Multiresolution time-frequency searches for gravitational wave bursts

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Presentation Outline

- Multiresolution analysis
- The discrete wavelet transform
- The discrete Q transform
- Gaussian white noise statistics
- Selection of events
- Linear predictor error filters
- Analysis Pipeline
- Simulated gravitational wave data
- Burst detection efficiencies
- Black hole mergers

Multiresolution Analysis

- Optimal time-frequency signal to noise ratio

$$\rho^2 = \int_0^\infty \frac{2|\tilde{h}(f)|^2}{S_h(f)}\, df \simeq \frac{h^2_{\text{rss}}}{S_h(f_c)}$$

- Only obtained if measurement pixel matches signal
  - Maximal measurement of burst "energy"

$$h^2_{\text{rss}} = \int_{-\infty}^{+\infty} |h(t)|^2\, dt$$

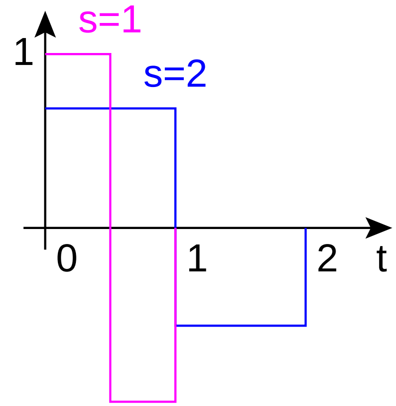
  - Minimize background energy
- Bursts are signals with $Q \lesssim 10$
- Tile the time frequency plane to maximize the detectability of bursts with a particular Q

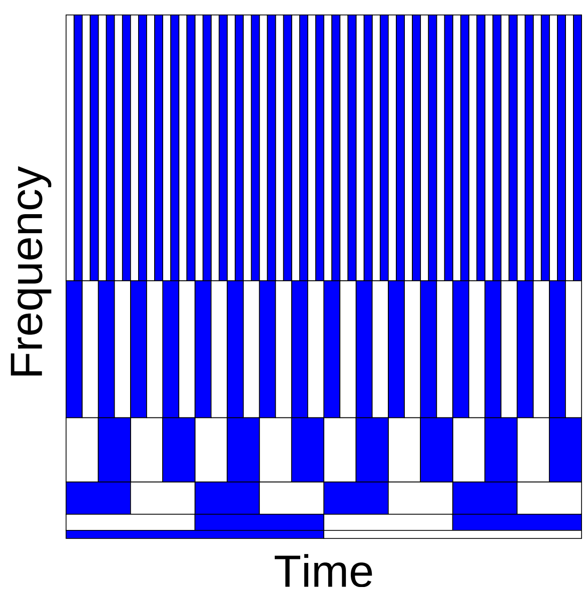
Discrete Dyadic Wavelet Transform

Project $x[n]$ onto time-shifted and scaled wavelets.

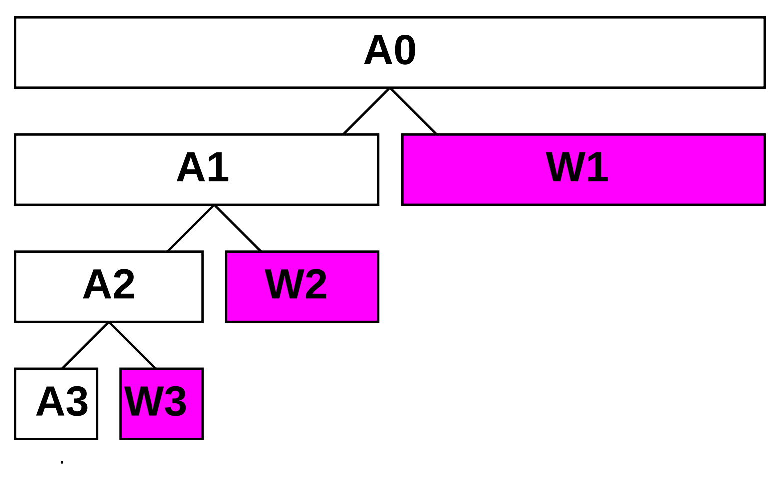
$$X_W[m,s] = \sum_{n=0}^{N-1} x[n] \frac{1}{\sqrt{2^s}} \psi\left(\frac{(n-m)T}{2^s}\right)$$

Haar Wavelet



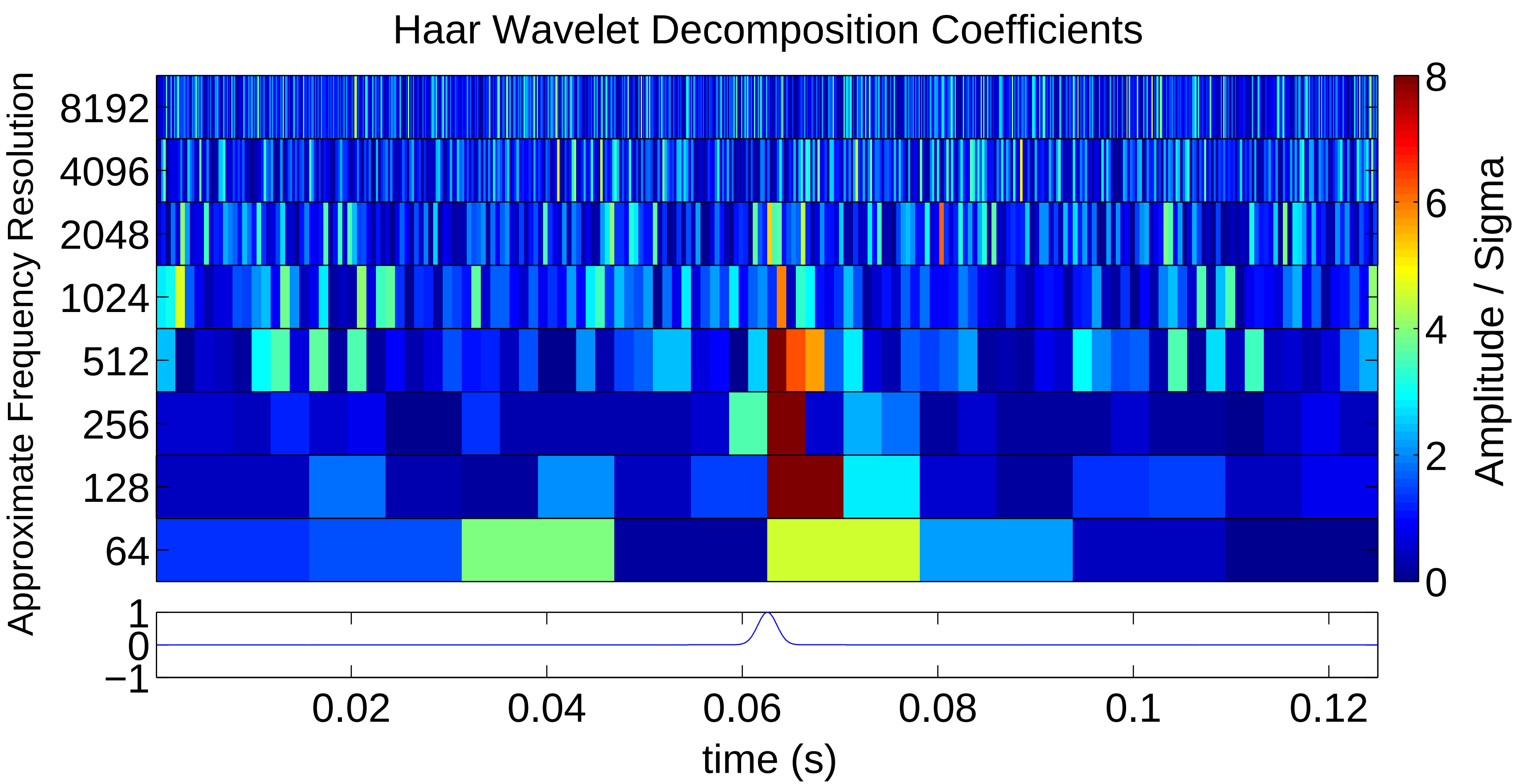


Dyadic Wavelet Decomposition Tree



DWT Example

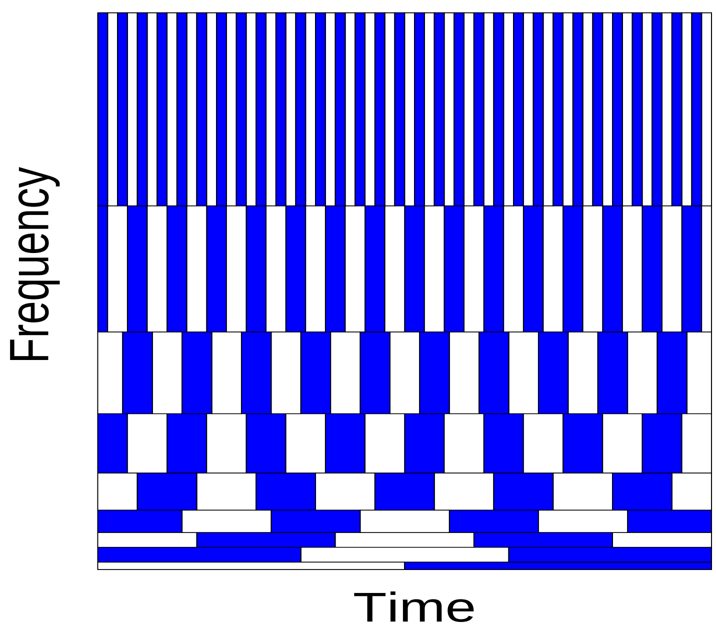
Discrete Haar wavelet decomposition for simulated burst

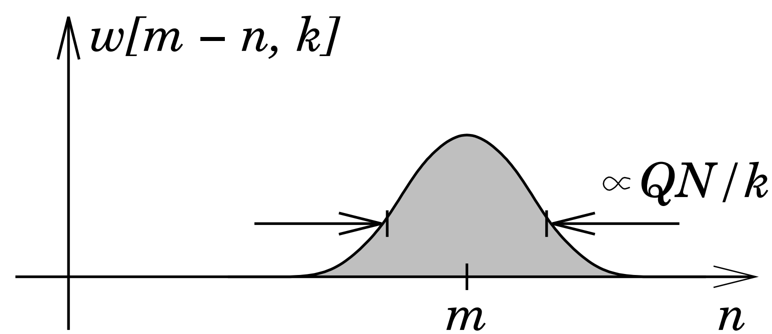


Discrete Q Transform

Project $x[n]$ onto time-shifted windowed sinusoids, whose widths are inversely proportional to their center frequencies.

$$X_Q[m,k] = \sum_{n=0}^{N-1} x[n] e^{-i2\pi nk/N} w[m-n,k]$$





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FAST Q Transform

Efficient computation is possible in frequency domain.

$$X_Q[m,k] = \sum_{l=0}^{N-1} \tilde{X}[l+k]\tilde{W}[l,k]e^{-i2\pi ml/N}$$

- One time FFT of signal: $\tilde{X}[l]$
- Frequency domain window: $\tilde{W}[l]$
- Inverse FFT for each frequency bin
  - Only for frequency bins of interest
  - Only for samples in proximity of window
  - Length determines overlap in time

Fast dyadic wavelet transform is also possible.

DQT Energy Conservation

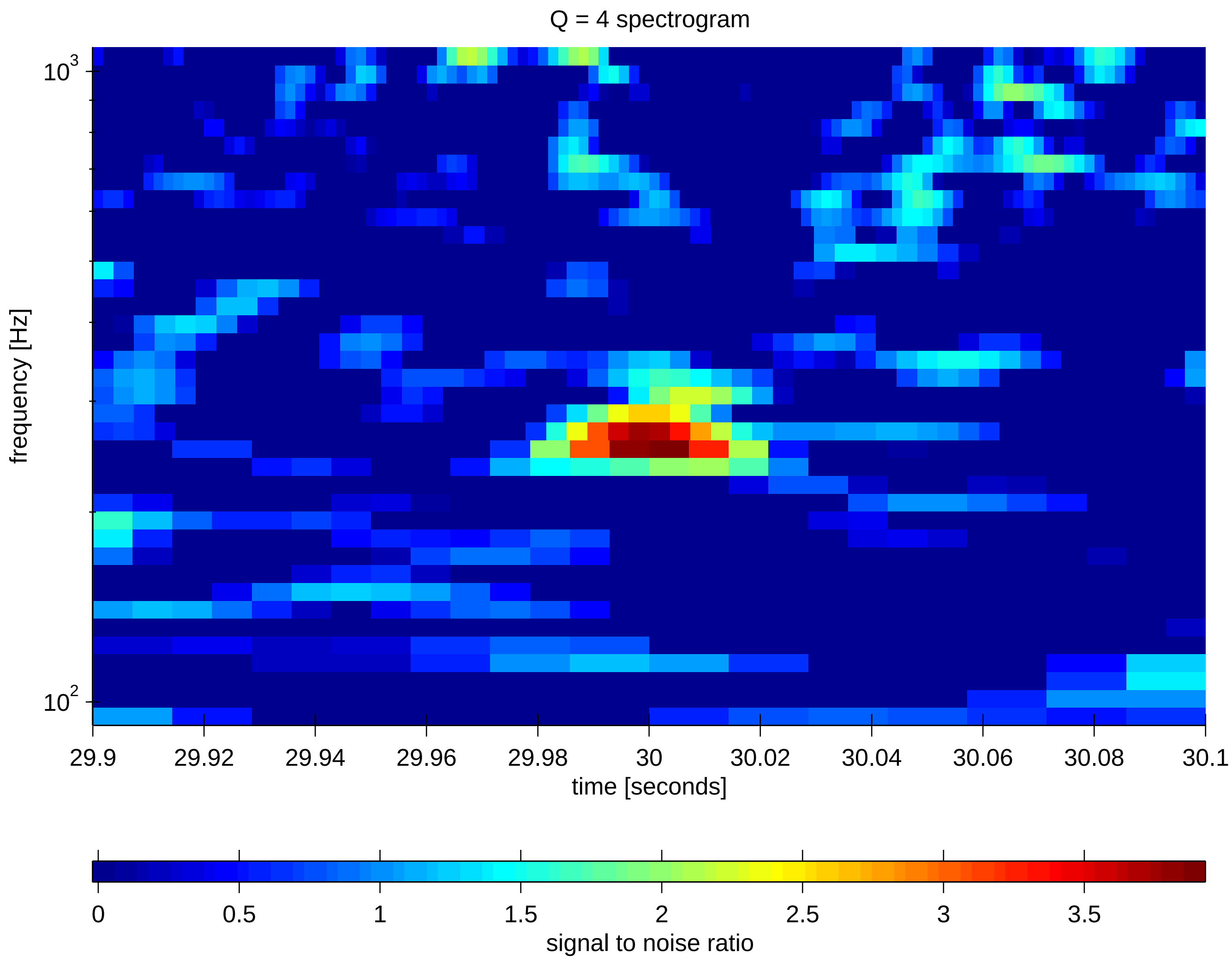
The window normalization is chosen to obey a generalized Parseval's theorem.

$$\frac{f_s}{N^2} \sum_{m=0}^{N-1} \sum_{k=0}^{N-1} \left| X_Q[m,k] \right|^2 = \frac{1}{N} \sum_{n=0}^{N-1} \left| x[n] \right|^2 = \sigma_x^2$$

The square root of the reported pixel energy yields the sum of the background noise amplitude spectral density and the signal root sum square in units of $\text{Hz}^{-1/2}$.

DQT Example

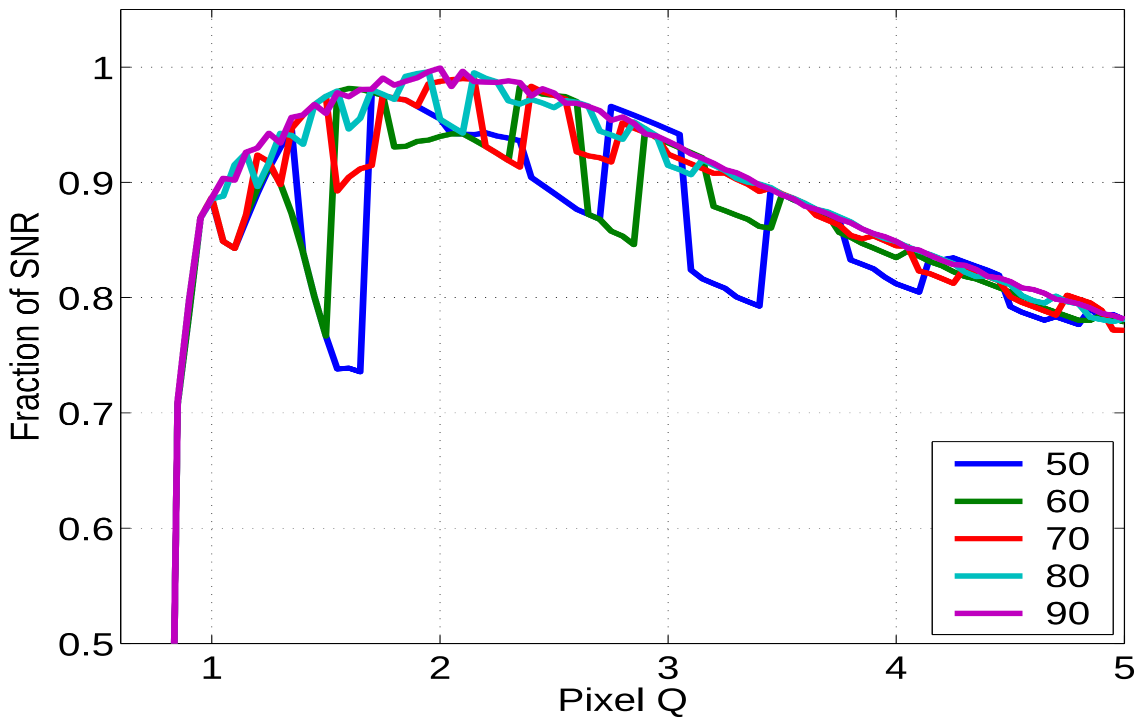
Simulated sine-gaussian gravitational wave burst



SNR Loss due to Pixel Mistmatch

Mismatch between a signal and the nearest time frequency pixel will result in a loss in measured signal to noise ratio.

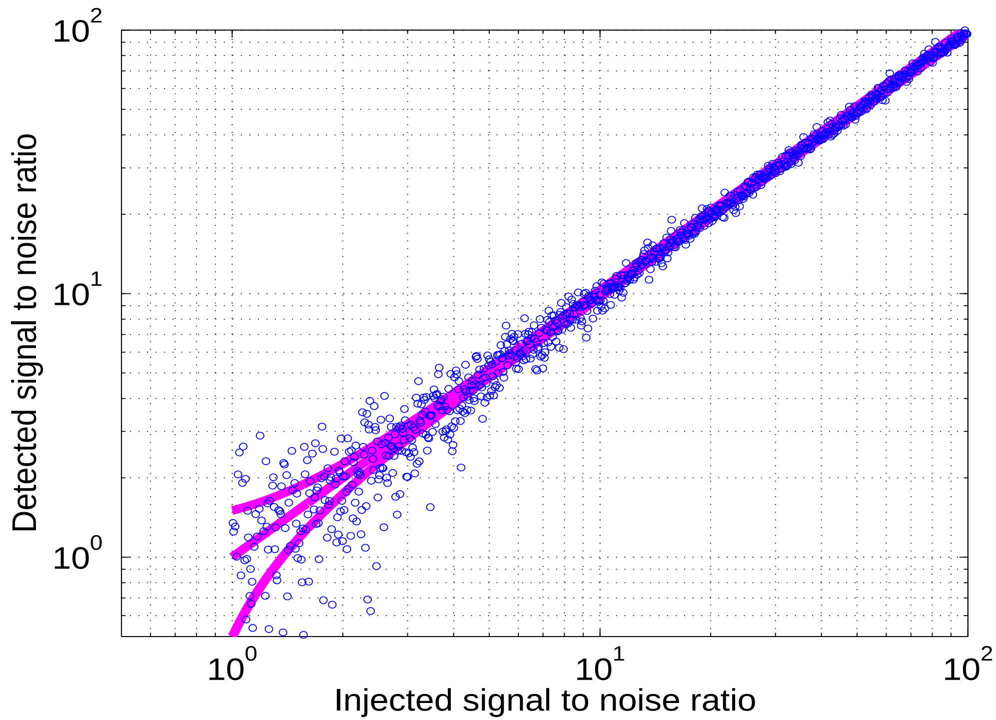
(Sine-Gaussian burst and white Gaussian noise)



This is similar to the problem of selecting discrete template banks in a matched filtering analysis.

Optimal SNR Measurement Accuracy

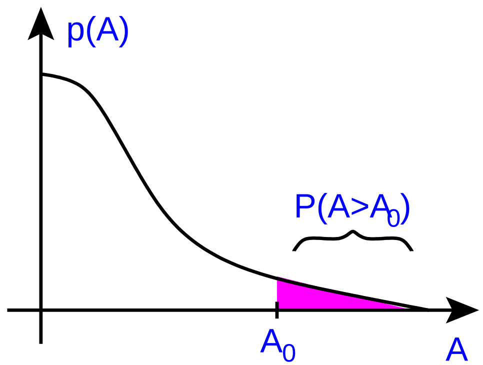
Optimal pixel match allows accurate measurement (Sine-Gaussian burst and white Gaussian noise)



Error due to statistical fluctuation in background noise and error in mean background energy measurement.

White Noise Statistics

DWT: Gaussian

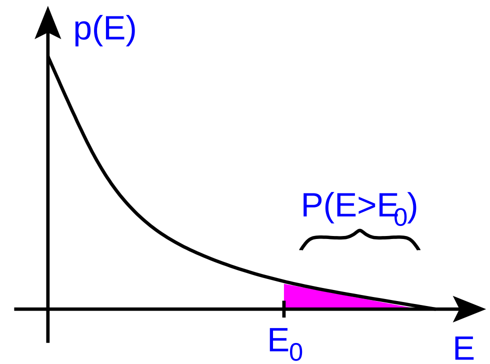


Significance:

$$P(A) = \text{erfc}\left(\frac{A}{\sqrt{2}\,\sigma_A}\right)$$

$$\text{SNR} = \left(\frac{E - \langle E \rangle}{\langle E \rangle}\right)^{1/2}$$

DQT: Exponential



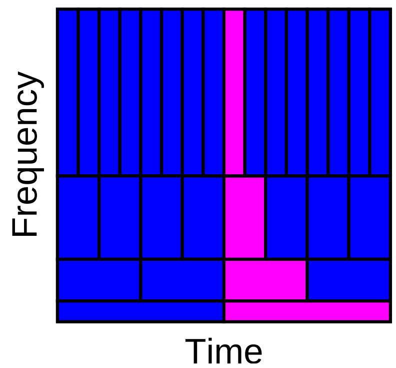
Significance:

$$P(E) = \exp\left(-\frac{E}{\langle E \rangle}\right)$$

$$\text{RSS} = (E - \langle E \rangle)^{1/2}$$

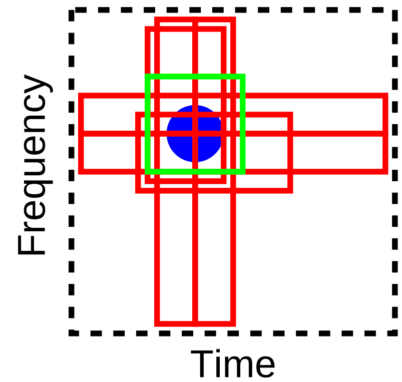
Event Selection

Wavelet Transform



- threshold on pixel significance
- select vertical "chains" of significant pixels

Q Transform



- threshold on pixel significance
- group overlapping pixels
- select most significant pixel in group

Linear Predictor Error Filter (LPEF)

- Linear Prediction: Assume each sample is a linear combination of the previous M samples.

$$\tilde{x}[n] = \sum_{m=1}^{M} c[m]x[n-m]$$

- Prediction Error: We are interested in the unpredictable signal content.

$$e[n] = x[n] - \tilde{x}[n]$$

- Training: Choose $c[m]$ to minimize the mean squared prediction error.

$$\sigma_e^2 = \frac{1}{N}\sum_{n=1}^{N} e[n]^2$$

LPEF Properties

- Linear least squares optimal filter problem
- Robust efficient algorithms exist to train and apply
  - Levinson-Durbin recursion
  - Produces minimum phase FIR filter
  - Frequency domain autocorrelation and filtering
- Zero-phase implementation exists
- Filter order, M, can compensate for features

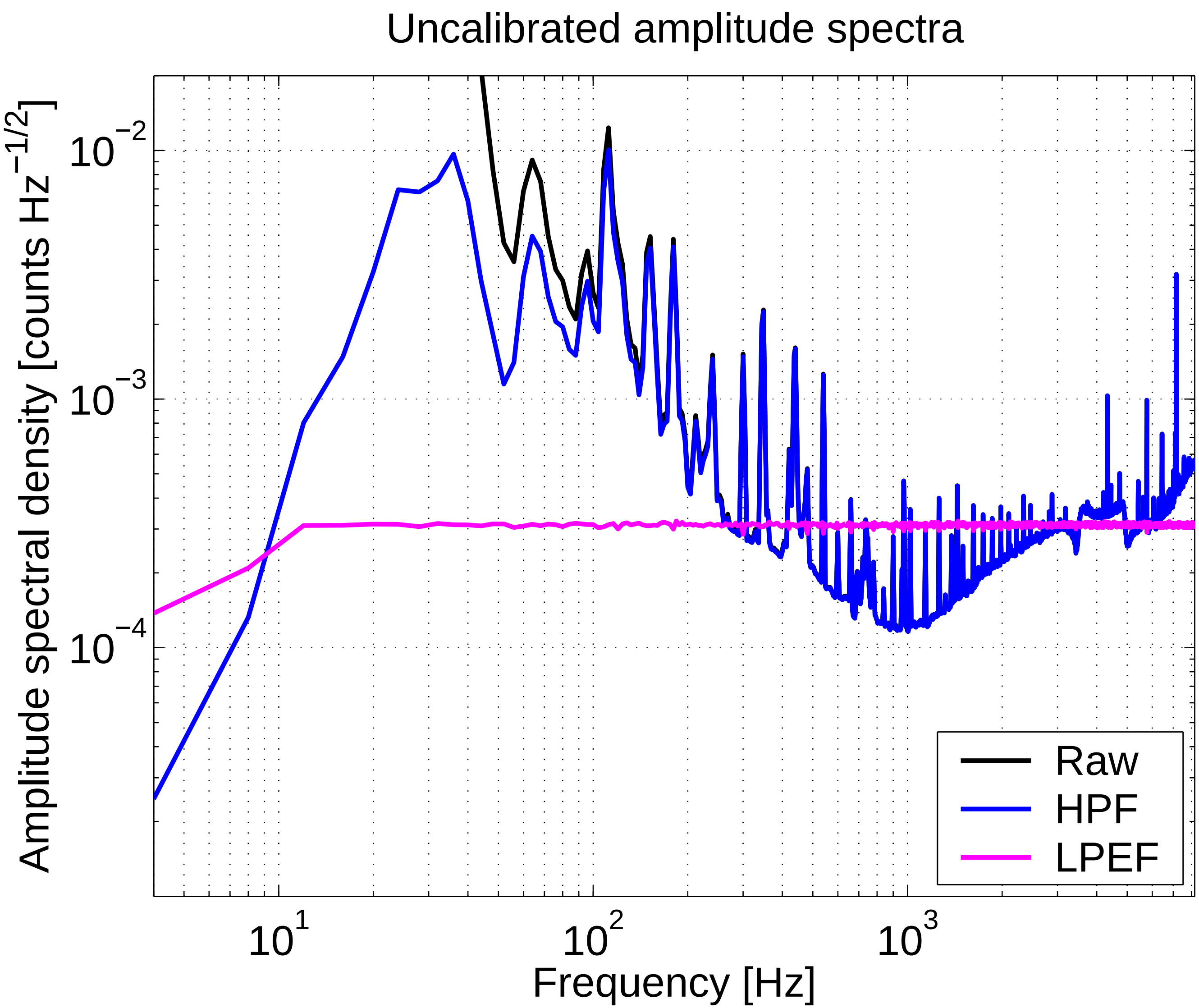
$$\Delta f \gtrsim f_s/M$$

- Training time, T can learn about features

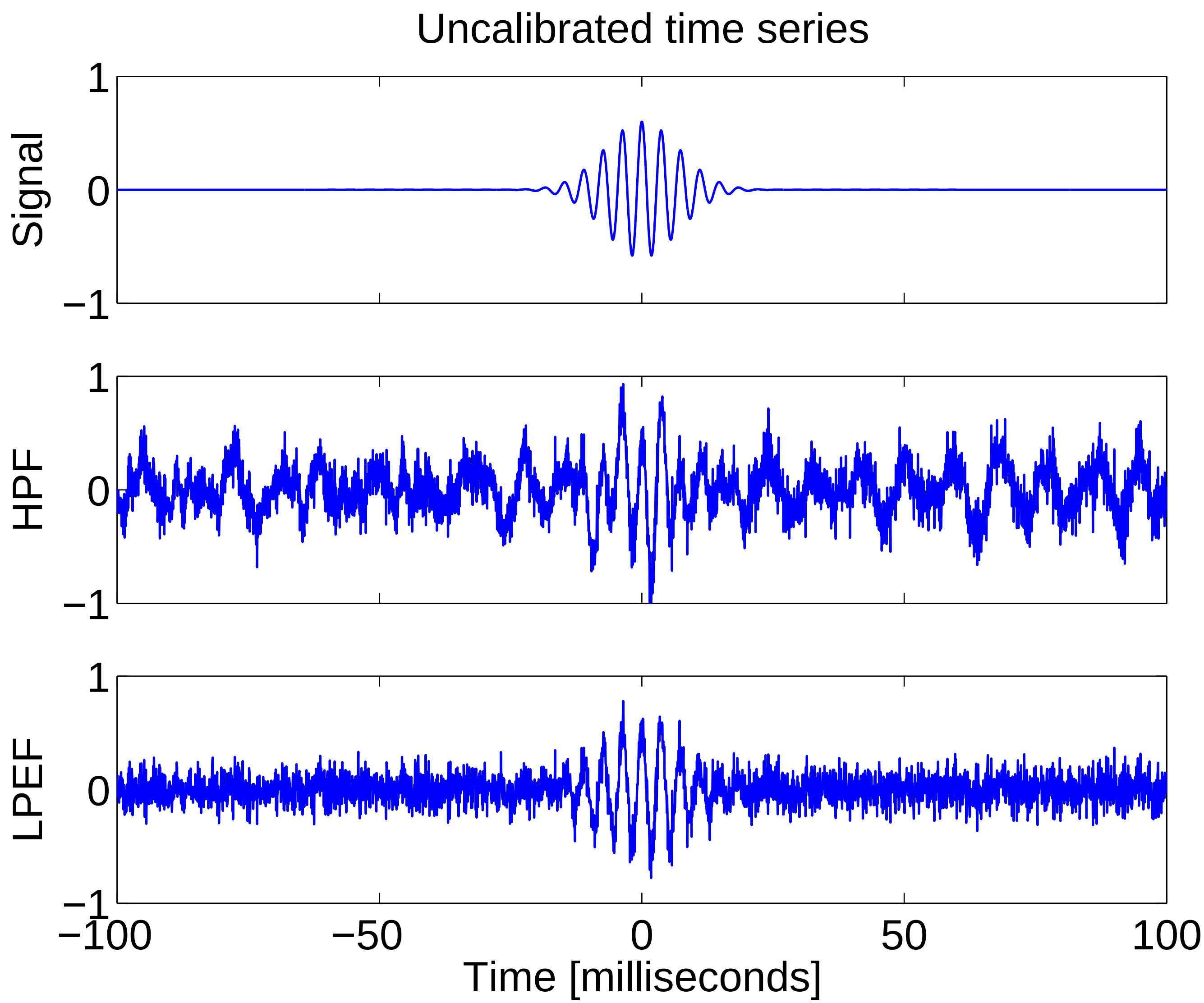
$$\Delta f \gtrsim 1/T$$

- Performance depends upon detector stationarity

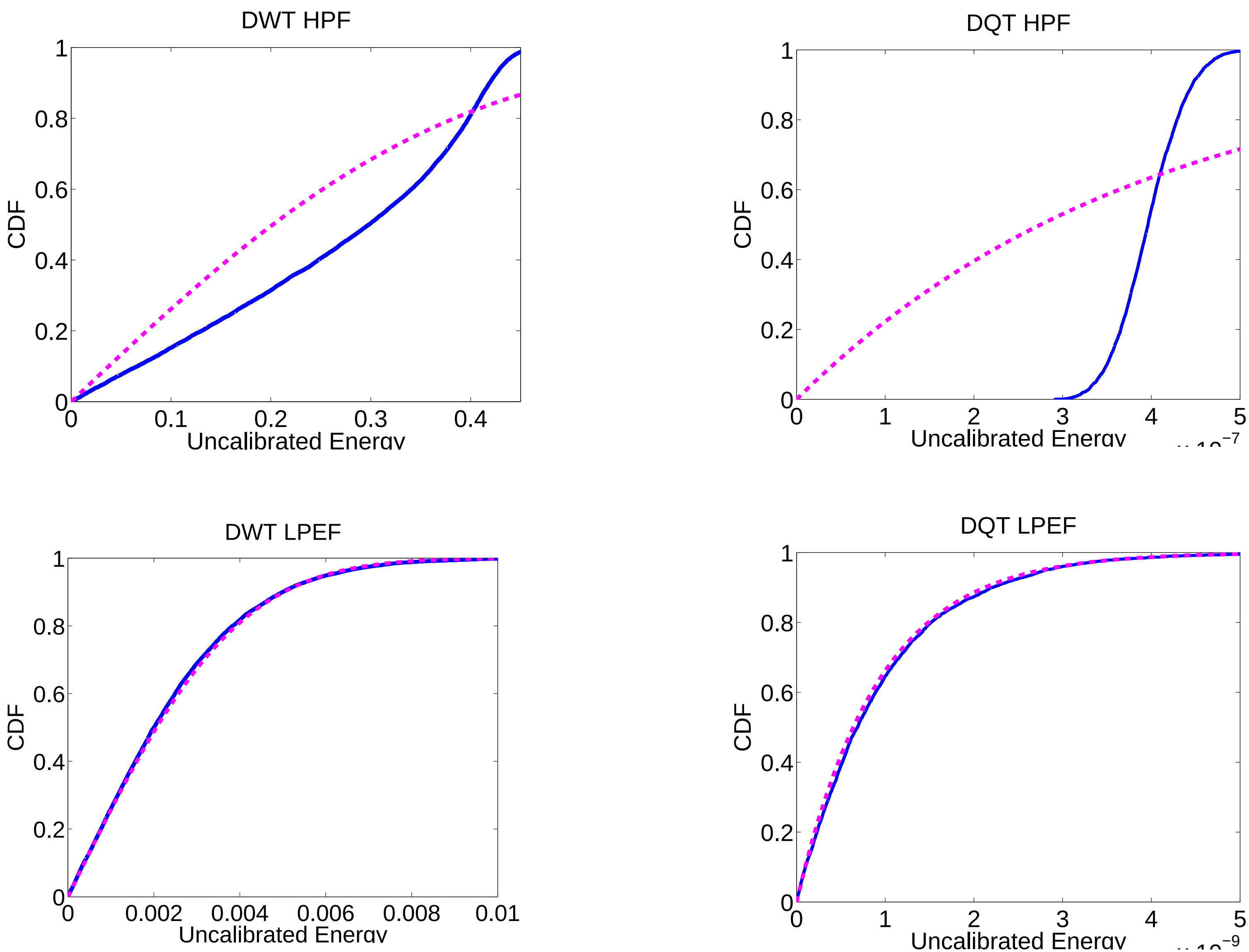
LPEF Example: Spectra



LPEF Example: Time Series



LPEF Example: Statistics



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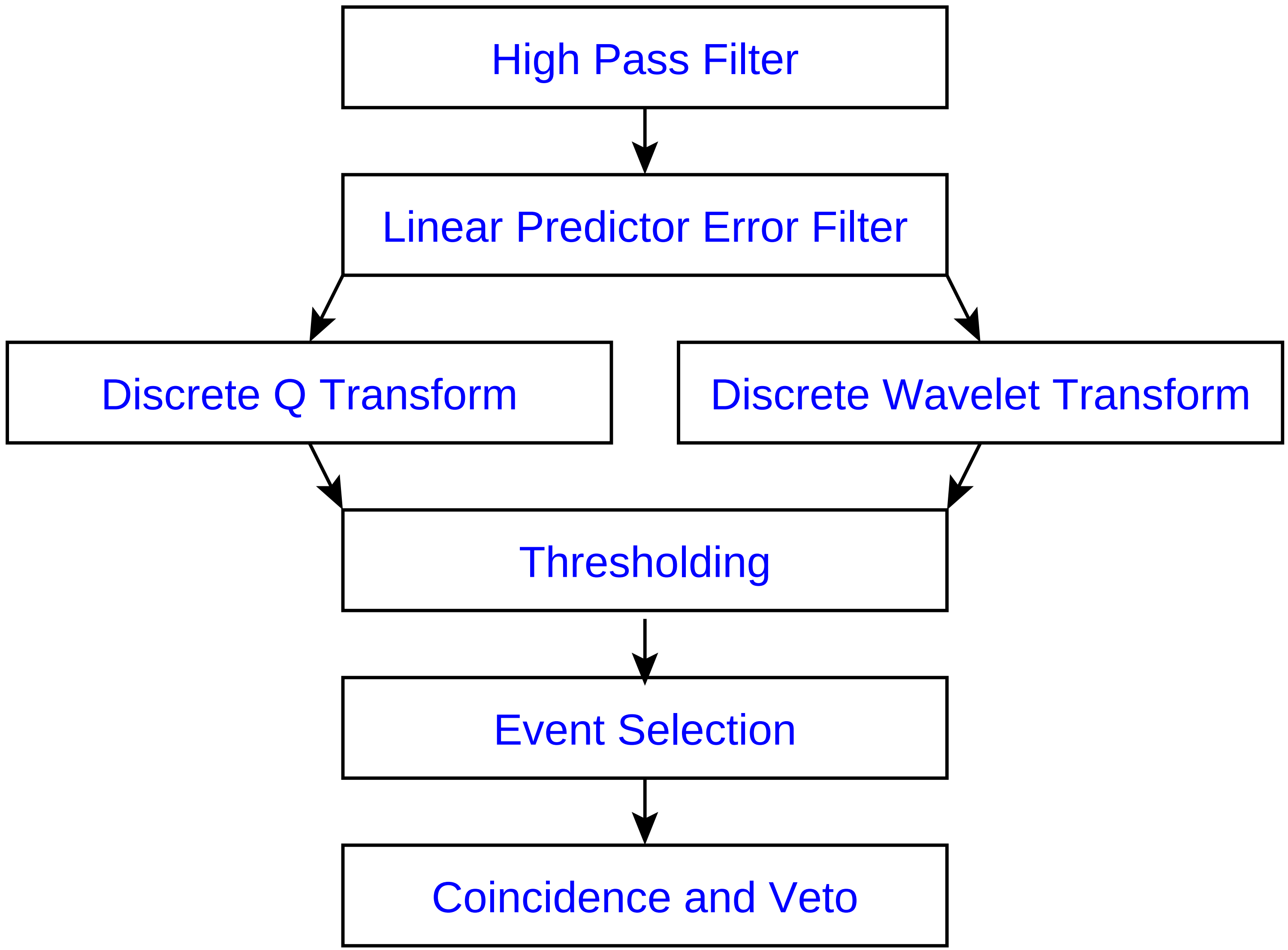
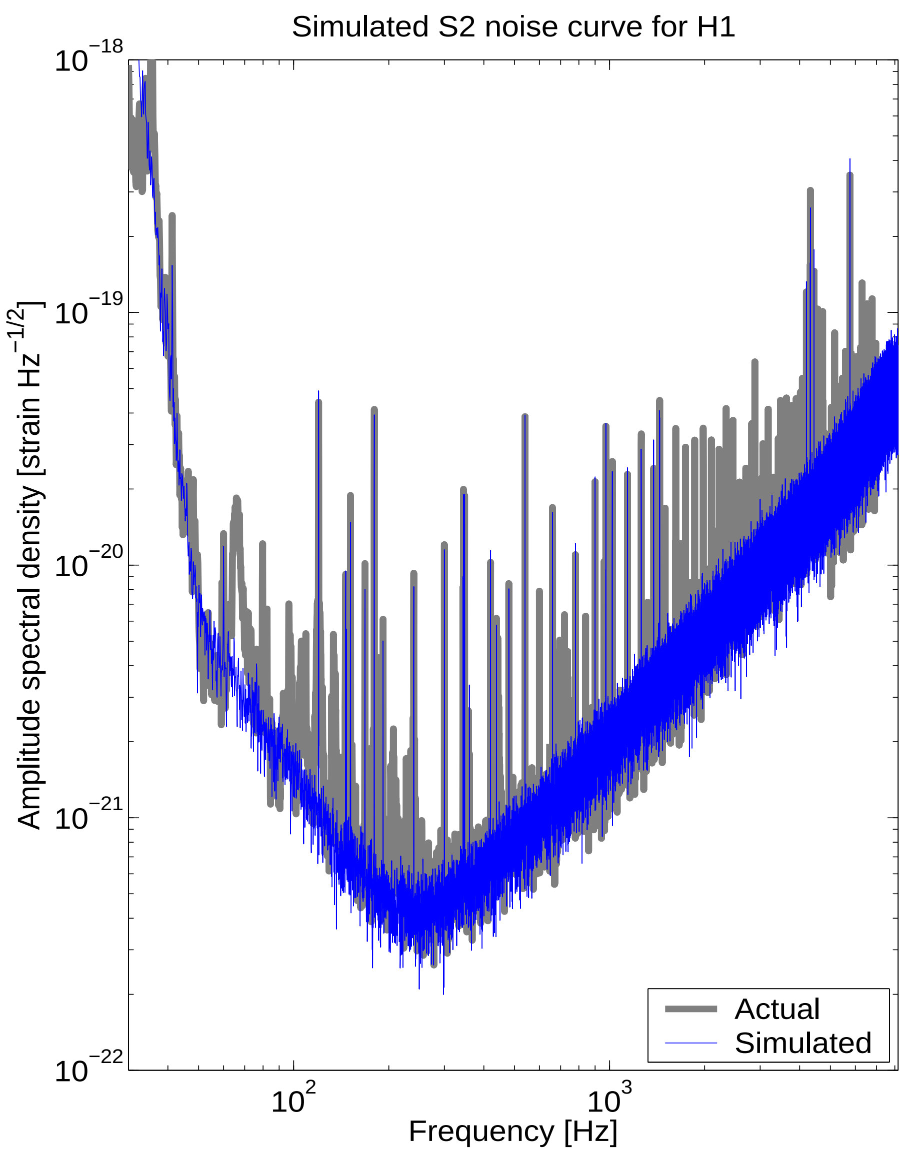
Data Analysis Pipeline



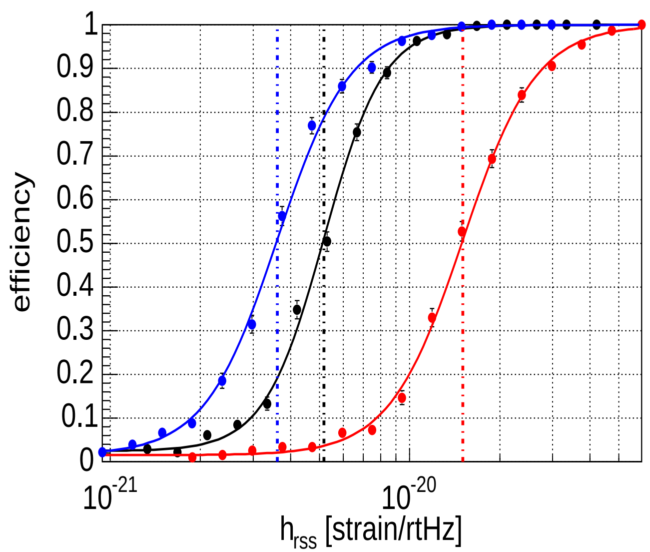
Simulated Gravitational Wave Data

- Simulated H1 noise for second LIGO science run
- Shaped gaussian white noise
- Included major lines
- Random injections
  - Gaussians
  - Sine-gaussians
- Caveat: No glitches



Preliminary Detection Efficiencies

Wavelet Transform



Gaussians

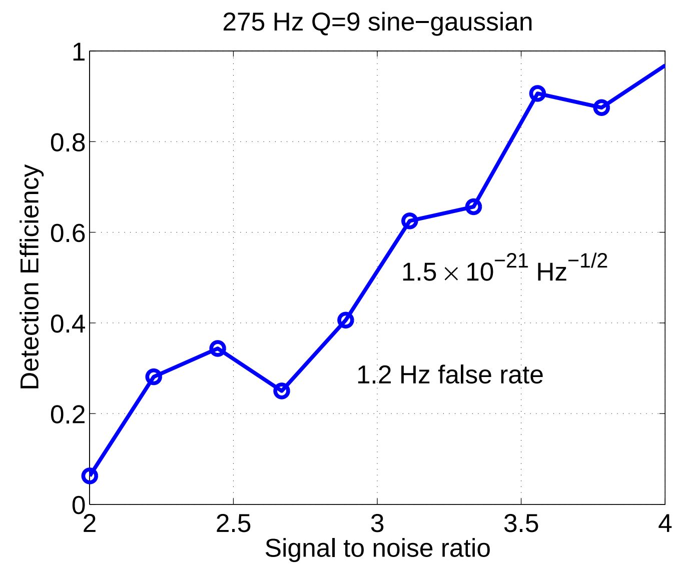
σ: 0.5, 1.0, 2.0 ms

RSS: 3.6, 5.2, 1.5 $\times 10^{-21}$

SNR: 4.6, 4.3, 4.5

false rate: 0.37 Hz

Q Transform



Sine-Gaussians

f: 275 Hz Q: 9

RSS: 1.5 $\times 10^{-21}$

SNR: 3

false rate 1.2 Hz

Black Hole Merger Model

- Equal mass black holes with no spin
- Optimally oriented with isotropic emission
- Fraction of rest mass energy emitted, $\epsilon = 0.01$
- Detectable amplitude signal to noise ratio, $\rho = 5$
- Dimensionless Kerr spin parameter, $a = 0.9$
- Energy distributed uniformly in frequency between the ISCO and QNM frequencies.

$$f_{\mathrm{ISCO}} \simeq 2 \times 10^3 \left(\frac{M}{M_\odot}\right)^{-1} \mathrm{Hz}$$

$$f_{\mathrm{QNM}} \simeq 10^4 \left(\frac{M}{M_\odot}\right)^{-1} \mathrm{Hz}$$

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Black Hole Mergers Predicted Range

Predicted from published detector noise spectra for second LIGO science run and simple merger model.