Optimally Combining the Hanford Interferometer Strain Channels

Albert Lazzarini

LIGO Laboratory Caltech

S. Bose, P. Fritschel, M. McHugh, T. Regimbau,

K. Reilly, J.T. Whelan, S. Whitcomb,. B. Whiting

GWDAW Meeting

UW Milwaukee

19 December 2003

LIGO-G030686-01-Z

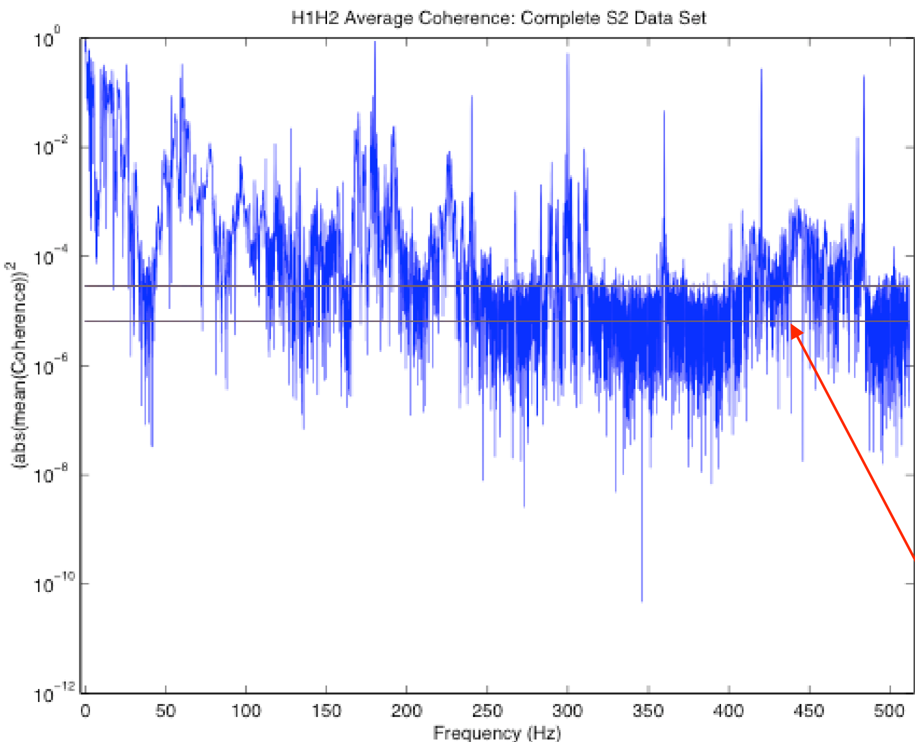
Motivation

- The S1 stochastic analysis exposed environmental correlations between H1 (4 km) and H2 (2 km) interferometers
  - » Precluded use of this measurement for setting an upper limit on the stochastic background
  - » Made combining the H1-L1 and H2-L1 results potentially tricky due to the known H1-H2 correlations
    - – H1-L1 and H2-L1 measurements made when the other interferometer was not operating may be added assuming no correlations between the measurements
      - see original Allen&Romano paper -- *PRD **59** (1999) 102001*
      - 2X measurements made during periods of 3X coincident operations in general cannot be combined in this way -- subject of this talk
      - see http://www.ligo.caltech.edu/docs/T/T030250-04.pdf

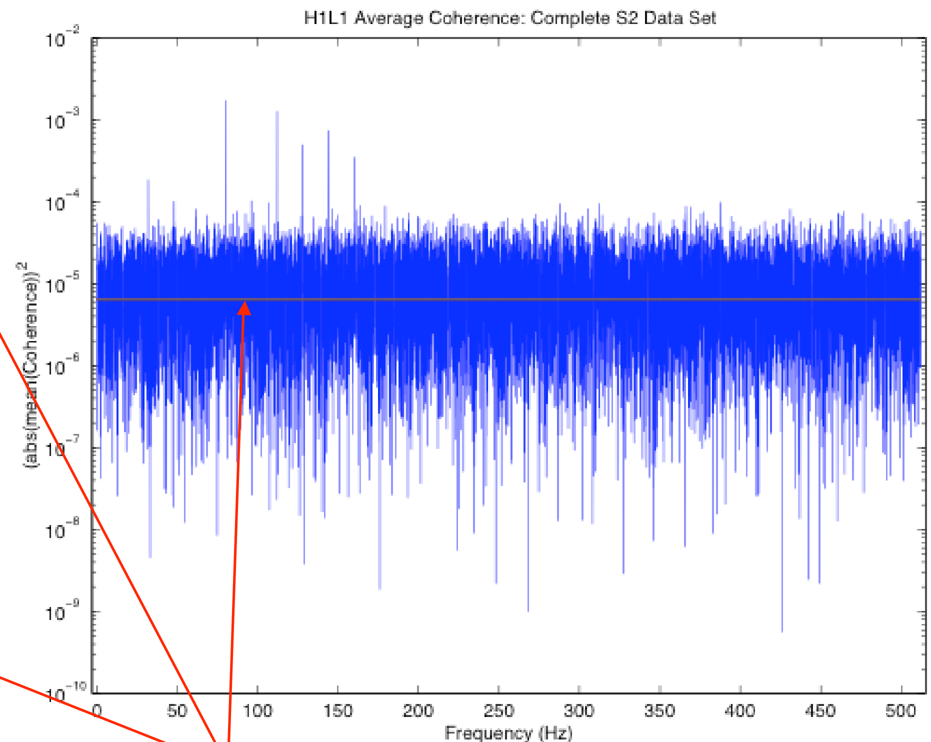
Optimally using the H1-H2-L1 data for stochastic background measurements

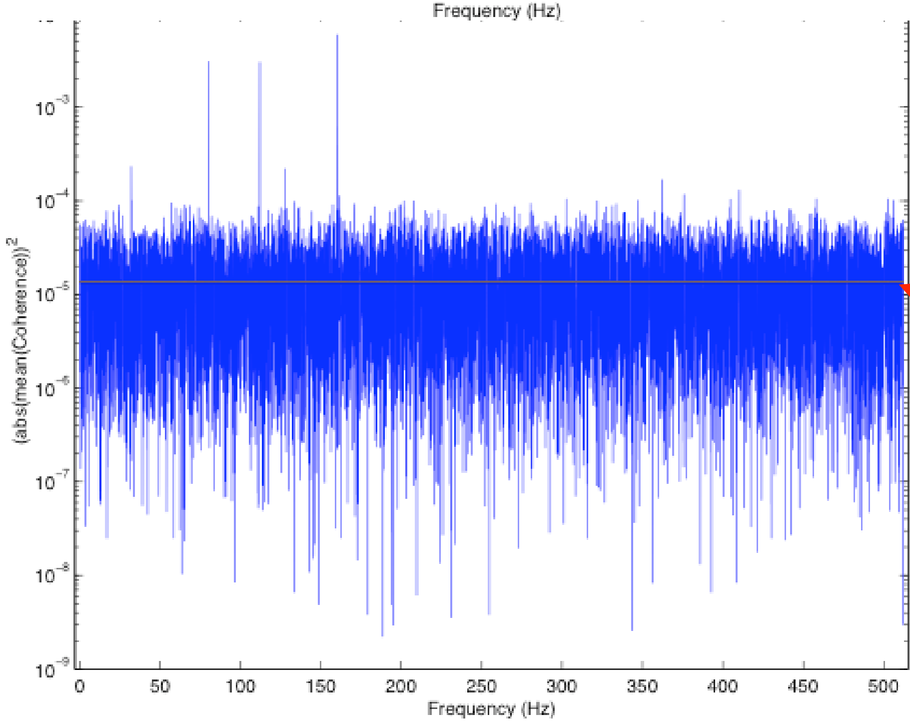
- Idea:
  - » Take advantage of the geometrical alignment and co-location of the two Hanford interferometers
    - – GW signature in two data streams *guaranteed* to be identically imprinted to high accuracy
    - – Coherent, time-domain mixing of the two strain channels possible
    - (i) Form an ***h*** pseudo-channel that is an efficient estimator of GW strain
    - (ii) Also form a ***null*** channel that cancels GW signature
      - Can be used to provide “off-source” background measurement
  - » Hanford *pseudodetector* ***h*** channel takes into account local instrumental and environmental correlations
  - » Then use the *pseudodetector* channels in the transcontinental cross-correlation measurement
  - » Naturally combines three interferometer datastreams to produce a *single* H-L estimate
- Assumes:
  - » No sources of broadband correlations between LIGO sites
    - – Supported by S1, S2 long-term coherence measurements*
      - * Except for very narrow lines related to GPS timing and DAQS
  - Local H1-H2 coherence is dominated by environment, instrumental noise
    - Supported by character, magnitude of the H1H2 coherence measurements during S1, S2
    - Turns out that so long as H1 and H2 calibrations are *accurate* linearly melding H1 + H2 does not affect GW component

Run-averaged Coherences - S2



S1





Theoretical levels for no correlation after integration time of S2

Optimal estimate of strain in the presence of instrumental correlations at Hanford

$$
\begin{aligned}
\tilde{s}_{H_1}(f) &= \tilde{h}(f) + \tilde{n}_{H_1}(f) \\
\tilde{s}_{H_2}(f) &= \tilde{h}(f) + \tilde{n}_{H_2}(f) \\
\langle \tilde{n}_i(f) \rangle = \langle \tilde{h}(f) \rangle &= 0 \\
\langle \tilde{n}_i^*(f)\, \tilde{h}(f) \rangle &= 0 \\
\langle \tilde{n}_i^*(f) \tilde{n}_j(f') \rangle &= P_{ij}(f)\delta(f-f')^{*} \\
&= \rho_{ij}(f)\sqrt{P_i(f)P_j(f)} \times \\
&\quad \delta(f-f') \\
P_{ii}(f) &:= P_i(f) \\
\langle \tilde{h}^*(f)\tilde{h}(f') \rangle &= P_\Omega(f)\delta(f-f') \\
P_\Omega(f) &\ll P_i(f) \\
\Gamma_{ij}(f) &:= |\rho_{ij}(f)|^2
\end{aligned}
$$

Not required for final conclusion to be correct (← $P_\Omega(f) \ll P_i(f)$)

*ignores bicoherence, etc.

- **Covariance matrix of raw signals**

$$
\begin{aligned}
\tilde{\mathbf{C}}_s(f)\delta(f-f') &= \\
&= \begin{bmatrix} \langle \tilde{s}^*_{H_1}(f)\tilde{s}_{H_1}(f')\rangle & \langle \tilde{s}^*_{H_1}(f)\tilde{s}_{H_2}(f')\rangle \\ \langle \tilde{s}^*_{H_2}(f)\tilde{s}_{H_1}(f')\rangle & \langle \tilde{s}^*_{H_2}(f)\tilde{s}_{H_2}(f')\rangle \end{bmatrix} \\
&= \left(\begin{bmatrix} P_{H_1}(f) & P_{H_1H_2}(f) \\ P_{H_2H_1}(f) & P_{H_2}(f) \end{bmatrix} + P_\Omega(f) \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} \right) \delta(f-f')
\end{aligned}
$$

- **C_s is dominated by diagonal elements P_{H1}, P_{H2}**
- **P_Ω appears in all four matrix elements**

Optimal estimate of strain in the presence of instrumental correlations at Hanford (2)

- Form linear combination of two interferometer signals:

$$\widetilde{s}_H(f) = \widetilde{\alpha}(f)\widetilde{s}_{H_1}(f) + (1 - \widetilde{\alpha}(f))\widetilde{s}_{H_2}(f)$$

- s_H is an *unbiased* estimate of h:

$$\langle \widetilde{h}^*(f)\ \widetilde{s}_H(f') \rangle \ = \ P_\Omega(f)\delta(f - f')$$

- Require s_H to have *minimum* variance:

$$\langle \widetilde{s}_H^*(f)\widetilde{s}_H(f') \rangle \ = \ P_H(f)\delta(f - f')$$

$$\frac{\partial P_H(f)}{\partial \widetilde{\alpha}(f)} = 0$$

- Solution

$$\widetilde{\alpha}(f) \ = \ \frac{P_{H_2}(f) - \rho_{H_1H_2}(f)\sqrt{P_{H_1}(f)P_{H_2}(f)}}{P_{H_1}(f) + P_{H_2}(f) - (\rho_{H_1H_2}(f) + \rho^*_{H_1H_2}(f))\sqrt{P_{H_1}(f)P_{H_2}(f)}}$$

$$P_H(f) \ = \ \frac{P_{H_1}(f)P_{H_2}(f)(1 - \Gamma(f))}{P_{H_1}(f) + P_{H_2}(f) - (\rho_{H_1H_2}(f) + \rho^*_{H_1H_2}(f))\sqrt{P_{H_1}(f)P_{H_2}(f)}}$$

Optimal estimate of strain in the presence of instrumental correlations at Hanford (3)

Limits:

III. For H1 and H2 the limiting design performance will have $P_{H_2}(f) = 4P_{H_1}(f)$ due to the $1:2$ arm length ratio,

$$\widetilde{\alpha}(f) \rightarrow \frac{2(2-\rho_{H_1H_2}(f))}{5-2(\rho_{H_1H_2}(f)+\rho^*_{H_1H_2}(f))} \quad \text{(A28)}$$

If the noise were either completely correlated $(\rho_{H_1H_2}(f) \rightarrow 1, \widetilde{\alpha}(f) \rightarrow 2)$ or anti-correlated $(\rho_{H_1H_2}(f) \rightarrow -1, \widetilde{\alpha}(f) \rightarrow \frac{2}{3})$, then it would be possible to exactly cancel the noise in the signals s_i. If the noise is uncorrelated $(\rho_{H_1H_2}(f) \rightarrow 0, \widetilde{\alpha}(f) \rightarrow \frac{4}{5})$, then the weighting of the signals from the two interferometers is in the ratio $4:1$, as expected.

No correlations:

$$\widetilde{s}_H(f) = \frac{P_{H_2}(f)\widetilde{s}_{H_1}(f)+P_{H_1}(f)\widetilde{s}_{H_2}(f)}{P_{H_1}(f)+P_{H_2}(f)}$$

$$P_H(f) = \frac{P_{H_1}(f)P_{H_2}(f)}{P_{H_1}(f)+P_{H_2}(f)}$$

NOTE -- $P_H(f)$ is <u>always</u> <u>less noisy</u> than the quieter instrument!

LIGO-G030686-01-Z

Optimal estimate of strain in the presence of instrumental correlations at Hanford (4)

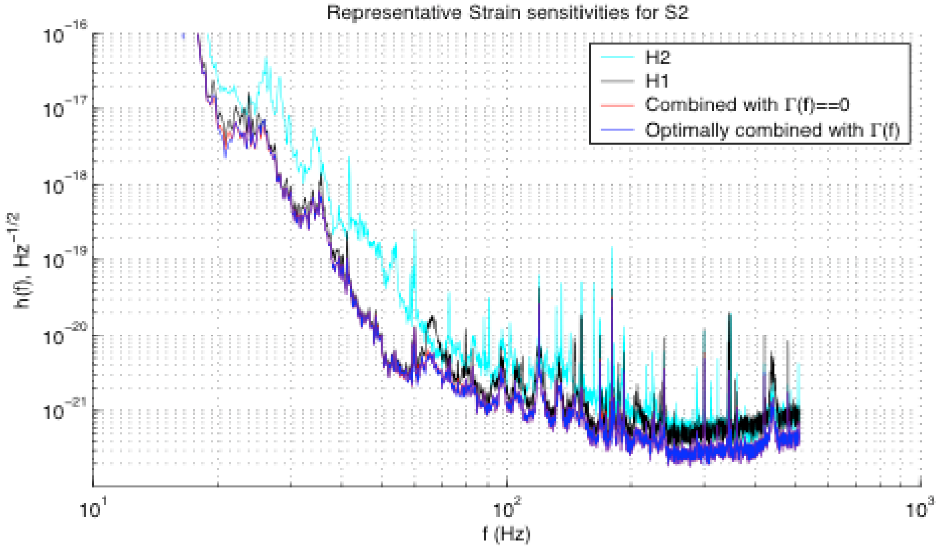
The correlation kernel for L1- H becomes (assuming $\Omega_{GW}(f)$ =const.):

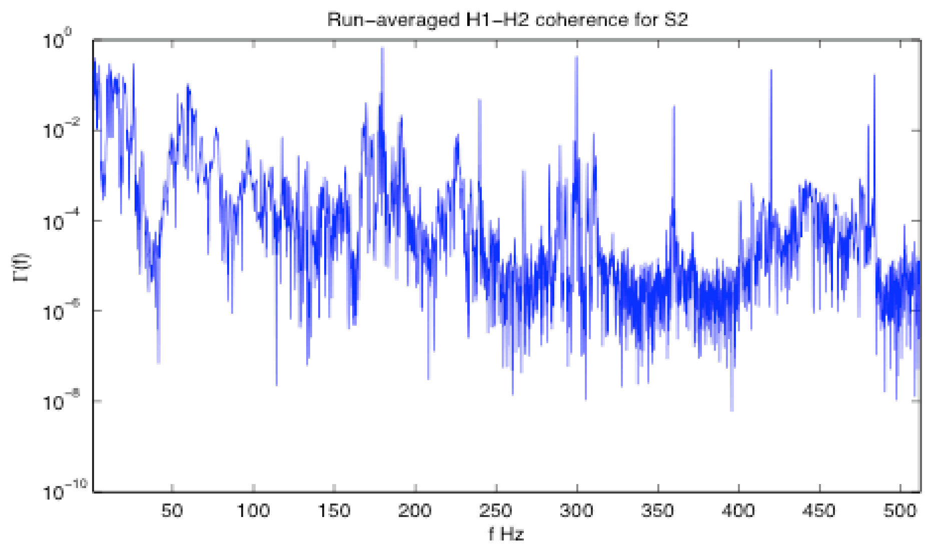
$$Y(f) \approx \frac{\gamma(|f|)s^*_{L_1}(f)\left(\left(P_{H_2}(f) - \rho_{H_1H_2}(f)\sqrt{P_{H_1}(f)P_{H_2}(f)}\right)\widetilde{s}_{H_1}(f) + \left(P_{H_1}(f) - \rho^*_{H_1H_2}(f)\sqrt{P_{H_1}(f)P_{H_2}(f)}\right)\widetilde{s}_{H_2}(f)\right)}{|f|^3 P_{L_1}(|f|)P_{H_1}(f)P_{H_2}(f)(1-\Gamma_{H_1H_2}(f))}$$

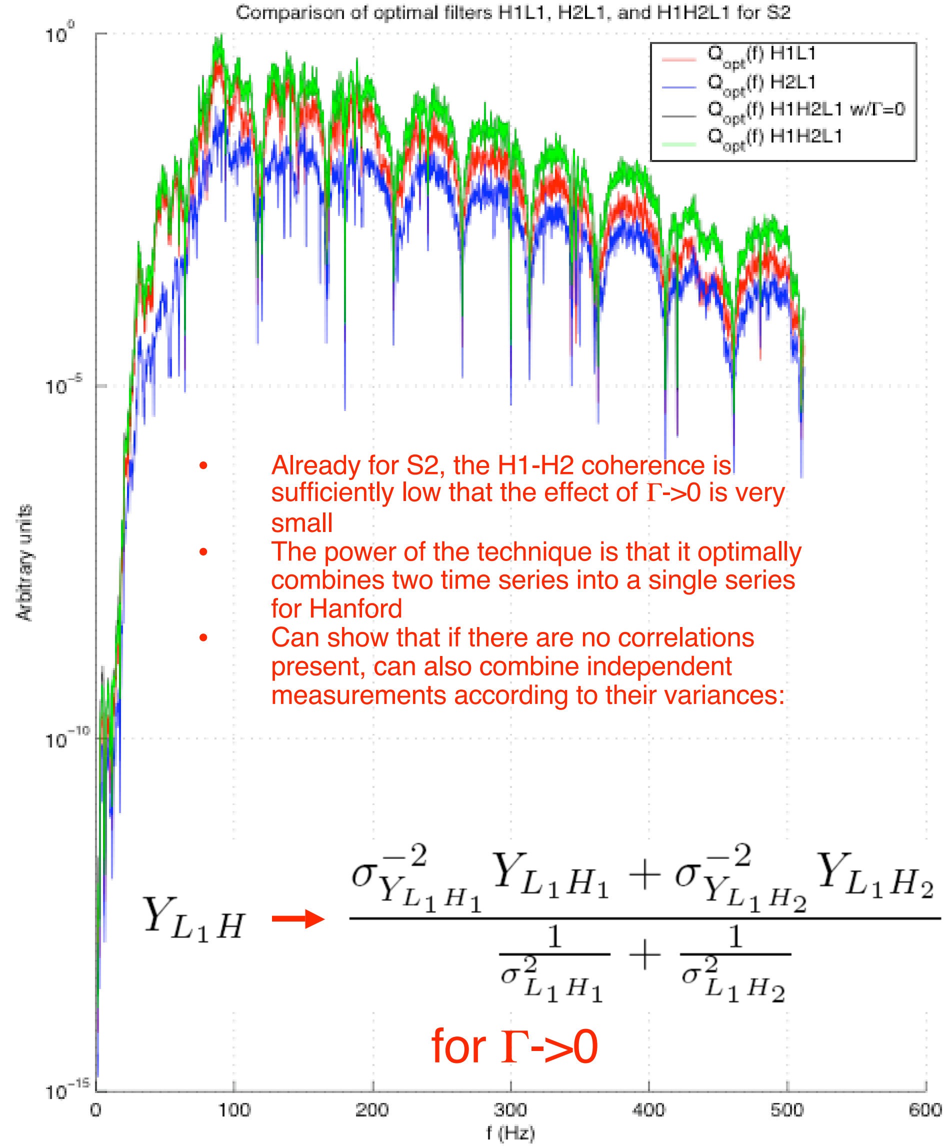
Implementation issues/details:

- Need to modify the correlation analysis to take in 3 interferometer channels, condition, etc.
- Γ(f) and ρ(f) should be calculated over the entire run -- read them in as frequency series, similar to R(f) data.
- Apply this ONLY to 3X data stretches

Results from S2 representative spectra







- Already for S2, the H1-H2 coherence is sufficiently low that the effect of Γ->0 is very small
- The power of the technique is that it optimally combines two time series into a single series for Hanford
- Can show that if there are no correlations present, can also combine independent measurements according to their variances:

$$Y_{L_1H} \rightarrow \frac{\sigma^{-2}_{Y_{L_1H_1}} Y_{L_1H_1} + \sigma^{-2}_{Y_{L_1H_2}} Y_{L_1H_2}}{\frac{1}{\sigma^2_{L_1H_1}} + \frac{1}{\sigma^2_{L_1H_2}}}$$

for Γ->0

Null GW channel derived from the two of Hanford strain channels

- Use s_H to cancel h in individual channels, $s_{H1,2}$

$$\widetilde{z}_{H_1}(f) = \widetilde{s}_{H_1}(f) - \widetilde{s}_H(f)$$
$$\widetilde{z}_{H_2}(f) = \widetilde{s}_{H_1}(f) - \widetilde{s}_H(f)$$

$$\widetilde{z}_{H_1}(f) = \big(1 - \widetilde{\alpha}(f)\big)[\widetilde{n}_1(f) - \widetilde{n}_2(f)]$$
$$\widetilde{z}_{H_2}(f) = \widetilde{\alpha}(f)[\widetilde{n}_1(f) - \widetilde{n}_2(f)]$$

$$\widetilde{\mathbf{C}}_z(f)\delta(f-f') = \begin{bmatrix} \langle \widetilde{z}^*_{H_1}(f)\widetilde{z}_{H_1}(f')\rangle & \langle \widetilde{z}^*_{H_1}(f)\widetilde{z}_{H_2}(f')\rangle \\ \langle \widetilde{z}^*_{H_2}(f)\widetilde{z}_{H_1}(f')\rangle & \langle \widetilde{z}^*_{H_2}(f)\widetilde{z}_{H_2}(f')\rangle \end{bmatrix}$$

$$= \langle(\widetilde{n}^*_1(f) - \widetilde{n}^*_2(f))(\widetilde{n}_1(f') - \widetilde{n}_2(f'))\rangle \begin{bmatrix} (1-\widetilde{\alpha}(f))(1-\widetilde{\alpha}^*(f)) & -\widetilde{\alpha}(f)(1-\widetilde{\alpha}^*(f)) \\ -\widetilde{\alpha}^*(f)(1-\widetilde{\alpha}(f)) & \widetilde{\alpha}(f)\widetilde{\alpha}^*(f) \end{bmatrix}$$

$$= \begin{bmatrix} (1-\widetilde{\alpha}(f))(1-\widetilde{\alpha}^*(f)) & -\widetilde{\alpha}(f)(1-\widetilde{\alpha}^*(f)) \\ -\widetilde{\alpha}^*(f)(1-\widetilde{\alpha}(f)) & \widetilde{\alpha}(f)\widetilde{\alpha}^*(f) \end{bmatrix} \times$$
$$\Big(P_{H_1}(f) + P_{H_2}(f) - (P_{H_1H_2}(f) + P_{H_2H_1}(f)\Big)\delta(f-f')$$

← NO P_Ω dependence !

LIGO-G030686-01-Z

Null GW channel derived from the two of Hanford strain channels (2)

- Diagonalization of C_z does not involve h
  - » C_z derived from single vector, $\{s_{H1}, s_{H2}\}$ -> one non zero eigenvalue (corresponds to power in signal z_H):

$$P_{zH}(f) = \left(P_{H_1}(f) + P_{H_2}(f) - (\rho_{H_1H_2}(f) + \rho^*_{H_1H_2}(f))\sqrt{P_{H_1}(f)P_{H_2}(f)}\right) \times \left(1 - \widetilde{\alpha}^*(f) - \widetilde{\alpha}(f) + 2\widetilde{\alpha}^*(f)\widetilde{\alpha}(f)\right)$$

  - » Corresponding eigenvector:

$$z_H(f) = -(s_{H_1}(f) - s_{H_2}(f))\,\widetilde{\alpha}(f)\sqrt{\frac{1 - \widetilde{\alpha}(f) - \widetilde{\alpha}^*(f) + 2\,\widetilde{\alpha}(f)\,\widetilde{\alpha}^*(f)}{\widetilde{\alpha}(f)\,\widetilde{\alpha}^*(f)}}$$

  - » $z_H \propto [s_{H1} - s_{H2}]$ x $g(\alpha(f))$
    - – filter function g reduces $Var(z_H)$ below $Var(s_{H1} - s_{H2})$

Null GW channel derived from the two of Hanford strain channels (3)

- For Γ-> 0,

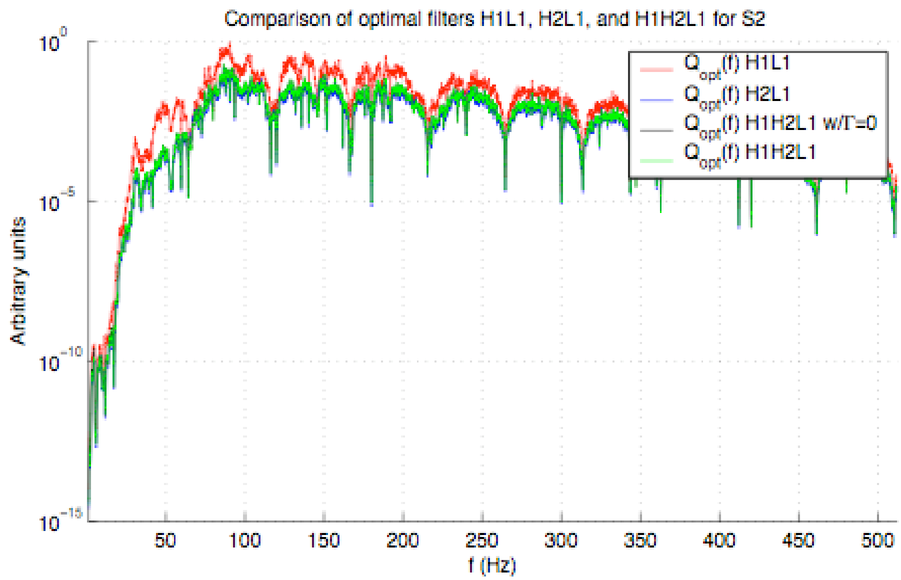
$$\tilde{z}_H(f) = (s_{H_2}(f) - s_{H_1}(f)) \frac{\sqrt{P^2_{H_1}(f) + P^2_{H_2}(f)}}{P_{H_1}(f) + P_{H_2}(f)}$$

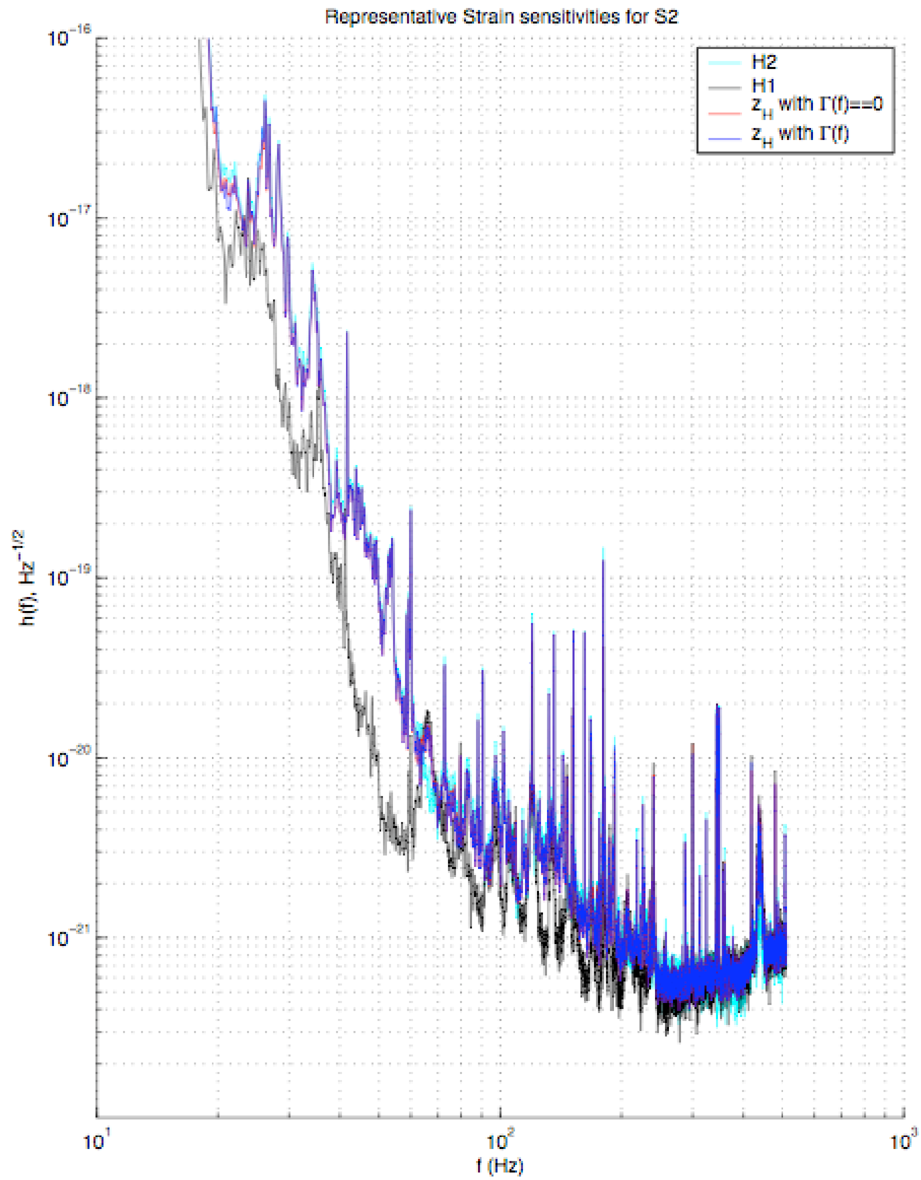
$$P_{z_H}(f) = \frac{P^2_{H_1}(f) + P^2_{H_2}(f)}{P_{H_1}(f) + P_{H_2}(f)} \leq P_{H_1}(f) + P_{H_2}(f)$$

$$\leq max[P_{H_1}(f), P_{H_2}(f)]$$

NOTE -- $P_{zH}(f)$ is <u>always</u> <u>less noisy</u> than the noisier instrument

Results from S2 representative spectra





Summary (1)

- It is possible to use the co-located/co-aligned H1/H2 interferometers in a fundamentally different manner than was done for S1
  - » We are a little smarter …
- An optimal estimate of h can be obtained that is robust against local instrumental correlations
  - » Allows a consistent manner of combining H1, H2, L1 datastreams to obtain a single best upper limit on Ω
  - » Reduces to standard expression for uncorrelated measurements

Summary (2)

- There exists a ***dual*** to s_H-- null channel -- z_H *designed* not to contain GW signature
  - » Can be used for "off-source" null measurement as a calibration for "on-source" measurement
    - – Analogous to rotated ALLEGRO+LLO technique
  - » Use of null channel can be generalized to other classes of searches
    - – e.g., run inspiral search over z_H -> if anything is seen, it can be used to veto same search over s_H
- Technique requires reasonably precise ***relative*** knowledge of H1, H2 calibrations
  - » Relative calibration errors between s_{H1}, s_{H2} will tend to average out in s_H
  - » Will tend to add in z_H
    - – Leads to ***leakage*** of h into z_H
    - – Relative calibration error +/- $\varepsilon(f)$ leakage into z_H:

      $\delta h(f) \sim 2\, \varepsilon(f)\, h$ in ***amplitude*** and $\delta P_\Omega(f) \sim 4\, |\varepsilon(f)|^2\, P_\Omega(f)$ in ***power***
  - » Event at threshold at ρ_* in s_H -> $2|\varepsilon|\,\rho_*$ in z_H
    - – For reasonable ε and ρ_*, signal in z_H will be at or below threshold.

Finis

Run-averaged Coherences - S1