_{flat}} \approx \frac{\alpha_{conc}}{\alpha_{flat}}$$

α = coupling

T = torque

For unstable coupling: $$\frac{\alpha^G_{conc}}{\alpha^G_{flat}} \approx \frac{1}{40}$$ in agreement with Sigg & Sidles

$$\frac{\alpha^{FTB}_{conc}}{\alpha^{FTB}_{flat}} \approx \frac{1}{247}$$ in agreement with Sav.&Vyatch.

Comparison between geometries $$T^G_{conc} \approx 1.1\, T^{FTB}_{conc} \qquad T^G_{flat} \approx 0.2\, T^{FTB}_{flat}$$

The *closer* to concentric, the *larger* the diffraction angle.

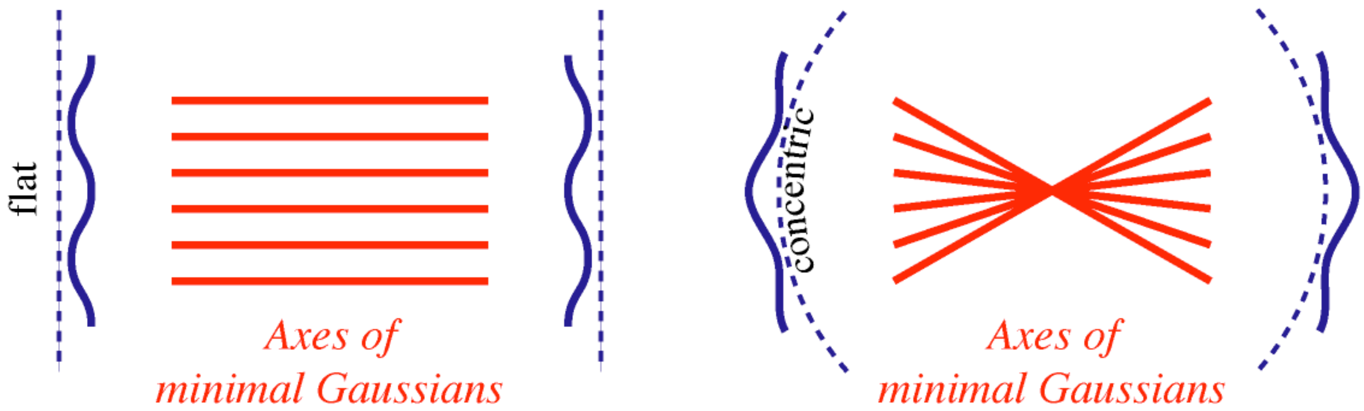
Two beams are related by Fourier Transform: Equivalence Relation Proof B

Yanbei Chen & Pavlin Savov mesa beams defined at the center of the cavity

- Savov and Vyatchanin: Two types of Mesa beams are supported by configurations with flat+h(r) and concentric spherical — h(r), respectively
- Bondarescu and Thorne: A continuum of Mesa beams are designed by overlapping minimal spreading Gaussian beams, from flat+h(r) to concentric spherical — h(r)

Formation of Nearly Flat Mesa Beam

Formation of Nearly Concentric Mesa Beam



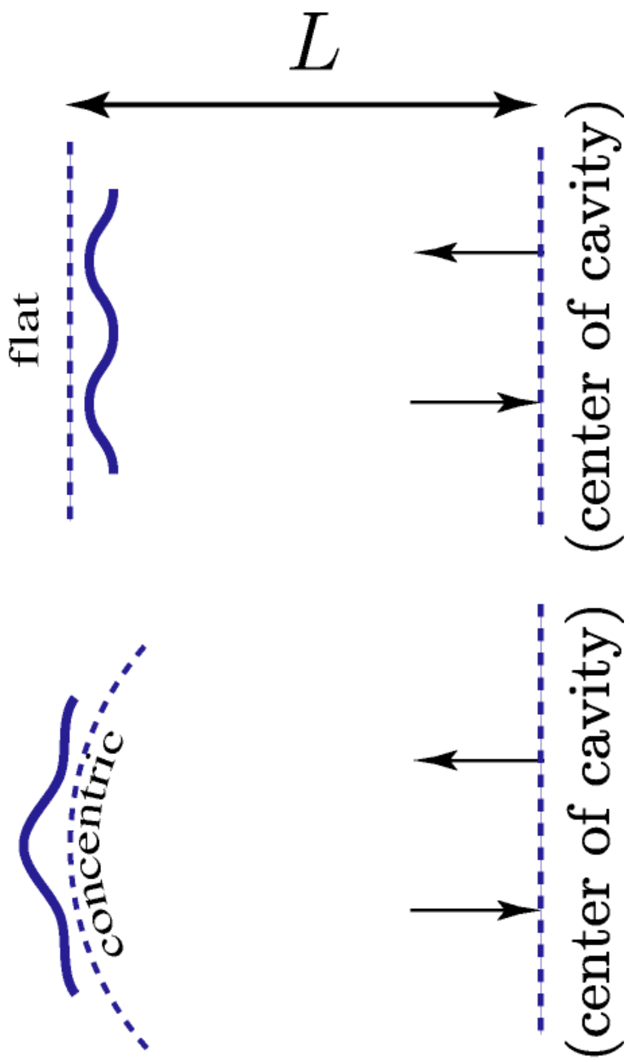
Gaussians being translated in $\mathbf{r}=(x, y)$ space, and linearly superimposed

Gaussians being translated in $\mathbf{k}=(k_x, k_y)$ space, and linearly superimposed

Gaussians have the same form in $\mathbf{r}$ and $\mathbf{k}$ spaces

Propagation Operators in Dual Configurations Equivalence Relation Proof B

Yanbei Chen & Pavlin Savov from the center of the cavity to the mirror and back



$$\mathcal{L}_A = \mathcal{G} e^{+2ikh(\vec{r})} \mathcal{G}$$

$$\mathcal{L}_B = \mathcal{G} e^{2ik\left[\vec{r}^2/L - h(\vec{r})\right]} \mathcal{G}$$

$\mathcal{G}$: Free propagation for a length of L

$h(\vec{r})$: Mirror height; we require $h(\vec{r}) = h(-\vec{r})$.

Relation between the two propagators Equivalence Relation Proof B

Yanbei Chen & Pavlin Savov Eigenstates and eigenvalues

- Calculation shows a mapping between propagators

$$\mathcal{P}\mathcal{L}_A^* = -e^{4ikL}\mathcal{F}^{-1}\mathcal{L}_B\mathcal{F}$$

$\mathcal{P}$: Parity Operation (flips up and down)

$\mathcal{F}$: $[\mathcal{F}u](\vec{r}) = \frac{k}{2\pi L}\int d^2\vec{r}'\, e^{-ik\vec{r}\cdot\vec{r}'/L} v(\vec{r}')$

- This implies a mapping between eigenstates (their complex-amplitude distributions at the center of the cavity):

Given an eigenstate u_A of $\mathcal{L}_A$: $\mathcal{L}_A u_A = \lambda_A u_A\,,\quad \mathcal{P}u_A = (-1)^p u_A\,,$

(note that $\mathcal{P}\mathcal{L}_{A,B} = \mathcal{L}_{A,B}\mathcal{P}$)

$(\mathcal{F}u_A^*)$ is an eigenstate of $\mathcal{L}_B$: $\mathcal{L}_B(\mathcal{F}u_A^*) = \underbrace{(-1)^{p+1}e^{-4ikL}\lambda_A^*}_{\text{new eigenvalue}}(\mathcal{F}u_A^*)$

Proposal of nearly concentric non-spherical mirrors as an alternative to Advanced LIGO baseline (Agresti,Bondarescu, Chen,D'Ambrosio,Desalvo,Savov,Thorne,Vyatchanin,…)

- Integration of thermal noise with different beam profiles
- Simulations of Advanced LIGO with both configurations
- Experience with non spherical cavities (Caltech prototype)
- Comparison with different alternatives (cryogenic design)
- Exploration of alternative numerical tools for simulations