First LIGO/GEO Upper Limits on Pulsar Gravitational Emissions

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For the Pulsar Upper Limits Working Group
of the LIGO Scientific Collaboration

PAC Meeting
June 5–6, 2003

Executive summary

- S1 science run took 3 weeks of data (Aug. 23 – Sep. 9, 2003) on 4 detectors (LIGO L1, H1, H2, and GEO600).

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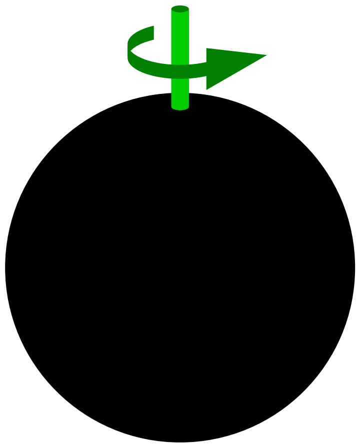
- For this pulsar, $h_0 < 1.4 \times 10^{-22}$ corresponds to ellipticity ratio (non-axisymmetry) $\epsilon < 1.1 \times 10^{-4}$.

Outline

I. Gravitational waves from pulsars
II. Frequentist analysis method
III. Bayesian analysis method
IV. S2 and beyond

GWs from pulsars

- Pulsars (spinning neutron stars) are known to exist!



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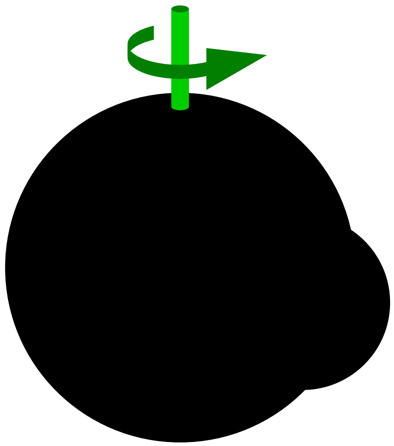

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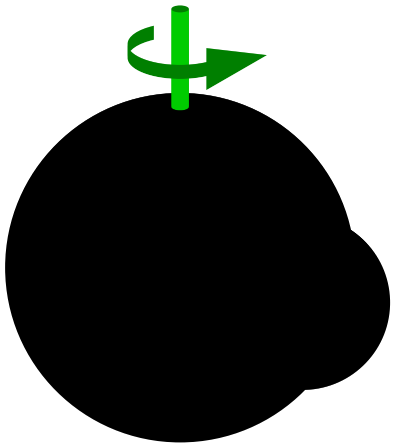

$$h(t) \quad = \quad h_0 \;\; \mathcal{A}(\phi_0, \iota, \psi;\; t) \;\; e^{i\Phi(t)}$$

$$\begin{aligned}
\mathcal{A}, \Phi \quad &= \quad \text{Amplitude and phase functions} \\
h_0 \quad &= \quad \text{intrinsic amplitude} \propto \epsilon \\
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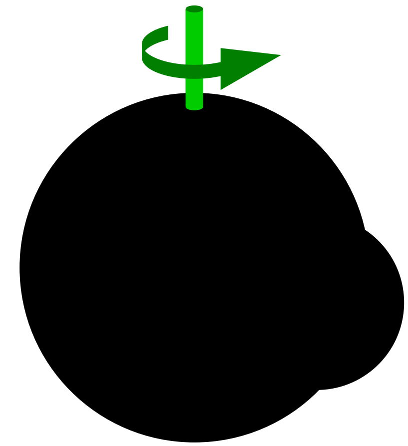

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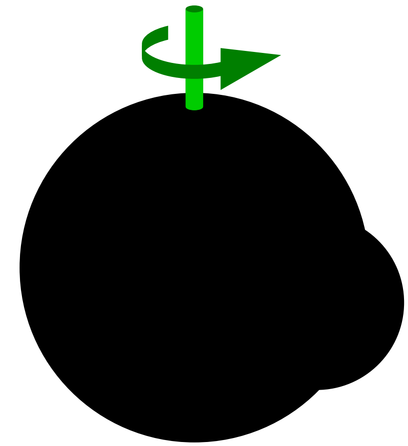

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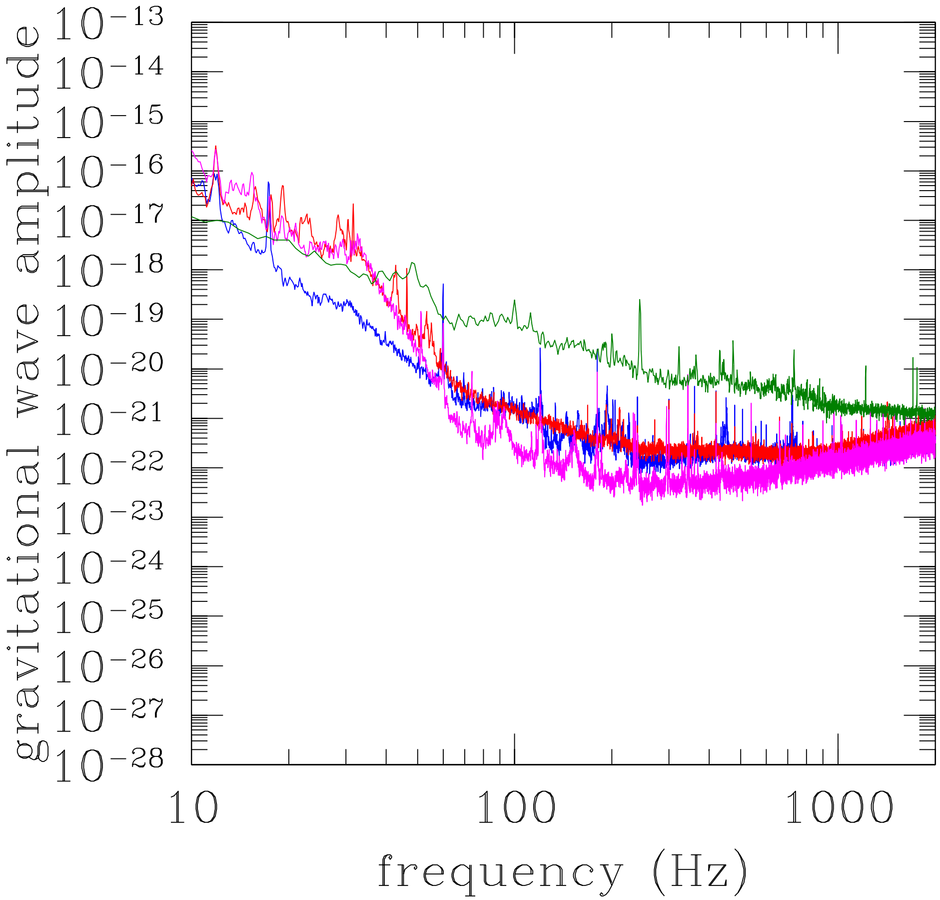
- $$\text{(Signal-to-noise)}^2 \sim \int_0^T \frac{h^2(t)}{S_h(f_{\rm gw})}\, dt$$

GWs from pulsars

Instrumental sensitivity during S1 (1% false alarm thresh., 408 hours):



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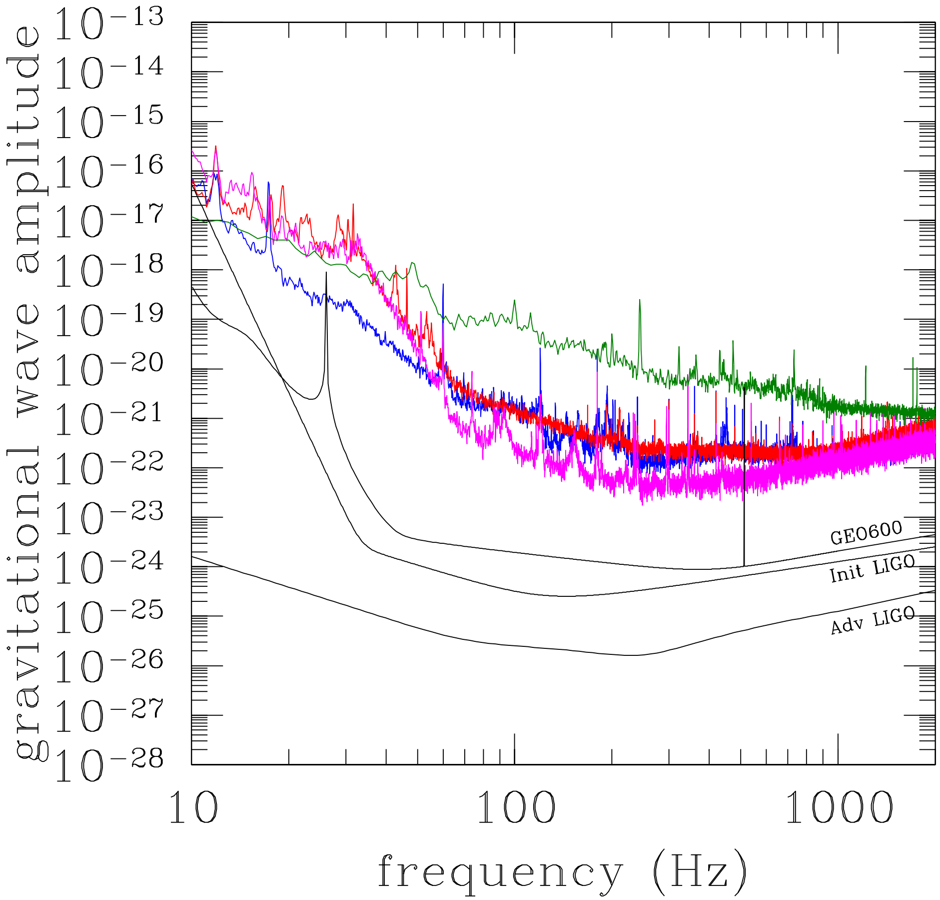

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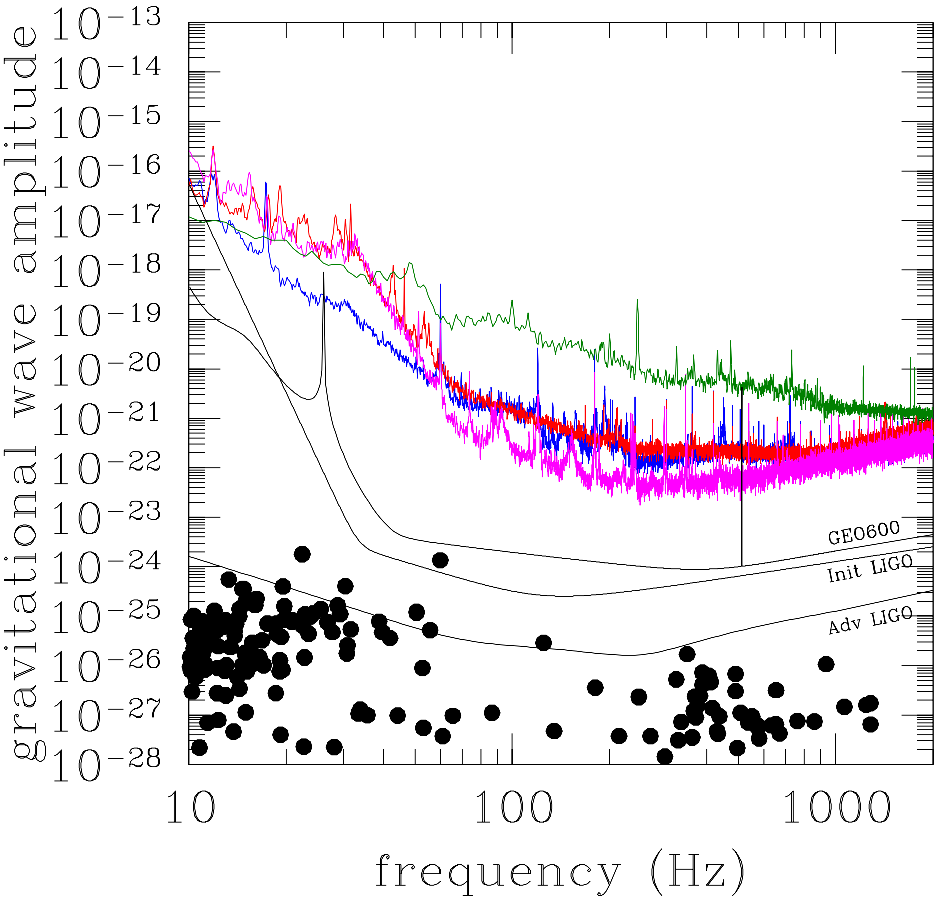

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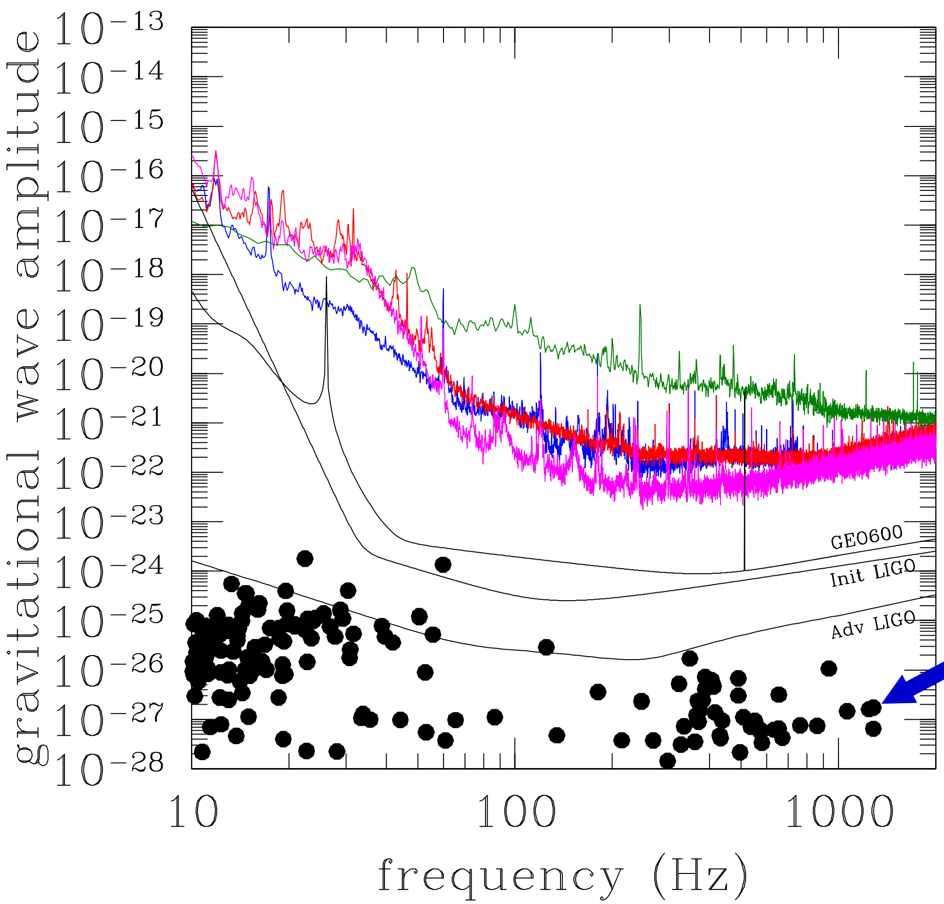

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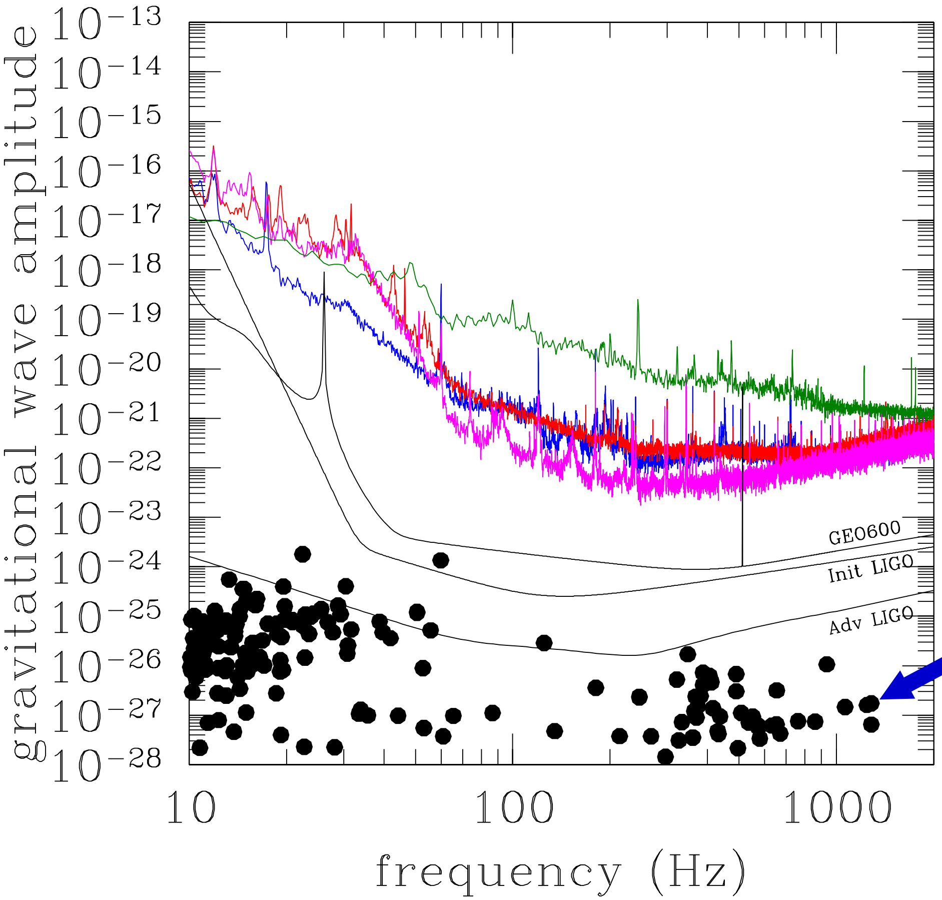

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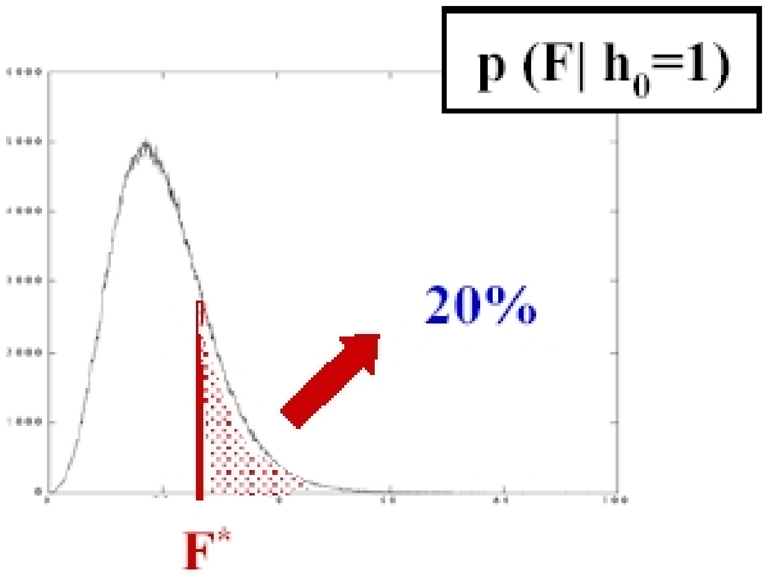

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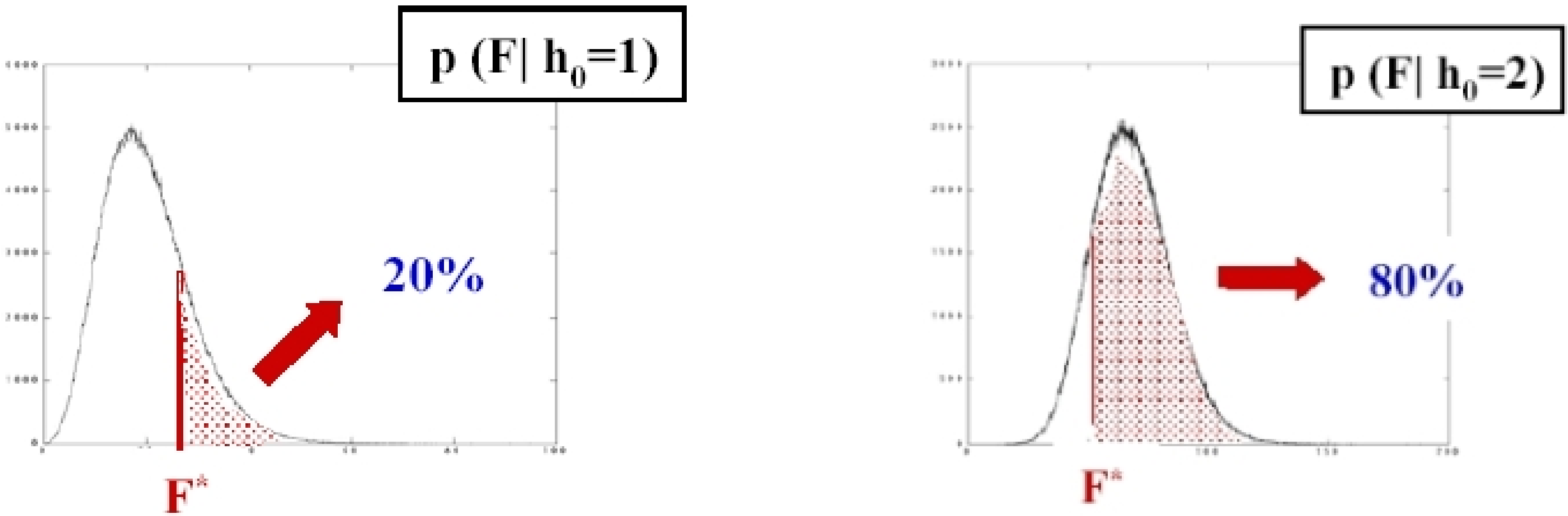

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- 95% frequentist upper limit h_{95}^* is the value such that, for repeated trials with a signal $h_0 > h_{95}^*$, we would obtain $\mathcal{F} > \mathcal{F}^*$ more than 95% of the time:

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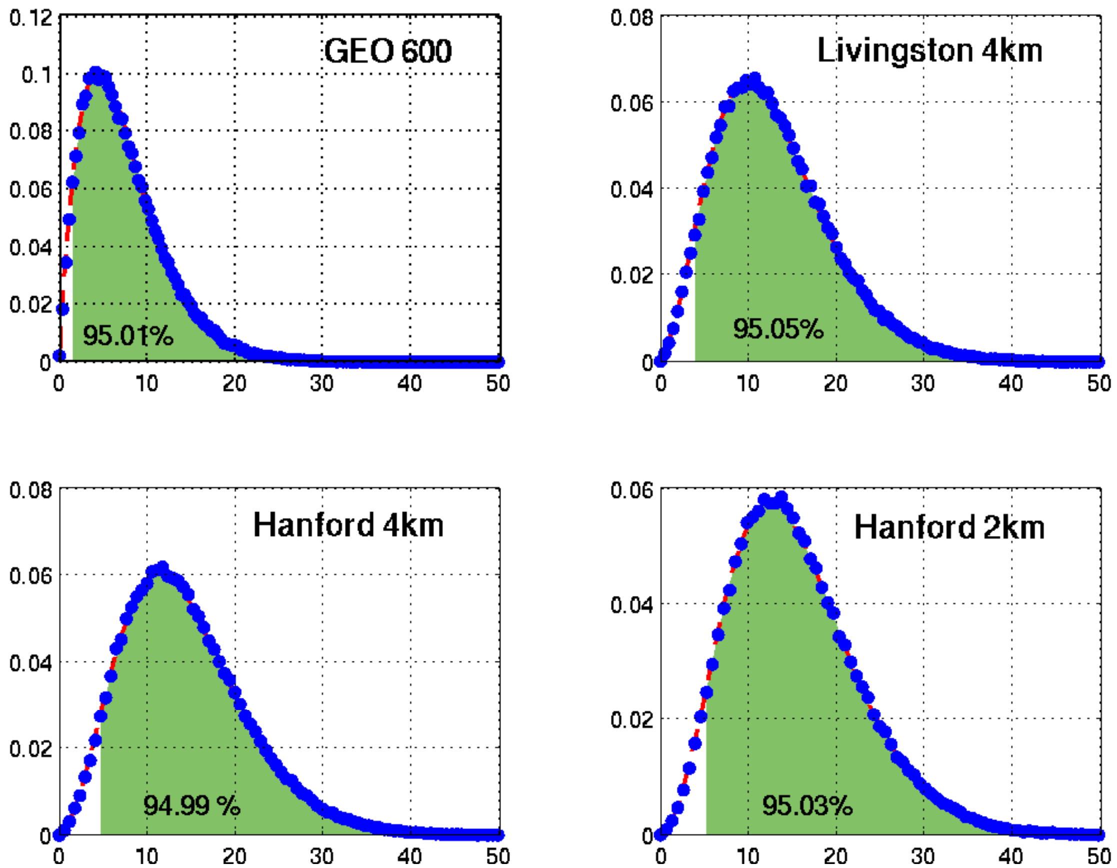
- Extra detail: Compute $p(\mathcal{F}|h_0)$ via signal injection, using *worst possible* orientation ψ, ι. This gives a *conservative* upper limit.

Frequentist analysis

Probability distributions:



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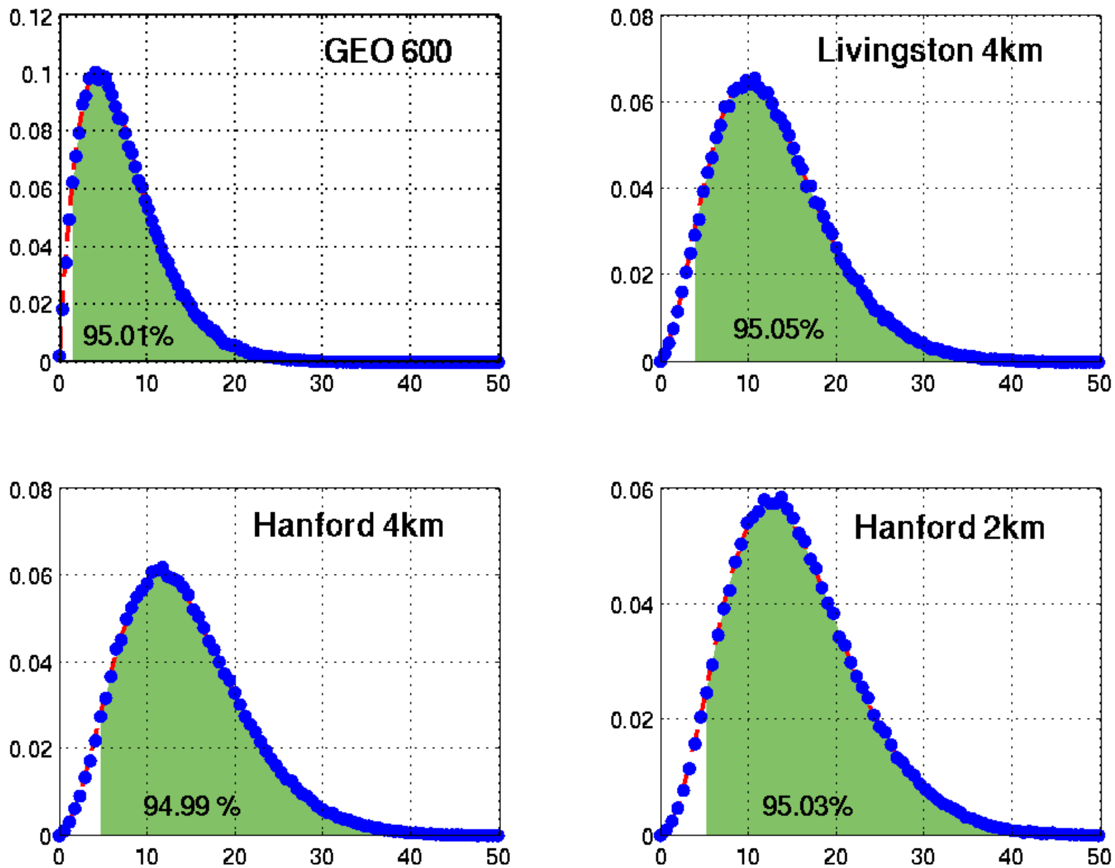

- 95% upper limits h^*_{95}:

GEO	(600m)	1.9×10^{-21}
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H1	(4km)	6.4×10^{-22}
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- Actual h_0 has a 95% propability of lying below h_{95}, defined by:

$$0.95 = \int_0^{h_{95}} dh_0 \;\; p(h_0|\text{data})$$

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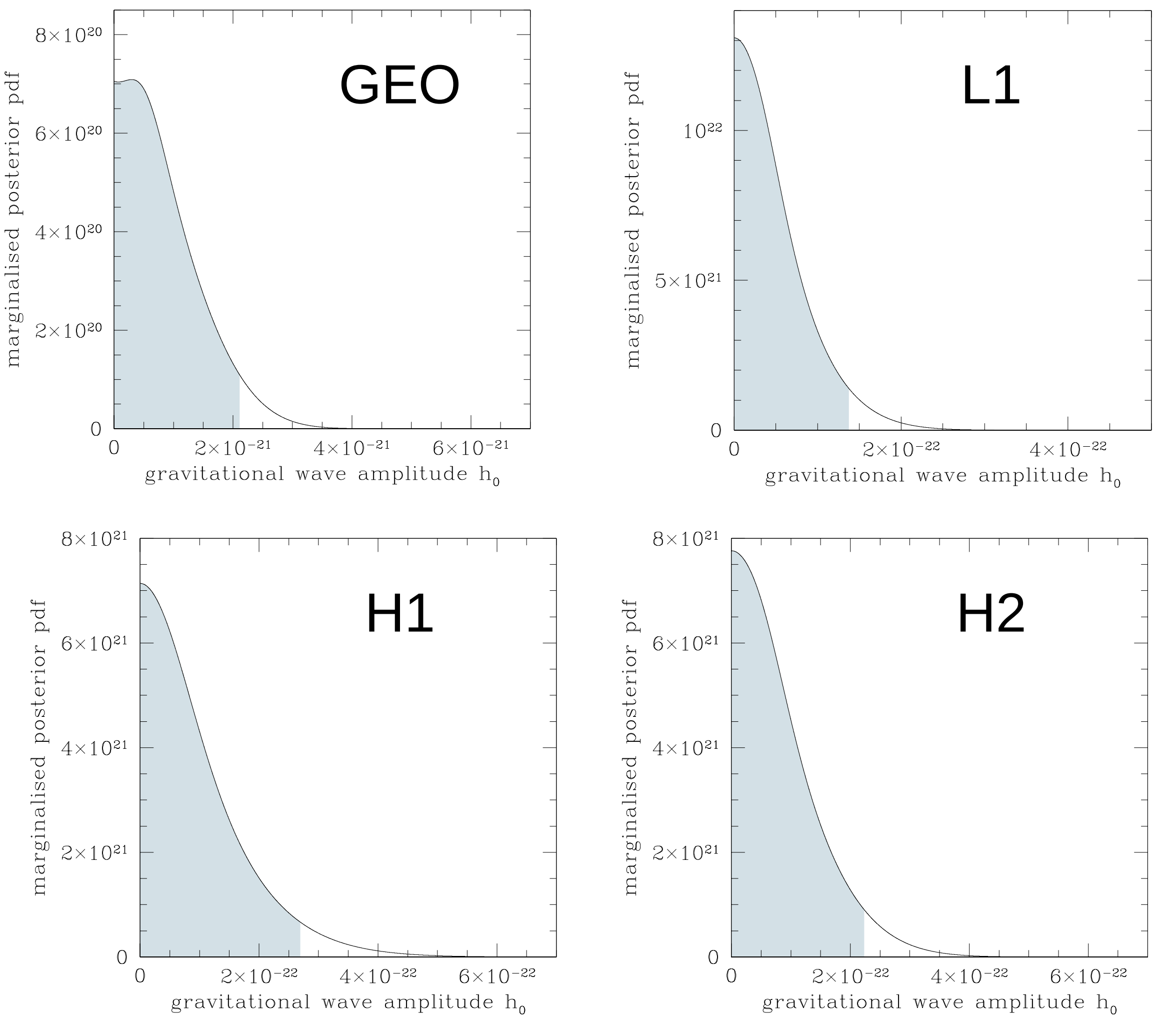
Posterior probability distributions:



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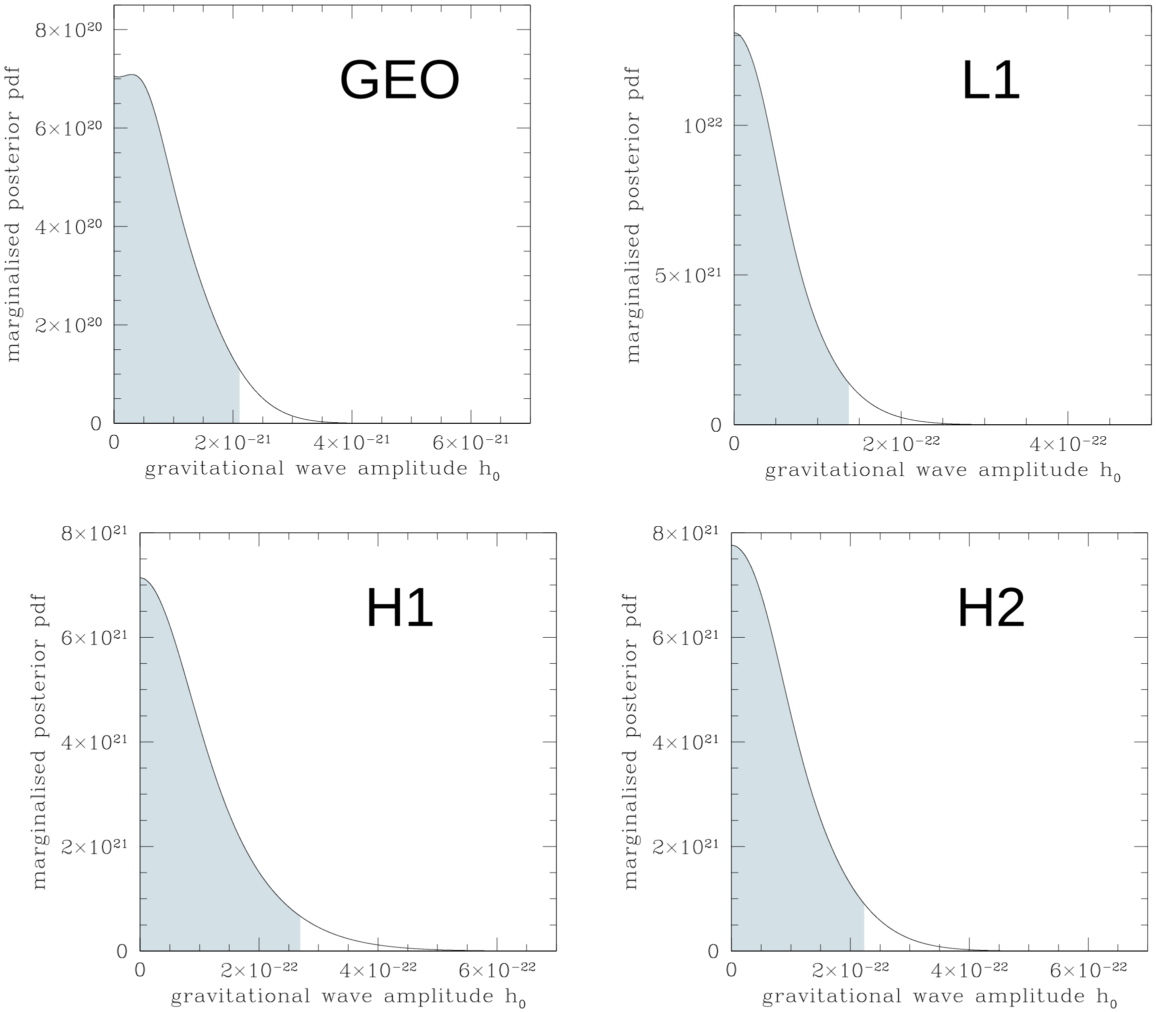

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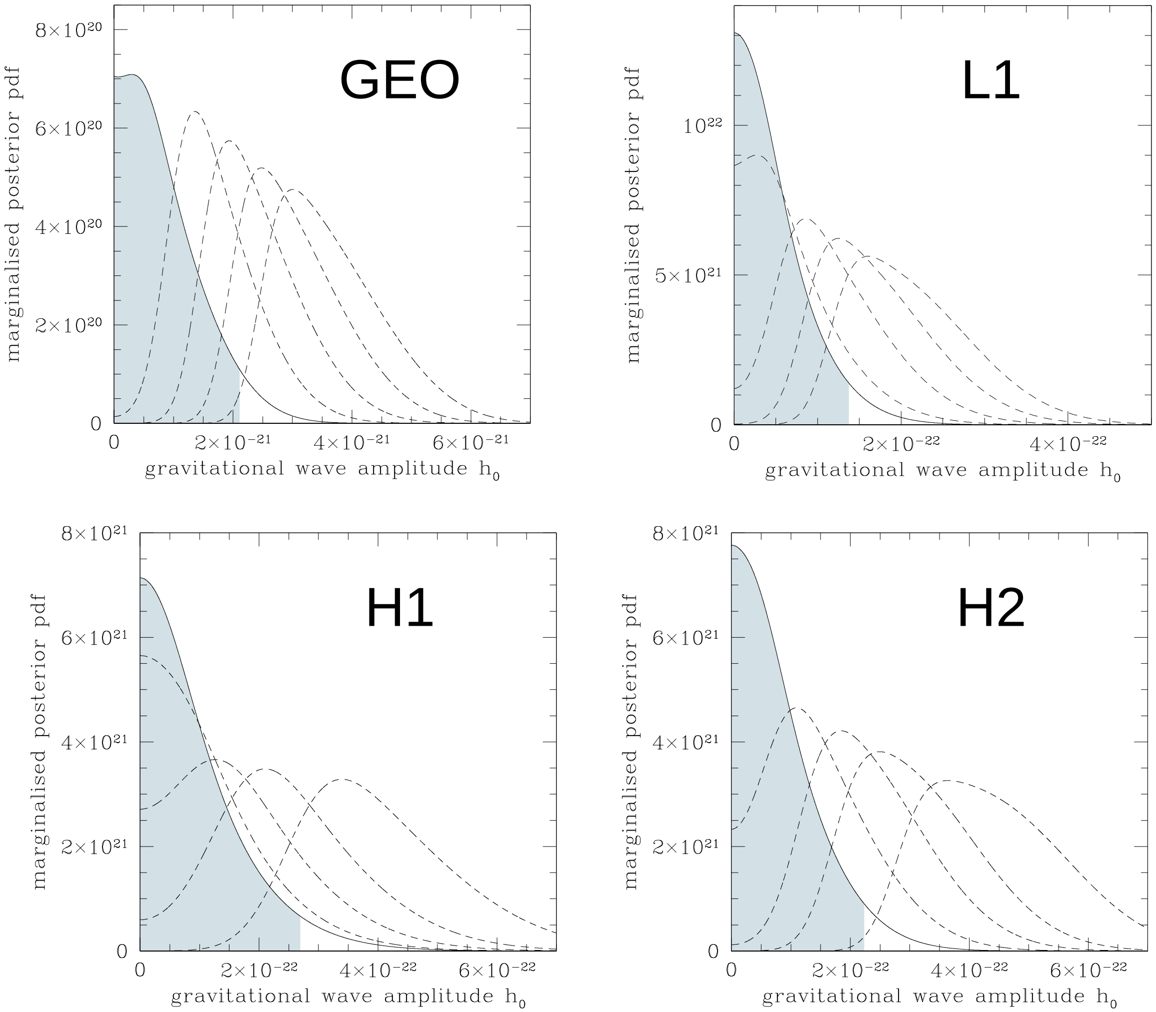

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- Can inject simulated signal to see how PDF changes.

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- Much of the discrepancy comes from how these methods handle ϕ_0, ι, ψ:
  - ⋆ Frequentist: Assume worst-case ϕ_0, ι, ψ
  - ⋆ Bayesian: Marginalize over ϕ_0, ι, ψ

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- Beyond S2:
  - ⋆ All-sky, all-frequency surveys.
  - ⋆ Progress to making actual detections! (Advanced LIGO)