LIGO's Ultimate Astrophysical Reach

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LIGO Seminar

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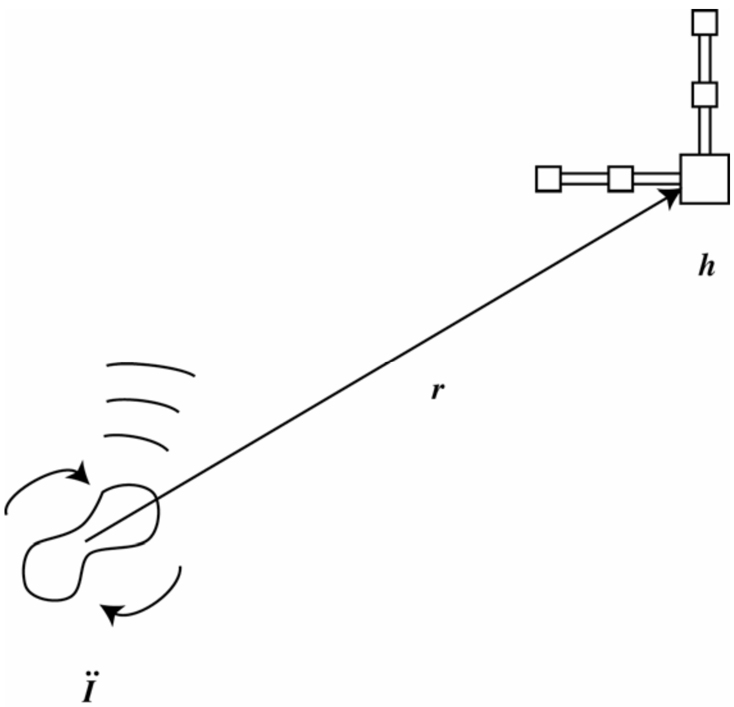
Range of Gravitational Radiation



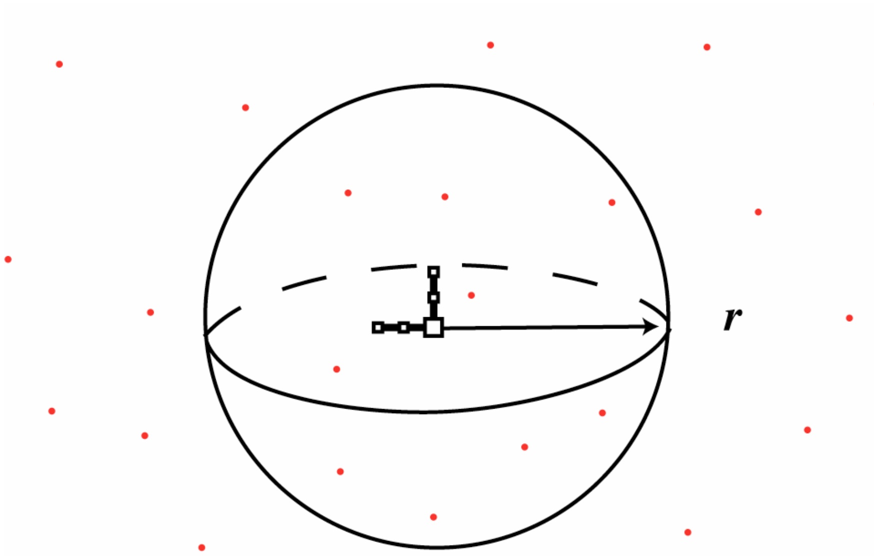
- Energy density must fall off as $1/r^2$.
- Energy density is the square of the strain amplitude *h*.
- Amplitude falls off as 1/r.

$$h_{\mu\nu} = \frac{2G}{c^4}\frac{\ddot{I}_{\mu\nu}}{r}$$

- Therefore, the *range* of a detector that is sensitive to a given strain *h* scales as 1/*h*

$$r \sim \frac{1}{h}$$

Event Rate vs. Range



- For isotropic distribution with density ρ, the number of sources included in radius r is given by

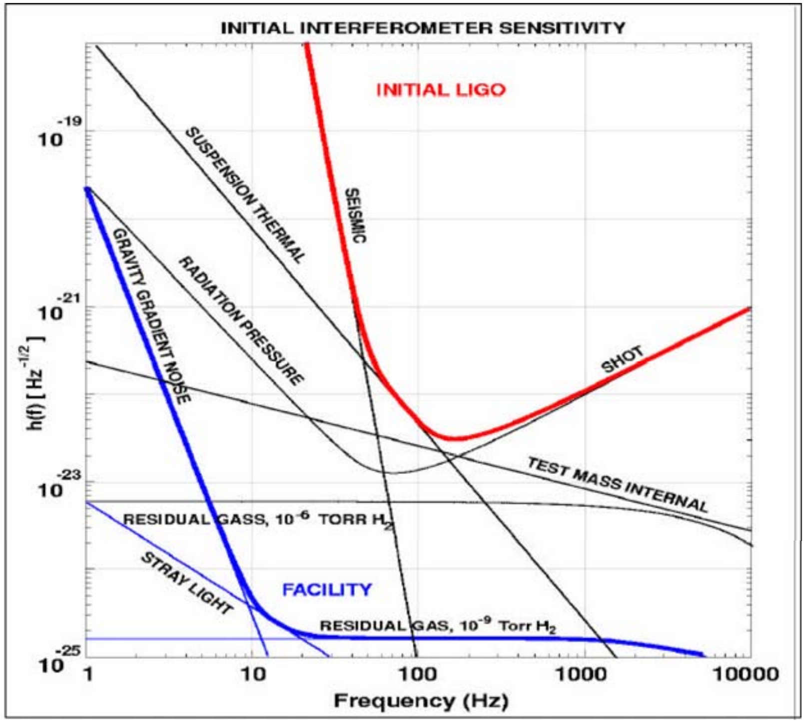
$$N = \rho\left(\frac{4}{3}\pi r^3\right)$$

- Event rate proportional to number of sources included in range, or

$$N \sim \left(\frac{1}{h}\right)^3$$

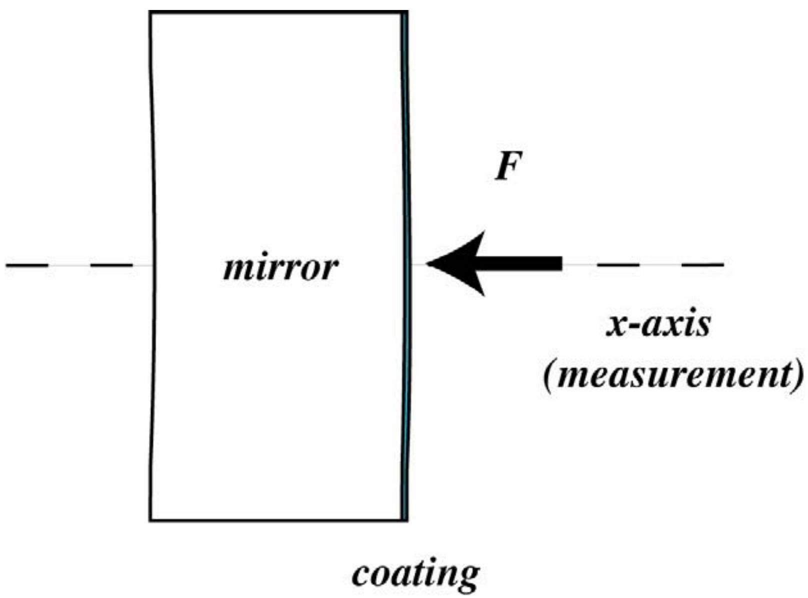
- Small reductions in detector noise floor h result in big increases in number of sources N within detector's range!

What will LIGO's range be?



- Need to know fundamental limits to *h*.
  - Seismic
  - Thermal
  - Shot
- Thermal noise limits *h* at the lowest levels, determines ultimate reach of detector.
- Original (SRD) curve was dominated by suspension thermal noise, but that assumed viscous damping. This estimate has since been superseded by a newer understanding of thermal noise (structural damping).
- Newer estimates lower suspension thermal noise, reveal test mass noise.
- New estimates show mirror thermal noise dominating at lowest noise levels.

Mirror thermal noise



- Fluctuation-dissipation theorem relates noise spectrum to losses.

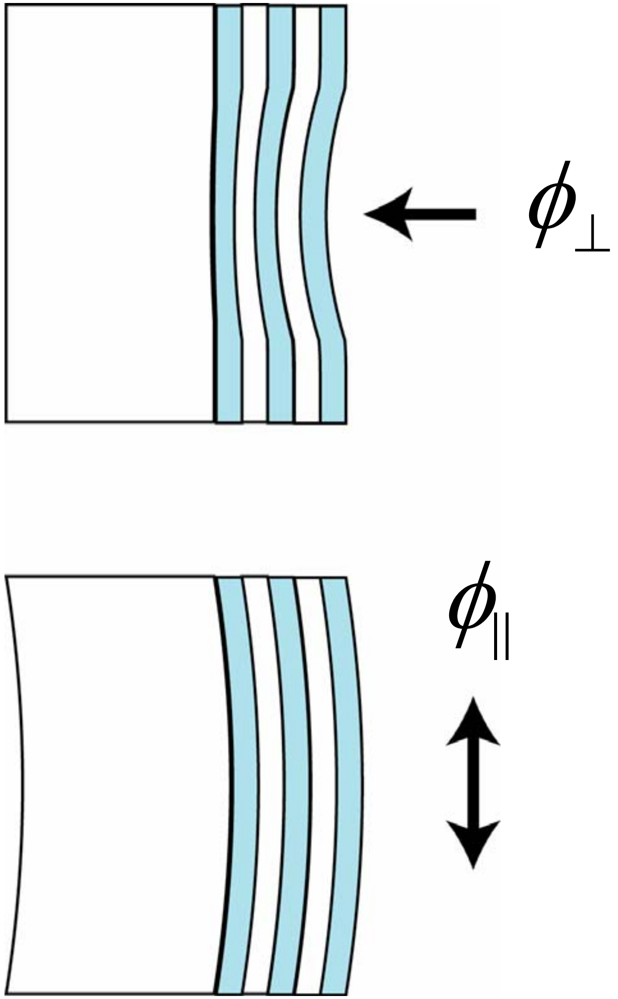
$$S_x(f) = \frac{k_B T}{\pi^2 f^2} \mathrm{Re}\left\{ \frac{\tilde{x}(f)}{F e^{i2\pi f t}} \right\}$$

- Structural damping loss
  - Substrate thermal noise
  - Coating thermal noise
- Thermoelastic damping loss (Braginsky noise)
  - Substrate thermoelastic noise
  - Coating thermoelastic noise

Calculating mirror thermal noise for different mechanisms

- Substrate thermoelastic noise
  - Need to know mirror material's bulk thermomechanical properties: thermal expansion coefficient, thermal conductivity, etc.
  - Well known parameters available in the literature. Can calculate from first principles.
- Substrate thermal noise, structural damping
  - Need to know substrate loss angle, or mirror Q (expect frequency independent)
  - Have to measure. Can't calculate from first principles, but measurement is (relatively) easy.
- Coating thermoelastic noise
  - Need to know coating thermomechanical properties, which may differ substantially from those of the same materials in bulk.
  - Preliminary measurements done. Estimates predict this won't be an issue even for AdLIGO.
- Coating thermal noise, structural damping
  - Coating loss angle (also expect frequency independent)
  - Have to measure. Can't calculate from first principles.
  - This is expected to be the limiting noise source!

Coating thermal noise: structural damping losses



- Coating is a stack of alternating dielectric materials, anisotropic and complicated!
- Fluctuation-dissipation theorem can deal with this complication, but…
- Need to know losses for strains (distortions) in the same direction that the laser beam senses, perpendicular to the coating-substrate interface.
- Can measure losses for parallel distortions by measuring ringdown of body modes, comparing with uncoated mirror.
- Are they the same? Different?
- Direct measurement would be definitive, but we need to have a *predictive* model for designing AdLIGO.

How to calculate coating thermal noise?

- Full theory, including coating anisotropy, different mechanical properties of substrate and coating:

$$S_x(f) = \frac{2k_B T}{\pi^{3/2} f} \frac{1-\sigma^2}{wY} \left\{ \frac{1}{\sqrt{\pi}} \frac{d}{w} \frac{1}{YY'(1-\sigma'^2)(1-\sigma^2)} \right.$$

$$\times \left[Y'^2 (1+\sigma)^2 (1-2\sigma)\phi_{\|} + YY'\sigma'(1+\sigma)(1+\sigma')(1-2\sigma)(\phi_{\|} - \phi_{\perp}) \right.$$

$$\left. \left. + Y^2 (1+\sigma')^2 (1-2\sigma')\phi_{\perp} \right] \right\}$$

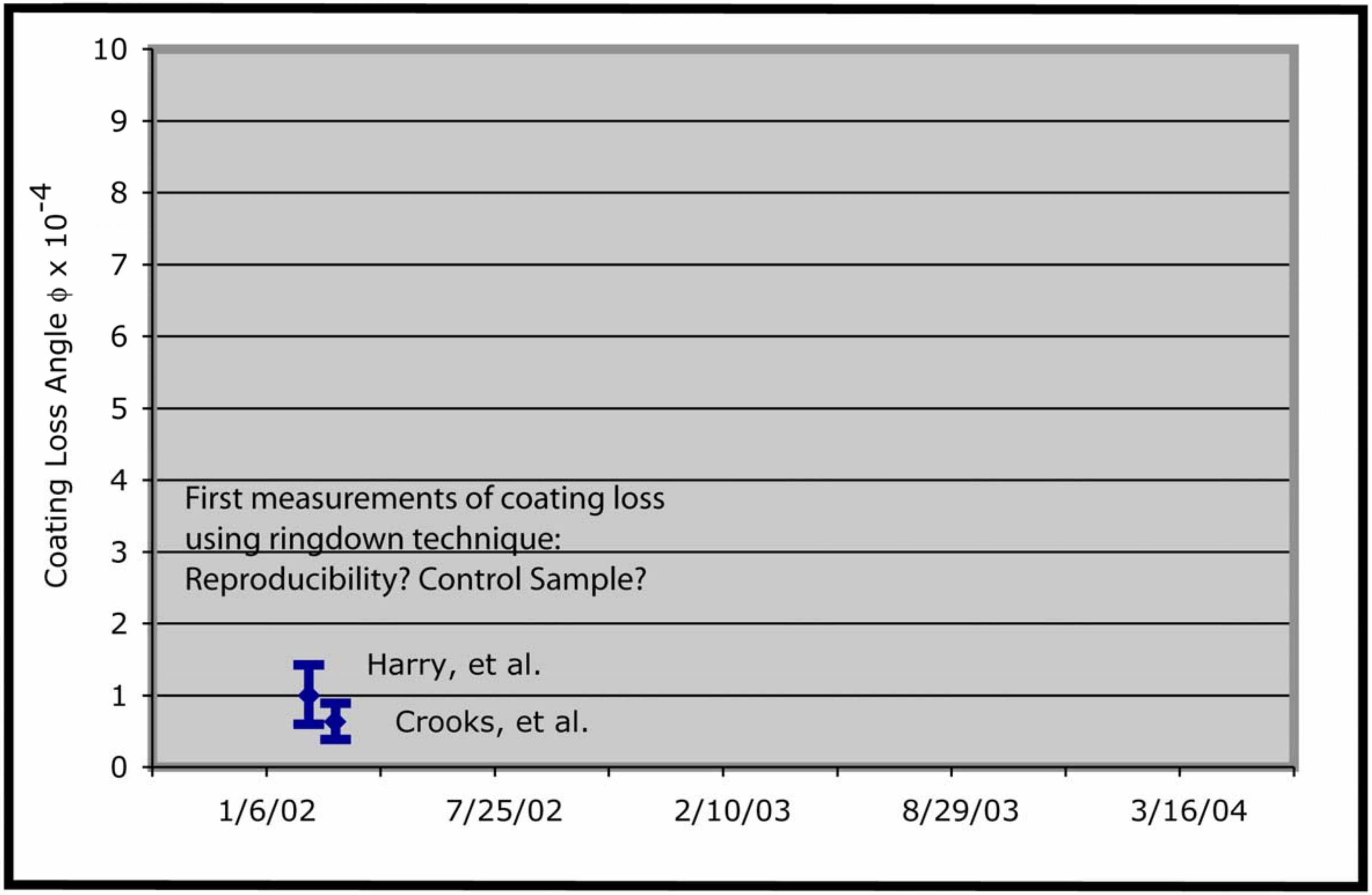
- One approximation: Neglect Poisson's ratio. Expect loss of accuracy of ~30%.

$$S_x(f) \approx \frac{2k_B T}{\pi^{3/2} f} \frac{1}{wY} \left\{ \frac{1}{\sqrt{\pi}} \frac{d}{w} \left(\frac{Y'}{Y}\phi_{\|} + \frac{Y}{Y'}\phi_{\perp} \right) \right\}$$

- Another approximation: Neglect anisotropy in coating, different parameters from substrate. Very simple formula, but how accurate?

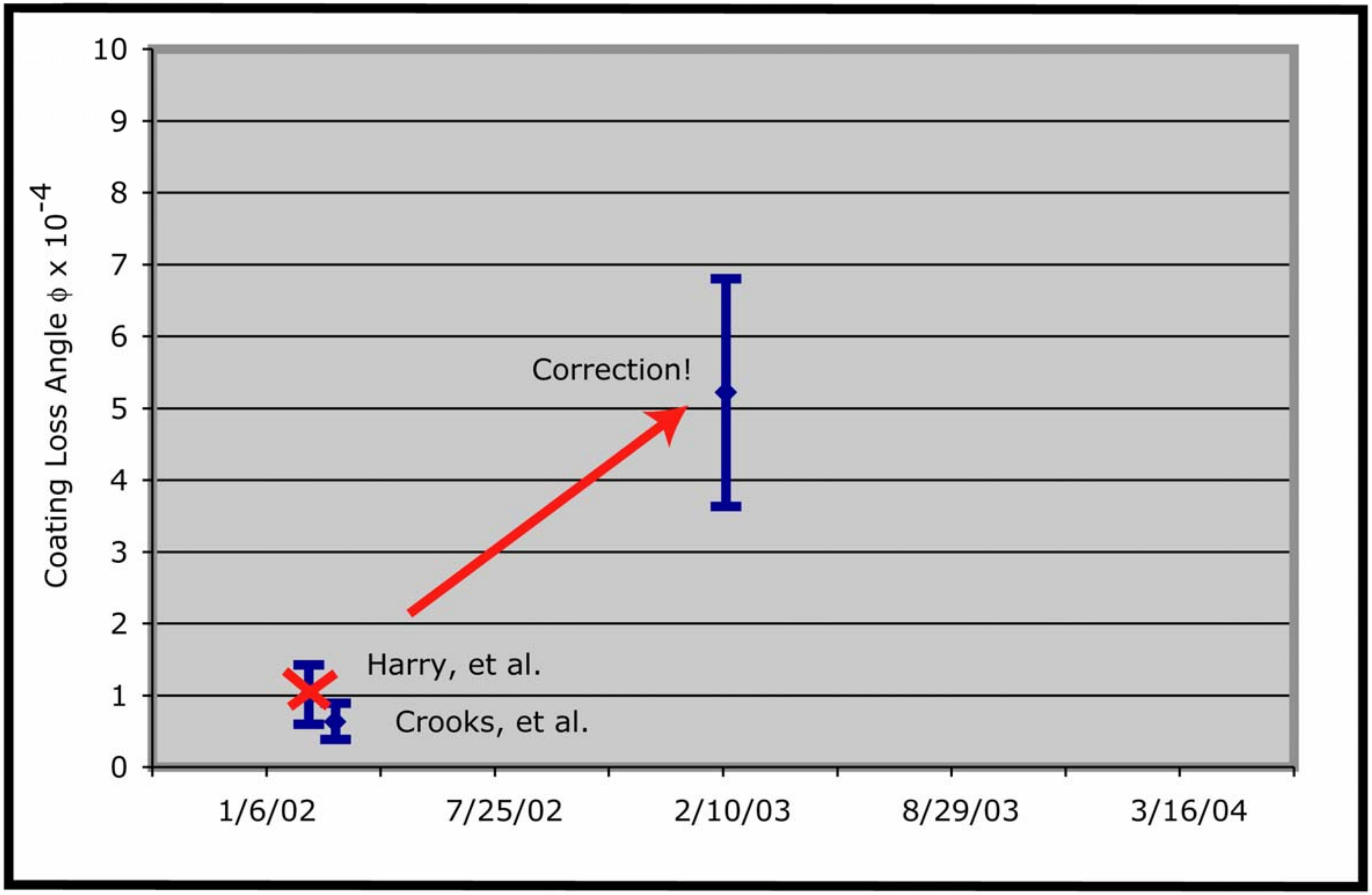
$$S_x(f) \approx \frac{2k_B T}{\pi^{3/2} f} \frac{(1-\sigma^2)}{wY} \left\{ \frac{2}{\sqrt{\pi}} \frac{d}{w} \left(\frac{1-2\sigma}{1-\sigma} \right) \phi \right\}$$

What is the coating loss?



LIGO

What is the coating loss?



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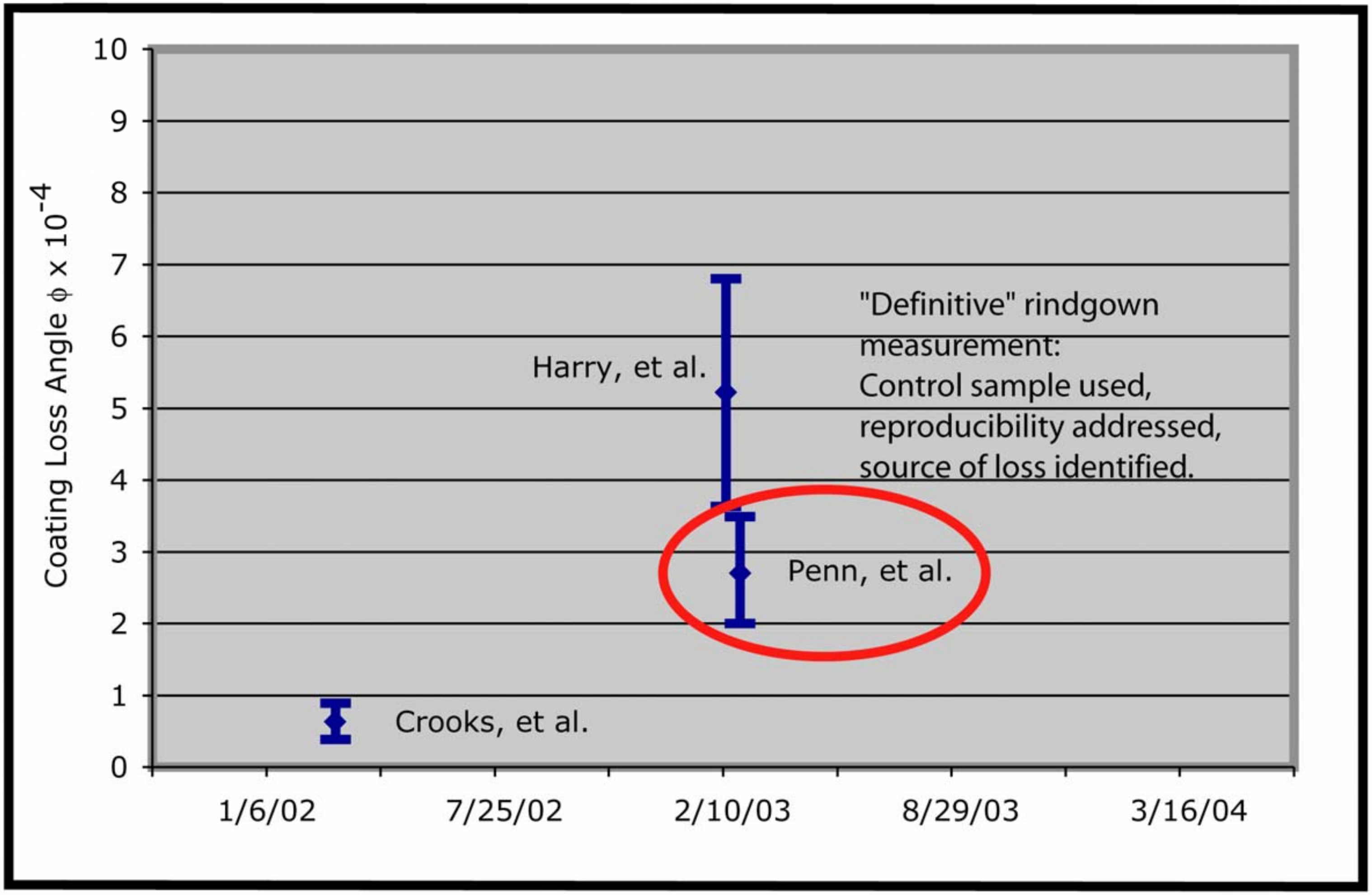

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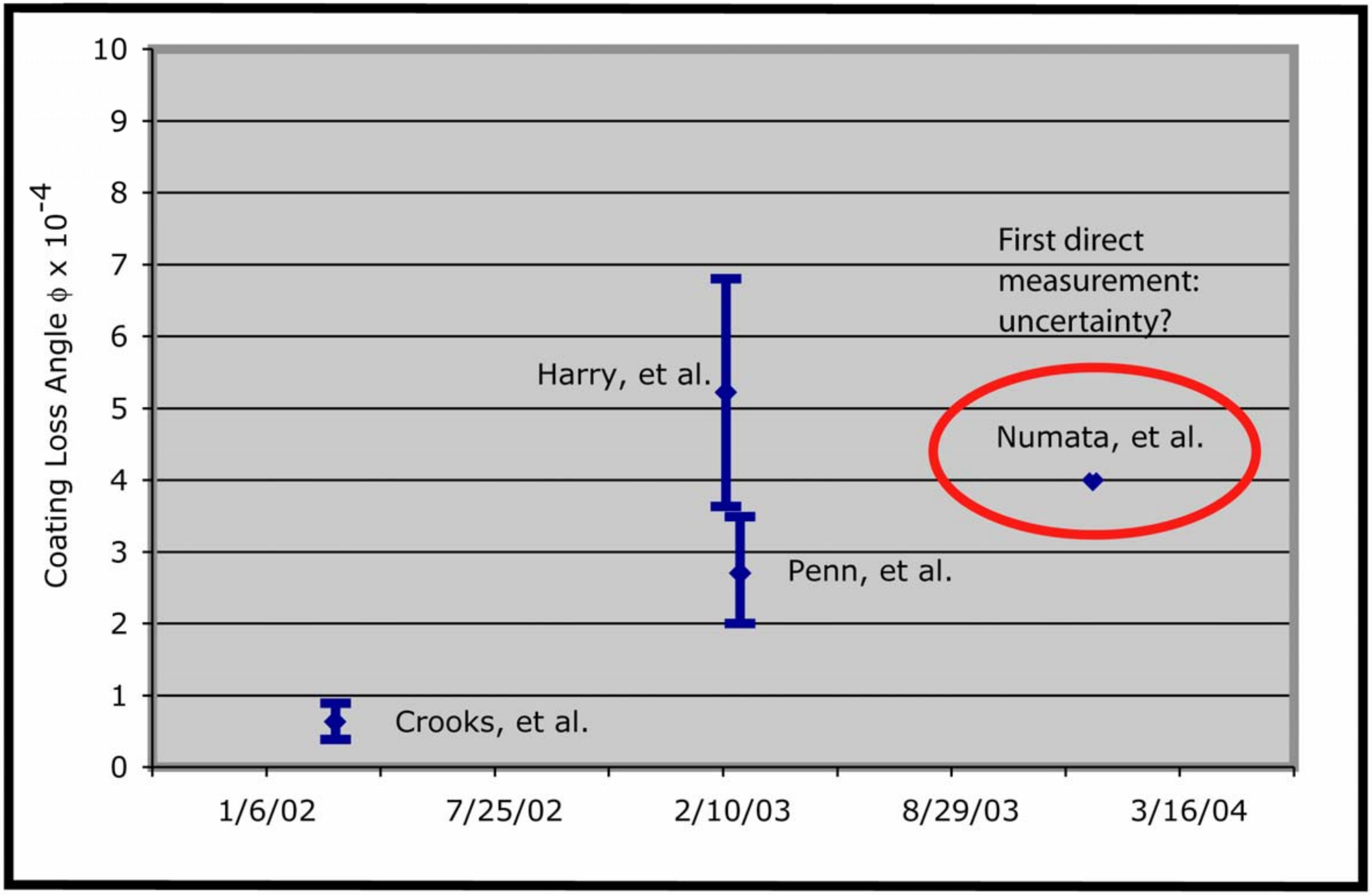
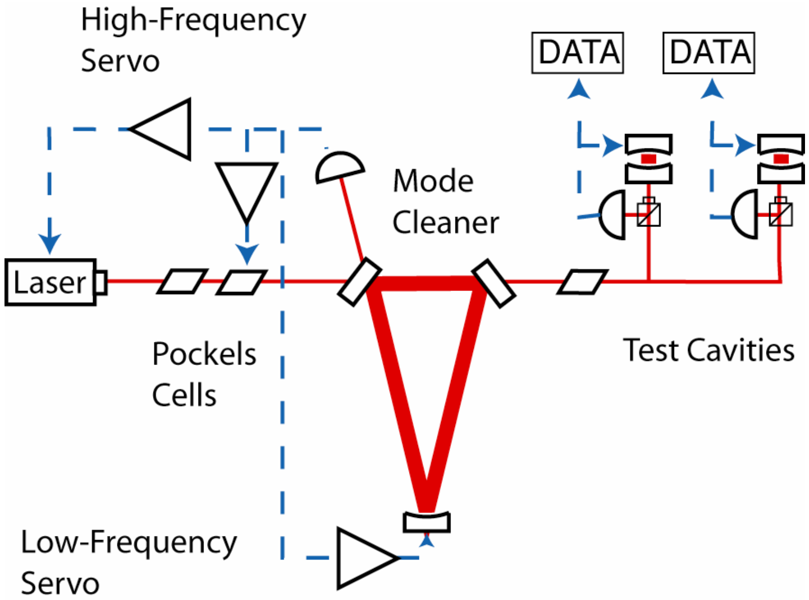
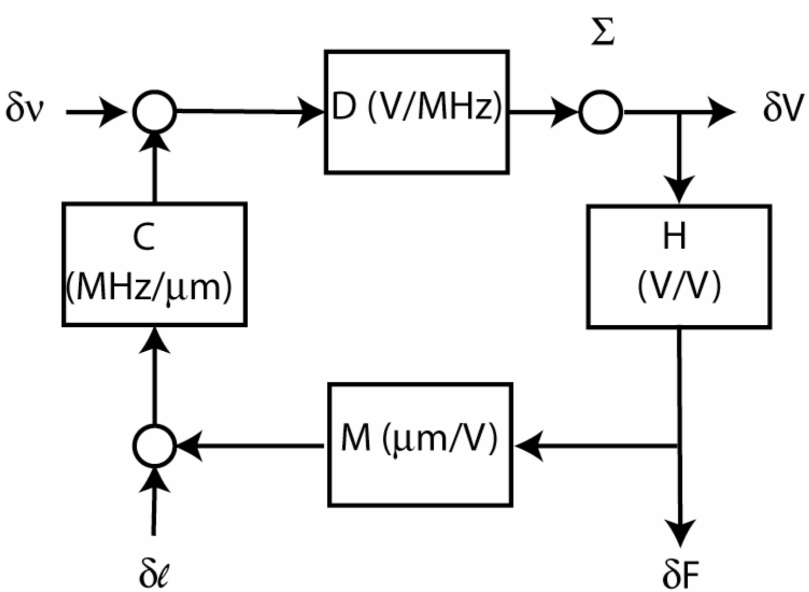


Thermal Noise Interferometer (TNI): Direct Measurement of Mirror Thermal Noise



- Short arm cavities, long mode cleaner (frequency reference) reduce laser frequency noise, relative to test cavity length noise.
- Measurement made as relevant to LIGO, AdLIGO as possible.
- Want to measure thermal noise at as low a level as possible in a small interferometer.
  - Low-mechanical-loss substrates: Fused Silica, Sapphire
  - Silica-Tantala coatings
  - Largest practical spot size

TNI Calibration



- Extract length noise from error signal

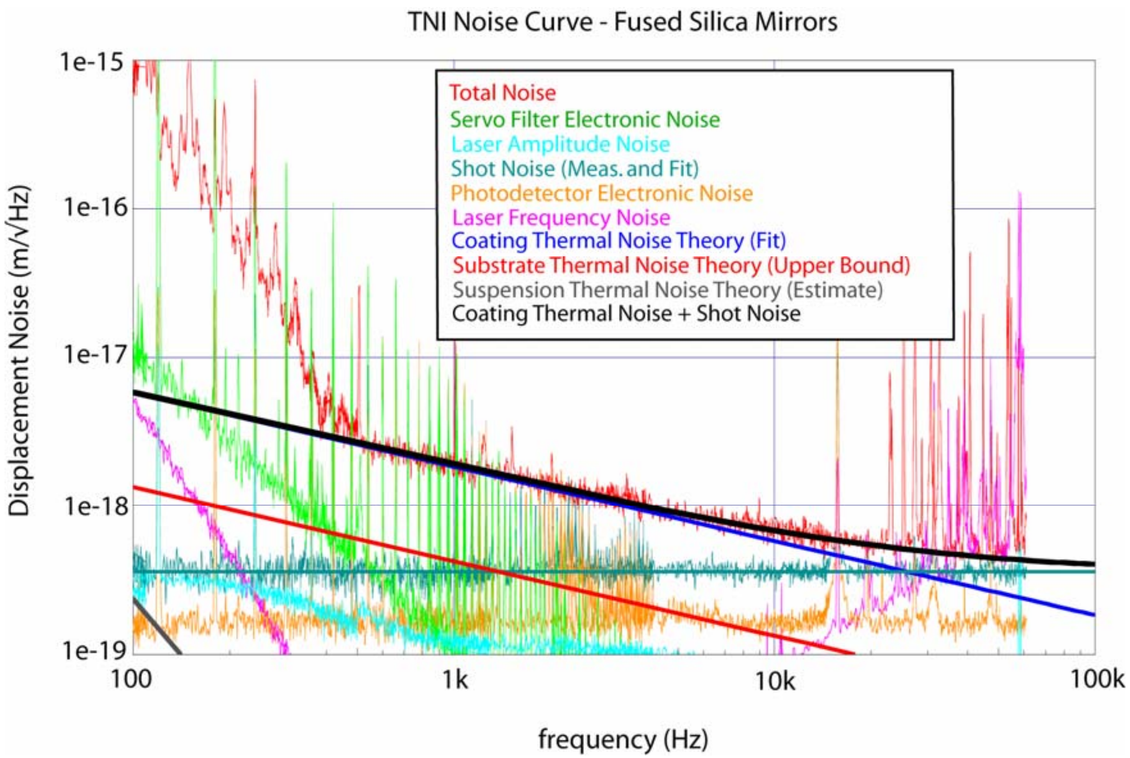
$$\delta \bullet = \frac{1 + DHMC}{DC} \delta V$$

- Must know each transfer function accurately!
- Electronic transfer function *H* specified by design, verified by direct measurement.
- Conversion factor *C*

$$C = \frac{\nu}{L}$$

- Discriminant *D* and mirror response *M* each measured two different ways.
- Additional tests localize noise within the test cavities.
  - Scaling with laser power
  - Scaling with modulation depth

TNI direct measurement of coating thermal noise



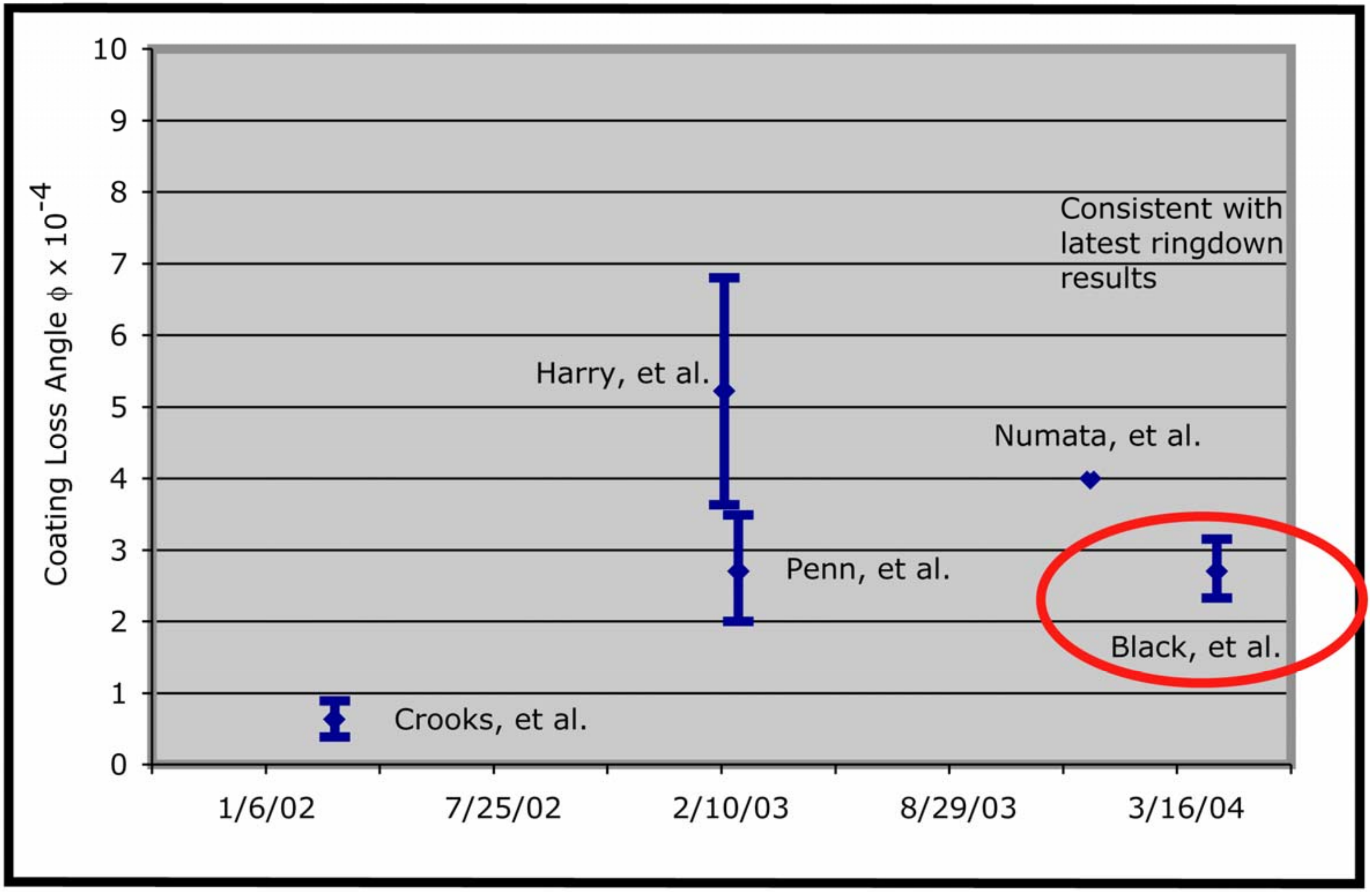
- Silica-tantala coatings on fused silica substrates
- Multiple calibrations performed.
- Noise source (in thermal noise band) localized inside cavities
- Assuming isotropic model, coating loss angle agrees with Penn, et al. ringdown measurement:

$$\phi = (2.7 \pm 0.3) \times 10^{-4}$$

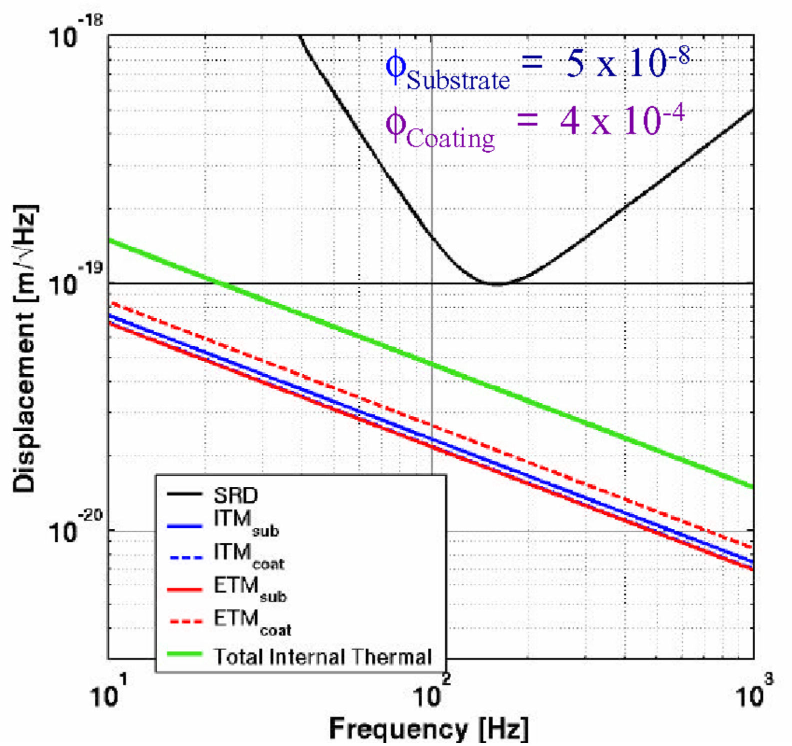
- Assuming anisotropic model,

$$\phi_{\parallel} = 2.7 \times 10^{-4} \Rightarrow \phi_{\perp} = (0 \pm 0.6) \times 10^{-4}$$

What is the coating loss?



What does this mean for the ultimate astrophysical reach of LIGO-I?



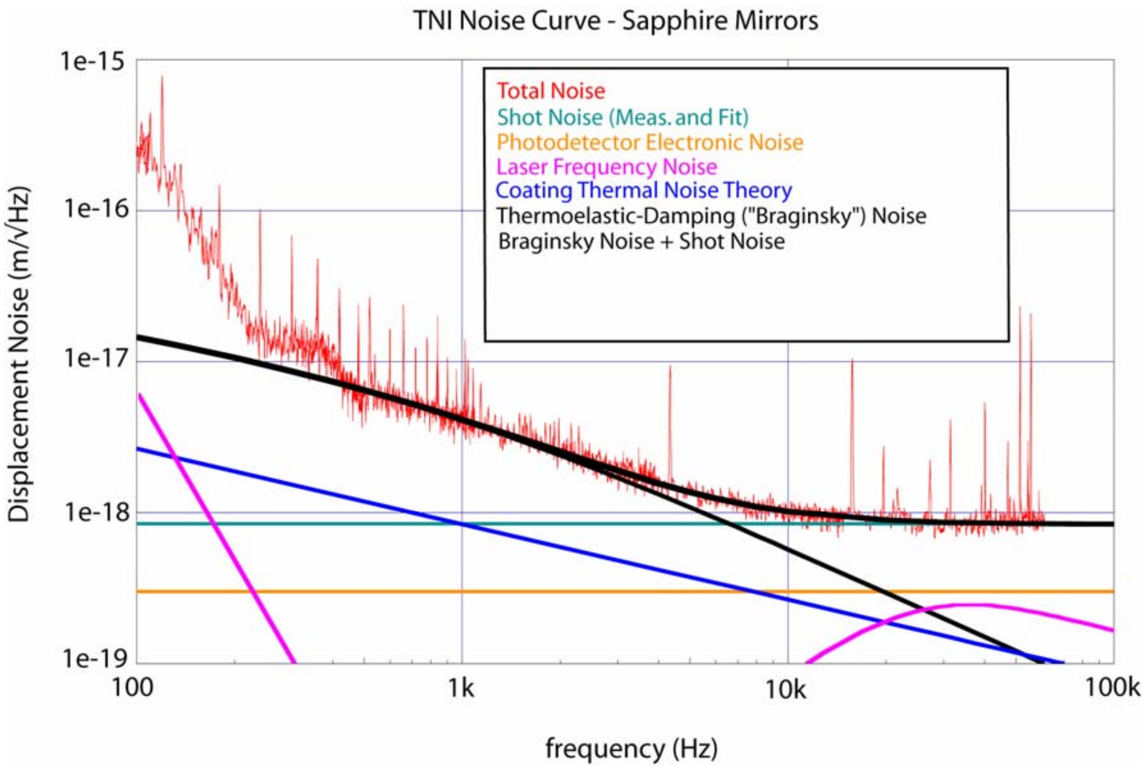
–Figure credit: Rana Adhikari

- Not much that we didn't already know.
- Coating thermal noise does dominate at lowest levels, and we expect it to be ~2x lower than the original SRD estimate, but…
- Substrate thermal noise is close behind! Change of coating phi of 4e-4 to 2.7e-4 doesn't change the total noise level very much.
- In any case, LIGO-I's mirrors are already installed. Can't do much about the noise floor now.
- However…

What does this mean for Advanced LIGO?

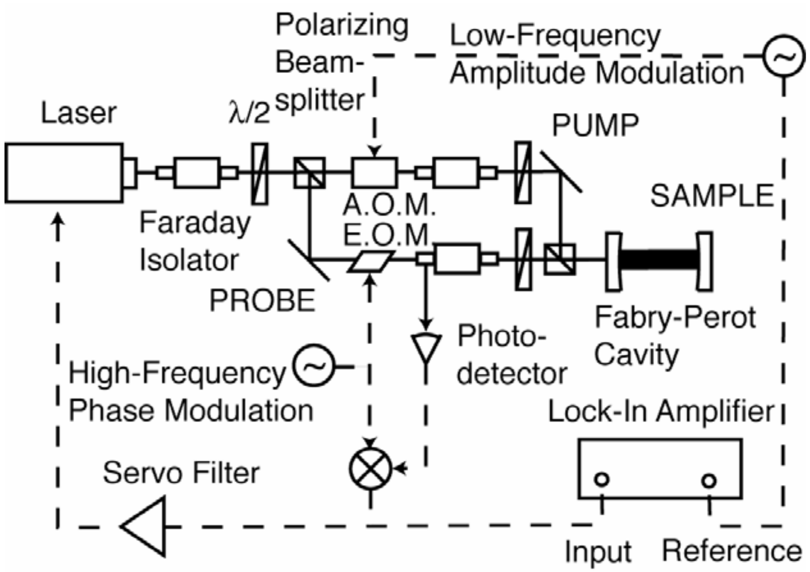
- Need lower-loss coatings for AdLIGO than Silica-Tantala.
- Losses in candidate coatings can be measured via ringdown method, final candidate verified by direct measurement in the TNI.
- Consistency between ringdown results and direct measurement validates our process of measuring the coating loss, development program for AdLIGO coatings.

Sapphire



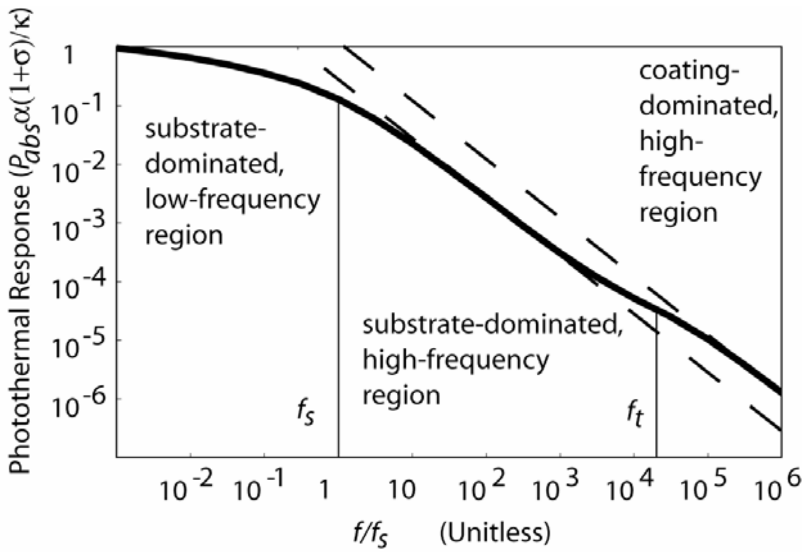
- Noise floor in Sapphire dominated by Substrate Thermoelastic noise.
- Parameters
  - α = 2.7e-6 K^{-1}
  - κ = 44 W/mK
- Numerical error in existing theory initially gave unexpected parameters
  - Cerdonio, et al., Phys. Rev. D 63 (8), 082003 (2001)
- Braginsky model validated in Sapphire - First measurement in AdLIGO candidate substrate material
- But what is the coating thermal noise on a Sapphire substrate?

Photothermal experiment:
Measuring coating thermomechanical properties



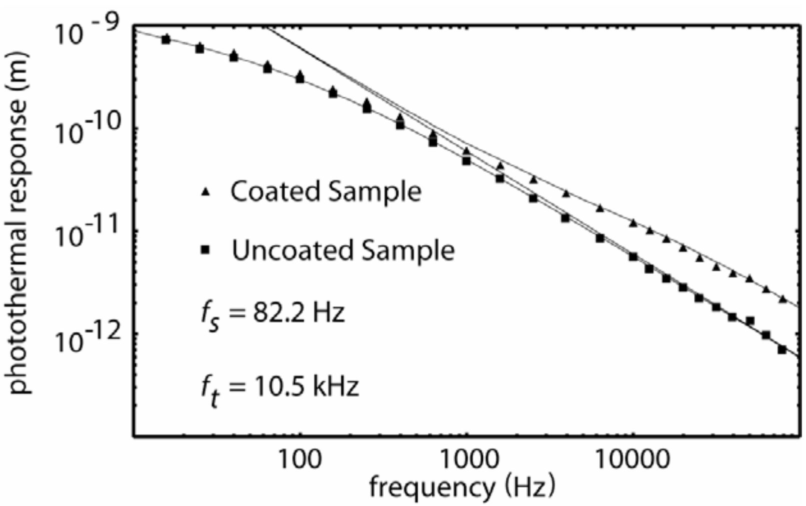
- Tabletop interferometer measures thermomechanical properties of mirrors in a Fabry-Perot cavity.
- Two cross-polarized beams at the same frequency resonate inside the cavity.
  - One, the Pump beam, drives the photothermal response in the cavity
  - The other, the Probe beam, measures the resulting length change in the cavity

Photothermal experiment: Interpreting the photothermal response



- Three distinct regimes:
- Low frequency - Thermal diffusion wavelength (penetration depth) greater than laser spot size, coating thickness
  - In this case, the response is dominated by the substrate, with a characteristic frequency dependence.
- Medium frequency - Thermal diffusion wavelength smaller than laser spot size, but still greater than coating thickness
  - Here, the response is still dominated by the substrate, but the frequency dependence is different from the low-frequency case.
  - Substrate thermal conductivity determines transition frequency.
- High frequency - Coating dominates
  - Transition frequency gives coating thermal conductivity.
  - High-frequency response gives coating thermal expansion coefficient.

Photothermal experiment: first results



- Silica-Tantala coating on Sapphire substrate
- Observe expected behavior
- Simple theory interpolating between asymptotic regions fits data reasonably well.
- Can extract thermal expansion coefficients, conductivities from the data, but…
- Theory of the photothermal response is not yet well enough developed to specify these parameters to better than ~ factor of 2.
- Complimentary measurement:
  - Ringdown measurement as a function of frequency, including thermoelastic loss
  - Crooks, Cagnoli, Fejer, et al., Class. Quantum Grav. 21, S1059-S1065 (2004)

Conclusions

- The astrophysical reach of an interferometric gravitational wave detector depends strongly on its strain sensitivity. Small improvements in sensitivity are expected to produce big gains in event rate.
- Thermal noise is expected to limit the strain sensitivity of both LIGO and AdLIGO at the lowest levels, thus setting the ultimate astrophysical reach.
- Because the event rate depends so strongly on the strain sensitivity, it behooves us to understand, with confidence and precision, the thermal noise that limits the performance of our detectors.
- Coating thermal noise affects LIGO-I, but not much. It affects AdLIGO much more, and we need to find a better coating than we now have for that detector.
- Our process of measuring the coating loss via ringdown, then predicting the noise floor based on that measurement, appears to be solid.
- Our prediction for thermoelastic noise (Braginsky noise) in Sapphire substrates appears to be accurate.
- Our understanding of thermoelastic noise in coatings is in development.