How optimal are wavelet TF methods?

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- **Introduction**
- **Time-Frequency analysis**
- **Comparison with optimal filters**
- **Example with BH-BH merger**
- **Summary**

Introduction

- **Match filter – optimal detection of signal of known form *m(t)* (*M(ω)*)**

$$\left(\frac{S}{N}\right)^2 = \frac{1}{2\pi}\int_{-\infty}^{\infty}\frac{|M(\omega)|^2}{P_n(\omega)}\,d\omega,$$

(Wainstein, Zubakov)

- **Many GW waveforms (like mergers, SN,..) are not well known, therefore other search filters are required.**
- **Excess power filters:**
  - **band-pass filter** **(Flanagan, Hughes: gr-qc/9701039v2 1997)**

$$\varepsilon = \left(\frac{SNR_{BP}}{SNR_{MF}}\right)^{1/2} \approx \frac{1}{\sqrt{2\tau\Delta f}}$$

Δf –filter bandwidth
τ - signal duration

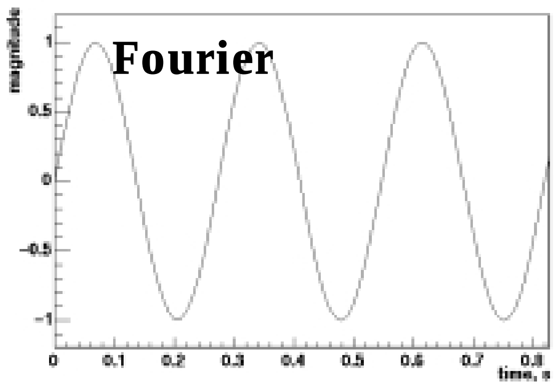
ε for BH-BH mergers ~ 0.2-0.5

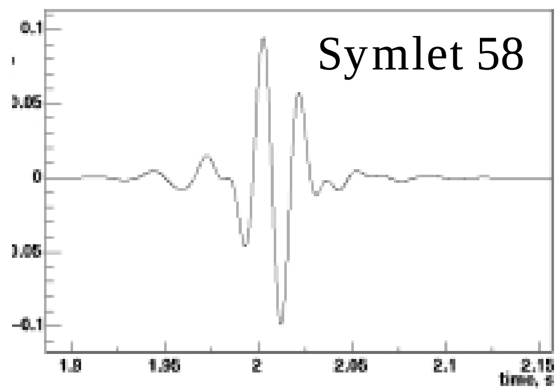
  - **Excess Power: (Anderson et al., PRD, V63, 042003)**
- **What is ε for wavelet time-frequency methods (like WaveBurst ETG)?**

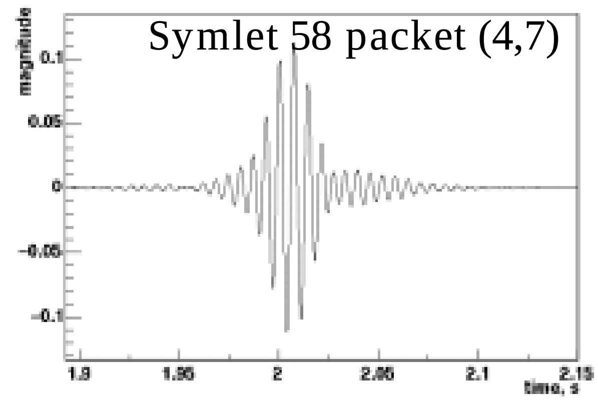
Time-Frequency Transform

- **TF decomposition in a basis of (preferably orthonormal) waveforms *{Ψ(t)}* - "bank of templates"**

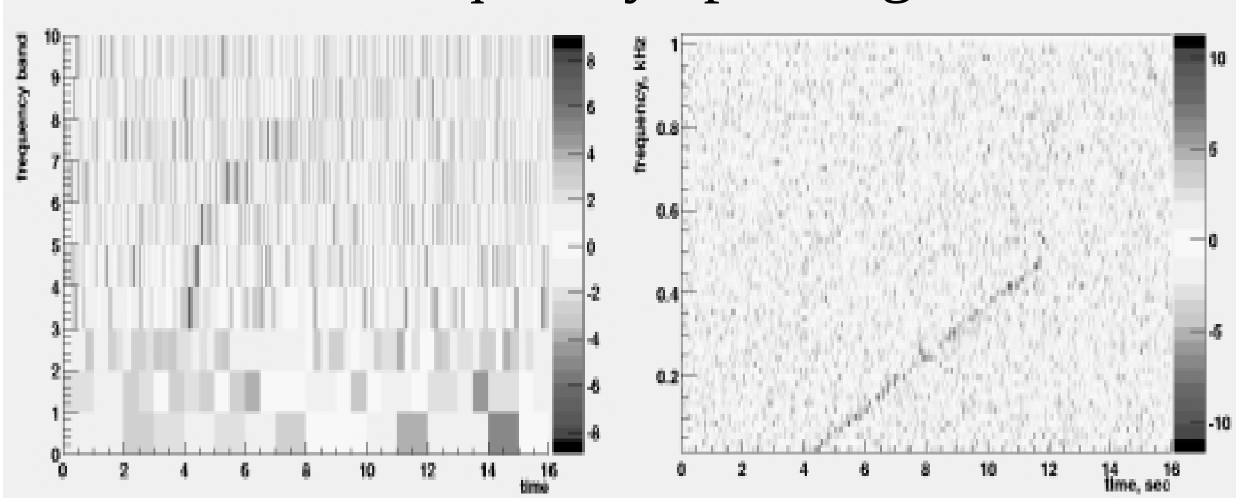
wavelet - natural basis for bursts







time-frequency spectrograms



Time-Frequency Analysis

- **Analysis steps:**
  - **Select "black" pixels by setting threshold x_p on pixels amplitude**
    The threshold x_p defines black pixel probability p
  - **cluster reconstruction – construct an "event" out of elementary pixels**
  - **Set second threshold(s) on cluster strength**
- **Match filter, if burst matches one of the basis functions (template)**

$$\left(\frac{S}{N}\right)^2_{opt} = x^2$$

σ– noise rms per pixel
$x = w/\sigma$ – wavelet amplitude / σ

- **If basis is not optimal for a burst, its energy will be spread over some area of the TF plot**

$$\left(\frac{S}{N}\right)^2_{TF} = \frac{\sigma^2}{\sigma_k^2} \cdot \sum x_i^2$$

$$\varepsilon \approx \sqrt{\frac{\sum x_i^2}{x^2}}\,\frac{\sigma}{\sigma_k} \approx \frac{1}{\sqrt{k}}$$

σ_k – noise rms per k pixels

statistics of filter noise

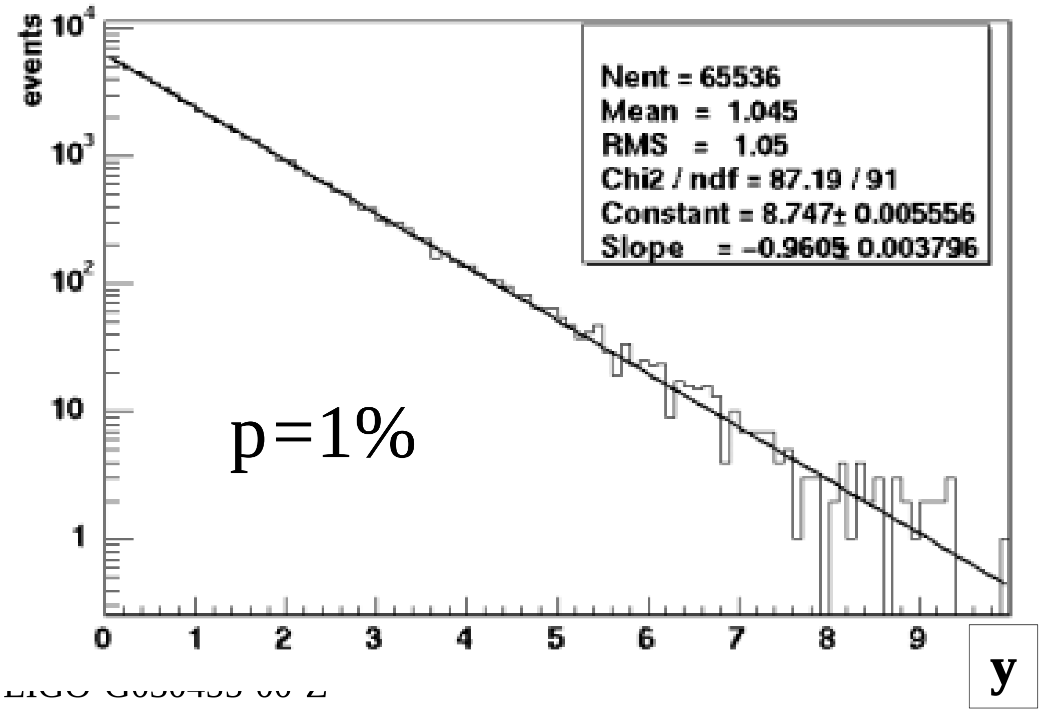
- **assume that detector noise is white, gaussian**
- **after black pixel selection (|*x*|>*x*$_p$)→ gaussian tails**

$$y = \frac{x^2 - x_p^2}{2}, \quad pdf(y) \approx e^{-\alpha y}, \quad \alpha = \left(1 + x_p^{-2}\right)^{-1}$$

- **sum of k (statistically independent) pixels has *gamma* distribution**

$$y_k = \tfrac{1}{2}\sum^{k}\left(x_i^2 - x_p^2\right)$$

$$pdf(y_k) = \frac{y_k{}^{k-1} e^{-y_k}}{\Gamma(k)}$$



z-domain

- ***cluster confidence:*** ***z = -ln*(survival probability)**

$$z(y_k) = -\ln\left(\frac{1}{\Gamma(k)} \int_{y_k}^{\infty} x^{k-1} e^{-x} dx \right)$$

- **noise *pdf(z)* is exponential regardless of *k*.**
- **control false alarm rate with set of thresholds z_t(k) on cluster strength in *z*-domain**

$$f_{alarm} = \sum_k e^{-z_t(k)} f_k$$

cluster rates

- **“canonical” threshold set**

$$z_t(k) = z_0 - \ln(k) \longrightarrow f_{alarm} = p \cdot f_{sampling} \cdot e^{-z_0}$$

data rate

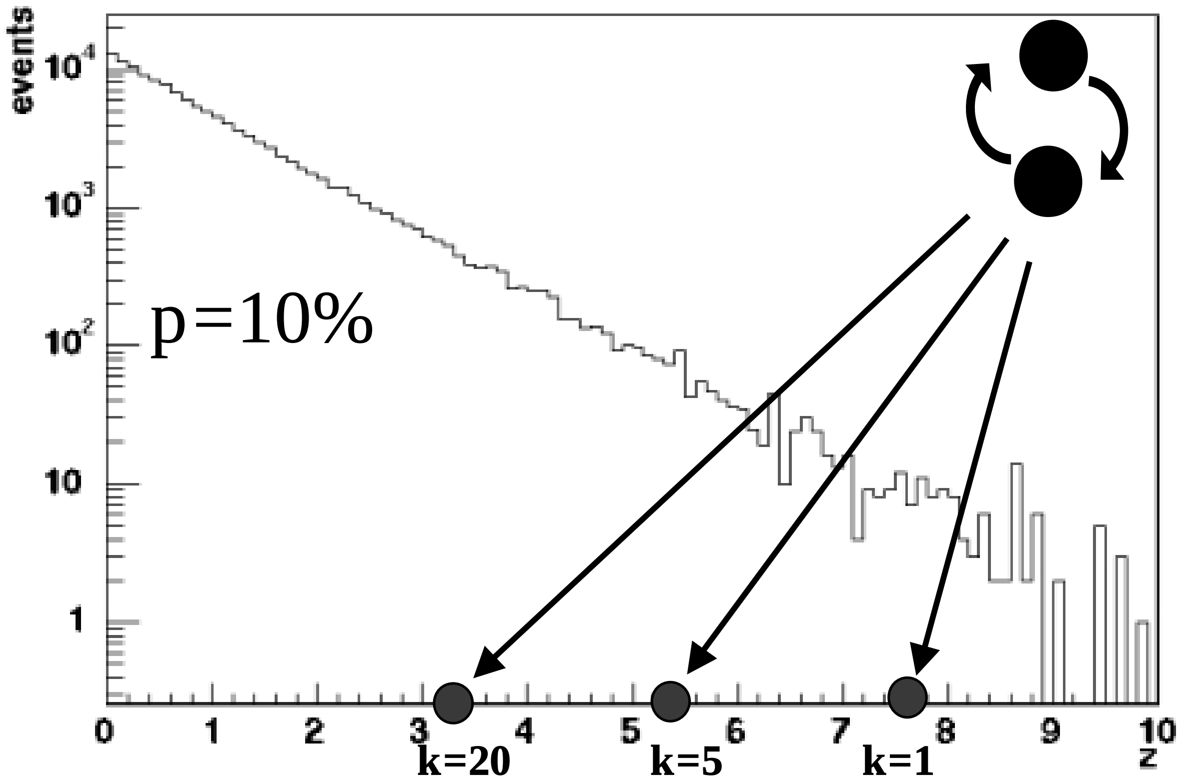
effective distance to source

- **given a source *h(t)*, the filter response in z-domain is different depending on how good is approximation of *h(t)* with the basis functions *{Ψ(t)}***
- **d_1 - distance to "optimal" source (k=1)**
- **d_k – distance to "non-optimal" source with the same z-response**

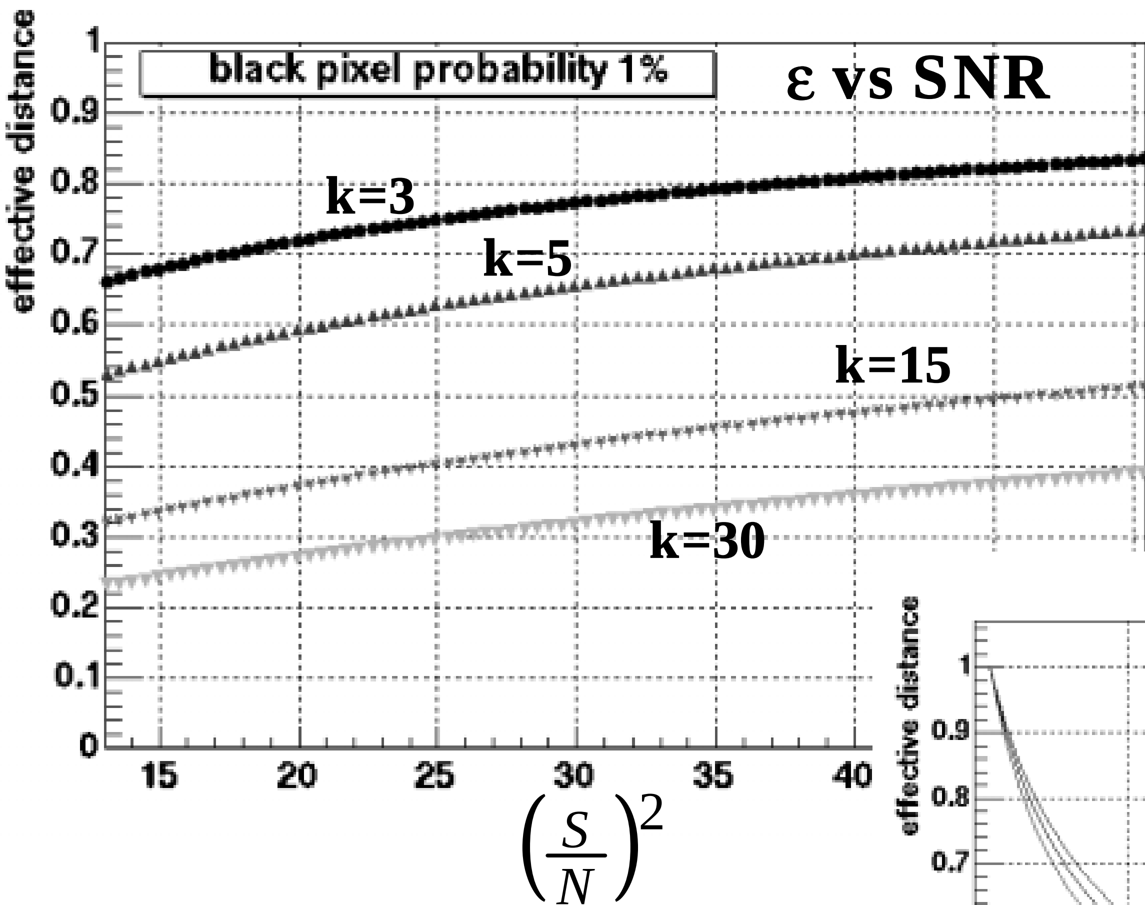
effectiveness:

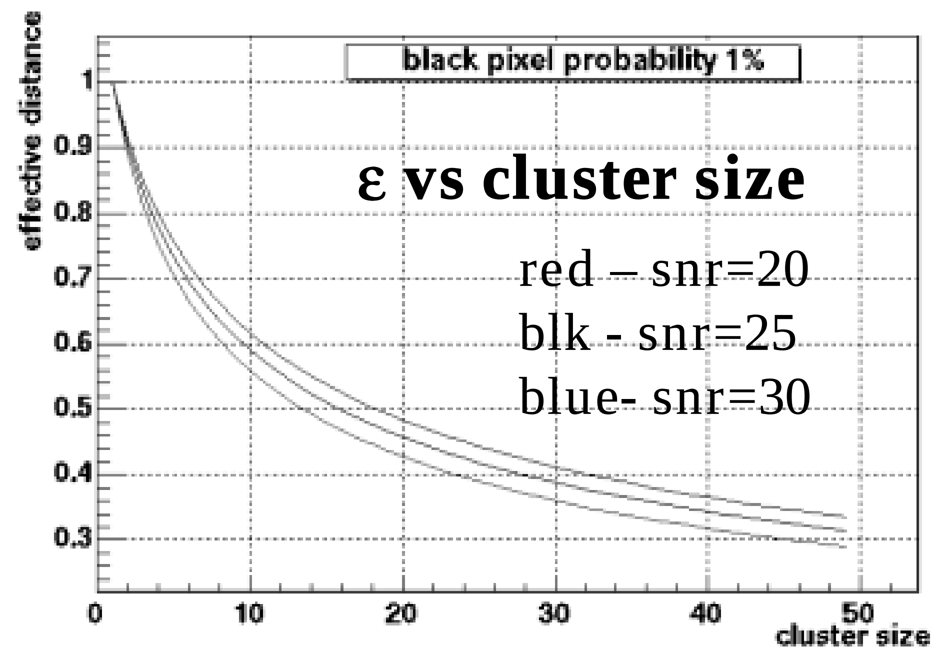
$$\varepsilon = d_k / d_1$$

same significance & false alarm rate as for MF



effective distance(snr,k)



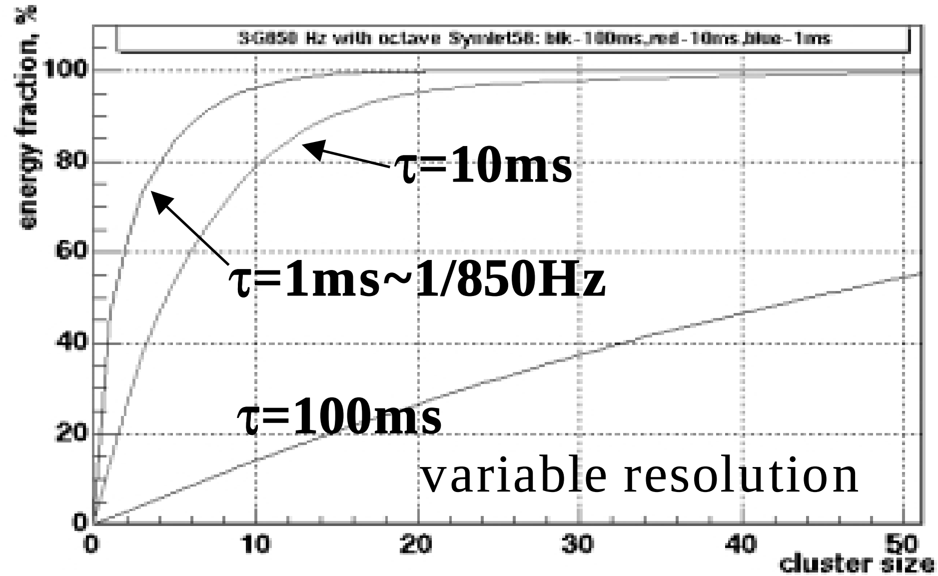


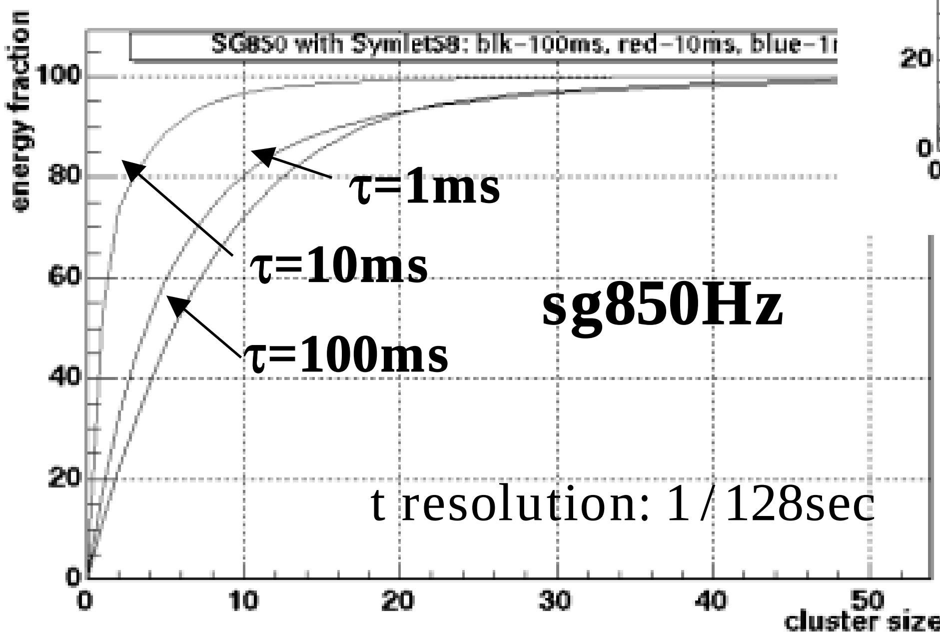
cluster size

- **select transforms that produce more compact clusters**

  resolution, properties of wavelet filters, orthogonality

octave resolution:
good if $f_c \sim 1/\tau$ →





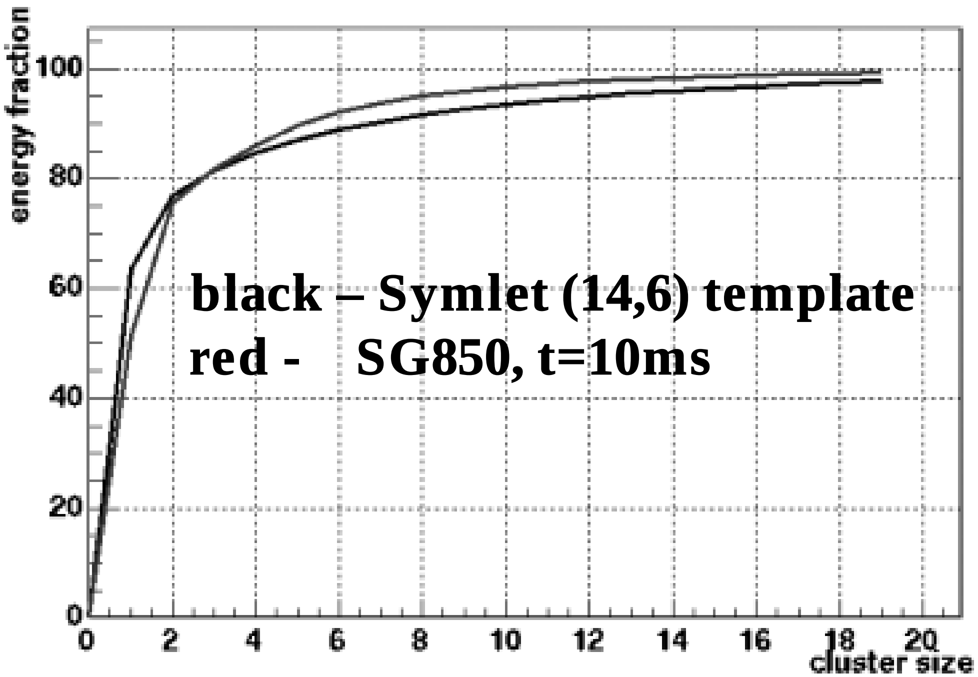
← **const resolution:**
more robust

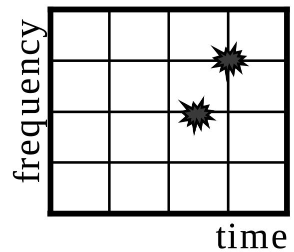
to cover larger parameter space the analysis could be conducted at several different resolutions

response to “templates” $\{\Psi(t)\}$

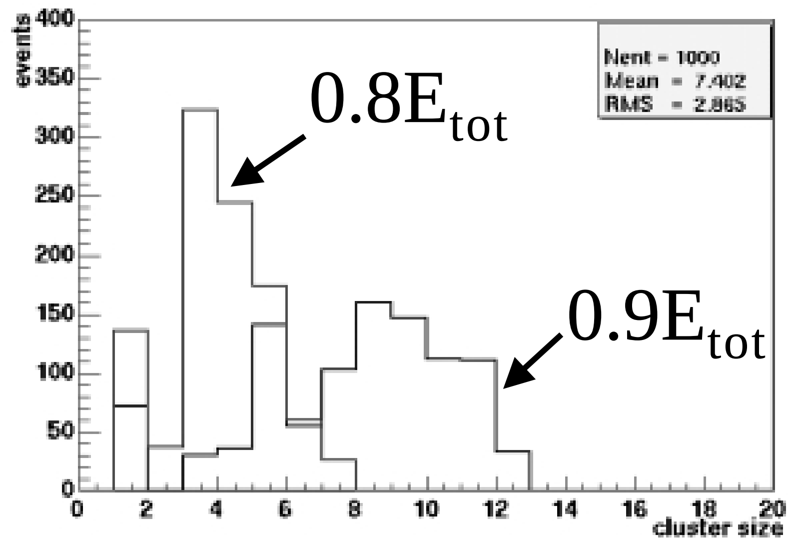
- $h(t+\delta t)=\Psi_i(t)$, $0<\delta t<\Delta t$, Δt – time resolution of the $\{\Psi(t)\}$ grid

time resolution 1/128 sec





optimal
“quasi-optimal”



- **Average cluster size of ~5 at optimal resolution.**
- **Doesn’t make sense to look for 1-pixel clusters**

BH-BH mergers

- **BH-BH mergers** **(Flanagan, Hughes: gr-qc/9701039v2 1997)**

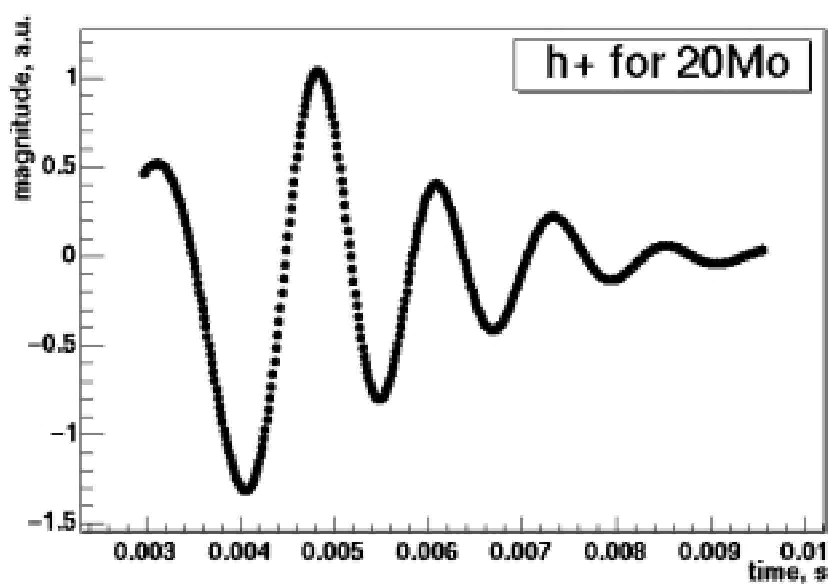
start frequency: $$f_{start} \approx \left(\frac{0.02}{M}\right) = 205Hz \cdot \left(\frac{20M_o}{M}\right)$$

duration: $$\tau \approx 50M = 5ms \cdot \left(\frac{M}{20M_o}\right)$$

bandwidth: $$\Delta f \sim f_{qnr} \approx \left(\frac{0.13}{M}\right) = 1300Hz \cdot \left(\frac{20M_o}{M}\right)$$

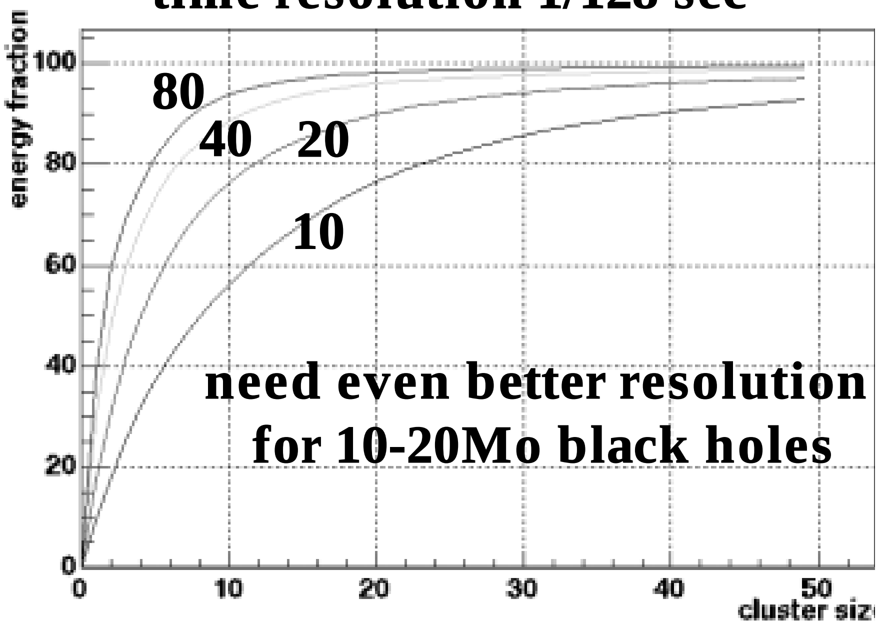
- **BH-BH simulation**

(J.Baker et al, astro-ph/0202469v1)

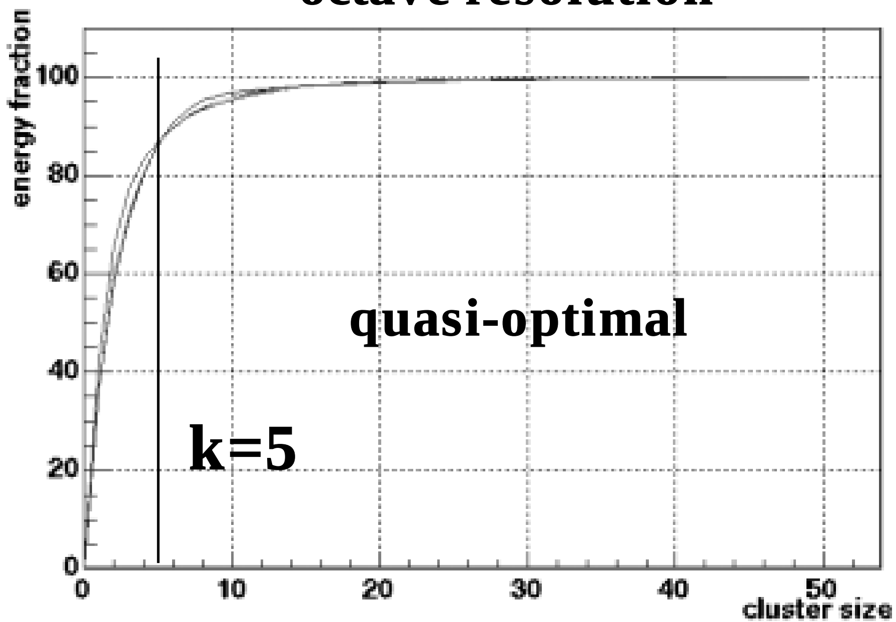


response to simulated BH-BH mergers

time resolution 1/128 sec



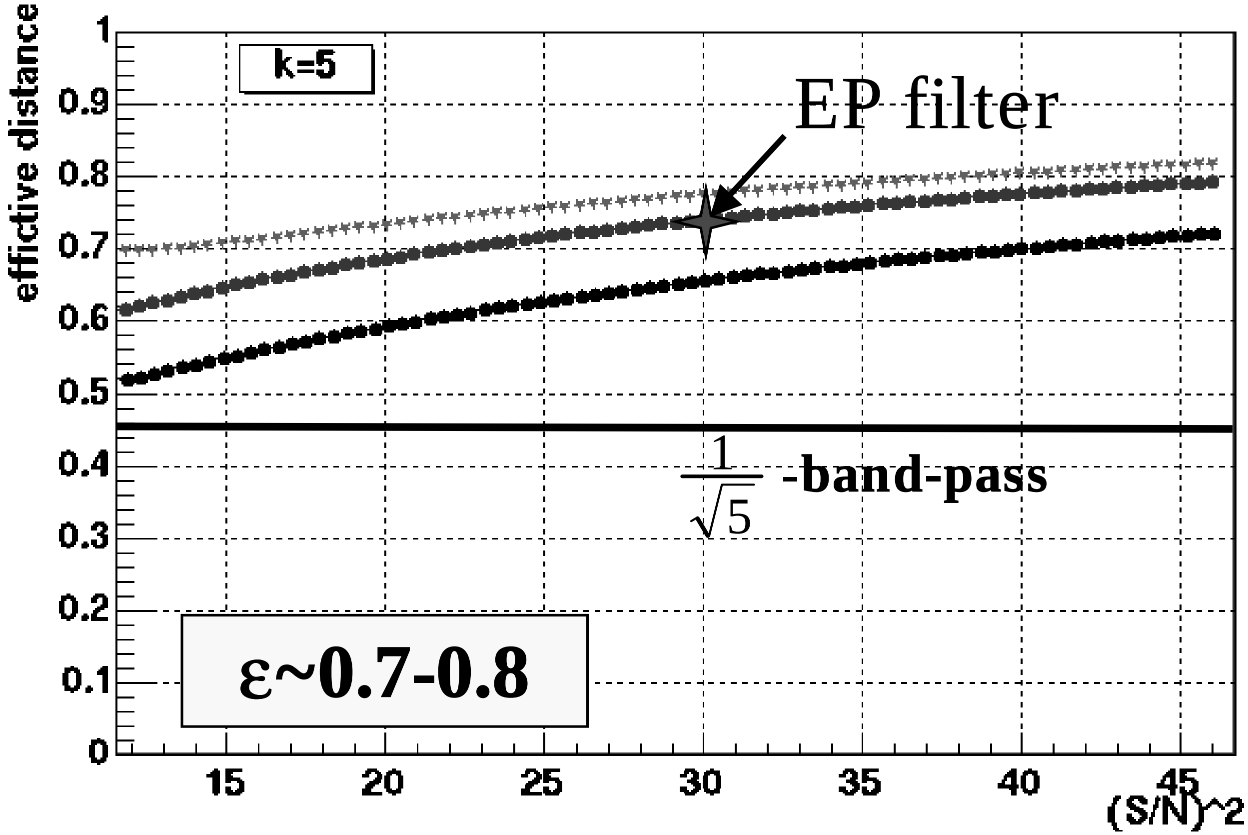
octave resolution



- **resolution should be >=10ms**
- **If proven by theory, that for BH-BH mergers** $f_{merger} \sim 1/\tau$**,**
  it allows *a priori* selection of a "quasi-optimal" basis

ε for BH-BH mergers

k>1,p=10%
p=10%
p=1%



- **ways to increase ε**
  - ➢ **higher black pixel probability**
  - ➢ **ignore small clusters (k=1,2), which contribute most to false alarm rate and use lower threshold for larger clusters.**

Summary

- **wavelet and match filter are compared by using a simple approximation of the wavelet filter noise.**
- **filter performance depends on how optimal is the wavelet resolution with respect to detected gravity waves.**
- **filter performance could be improved by increasing the black pixel probability and by ignoring small (k=1,2) clusters**
- **expected efficiency for BH-BH mergers with respect to match filter: 0.7-0.8**