Possible Eccentric Black-hole Binary Mergers and their Implications to LIGO detections

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Outline

- Overview
  - Globular clusters
  - Binary black hole (BH) mergers

- Eccentricity evolution of merger systems under Kozai Mechanism
- Implications to LIGO GW detections

Globular Clusters

M_{BH}~4000 Msun 20,000 Msun

(stolen from the STScI web site)

Globular Clusters

- Excellent birthplace for BH binaries
  - after 10^7 yrs, > 20-25 Msun stars evolved into BHs :
  - BHs are the heaviest objects left

    2-body relaxation, energy equipartition, mass segregation:
  - BHs sink to the core
- BH binaries form in the core
  - 3-body relaxation process
- LIGO sources if merge within Hubble time

- Binary Merger Time Scale

$$\tau_{\rm GR} \approx 6 \times 10^{10} \frac{(a/{\rm AU})^4 (\epsilon/0.01)^{3.5}}{(m_0 + m_1) m_0 m_1 / {M_\odot}^3} \quad {\rm yr},$$

* a: semi-major axis, ε = 1-e^2 , m_0, m_1: masses

* quadrupole approximation for gravitational radiation

* Need to reduce a or ε to have T_{GR} < Hubble time

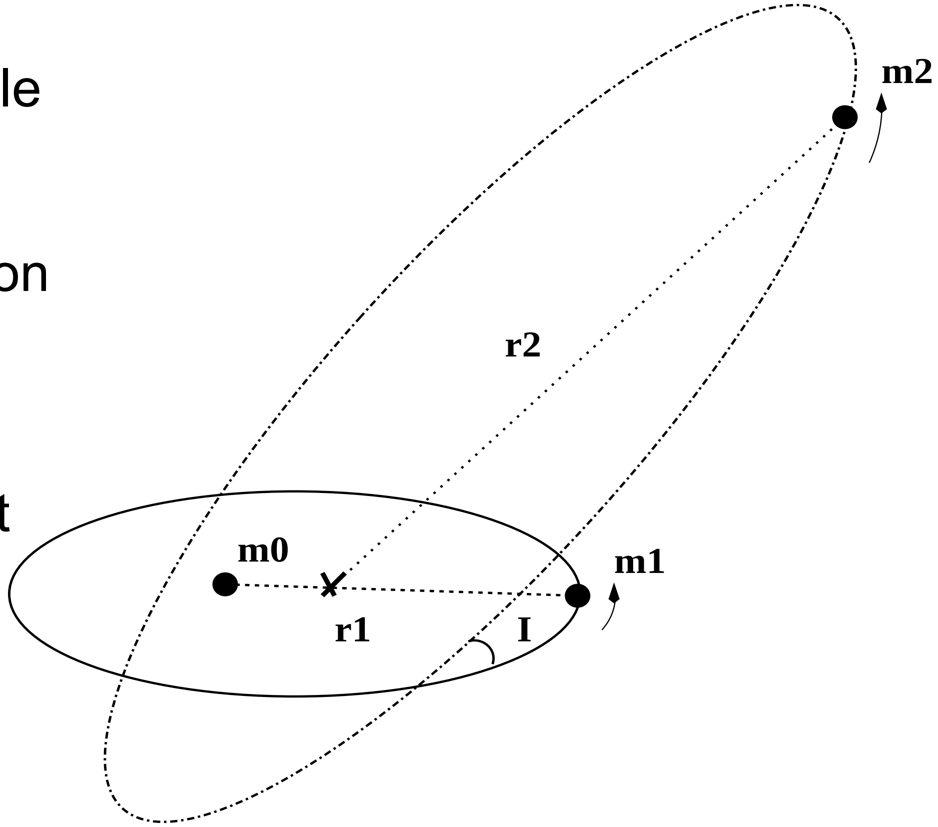
Will BH Binaries Merge within GCs ?

- Binary-single interaction
  - throws out most BH binaries
  - ~ 8 % retained within lifetime
  - recoil velocity associated with hardening
  - major contribution to current BH-BH merger even rate
- Binary-binary interactions
  - produce hierarchical triple systems
  - ~ 20-50 %
- Kozai mechanism in triple systems
  - drives inner binaries to extreme eccentricity
  - shortens $T_{GR,}$
  - ~70 % inner binaries could merge successfully
    - before interrupted by interactions with field stars
    - potentially important LIGO sources
  - Subsequent merger ==> formation of Intermediate mass BHs

(Miller & Hamilton 2002)

Kozai Mechanism

- Operate in hierarchical Triple systems ($r_2 >> r_1$)
  (Kozai 1962)
- Orbital averaged perturbation
  - $H=H_0 + H'_1 + H'_2$
  - Equations of motion
  - $T_{evol} >> P_{orb}$
- Oscillation of e_1 and g_1 w/ t
  - extreme e_1 for $I_0 \sim I_c$
  - e = 1 in classical limit
- a_1, a_2, e_2 fixed
  - ($E_{1,2}$, $|J_2|$ conserved)
- Cyclic exchange of angular momentum between inner and outer binaries



These merger systems are associated with extremely high eccentricities !

Why Study Eccentricities

- High e systems were not expected for LIGO detections
  - Gravitational radiation reaction is very efficient in circularizing the orbit
    - » e.g., Hulse Taylor NS-NS system
  - Current effort has been focused on GWs from circular orbits
- Circular templates might not be good enough for optimal detections of high -e systems
- Eccentricity distribution of these systems in LIGO band

Our Work

- Study evolution of individual system
  - equations of motion, estimate ε_{min}
  - $f^{m}_{GW} \sim (a\varepsilon)^{-1.5}$
- Find parameter space for successful mergers
  - consider a, e0, a1,a2,I0, g10, m's
  - merge before disrupted by a field star
- Derive eccentricity distribution in LIGO frequency band
  - f^{m}_{GW} =10, 40, 200 Hz

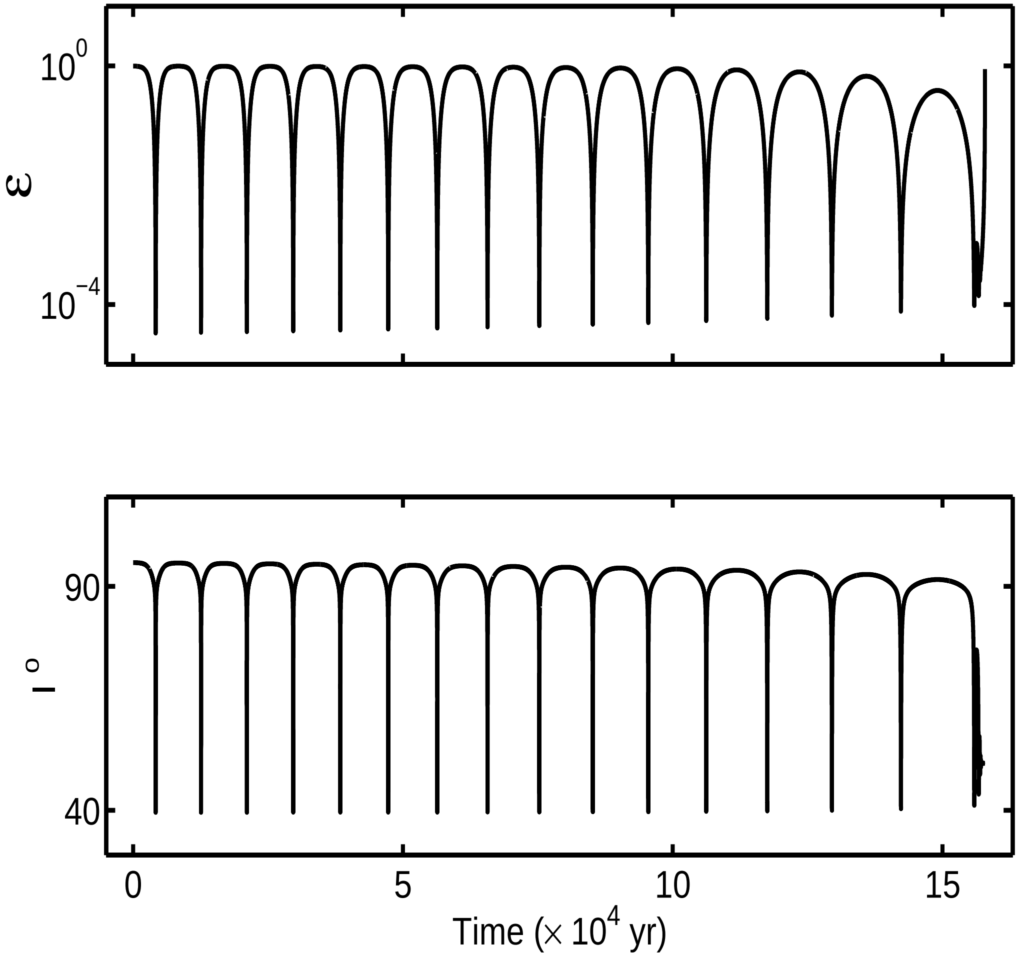
II. Evolution of Individual Triple System

Three competing effects

- Kozai Mechanism
    - eccentricity enhancing
- Gravitational radiation reaction (GR effect)
    - extract energy and angular momentum
    - orbital decay
    - circularizing
    - important near ε_{min} , negligible otherwise
    - rapid transition once GR effect dominates
- Post-Newtonian periastron precession (PN effect)
    - mess up the phase relation
    - introduce fast oscillations to destroy Kozai cycle

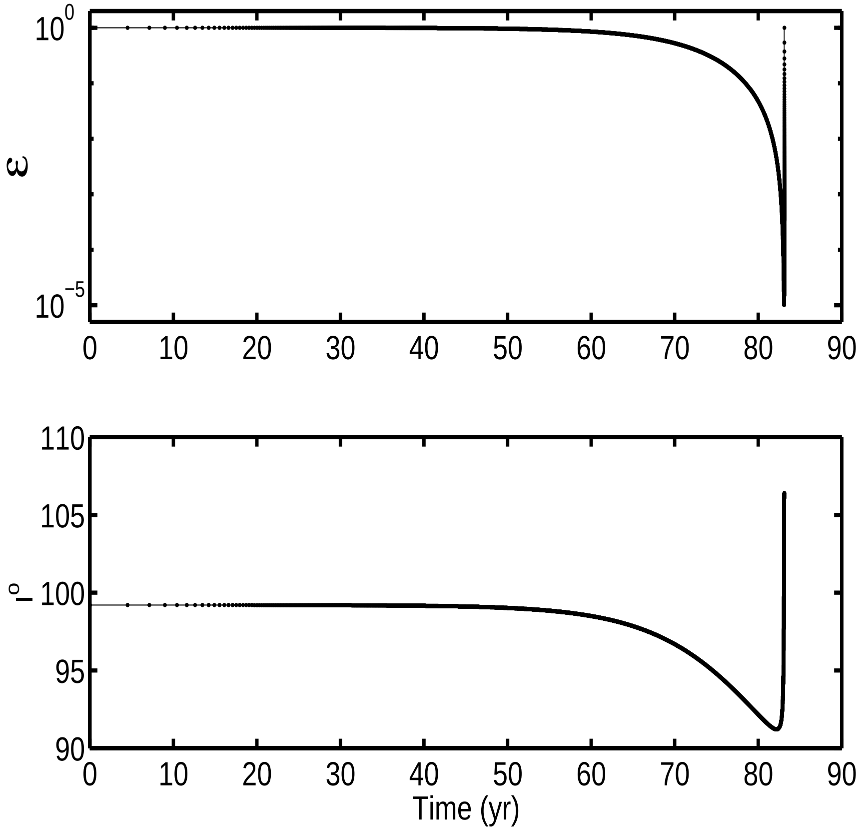
I: Merge after many Kozai cycles

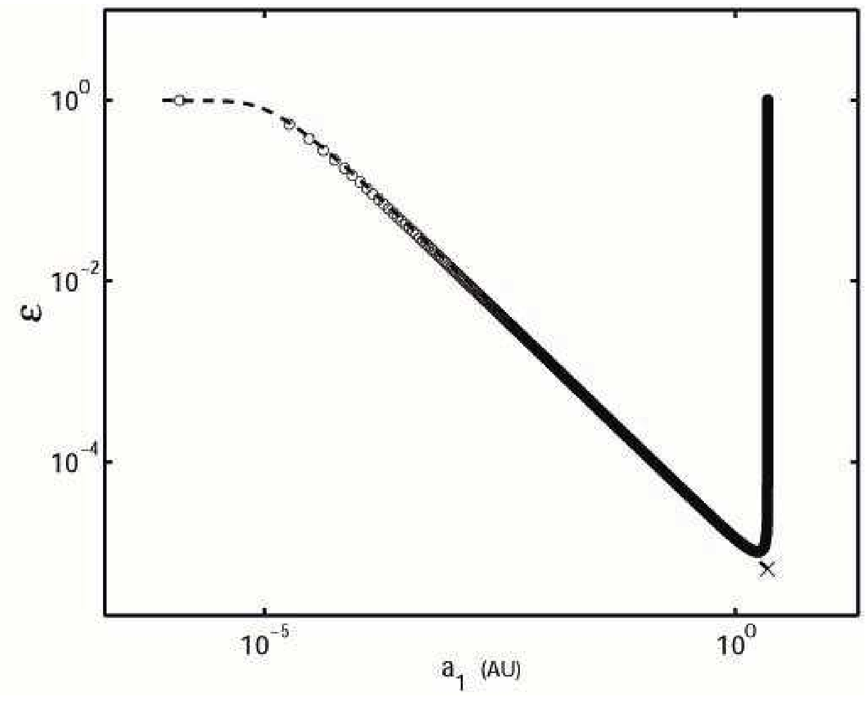
- Integrated from the ODE equations
- Typical case that the PN effects dominates before the GR effect
- System spends most time at low eccentricities
- Gradual change in the beginning
- Fast oscillations by PN effects
- GR effect dominates near ε_{min}
- Fast transition



* $\varepsilon=1-e^2$

II. Merge within one Kozai cycle





- I_0 ~I_c
- x--- : predicted evolution
- typical case of extreme eccentricity

Parameter Space

- mass: 10 M_{sun}
    - 3-20 Msun for known galactic stellar mass BHCs
- a_1 =0.2 - 30 AU
    - lower limit: not kicked out of GC
    - higer limit: not disrupted by field stars
- a_2/a_1 =[3,5,10,20,30]
    - a_2/a_1 >3: required by the stability of the triple system
- e_{10},e_{20} =0.01-0.901

- g_{10}=0-90^o , uniform I_0

Requirement for Successful Mergers

* the system should have enough time to reach extreme eccentricity
* merge should occur before the system is disrupted by encounters with field stars

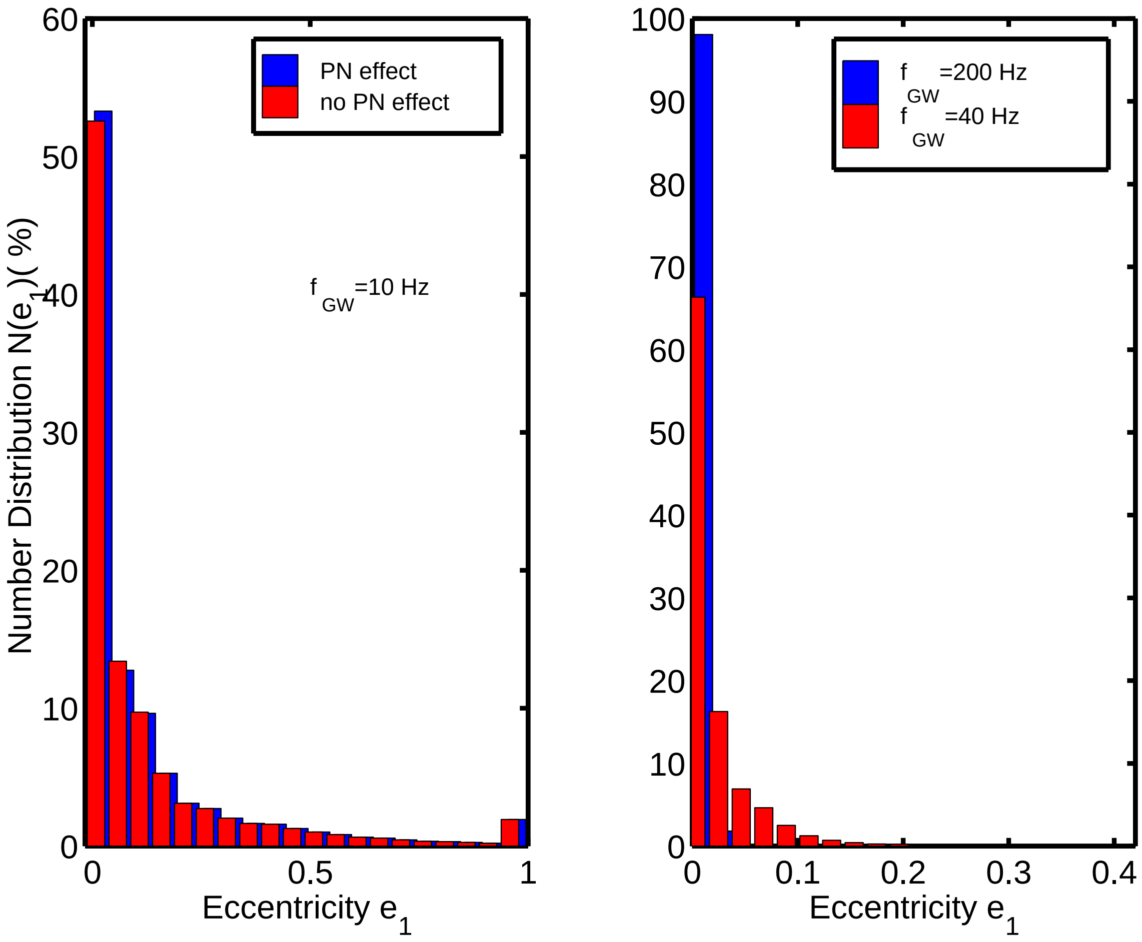
$$\tau_{\rm evol} < \tau_{\rm enc},$$
$$\frac{\tau_{\rm GR}(a_1, \epsilon_{\rm min})}{\sqrt{\epsilon_{\rm min}}} < \tau_{\rm enc}.$$

The time scale for disruption (the same as the stellar encounter time scale) is given as

$$\tau_{\rm enc} \approx 6 \times 10^5 n_6^{-1} \frac{\rm AU}{a_2} \frac{10 M_\odot}{M_2} \ {\rm yr},$$

where the number of stars in the globular cluster is $N = 10^6 n_6$.

III. Eccentricity Distribution


Number Distribution N(e1)(%)
60
50
40
30
20
10
0
0
0.5
1
Eccentricity e1
PN effect
no PN effect
fGW=10 Hz
100
90
80
70
60
50
40
30
20
10
0
0
0.1
0.2
0.3
0.4
Eccentricity e1
fGW=200 Hz
fGW=40 Hz

Conclusion

- At 10 Hz:
  e >0.1 : 30 %,
  e~1 : 2 %

  Eccentricity might be important for advanced LIGO

- At 40 Hz:
  e<0.2
- At 200 Hz:
  e<0.04

  * Consistent with $e \sim f^{m}_{GW}{}^{-19/18}$
  * Eccentricity is probably irrelevant for initial LIGO for this type of mergers. But it might be important for advanced LIGO