# Combining the Two Hanford-Livingston Stochastic Background Limits in the Presence of Local Environmental Correlations Between the Hanford Interferometers

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***LSC Meeting at LIGO Hanford Observatory***

***10 - 13 November 2003***

*LIGO-G030553-03-E*

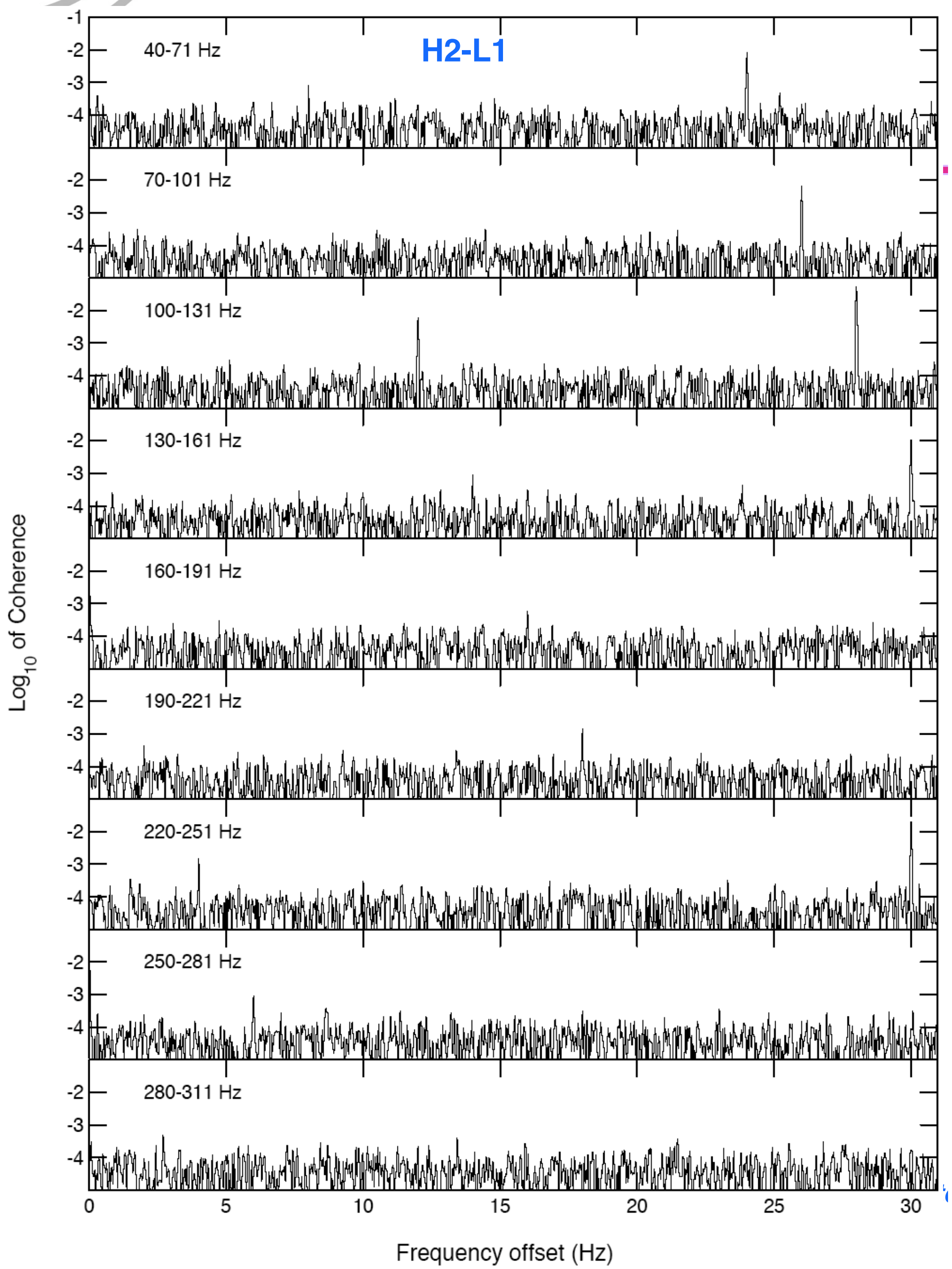
# Motivation

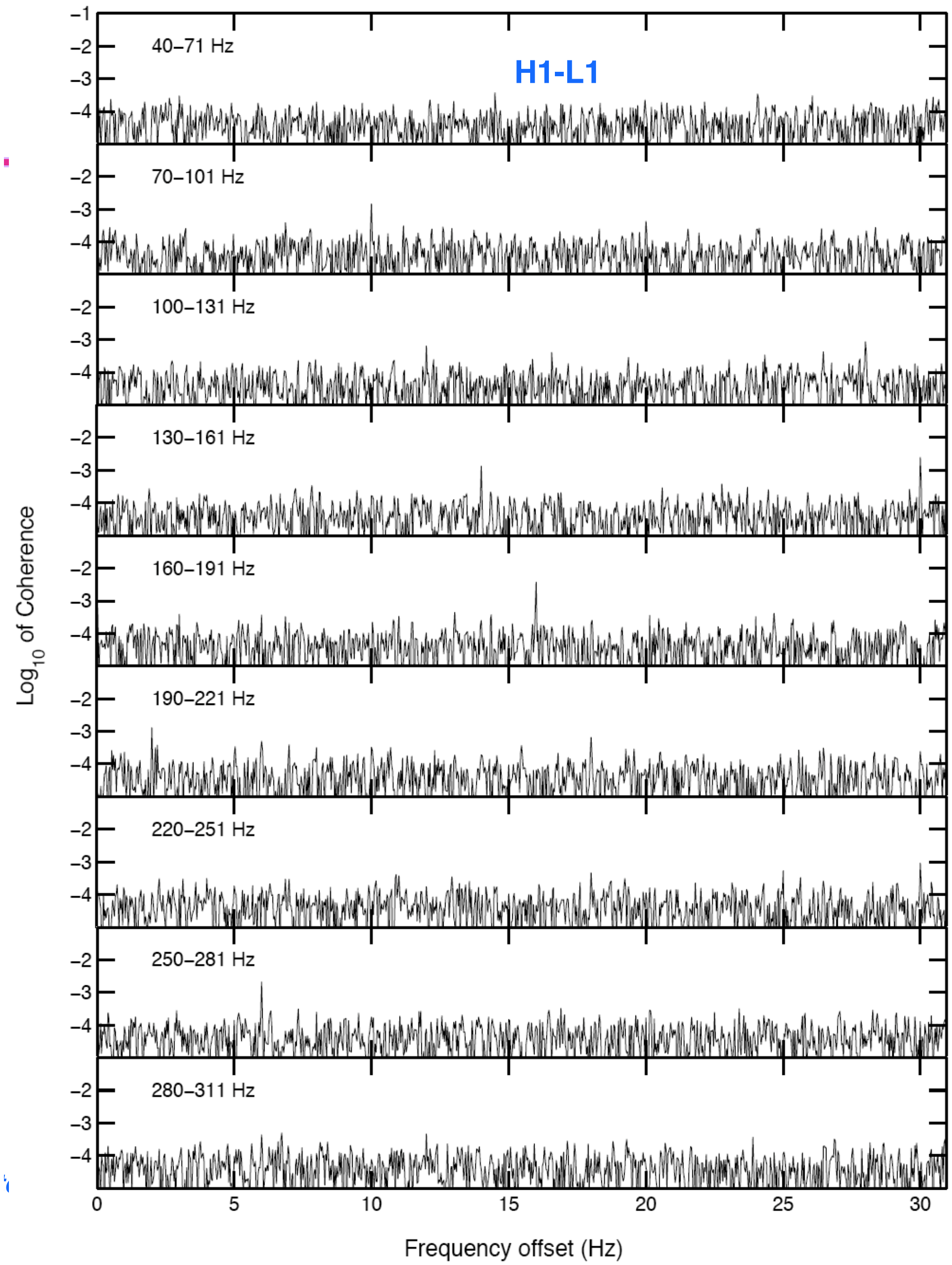
- The S1 stochastic analysis exposed environmental correlations between H1 and H2
  - » Precluded use of this measurement for setting an upper limit on the stochastic background
  - » Made combining the H1-L1 and H2-L1 results potentially tricky due to the known H1-H2 correlations
    - – H1-L1 and H2-L1 measurements made when the other interferometer was not operating may be added assuming no correlations between the measurements
      - see original Allen&Romano article, also T. Regimbau note in stochastic e-log 02 Nov 2003: http://www.ldas-sw.ligo.caltech.edu/ilog/pub/ilog.cgi?group=stochastic
    - – 2X measurements made during periods of 3X coincident operations in general cannot be combined in this way -- subject of this talk
      - see http://www.ligo.caltech.edu/docs/T/T030250-01.pdf
  - » Formulation discussed during recent face-to-face meeting at MIT

# Optimally using the H1-H2-L1 data for stochastic background measuremnts

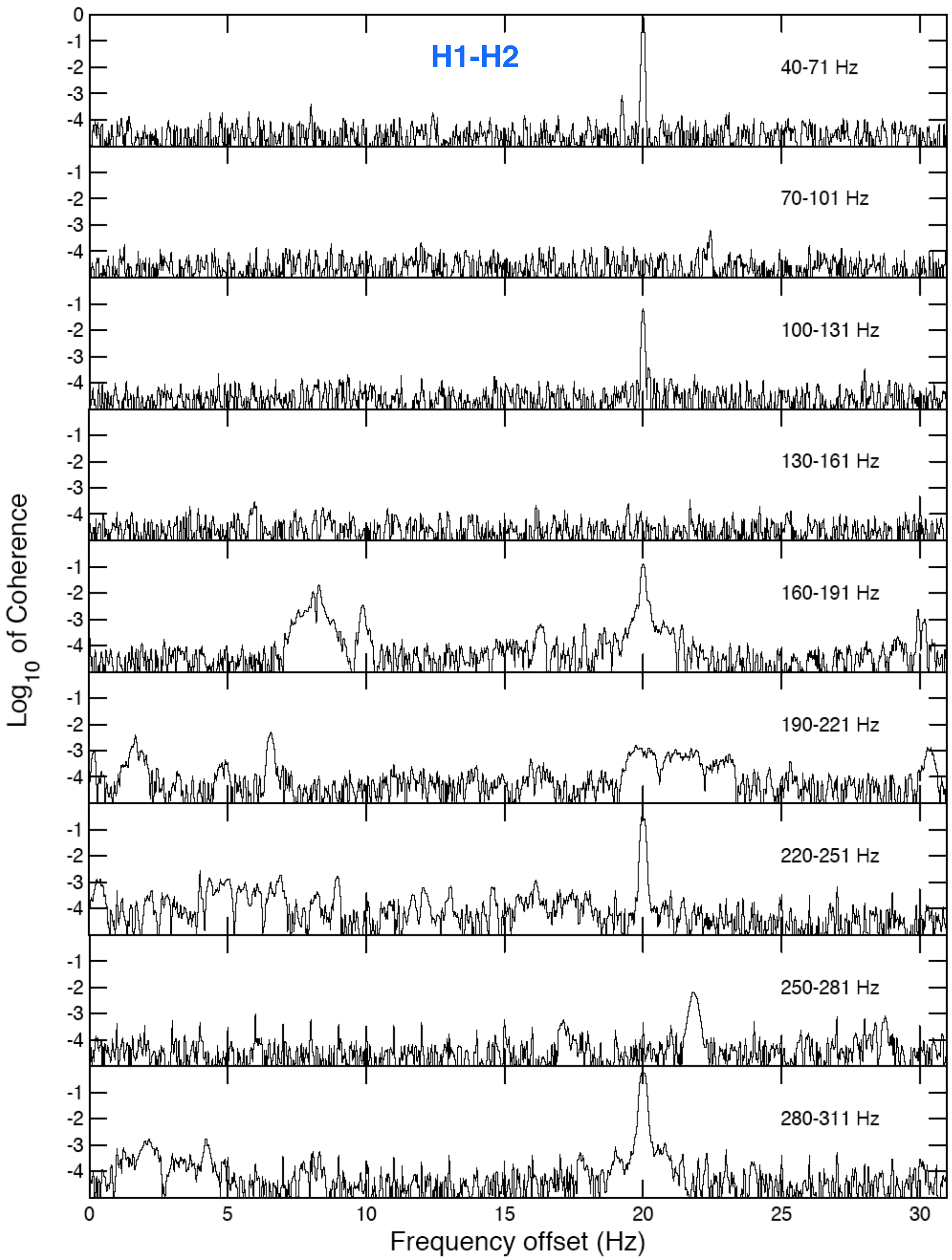
- Idea:
  - » First form a Hanford *pseudodetector* data set that takes into account local instrumental and environmental correlations
  - » Then use the *pseudodetector* data in the transcontinental cross-correlation measurement
  - » Naturally combines three interferometer datastreams to produce a *single* H-L estimate
- Assumes:
  - » No sources of broadband correlations between LIGO sites
    - – Supported by S1, S2 long-term coherence measurements*
      - * Except for very narrow lines related to GPS timing and DAQS
  - Local H1-H2 coherence is dominated by environment, instrumental noise
    - Supported by character, magnitude of the H1H2 coherence measurements during S1, S2
    - Turns out that so long as H1 and H2 calibrations are *accurate* linearly melding H1 + h2 does not affect GW component

# Run-averaged Coherences - S1



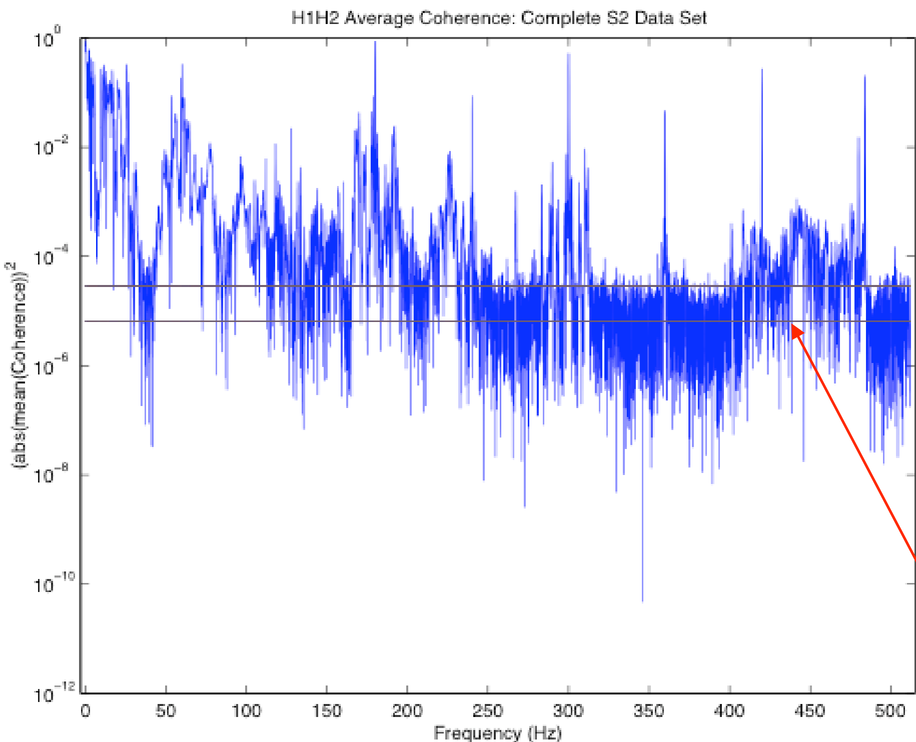



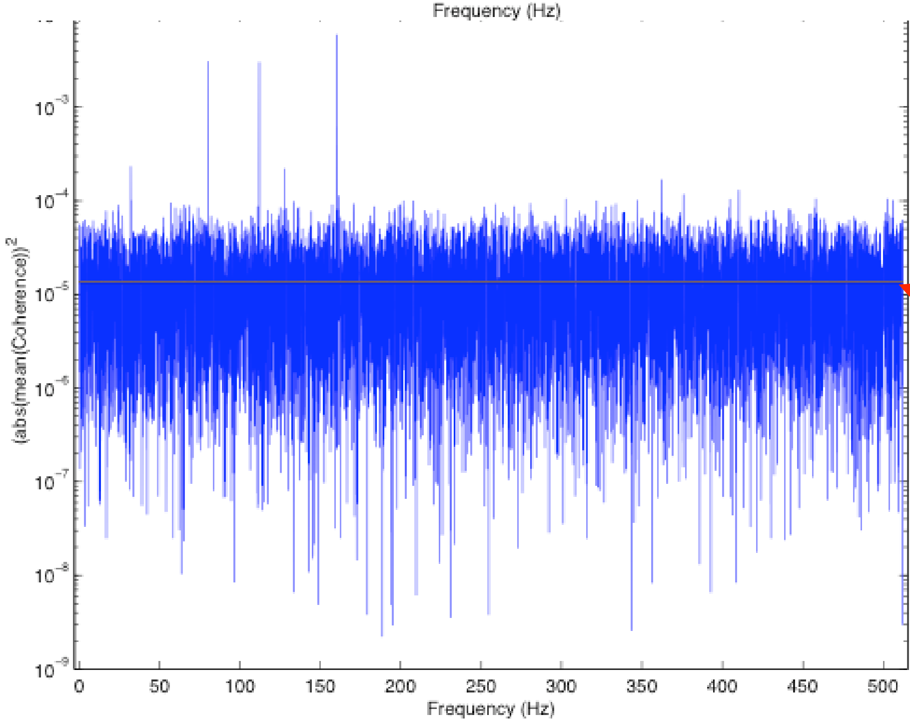
# Run-averaged Coherences - S1

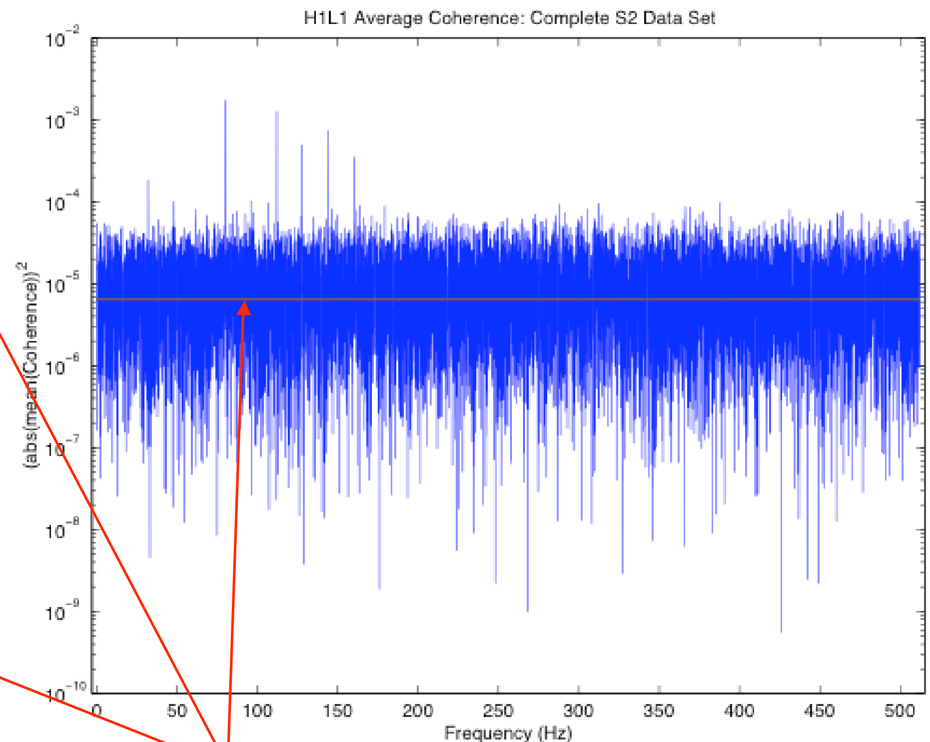


# Run-averaged Coherences - S2

S1




**Theoretical levels for no correlation after integration time of S2**

# Optimal estimate of pseudostrain in the presence of instrumental correlations at Hanford

$$\begin{aligned} s_{H1}(t) &= h(t) + n_{H1}(t) \\ s_{H2}(t) &= h(t) + n_{H2}(t) \end{aligned} \quad \Rightarrow \quad \begin{aligned} \widetilde{s}_{H1}(f) &= \widetilde{h}(f) + \widetilde{n}_{H1}(f) \\ \widetilde{s}_{H2}(f) &= \widetilde{h}(f) + \widetilde{n}_{H2}(f) \end{aligned}$$

$$\begin{aligned} \langle \widetilde{n}_i(f) \rangle = \langle \widetilde{h}(f) \rangle &= 0 \\ \langle \widetilde{n}_i^*(f)\, \widetilde{h}(f) \rangle &= 0 \\ \langle \widetilde{n}_i^*(f) \widetilde{n}_j(f') \rangle &= P_{ij}(f)\delta(f-f')^{*} \\ &= \rho_{ij}(f)\sqrt{P_i(f)P_j(f)} \times \\ &\quad \delta(f-f') \\ P_{ii}(f) &:= P_i(f) \\ \langle \widetilde{h}^*(f)\widetilde{h}(f') \rangle &= P_\Omega(f)\delta(f-f') \\ P_\Omega(f) &\ll P_i(f) \\ \Gamma_{ij}(f) &:= |\rho_{ij}(f)|^2 \end{aligned}$$

Not required for final conclusion to be correct (refers to $P_\Omega(f) \ll P_i(f)$)

*ignores bicoherence, etc.

# Optimal estimate of pseudostrain in the presence of instrumental correlations at Hanford (2)

- Form linear combination of two interferometer signals:

$$\widetilde{s}'(f) = \widetilde{\alpha}(f)\widetilde{s}_1(f) + \widetilde{\beta}(f)\widetilde{s}_2(f)$$

- Require *s'* to be an *unbiased* estimate of h:

$$\begin{aligned} \langle \widetilde{h}^*(f)\ \widetilde{s}'(f') \rangle &= P_\Omega(f)\delta(f - f') \\ &\rightarrow\ \widetilde{\alpha}(f) + \widetilde{\beta}(f) = 1 \end{aligned}$$

- Require *s'* to have *minimum* variance:

$$\begin{aligned} Var(s') &:= P_{s'}(f) \\ \langle \widetilde{s}'^*(f)\widetilde{s}'(f') \rangle &= P_{s'}(f)\delta(f - f') \end{aligned}$$

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# Optimal estimate of pseudostrain in the presence of instrumental correlations at Hanford (3)

$$
\begin{aligned}
P_{s'}(f) &= |\widetilde{\alpha}(f)|^2 P_1(f) + |1-\widetilde{\alpha}(f)|^2 P_2(f) + \\
&\Big(\widetilde{\alpha}^*(f)(1-\widetilde{\alpha}(f))\rho_{12}(f) + \\
&\widetilde{\alpha}(f)(1-\widetilde{\alpha}^*(f))\rho_{12}^*(f)\Big) \times \\
&\sqrt{P_1(f)P_2(f)}
\end{aligned}
$$

- Minimize $P_{s'}$:

$$
\begin{pmatrix} \frac{\partial P_{s'}(f)}{\partial \widetilde{\alpha}(f)} \\ \frac{\partial P_{s'}(f)}{\partial \widetilde{\alpha}^*(f)} \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}
$$

# LIGO Optimal estimate of pseudostrain in the presence of instrumental correlations at Hanford (4)

Result for □:

$$\widetilde{\alpha}(f) \;=\; \frac{P_2(f) - \rho_{12}(f)\sqrt{P_1(f)P_2(f)}}{P_1(f) + P_2(f) - (\rho_{12}(f) + \rho_{12}^*(f))\sqrt{P_1(f)P_2(f)}}$$

and for $P_{s'}$ :

$$P_{s'}(f) \;=\; \frac{P_1(f)P_2(f)(1 - \Gamma(f))}{P_1(f) + P_2(f) - (\rho_{12}(f) + \rho_{12}^*(f))\sqrt{P_1(f)P_2(f)}}$$

# LIGO Optimal estimate of pseudostrain in the presence of instrumental correlations at Hanford (6)

Limits:

I. If $\rho_{12}(f) \to 0$: Then $\widetilde{\alpha}(f)$ becomes,

$$\widetilde{\alpha}(f) \to \frac{P_2(f)}{P_1(f) + P_2(f)}$$

IIa. If $P_1(f) \to P_2(f)$: Then $\widetilde{\alpha}(f)$ is independent of $P(f)$,

$$\widetilde{\alpha}(f) \to \frac{1 - \rho_{12}(f)}{2 - (\rho_{12}(f) + \rho^*_{12}(f))}$$

IIb. If $\rho_{12}(f) \to 1$, then $\rho_{12}(f) = \rho^*_{12}(f) = \sqrt{\Gamma(f)}$. If also $P_1(f) \to P_2(f)$, then $P_{s'}(f) \to P_1(f)$:

$$P_{s'}(f) = \lim_{\Gamma(f)\to 1} \frac{P_1(f)}{2} \frac{1 - \Gamma(f)}{1 - \sqrt{\Gamma(f)}} = P_1(f)$$

# Optimal estimate of pseudostrain in the presence of instrumental correlations at Hanford (7)

Limits :

III. For H1 and H2 the limiting design performance will have $P_2(f) = 4P_1(f)$ due to the $1:2$ arm length ratio,

$$\widetilde{\alpha}(f) \rightarrow \frac{2(2-\rho_{12}(f))}{5-2(\rho_{12}+\rho_{12}^*)}$$

If the noise were either completely correlated ($\rho_{12} \rightarrow 1, \widetilde{\alpha}(f) \rightarrow 2$) or anti-correlated ($\rho_{12} \rightarrow -1, \widetilde{\alpha}(f) \rightarrow \frac{2}{3}$), then it would be possible to exactly cancel the noise in the signals $s_i$.

# Optimal estimate of pseudostrain in the presence of instrumental correlations at Hanford (8)

## The correlation kernel for L1- H becomes (assuming □$_{GW}$(f) =const.):

$$Y(f) \approx \frac{\gamma(|f|)s^*_{L_1}(f)\left(\left(P_{H_2}(f) - \rho_{H_1H_2}(f)\sqrt{P_{H_1}(f)P_{H_2}(f)}\right)\widetilde{s}_{H_1}(f) + \left(P_{H_1}(f) - \rho^*_{H_1H_2}(f)\sqrt{P_{H_1}(f)P_{H_2}(f)}\right)\widetilde{s}_{H_2}(f)\right)}{|f|^3 P_{L_1}(|f|)P_{H_1}(f)P_{H_2}(f)(1-\Gamma_{H_1H_2}(f))}$$

## Implementation issues/details:

- Noise properties of a 4x product of spectra
- Need to modify the correlation code to take in 3 interferometer channels, condition, etc.
- □(f) and □(f) should be calculated over the entire run -- read them in as frequency series, similar to R(f) data.
- Apply this ONLY to 3X data stretches

# Results from S2 representative spectra