Mexican-Hat (Flat-Topped) Beams for Advanced LIGO

LIGO-G030137-00-Z

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Talk by Kip at LSC Meeting

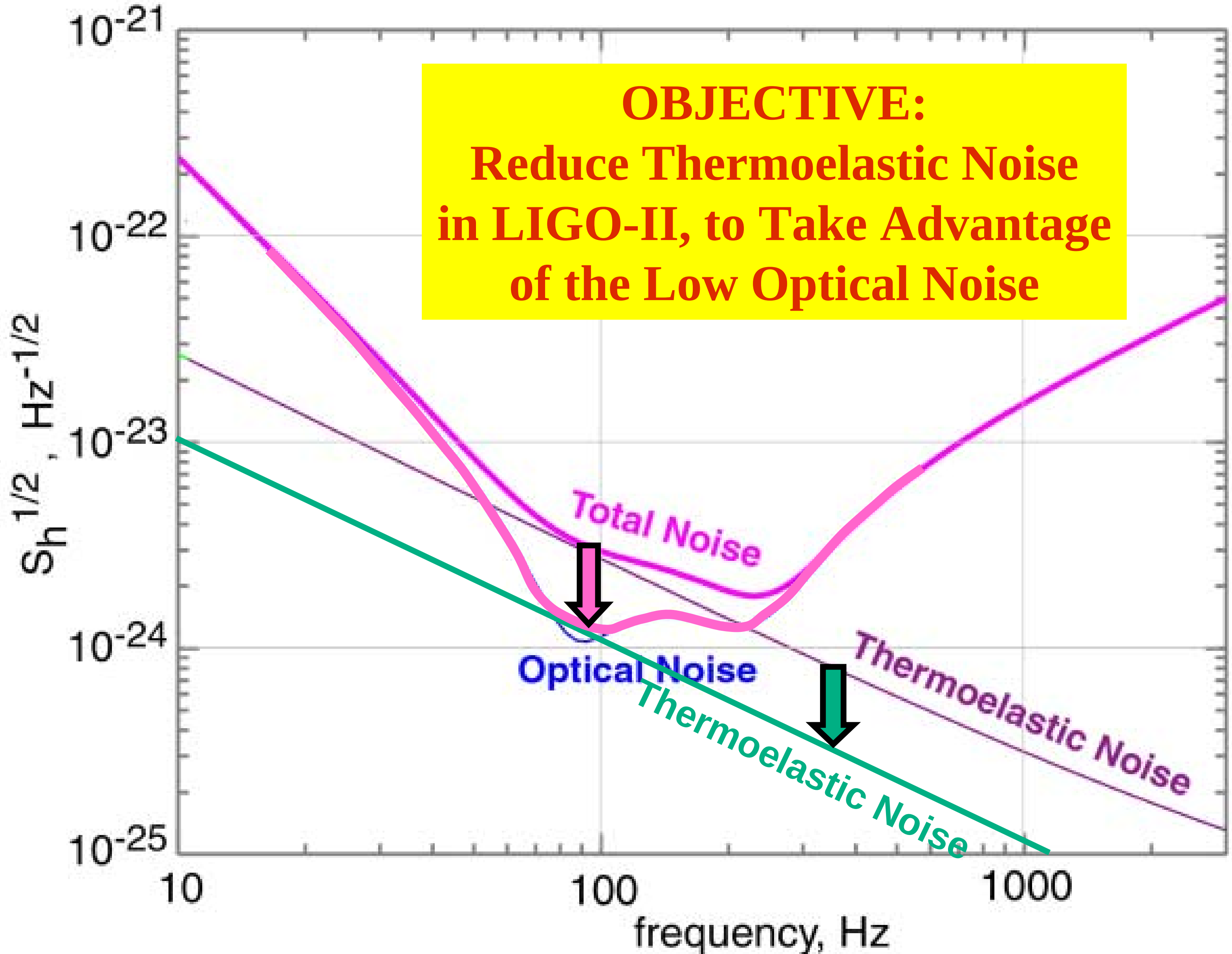
LIGO-Livingston, 19 March 2003

For details, see:

1. Partial draft of a paper for Phys Rev D: beamreshape020903.pdf available at **http://www.cco.caltech.edu/~kip/ftp**
2. LIGO Report **T030009-00-R**

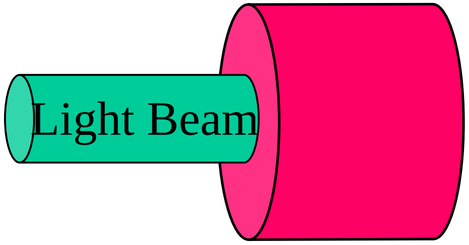
CONTEXT AND OVERVIEW

Sapphire Mirror Substrates

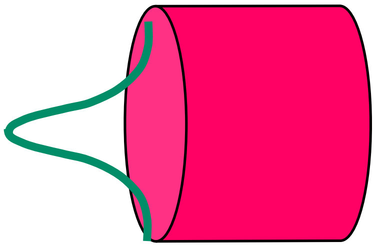


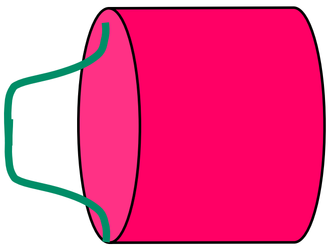
MOTIVATION FOR FLAT-TOPPED BEAMS

- On timescale ~0.01 secs, random heat flow
  => hot and cold bumps of mean size ~0.5 mm
- Hot bumps expand; cold contract
- Light averages over bumps
- Imperfect averaging => ***Thermoelastic noise***



- Gaussian beam averages over bumps much less effectively than a flat-topped beam.

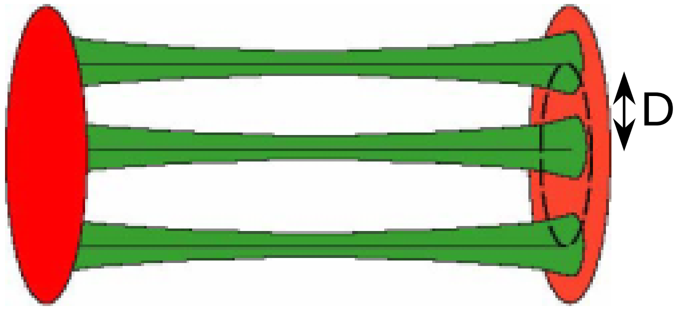




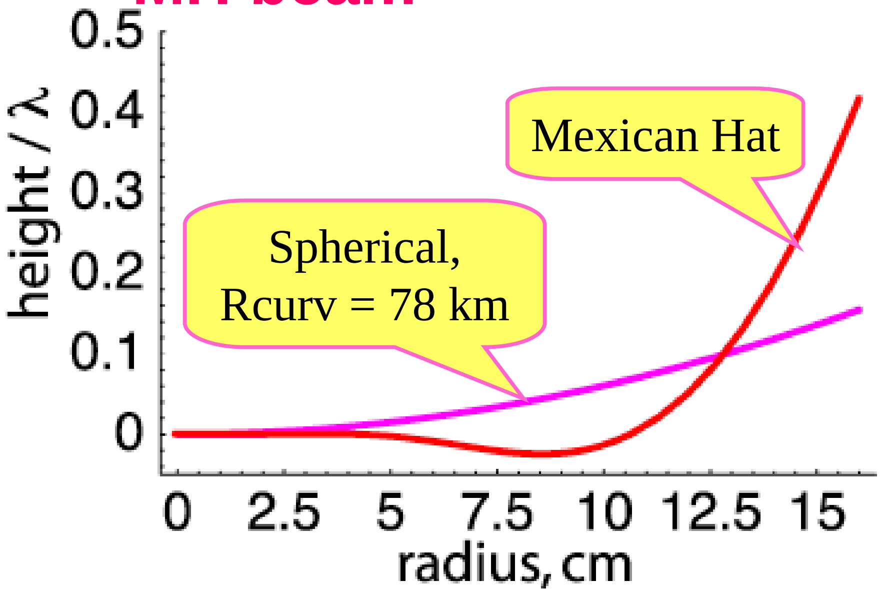
- The larger the beam, the better the averaging.
  - Size constrained by diffraction losses

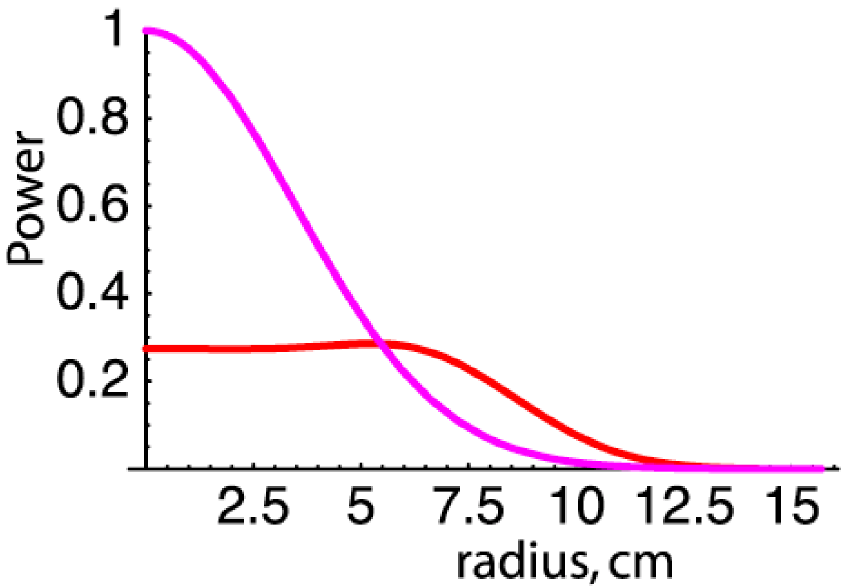
MEXICAN-HAT [MH] BEAM
(a near optimal type of flat-topped beam)

- **MH beam shape:**
  - Superposition of minimal-spreading Gaussians -- axes uniformly distributed inside a circle of radius D
  - Choose D so diffraction losses are 10 ppm



- **MH mirror shape: matches phase fronts of MH beam**





THERMOELASTIC NOISE REDUCTION WITH MH BEAMS & BASELINE TEST MASSES [O'S, S, V]

- We recommend: **coat baseline mirrors out to edge** (or edge minus 1 mm) rather than current edge minus 8 mm
  - Permits **increase of Gaussian beam radius** on ITM and ETM faces **from 4.23 cm to 4.49 cm** (at fixed diffraction loss)
  - Reduces power spectral density of Thermelastic noise by $S_h^{Gauss4.49} / S_h^{Gauss4.23} = 0.856$ (14%)
  - Increases network range for NS/NS binaries from 300 Mpc to 315 Mpc; **increases event rate by $(315/300)^3 = 1.16$**
- Switching from these Gaussian beams to **MH beams** with same diffraction loss (about 10 ppm per bounce):
  - **Reduces TE noise by $S_h^{MH} / S_h^{Gauss4.49} = 0.34$**
  - If coating thermal noise is negligible: **increases NS/NS network range from 315 Mpc to 431 Mpc**; **increases event rate by $(413/315)^3 = 2.6$**
- (Greater gains with conical test masses)

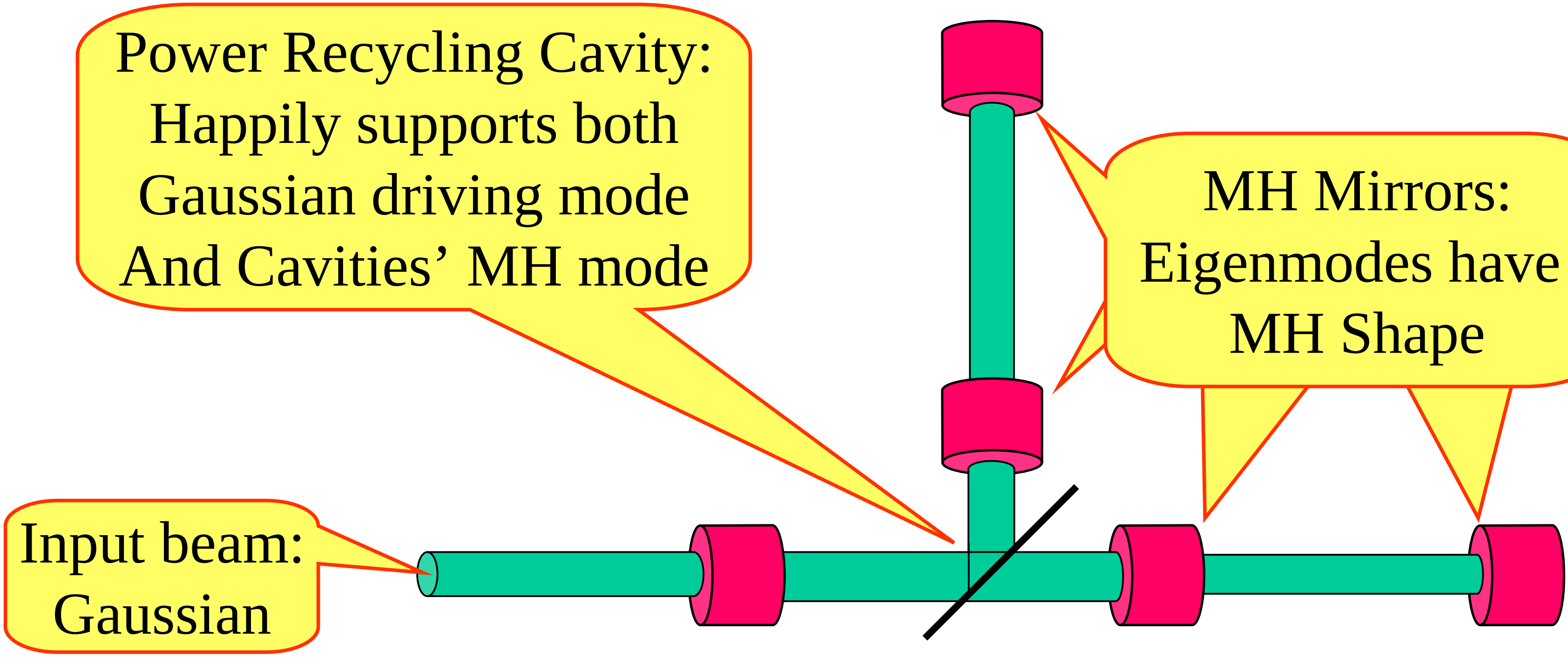
Practical Issues: Tilt, Displacement, Figure Errors

- **Compare two configurations:**
  - ***Baseline*** Gaussian-Mode Interferometer
    - Mirror radius R= 15.7 cm
    - Gaussian beam radius ro = 4.23 cm
    - Diffraction losses (per bounce in arms) L0 = 1.9 ppm
  - ***Fiducial MH*** (Mexican-Hat) Interferometer
    - Mirror radius R = 16 cm
    - MH beam radius D = 10.4 cm
    - Diffraction losses (per bounce in arms) L0 = 18 ppm
  - [Conservative comparison]

- **Three sets of analysis tools:**
  - Arm-cavity ***eigenequation*** **O'Shaugnessy, Strigin, Vyatchanin**
    - + 1st & 2nd order ***perturbation theory*** [-> mode mixing]
  - FFT simulation code - **D'Ambrosio**
  - Geometric optics on recycling cavities - **Kip**

Driving MH Arm Cavities by Gaussian Beam

- Optimal driving beam: Gaussian, beam radius $r_{od} = 6.92$ cm
- Overlap with arm cavities' MH Eigenmode:
  - $\left| \iint u^*_{MH}\, u_{Gd}\, dArea \right|^2 = 0.940$
- So **94 per cent** of Gaussian driving light gets into arm cavities



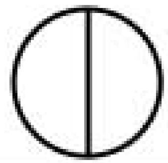
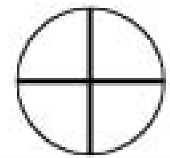
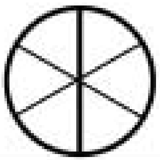
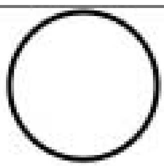
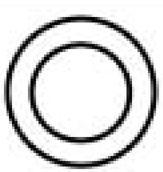
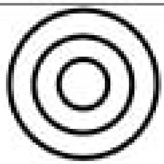
Arm-Cavity Parasitic Mode Frequencies

- **Baseline Gaussian:** $\Delta\omega = \omega - \omega_{fund}$ = (integer) x 0.099 x $\pi c/L$
- **Fiducial MH:** ***[O'Shaugnessy; Strigin & Vyatchanin]***

FSR

Azimuthal Nodes

Radial Nodes

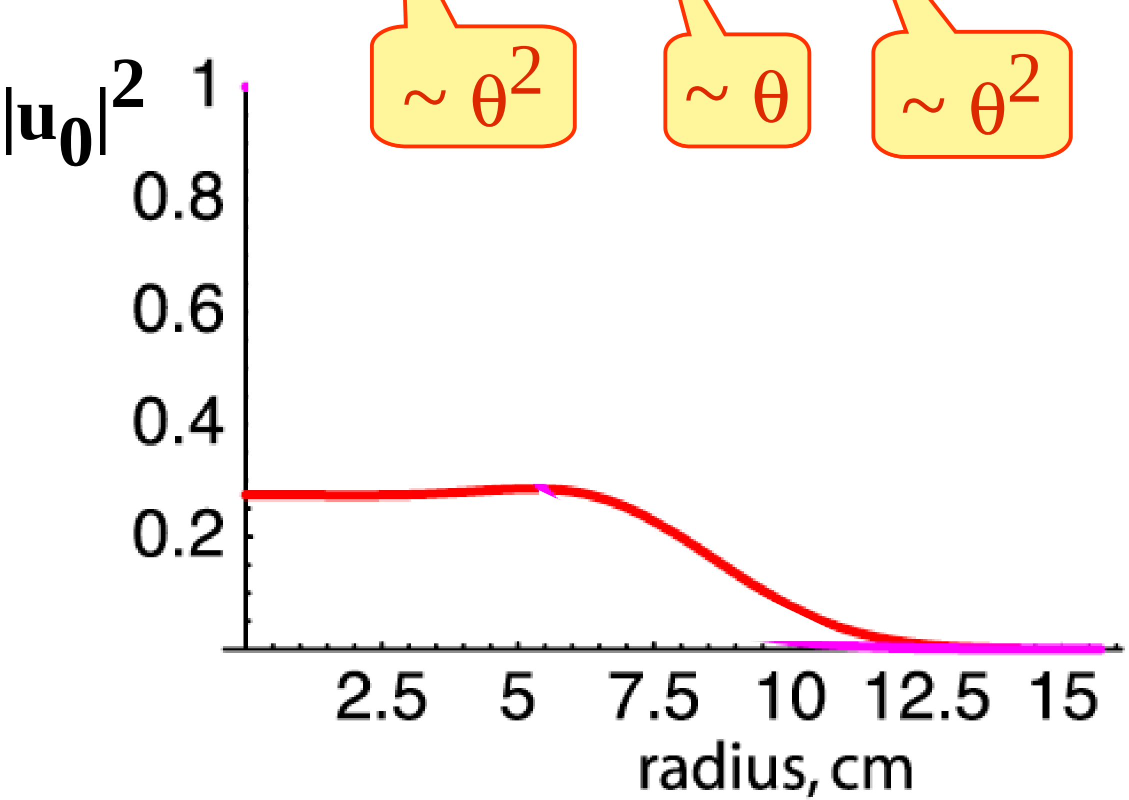
$\frac{\Delta\omega}{\pi c / L}$				
	0	0.0404	0.1068	0.1943
	0.1614	0.2816	0.4077	-0.4581
	0.4303	-0.4140	-0.2570 (X)	-0.0812 (X)
	-0.2330 (X)	-0.0488 (X)	0.1406 (X)	(X)

Excited By Tilt

X → indicates diffraction losses per bounce > 1%

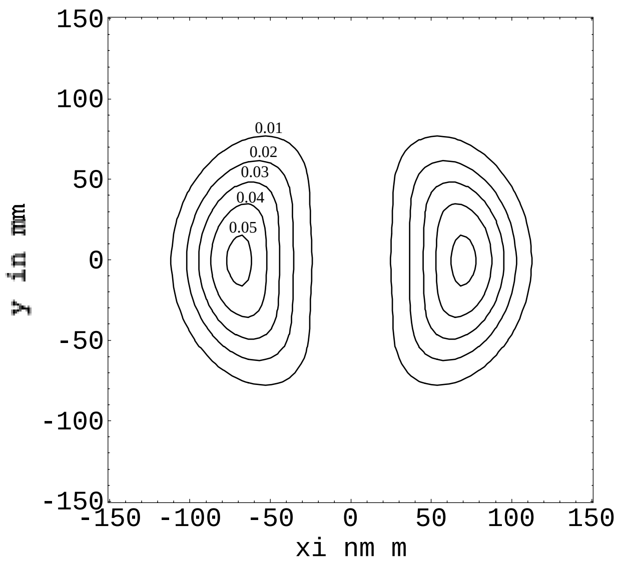
Arm-Cavity Tilt-Induced Mode Mixing

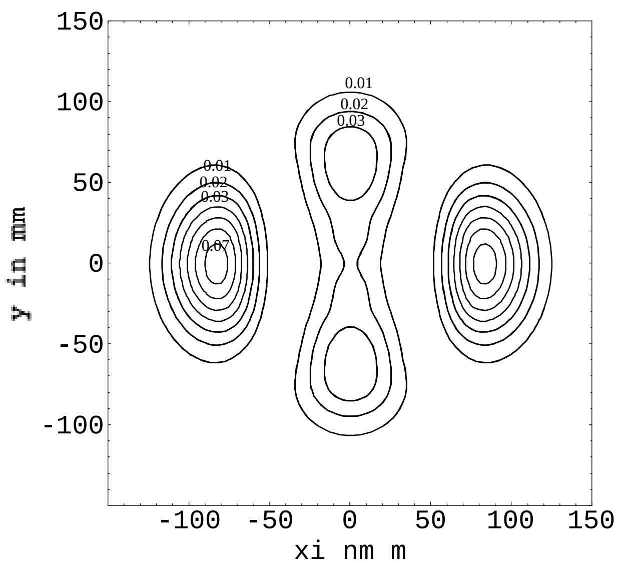
- Tilt arm-cavity ETM through an angle θ
- Mode mixing:
  - $u'_0 = (1-\alpha_1^2/2)u_0 + \alpha_1\, u_1 + \alpha_2\, u_2$



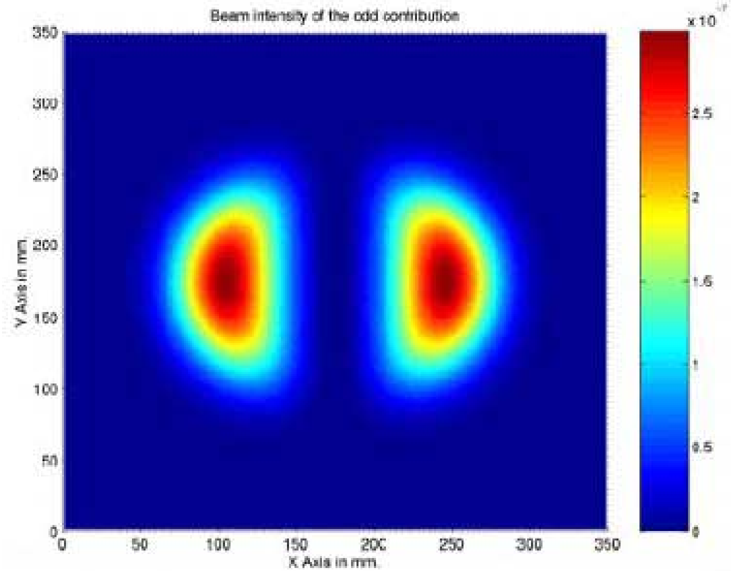
The Admixed Parasitic Modes

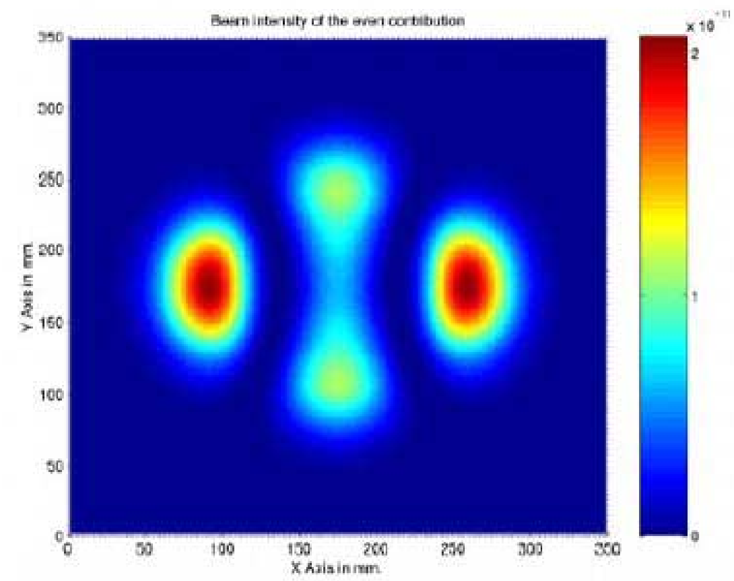
- Eigeneqn + Pert'n Theory: O'Shaugnessy





- FFT Simulations: d'Ambrosio





Tilt of One ETM
[d'Ambrosio & O'Shaughnessy

- **Tilt one ETM through an angle θ**
  - $\theta_8 = \theta/10^{-8}$ rad
- **Mode mixing:**
  - $u'_0 = (1-\alpha_1^2/2)u_0 + \alpha_1 u_1 + \alpha_2 u_2$

Baseline Gaussian-Beam Cavity

- $\alpha_1 = 0.0064\ \theta_8$

MH Cavity

- $\alpha_1 = 0.02272\ \theta_8$
- $\alpha_2 = 0.00016\ \theta_8^2$

MH Cavity has same α_1 as Baseline Gaussian if tilt is controlled 3.5 times better

- **Diffraction Losses**
  - Fractional increase due to tilt: $0.004\ \theta_8^2$
- **Arm Cavity Gain**
  - Fractional decrease due to tilt: $0.00057\ \theta_8^2$
- **Dark Port Power**
  - $P1 = 480$ ppm θ_8^2
  - $P0 = 0.26$ ppm θ_8^4
  - $P2 = 0.024$ ppm θ_8^4
- **4x larger for 4 r**

Arm-cavity tilts

Not a serious issue

Displacement of One ETM [O'Shaugnessy]

- **Displace one ETM transversely through distance s**
  - $s_{mm} = s/1mm$
- **Mode mixing**
  - $u'_0 = (1-\zeta_1^2/2)u_0 + \zeta_1 u_1 + \zeta_2 u_2$

Baseline Gaussian-Beam Cavity

- $\zeta_1 = 0.008\ s_{mm}$

MH Cavity

- $\zeta_1 = 0.010\ s_{mm}$

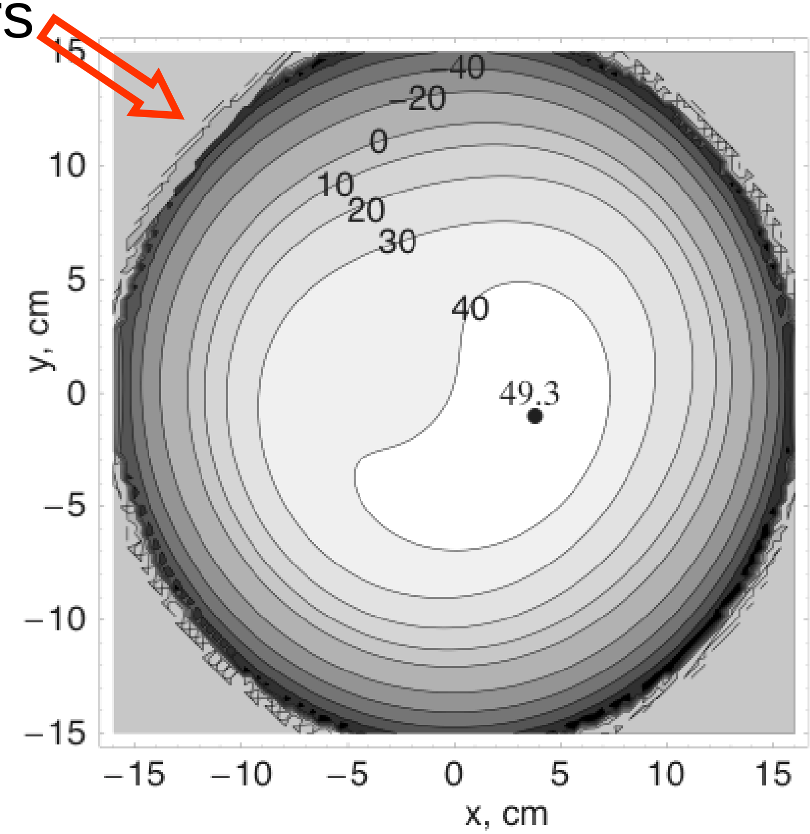
Negligible difference Between MH arm cavity And baseline Gaussian

Arm-Cavity Mirror Figure Errors

[d'Ambrosio, O'Shaugnessy, Billingsley]

- Foundation for analysis: Billingsley's "worst-case" figure error [measured map of a LIGO-I beamsplitter substrate]
- Height error δz_{wc} in nanometers
- We scale down by ε:

  $\delta z = \varepsilon \, \delta z_{wc}$
- Fiducial value: $\varepsilon = 0.2$ <=> peak-to-valley errors in innermost 10 cm: $\Delta z = 6\text{nm}$



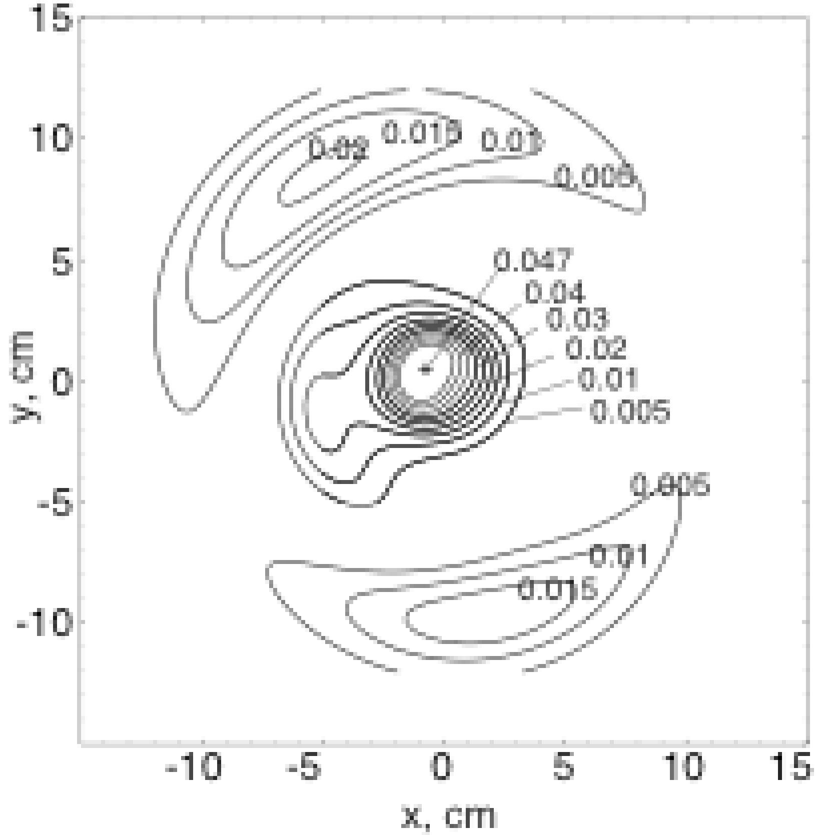
Mode Mixing by Figure Errors with Compensating Tilt [d'Ambrosio & O'Shaughnessy]

MH Mirrors:

- $u'_0 = (1-\beta_1^2/2)v_0 + \beta_1 v_1$
- Fractional power in parasitic mode; both mirrors deformed:

  $$2|\beta_1|^2 = 800\text{ppm } (\Delta z/6\text{nm})^2$$

- Parasitic dark-port power with four deformed mirrors: $0.0015\ (\Delta z/6\text{nm})^2$



Power in Parasitic mode:

$|\beta_1 v_1|^2$ for fiducial Δz = 6 nm

Influence of mirror figure errors On MH arm-cavity modes appears Not to be a serious problem

Fractional Increase in Thermoelastic Noise due to Figure Errors

[O'Shaughnessy, Strigin, Vyatchanin]

- $\delta S_h / S_h = 0.14$ (Δz/6 nm) when all four mirrors have uncorrelated figure errors
- In actuality, we may expect $\Delta z \sim 2$nm or less, so $\delta S_h / S_h =$ 0.05 or less

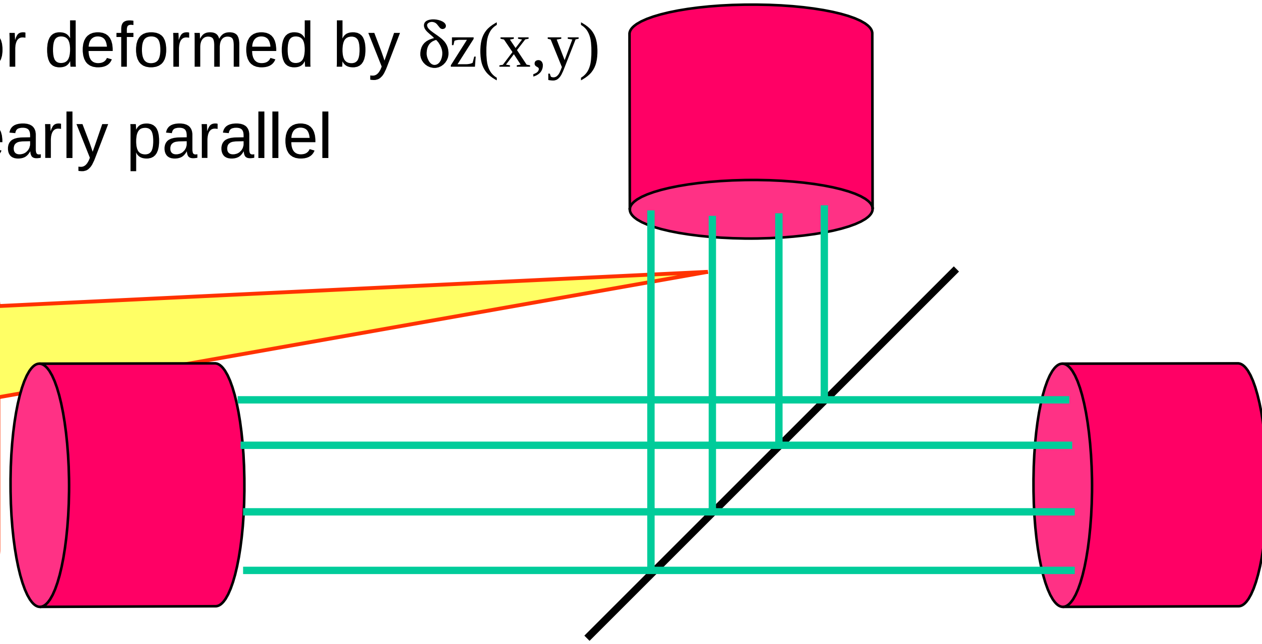
Recycling Cavities: General Considerations

- **Number of one-way trips in cavity for 95% power decay ≈ Finesse = F [in PR cavity: sidebands; in SR: signal]**
  - LIGO-I PR cavity: F~ 120
  - AdLIGO PR cavity: F ~ 50
  - Ad LIGO SR cavity: F ~ 80 for standard broadband,
    F ~ {360, 800} narrowbanded at {500 Hz, 1000 Hz}
- **Fresnel length for light that travels distance F x (cavity arm length, *l*):** $r_F = (\lambda F l)^{1/2} \approx 3 \text{ cm } (F/80)^{1/2}$
  - r_F << (beam 95% diameter) ≈ 15cm baseline, 20cm MH
  - r_F ~ 1/2 (5cm scale on which mirror shapes vary)
  - ⇨ little diffractive coupling; geometric optics fairly good
  - Cavities highly degenerate
  - MH beam will happily resonate in cavities designed for MH mirrors, and conversely

Power Recycling Cavity with RF Readout:

- Suppose PR mirror deformed by $\delta z(x,y)$
- Light's rays are nearly parallel to optic axis

Independent rays; experience phase shifts
$\delta\phi = k\,(F/\pi)\,\delta z(x,y)$
$= (2F/\lambda)\,\delta z(x,y)$



- Sideband light emerges from cavity with $u \sim u_o \exp[i\delta\phi] \approx u_o + i\,(2F/\lambda)\,\delta z(x,y)\,u_o$
  – (Fraction of power in parasitic modes)
  = (fractional increase in shot noise if no output mode cleaner)
  = $< u_o\,,\,[(2F/\lambda)\,\delta z(x,y)]^2\,u_o >$ if one mirror deformed
- All mirrors deformed in uncorrelated way: multiply by 2
  – $\delta S_h / S_h = 2\,(2F/\lambda)^2 < \delta z^2 >$

Power Recycling Cavity with RF Readout; No output Mode Cleaner

- **Shot-noise increase:** $\delta S_h / S_h = 2\,(2F/\lambda)^2 < \delta z^2 >$
- **Mirror tilt:** $\delta z = \theta\, r \cos\varphi$; $\delta S_h / S_h = (2F/\lambda)^2 < r^2 > \theta^2$
  - For 1% shot noise increase, tilt must be constrained to:
  - **LIGO-I:** $< r^2 > = (2.6\ \mathrm{cm})^2$,
    . so $\theta_8 < 1.6$ [good agreement with Fritschel et al]
  - **AdLIGO, baseline Gaussian:** $< r^2 > = (4.70\ \mathrm{cm})^2$, so $\theta_8 < 2.5$
  - **AdLIGO, MH beams:** $< r^2 > = (6.95\ \mathrm{cm})^2$, so $\theta_8 < 1.5$
- **Mirror figure error:** $< \delta z^2 > = 1/8\ \Delta z^2$
  - Δz = peak-to-valley height variations inside radius ~7.5 cm (baseline Gaussian); ~10 cm (MH mirrors)
  - For 1% shot noise increase, need
  - **LIGO-I: $\Delta z < 0.8$ nanometers!**
  - **AdLIGO, baseline or MH: $\Delta z < 2$ nanometers**

Signal Recycling Cavity

- Most serious constraint on SR mirror height error $\delta z(x,y)$ and ETM height error comes from spatially dependent phase shift put onto light as it passes through SR cavity.
- Resulting **constraint on tilts** θ_8 for 1% shot-noise increase:
  - **Broadband: Baseline $\theta_8 < 2.4$; MH $\theta_8 < 1.6$**
  - **Narrowband @ 500 Hz: Baseline $\theta_8 < 1.1$; MH $\theta_8 < 0.7$**
  - **Narrowband @ 1000 Hz: Baseline $\theta_8 < 0.6$; MH $\theta_8 < 0.4$** (more likely **~1.1 & ~0.7** due to breakdown of geometric optics)
- Constraint on **Figure Error** inside radius ~7.5 cm (baseline) and ~10 cm (MH)
  - **Broadband: $\Delta z < 2$ nm**
  - **Narrowband @ 500 Hz: $\Delta z < 1$ nm**
  - **Narrowband @ 1000 Hz: $\Delta z < 0.5$ nm** (more likely **1 nm** due to breakdown of geometric optics)

Conclusions

- Switching to MH beams at fixed diffraction loss will reduce the power spectral density of thermoelastic noise by about a factor 3.
- Arm-cavity constraints on tilt are about 3.5 times tighter for MH than for baseline Gaussian -- but are no more severe than in LIGO-I, $\theta_8 < 1$
- Arm-cavity constraints on lateral displacement are about the same for MH and baseline Gaussian: ~ 1 mm
- Most serious constraints on tilt and figure error are from PR and SR cavities. These are marginally tighter for MH than for baseline Gaussian, but not as tight as for LIGO-I.

Recommendations to LSC

- Carry out FFT analysis of PR and SR cavities, to check my geometric optics analysis
- Maintain MH mirrors as an option for AdLIGO
  - Explore a suggestion by Bill Kells:
    - Switch from Gaussian to MH beams by changing only the ETMs
    - Keep PR, SR, and ITMs spherical
  - Explore control signals for MH interferometers

- Whether MH or not, consider reducing the degeneracy of the PR and SR cavities via a lens in entrance face of ITMs, which reduces the spot size on PR and SR mirrors to
  - Beam diameter ~ $r_F = (\lambda F \, l)^{1/2} \approx 3 \text{ cm} (F/80)^{1/2}$
    [large enough diameter that eigenmode is still fixed by arm cavities and MH and Gaussian both resonate, but barely so]

REFERENCES

1. **Partial draft of a paper for Phys Rev D:**

   beamreshape020903.pdf

   available at http://www.cco.caltech.edu/~kip/ftp

2. **LIGO Report T030009-00-R**