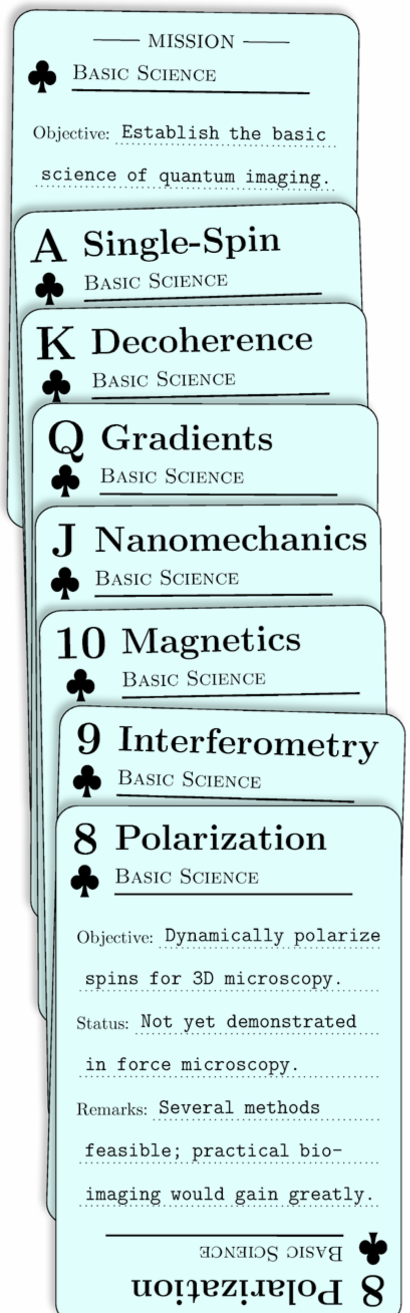
UW Quantum System Engineering Group

e2e Simulation Techniques in Quantum Microscopy and GW Interferometry

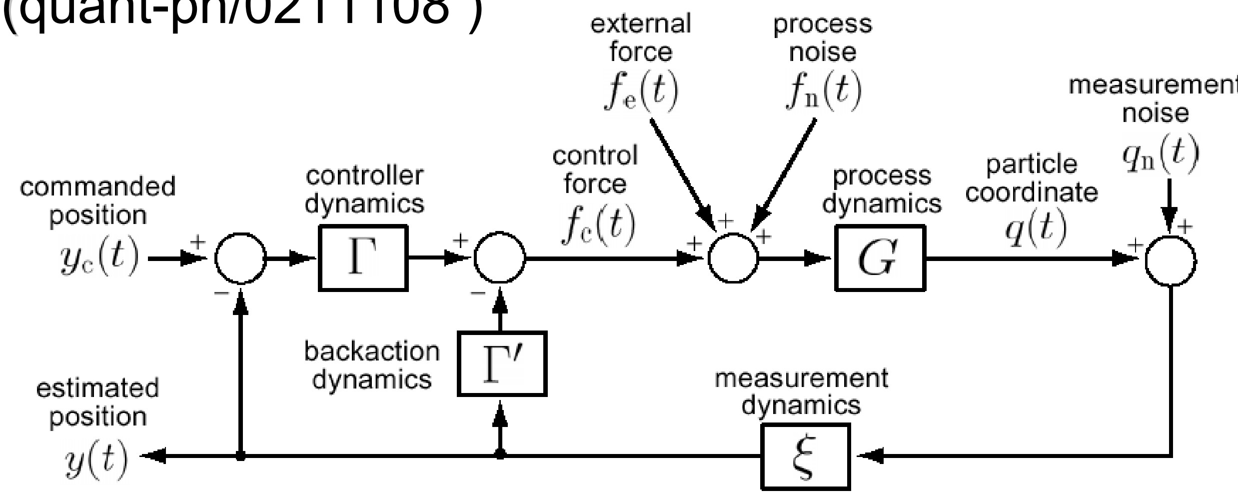
- Design Goal: **Get what we want to go from where we are**
- What we want downstream: **Hardware that works**
  - Optimally designed and optimally controlled
  - All the transfer function poles and zeros, in closed form
  - Autonomous control software (autodiagnostic and self-calibrating)
  - Reliable predictions for signal, noise, and SNR, in closed form
- What we've got upstream: **Classical response of the system**
  - The mechanical dynamics (suspension modes, frequencies, etc.)
  - The measured decoherence (Q's, noise temperatures, shot noise, etc.)
  - The optical scattering phases (finesses, Hermite-Gauss modes, etc.)
- Message I: **We've got enough to design our hardware!**
  - Enough for an automated quantum-mechanical e2e analysis
  - This is our primary UW/QSE goal for 2004
- Message II: **as presently designed**, **advLIGO will not work**
  - But LIGO/LSC could apply our methods to fix the advLIGO design

UW LSC Presentation
At: LIGO Hanford Laboratory
Date: November 12, 2003
Presenter: John Sidles
LIGO-G030635-00-Z



What We Want Downstream

- The control theory block diagram (quant-ph/0211108)



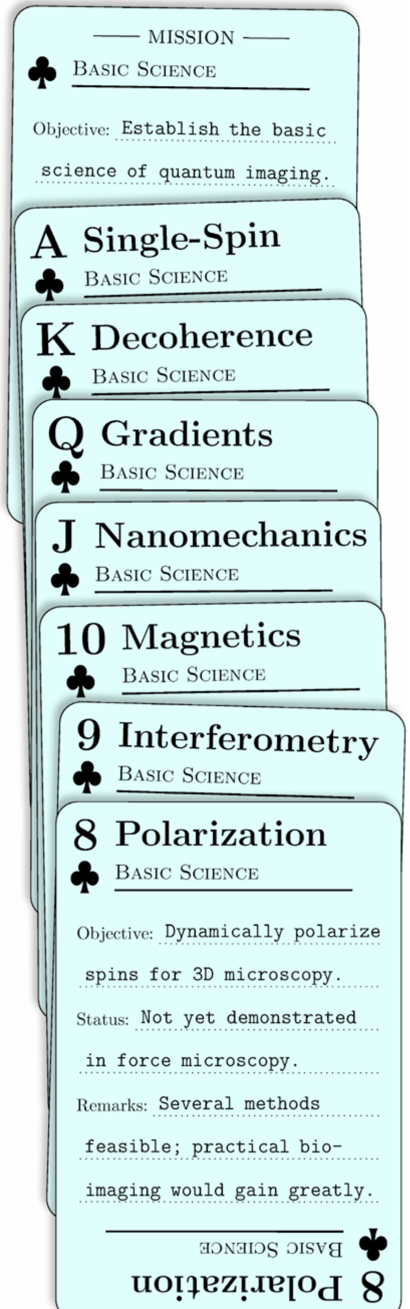
- The control theory closed-form result:

Signal: $$\langle \tilde{y}(\omega) \rangle = \frac{\tilde{\xi}(\omega)\tilde{G}(\omega)(\tilde{f}_e(\omega) + \tilde{\Gamma}(\omega)\tilde{y}_c(\omega))}{1 + \tilde{\xi}(\omega)\tilde{G}(\omega)\big(\tilde{\Gamma}(\omega) + \tilde{\Gamma}'(\omega)\big)}$$

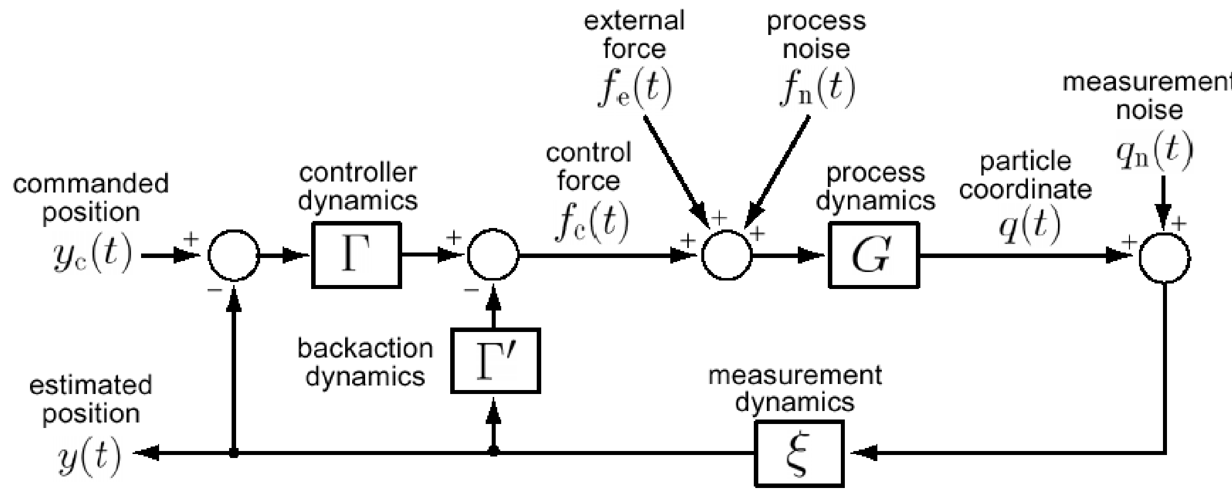
Noise: $$S_y(\omega) = \frac{\big(S_q(\omega) + |\tilde{G}(\omega)|^2 S_f(\omega)\big)|\tilde{\xi}(\omega)|^2}{\big|1 + \tilde{\xi}(\omega)\tilde{G}(\omega)\big(\tilde{\Gamma}(\omega) + \tilde{\Gamma}'(\omega)\big)\big|^2}$$

Including all the poles and zeros!

Working back to what we're got



- The control theory block diagram



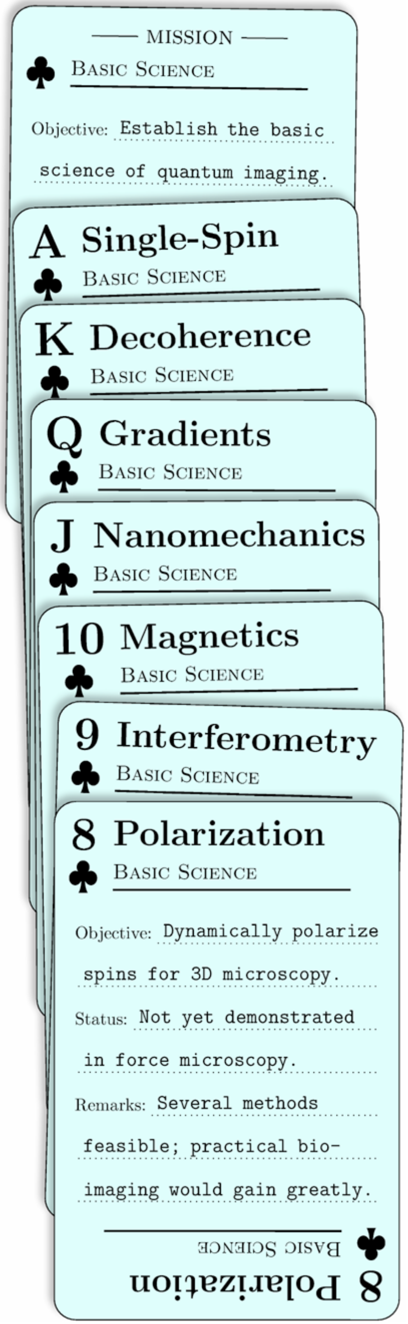
- A unique path integral equivalence:

$$P(y(t)|f_e(t), y_c(t)) = \left| \int [dq] \exp\left[\mathcal{A}(q, y, f_e, y_c)\right] \right|^2$$

what we measure — what we apply — integrate over all test mass trajectories — complex action

- This formalism known to Feynman, Mensky, and Caves, equivalent to that of Thorne, Braginsky, Buonanno

Working back to what we're got



$a_{\text{in}} \longrightarrow$ **linear optics**

$a_{\text{out}}(q(t)) \longleftarrow$ **linear optics**

- General optical kernels :

These two amplitudes, plus the photon detection statistics, completely determine both the system dynamics and the quantum noise.

$$a_{\text{out}}(q(t)) = a_{\text{in}}\left(1 + \int_{-\infty}^{\infty} dt'\,\alpha(t-t')q(t') + \int_{-\infty}^{\infty} dt' \int_{-\infty}^{\infty} dt''\,\beta(t-t', t-t'')q(t')q(t'')\right)$$

- Optical kernels for Fabry-Perot cavities:

sideband amplitude

$$\tilde{\alpha}(\omega) = \frac{2ike^{i\tau(2\omega_0+3\omega)}\sin^2\rho}{(e^{i\tau(2\omega_0+\omega)} - \cos\rho)(e^{i\tau(2\omega_0+\omega)}\cos\rho - 1)}$$

carrier amplitude

$$\tilde{\beta}(\omega, -\omega) = \frac{\tilde{\alpha}(\omega)\tilde{\alpha}(-\omega)\sin^2\rho}{2(\sin^2\rho - 2i\sin(2\tau\omega_0)\cos\rho)}$$

Now we're done. For linear systems (like advLIGO) the rest is just plugging in to the path integral.

Fixing advLIGO in 2004: Suggestions

- Switch advLIGO now to negative-*g* cavities
  - Positive-*g* cavities are grossly unstable
  - Negative-*g* cavities aren't much better!
- Calculate ASAP the e2e transfer function
  - Locate all the poles and zeros analytically
  - Prove the system is observable and controllable
  - Calculate noise injected by stabilizing control
- Then optimize the advLIGO design
  - Noise near SQL
  - 10 Hz roll-off
  - Controllable/observable
  - High optical power
  - Large beam diameter
  - Good seismic isolation

Find out: are these goals compatible?

- Be prepared to accept:
  - Very substantial revisions to present design
  - Far closer coupling of ISC, COC, and SUS
  - E2e analysis as the "One Ring" of advLIGO
  - A path integral script as the implementation of e2e analysis
  - Asking NSF for more money and time
- Be prepared to answer: **Does advLIGO make technical sense at this time?**

TABLE I: Cavity parameters and stiffness values for current LIGO and the advanced LIGO design.

parameter	LIGO	advLIGO	unit
P	15	830	kW
g_1	0.460	$\pm$ 0.927	–
g_2	0.726	$\pm$ 0.927	–
κ_{pendulum}	$\sim$ 0.41	$\sim$ 2.4	N·m
κ_{major}	-0.96	$\mp$301	N·m
κ_{minor}	0.25	$\pm$11.5	N·m

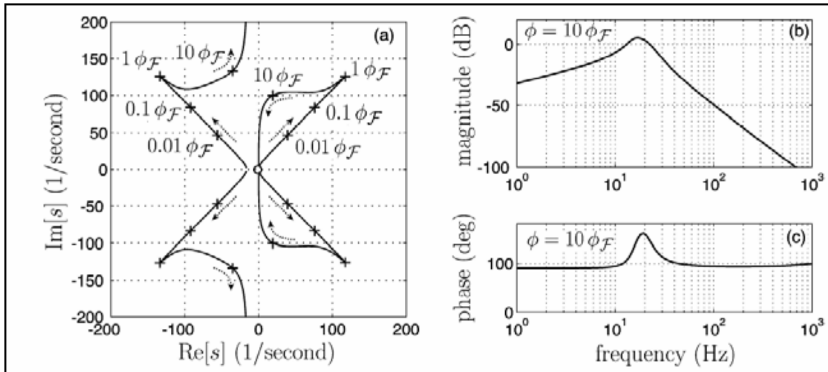
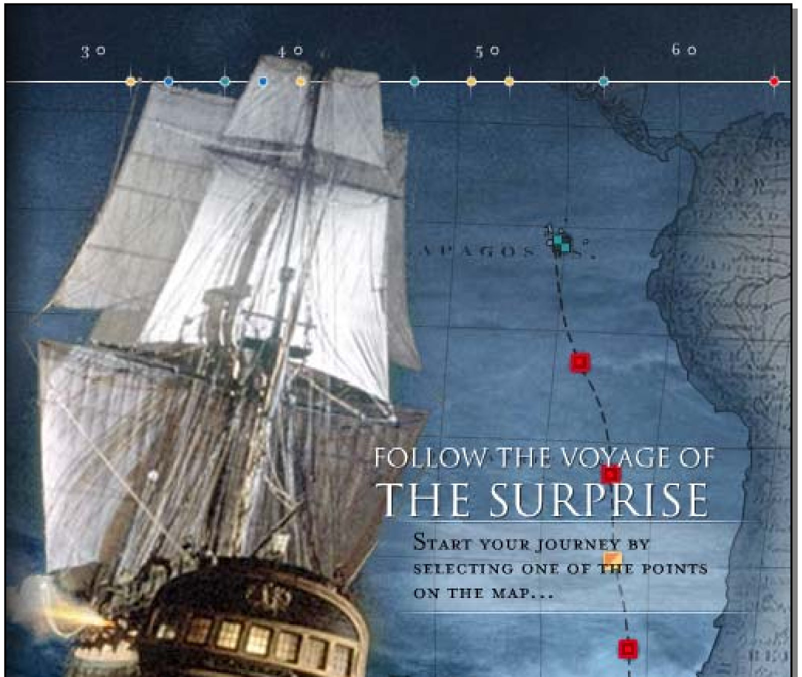


FIG. 4: Dynamical behavior of the test mass. (a) Poles of the transfer function $\tilde{T}(\omega)$ as the cavity tuning is varied over

The Voyage of Discovery!



Thanks for the adventure!
-- and --
there is not a moment to lose!