Super SQL Sensitivity via Optical Rigidity in Signal-recycling Laser Gravitational Antenna

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We found the set of parameters to obtain super SQL sensitivity without increasing laser pump due to optical rigidity introduced by signal recycling mirror. The gain of sensitivity is inverse proportional the bandwidth. Varying parameters one can goes from one minimum sensitivity curve to two minimum curve. It provides the additional possibility to control sensitivity of antenna "on line".

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Introduction

The use of optical (pondermotive) rigidity allows to obtain super SQL sensitivity in gravitational-wave detector.

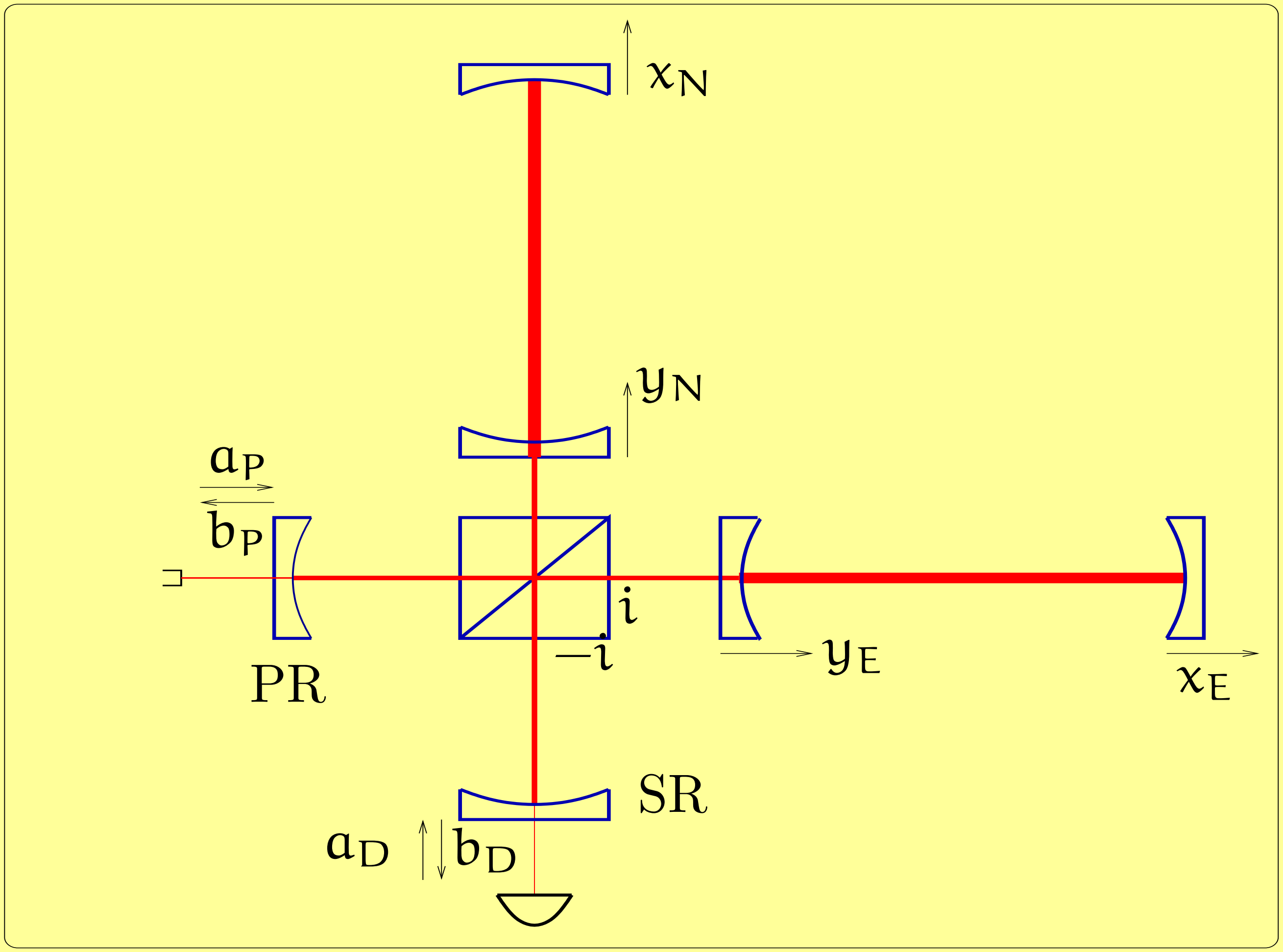
Optical rigidity can be created almost "for free" in the signal-recycled configurations of the laser gravitational-wave detectors.

This work is development of results of papers:

[1] A.Buonanno, Y.Chen, Physical Review D **64**, 042006 (2001).

[2] F.Ya.Khalili, Physics Letters A **288**, 251 (2001).

Avanced LIGO inteferometer



Notations and Approximations

$$\theta = e^{i\Omega L/c} \simeq 1 + \frac{i\Omega L}{c}, \quad \gamma = \frac{cT^2}{4L}, \tag{1}$$

$$\phi = \frac{(\omega_o + \Omega)l}{c} \simeq \frac{\omega_o l}{c}, \quad l \ll L \tag{2}$$

$$\gamma_0 = \frac{\gamma(1-\rho^2)}{1 + 2\rho\cos 2\phi + \rho^2} \ll \gamma, \tag{3}$$

$$\delta = \frac{2\gamma\rho\sin 2\phi}{1 + 2\rho\cos 2\phi + \rho^2}, \quad \Gamma = \gamma_0 - i\delta \tag{4}$$

Here γ is relaxation rate of alone FP cavity, δ is detuning introduced by SR mirror, γ_0 is relaxation rate of difference mode of interferometer. We assume that ϕ does not depend on Ω due to small length l of SR cavity: $l \ll L$.

Difference Position and Optical Rigidity

The output field b_D depends on difference position z of mirrors for which we have:

$$z \equiv \big(x_E - y_E\big) - \big(x_N - y_N\big) = \frac{F_{fl} + F_s}{Z(\Omega)}\,, \quad F_s = m\Omega^2 L\, h_s,$$

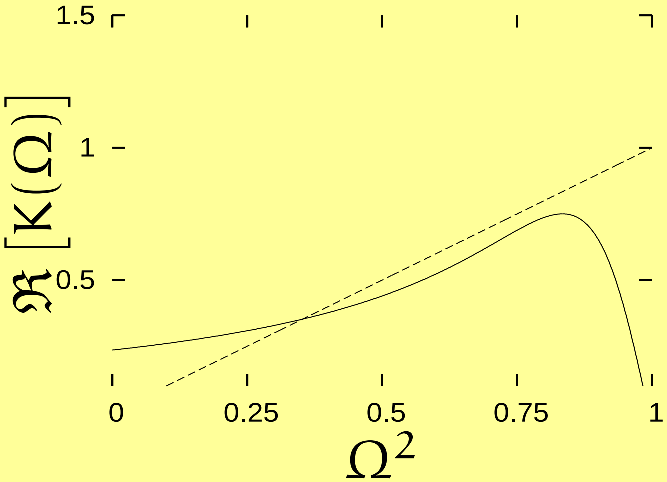
$$Z(\Omega) = -m\Omega^2 + K = \frac{-m\Omega^2\,\mathcal{M}}{\mathcal{D}}, \quad K(\Omega) = \frac{8I_o\omega_o\,\delta}{cL\,\mathcal{D}}\,, \tag{5}$$

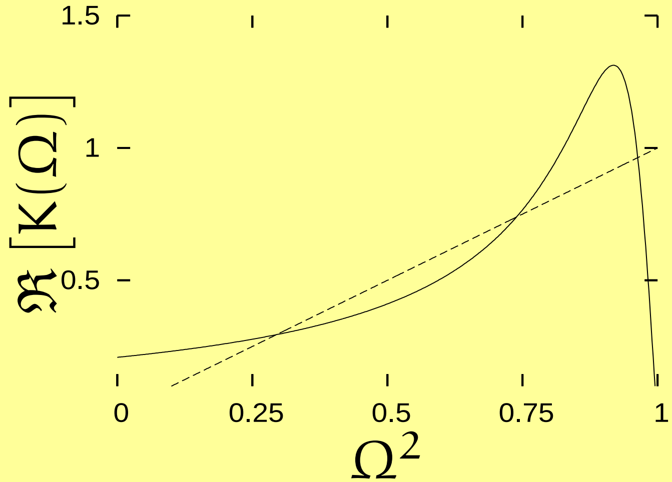
$$\mathcal{D} = \big(\Gamma - i\Omega\big)\big(\Gamma^* - i\Omega\big), \quad \mathcal{M} = \mathcal{D} - \frac{8I_o\omega_o\delta}{m\Omega^2 cL}$$

Here h_s dimentionless metric perturbation, $Z(\Omega)$ is mechanical impedance, $K(\Omega)$ is complex optical rigidity, I_o is optical power circulating inside FP cavity in each arm. F_{fl} is back action force produced by fluctuations of light pressure.

Frequency-dependent Optical Rigidity

Direct line corresponds to $m\Omega^2$

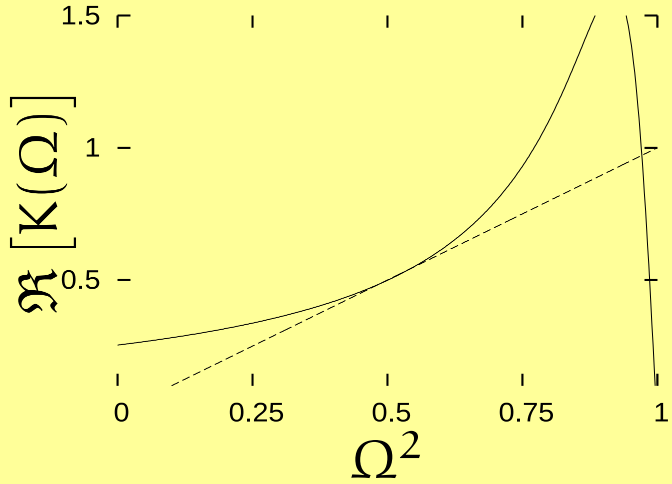


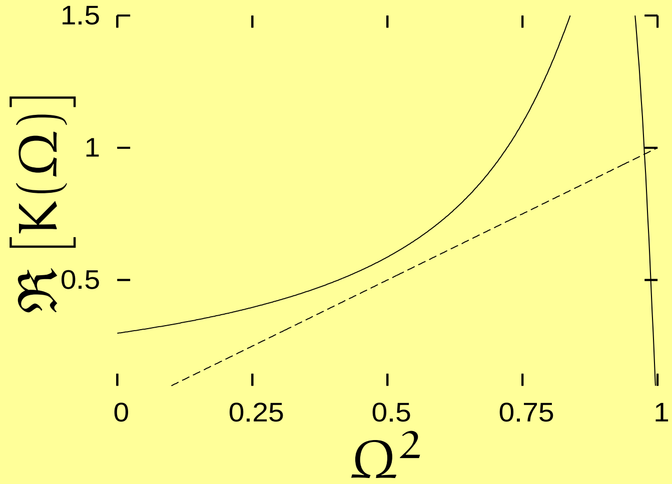


Large signal-recycling mirror transmittance. Only one zero of $\Re\big[Z(\Omega)\big]$.

Sub-critical pumping. $I_o < I_o^{crit}$: $\Re\big[Z(\Omega)\big]$ has three zeros. It provides two narrow minima in noise spectral density.

Frequency-dependent Optical Rigidity (continuation)





Critical pumping: $\mathrm{I_o} = \mathrm{I_o^{crit}}$. $\Re\big[Z(\Omega)\big]$ has one 2 fold zero. It provide one, but more wide minimum in the noise spectral density.

Super-critical pumping: $\mathrm{I_o} > \mathrm{I_o^{crit}}$.

Scketch of Calculations:

1. Calculation of output field $b_D(\Omega, z)$.
2. Calculation of fluctuational and regular (rigidity) forces acting on difference position z.
3. Calculation of z and substitution result into $b_D(\Omega, z)$ obtained in item 1.
4. Calculation of quadrature component of output field b_ζ, measured in balance homodyne scheme.
5. Calculation of spectral density of dimentionless metric S_h and its normalization in "SQL" units:

$$\xi^2(\Omega) = \frac{S_h(\Omega)}{h^2_{SQL}(\Omega_0)}, \quad h^2_{SQL}(\Omega_0) = \frac{8\hbar}{mL^2\Omega_0^2},$$

Ω_0 is working frequency, $\Omega_0 \simeq 2\pi \times 100\ s^{-1}$.

Quadrature component analysis

We can write down the sensitivity $\xi(\Omega)$ as following

$$\xi(\Omega) = \left|\frac{P}{Q}\right|, \tag{6}$$

$$P = \frac{1}{\Omega^2}\Big(\Omega^4 + \Omega^2(\gamma_0^2 - \delta^2) + J\gamma(\delta - \gamma_0 \sin 2\alpha) + \\ + i\gamma_0\Big(J\gamma(1+\cos 2\alpha) - 2\Omega^2\delta\Big)\Big) \tag{7}$$

$$Q = 2\sqrt{\frac{J\gamma\gamma_0}{\Omega_0^2}}\Big((\gamma_0 - i\Omega)\cos\alpha - \delta\sin\alpha\Big), \tag{8}$$

$$J = \frac{8I_o\omega_o}{m\gamma Lc}, \quad I_{SQL}(\Omega_0) = \frac{m\Omega_0^3 Lc}{8\omega_o}, \tag{9}$$

$$\alpha = \zeta + \frac{1}{2}\times\arg\left(\frac{e^{2i\phi}+\rho}{1+\rho e^{2i\phi}}\right). \tag{10}$$

ζ is homodyne angle.

We consider $P(\Omega)$ (coefficient Q is monotonic). Real part $\Re[P(\Omega)]$ has roots:

$$\Omega_{1,2}^2 = \frac{\delta^2 - \gamma_0^2}{2} \pm \sqrt{\left(\frac{\delta^2 - \gamma_0^2}{2}\right)^2 + J\gamma\big(\gamma_0 \sin 2\alpha - \delta\big)} \qquad (11)$$

The conditions for super SQL sensitivity:

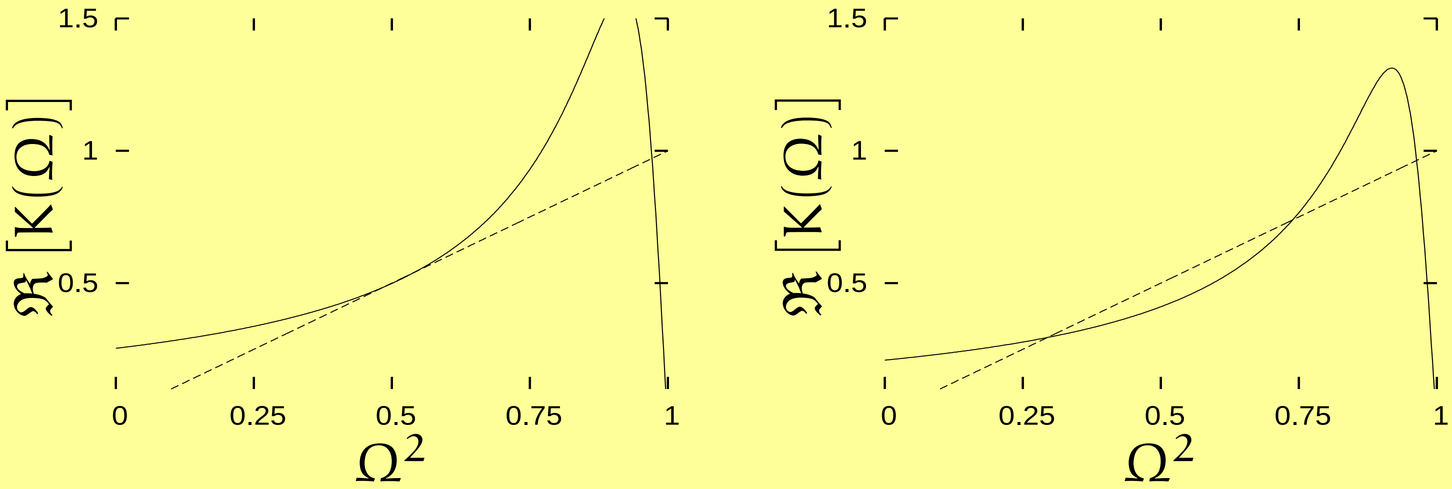
$$\Omega_0^2 \simeq \frac{\delta^2 - \gamma_0^2}{2}; \qquad (12)$$

One minimum of sensitivity curve:

$$0 = \left(\frac{\delta^2 - \gamma_0^2}{2}\right)^2 - J\gamma\left(\delta - \gamma_0 \sin 2\alpha\right);$$

Two minimums of sensitivity curve:

$$0 < \left(\frac{\delta^2 - \gamma_0^2}{2}\right)^2 - J\gamma\left(\delta - \gamma_0 \sin 2\alpha\right)$$



One minimum sensitivity for $\gamma_0 \ll \Omega_0$

$$\Omega_0 = \frac{\delta}{\sqrt{2}} \quad \text{and} \quad I_o = \frac{I_{SQL}(\Omega_0)}{\sqrt{2}}, \tag{13}$$

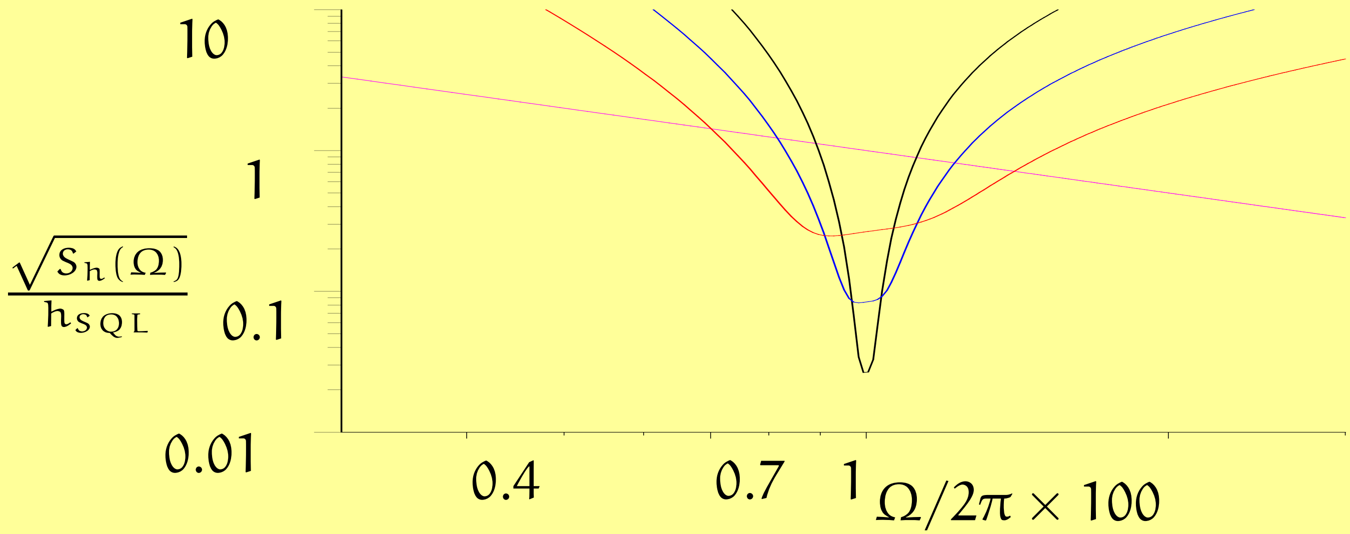
$$\xi_0 \equiv \xi(\Omega_0) \simeq \sqrt{\frac{\gamma_0\left(2-\cos^2\alpha\right)}{\sqrt{2}\,\Omega_0}}\,, \tag{14}$$

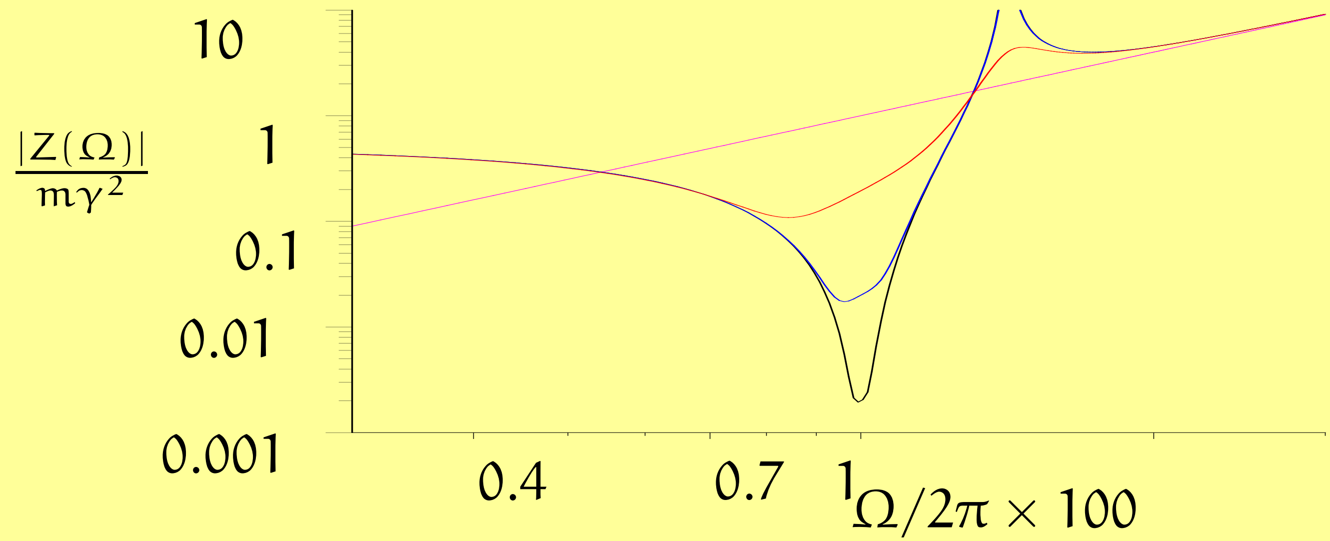
$$\frac{\Delta\Omega}{\Omega_0} \simeq \sqrt{\frac{\gamma_0\left(2-\cos^2\alpha\right)}{2\sqrt{2}\,\Omega_0}} \simeq \frac{\xi_0}{\sqrt{2}} \tag{15}$$

The mechanical impedance $Z(\Omega)$ has minimum (resonance) at the same frequency $\Omega = \Omega_0$:

$$Z(\Omega) \simeq \frac{m\left(\left(\Omega^2-\Omega_0^2\right)^2 + 2i\gamma_0\Omega^3\right)}{2\Omega_0^2 - \Omega^2 - 2i\gamma_0\Omega} \tag{16}$$

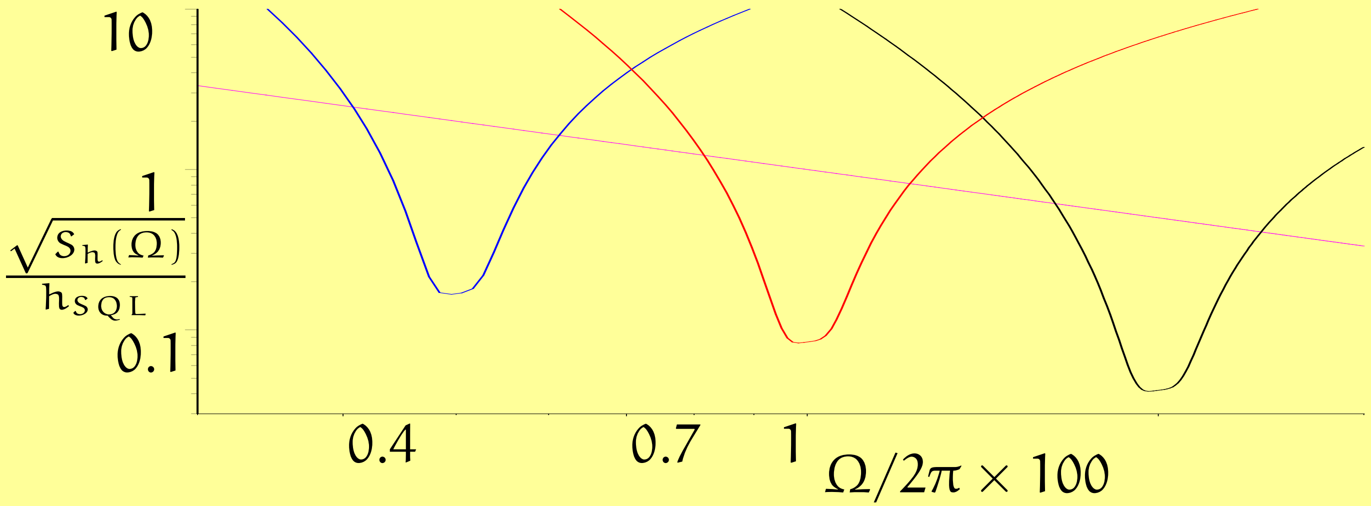
$I_o = I_{SQL}(\Omega_0)/\sqrt{2}, \quad \gamma_0/\Omega_0 = 0.1,\ 0.01,\ 0.001, \quad \alpha = 0$

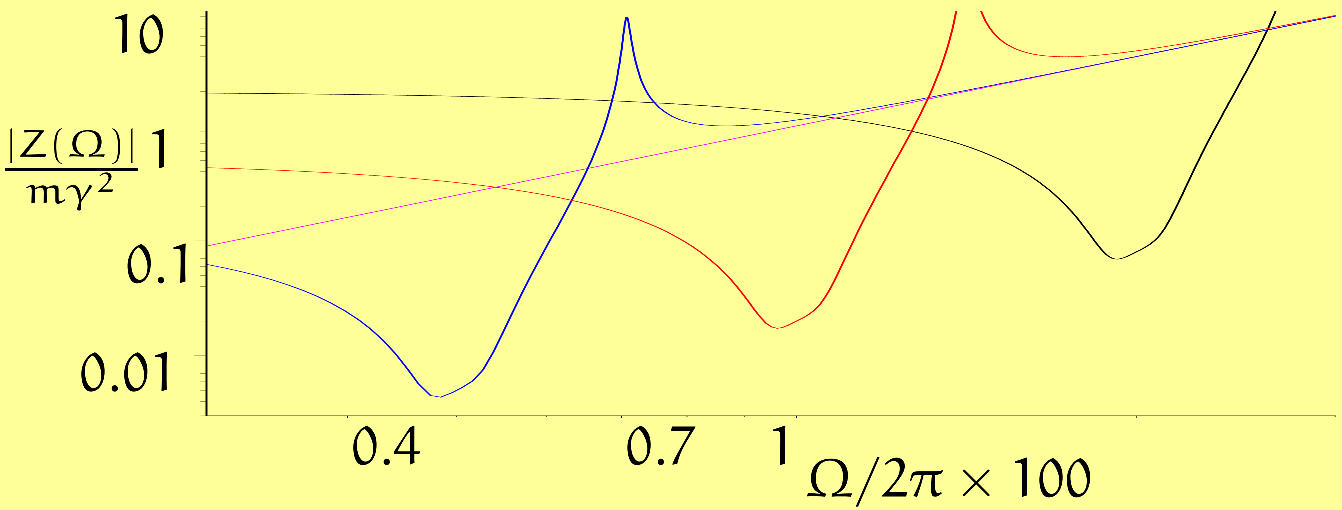




$\gamma_0/\Omega_0 = .01,\ \alpha = 0,\ \Omega_0 = (\pi,\ 2\pi,\ 4\pi) \times 100\ \mathrm{s}^{-1}$

$I_o = (0.125,\ 1,\ 8) \times I^o_{SQL}/\sqrt{2}, \quad I^o_{SQL} = I_{SQL}(\Omega = 2\pi \times 100\ \mathrm{s}^{-1})$





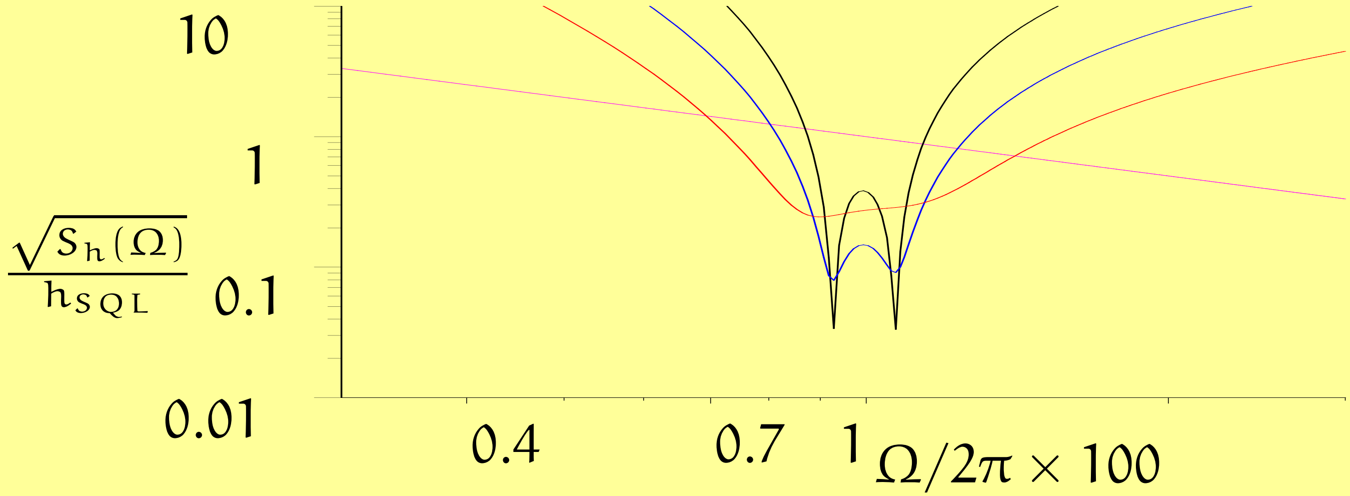
Two minimums sensitivity curve for $\gamma_0 \ll \Omega_0$

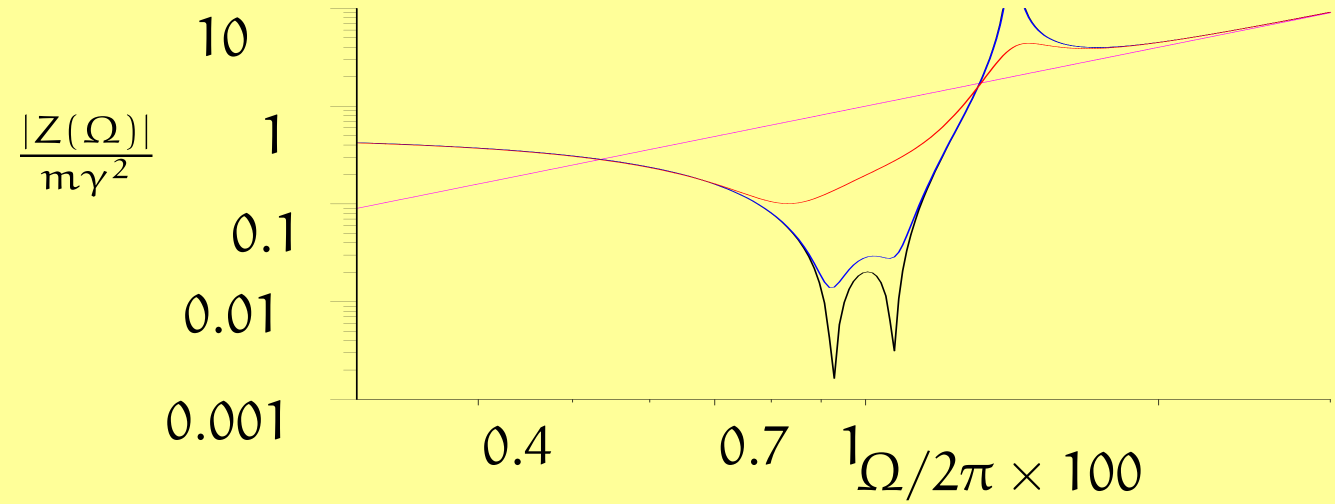
$$\Omega_0 = \frac{\delta}{\sqrt{2}}, \qquad (17)$$

$$I_o < \frac{I_{SQL}(\Omega_0)}{\sqrt{2}} \qquad (18)$$

Two close minima:

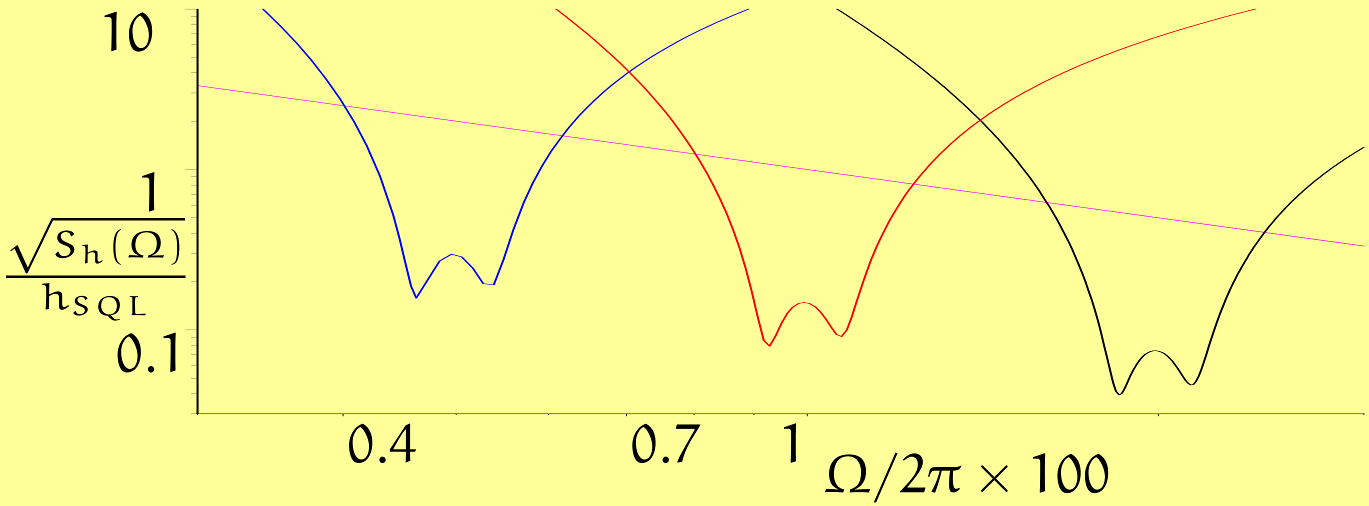
$I_o = 0.98 \times I_{SQL}(\Omega_0)/\sqrt{2}, \quad \gamma_0/\Omega_0 = 0.1,\ 0.01,\ 0.001, \quad \alpha = 0$

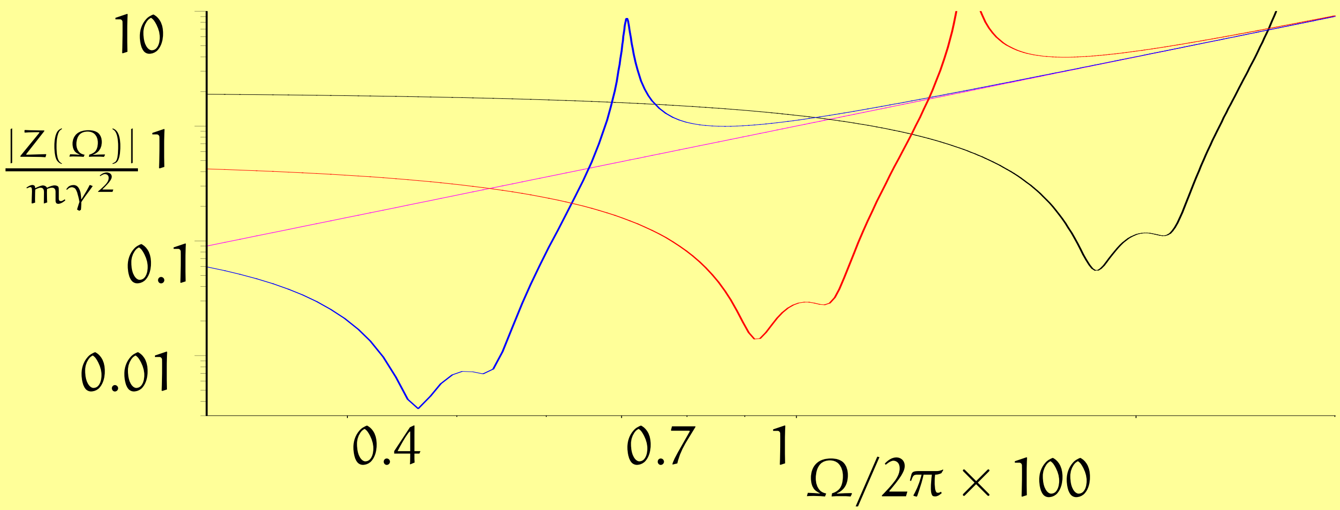




$\gamma_0/\Omega_0 = 0.01, \ \alpha = 0, \ \Omega_0 = (\pi, \ 2\pi, \ 4\pi) \times 100 \text{ s}^{-1}$

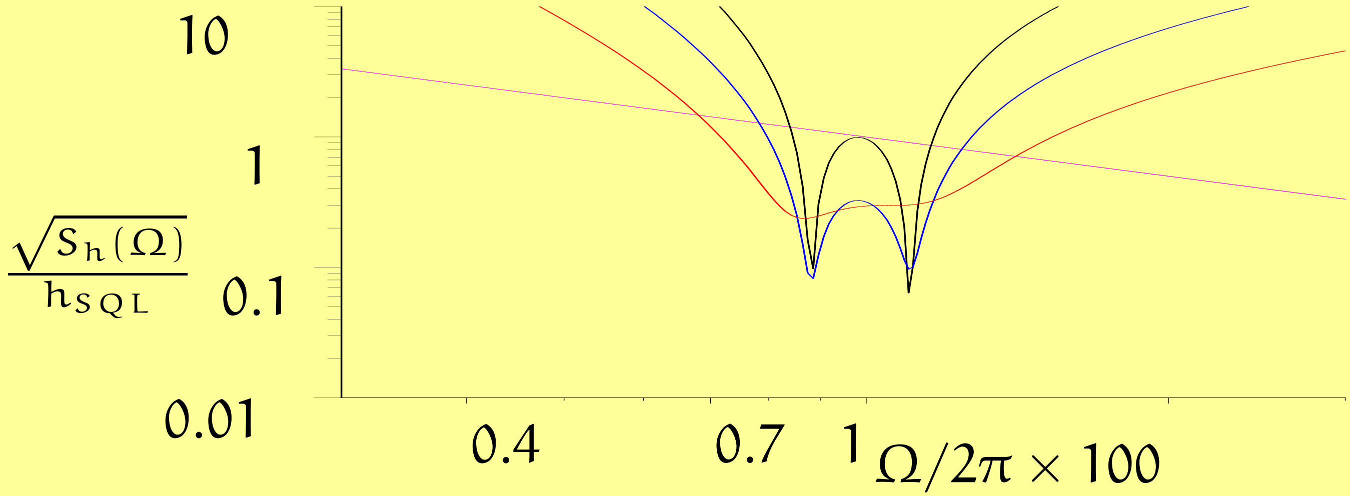
$I_o = (0.125, \ 1, \ 8) \times 0.98 \times I^o_{SQL}/\sqrt{2}$

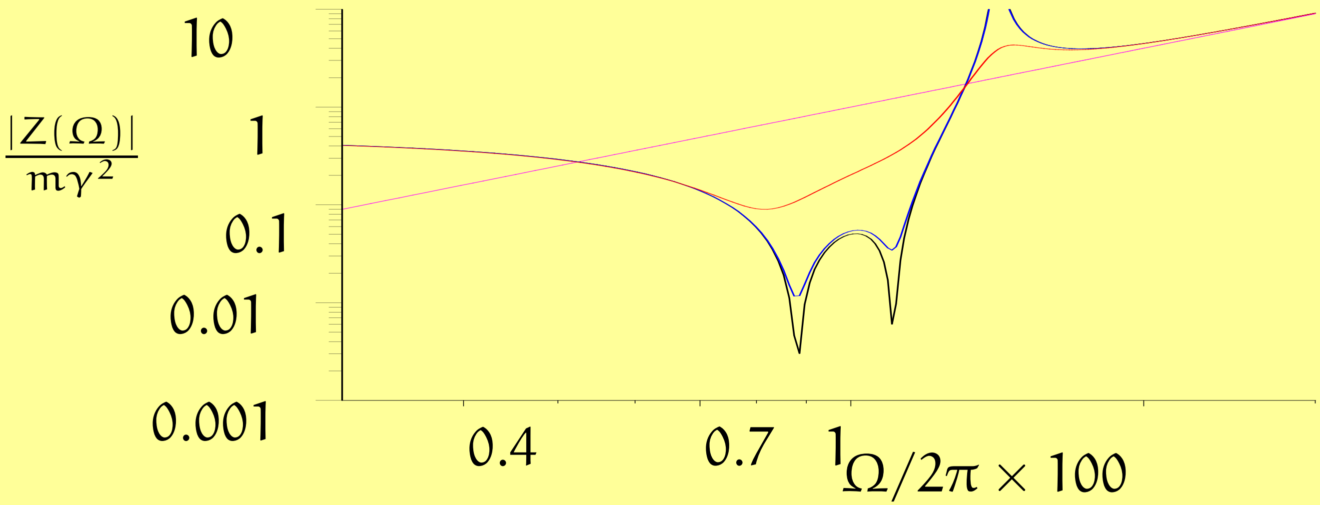




Two separate minima:

$I_o = 0.95 \times I_{SQL}(\Omega_0)/\sqrt{2}, \quad \gamma_0/\Omega_0 = 0.1,\ 0.01,\ 0.001, \quad \alpha = 0$





Comparision with paper of Buonanno and Chen (BC)

We find the regime of signal recycled interferometer which differs from regime discussed in BC[a]

1. Power is $\sqrt{2}$ times less than power in BC.
2. BC consider the case of relatively large total relaxation rate: $\gamma_0 \simeq 0.25 \times \Omega_0$ (relatively large transmittance of SR mirror: $\rho = 0.9$). Detuning is not optimal: $\delta \simeq 1.9 \times \Omega_0$.
3. Dependence on homodyne angle is practically absent — in BC sensitivity depends on homodyne angle.

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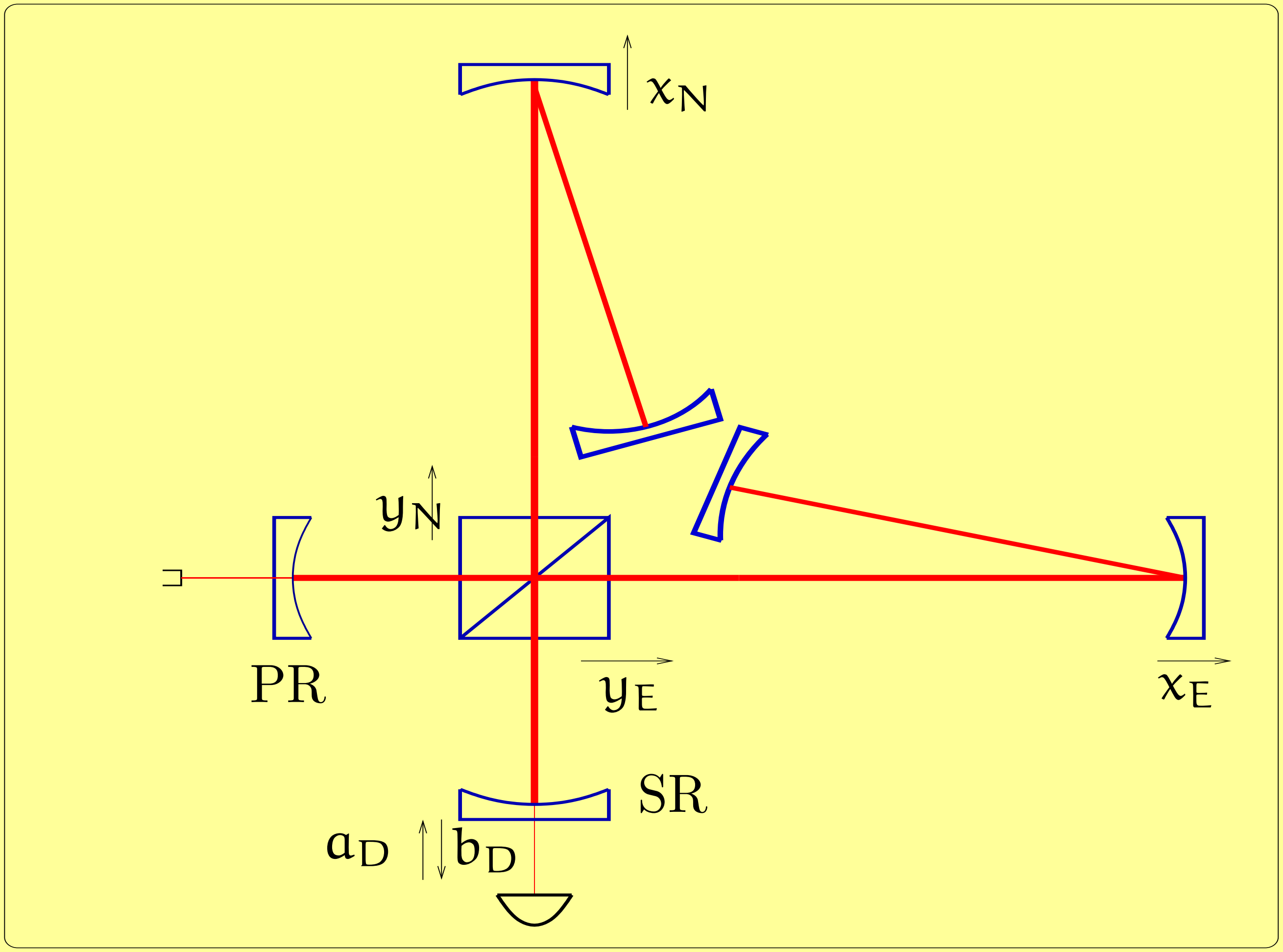
[a] A.Buonanno, Y.Chen, Physical Review D **64**, 042006 (2001).

GEO-600

$\gamma = c/L \simeq 2 \times 10^5\ \mathrm{s}^{-1}$. If $\gamma_0 \ll \Omega_0 \ll \gamma$ — our results are correct.



Modest value of power

For frequency $\Omega_0 = 2\pi \times 50\ \mathrm{s}^{-1}$ the optical power needed to circumvent SQL about 10 times is rather small ($I_{SQL} \sim \Omega^3$):

$$\text{LIGO}: \quad I_o = \frac{I^o_{SQL\,LIGO}}{8\sqrt{2}} \simeq 66\ \mathrm{kW}, \tag{19}$$

$$\text{GEO}: \quad I_o = \frac{I^o_{SQL\,GEO}}{8\sqrt{2}} \simeq 2.5\ \mathrm{kW} \tag{20}$$

Now in GEO the circulating in arms power is equal to 150 W and in final stage the power will be 5 kW.

Translations of parameters

"High-level" parameters γ_0, δ, I_o/I^0_{SQL} can be translated into "physical" parameters ρ, ϕ using expressions:

$$\frac{\gamma_0}{\delta} = \frac{1-\rho^2}{2\rho\sin 2\phi}\,, \qquad \frac{\delta}{\gamma} = \frac{2\rho\sin 2\phi}{1+2\rho\cos 2\phi+\rho^2}\,. \tag{21}$$

For GEO-600 $\gamma_{\rm GEO} = c/2L \simeq 2\times 10^5\,{\rm s}^{-1} \gg \delta \gg \gamma_0$. Hence $\phi \ll 1$, $1-\rho \ll 1$ and we have good approximation:

$$\phi \approx \frac{\delta}{\gamma}\,, \qquad 1-\rho \approx 2\,\frac{\gamma_0}{\delta}\,\frac{\delta}{\gamma}\,. \tag{22}$$

For the Advanced LIGO $\gamma_{\rm LIGO} = 2\pi\times 100\,{\rm s}^{-1} \simeq \delta \gg \gamma_0$, and we have to solve (21).

Table 1: Advanced LIGO: values of ρ, ϕ and I_o for $\gamma_0/\Omega_0 = 0.01$

$\Omega_0/2\pi$ (Hz)	$1-\rho$	ϕ	I_o (kW)
50	9.4×10^{-3}	0.62	66
100	9.4×10^{-3}	0.96	526
200	6.3×10^{-3}	1.23	421

Table 2: CEO-600: values of ρ, ϕ and I_o for $\gamma_0/\Omega_0 = 0.01$

$\Omega_0/2\pi$ (Hz)	$1-\rho$	ϕ	I_o (kW)
50	4.4×10^{-5}	0.0022	2.5
100	8.8×10^{-5}	0.0044	19.7
200	18×10^{-5}	0.0089	158

Conclusion

- We found the family of parameters to achieve super SQL sensitivity both for Advanced LIGO and GEO topology.
- We can get large sensitivity gain (small ξ_0) in narrow bandwidth or smaller gain but in wider bandwidth. The tuning may be produced "on line" — by variation of SR mirror position and/or value of circulating power.
- Super SQL sensitivity (gain 10) on frequency 50 Hz can be achieved at modest value of power: 66 kW for Adv. LIGO and 2.5 kW for GEO. It is intriguing and independent problem. The main difficulty is classical noise (brownian, thermoelastic, seismic) in mirrors on low frequencies.
- We do not discuss that optical rigidity produces negative damping. In pinciple this instability may be compensated by noise free feed back.