Statistical Issues in Upper Limit Setting: Stochastic Background

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What We Calculate

- M jobs, each split into 10 segments of $T = 90\,\mathrm{sec}$ each
- Measurement of correlations, scaled to represent Ω_{GW}:

$$\mathcal{Y}_{IJ} = \frac{1}{T}\int \tilde{h}_1^* \, Q \, \tilde{h}_2 \, df$$

- Average & std dev for Ith job:

$$\bar{\mathcal{Y}}_I = \frac{1}{10}\sum_{J=1}^{10} \mathcal{Y}_{IJ} \qquad \mathcal{S}_I = \sqrt{\frac{1}{9}\sum_{J=1}^{10} (\mathcal{Y}_{IJ} - \bar{\mathcal{Y}}_I)^2}$$

- Point estimate and std error for whole run (weighted avg):

$$\mathcal{Y} = \frac{\sum_{I=1}^{M} \lambda_I \bar{\mathcal{Y}}_I}{\sum_{I=1}^{M} \lambda_I} \qquad \mathcal{S} = \frac{1}{10}\frac{\sqrt{\sum_{I=1}^{M} \lambda_I \mathcal{S}_I^2}}{\sum_{I=1}^{M} \lambda_I}$$

2-Sided Frequentist Confidence Interval

- Value-neutral expression of overall meas & its statistical spread:

- Combined statement about "real" Ω_{GW} & instrumental Ω_{inst}

$$\mathcal{Y} - 1.65\,\mathcal{S} \leq \Omega_{\mathrm{inst}} + \Omega_{\mathrm{GW}} \leq \mathcal{Y} + 1.65\,\mathcal{S}$$

- "Safe" because
  1. Ω_{inst} could be negative
  2. positive lower limit not a "detection"

How to Set an Upper Limit?

- If we believe $\Omega_{\rm inst} \ll \mathcal{S}$, 90% frequentist UL is

$$\Omega_{\rm GW} \leq \mathcal{Y} + 1.28\,\mathcal{S}$$

- Problems:
  1. What does a negative value mean?
     (Means our assumptions are probably wrong!)
  2. Substantial $\Omega_{\rm inst}$ obscures physical results

$$\Omega_{\rm GW} \leq \mathcal{Y} - \Omega_{\rm inst} + 1.28\,\mathcal{S}$$

$\longrightarrow$ Need more information to gauge impact of $\Omega_{\rm inst}$

Sources of information on $\Omega_{\rm inst}$

- Large time shifts:
  - For non-coloc detectors, shift by many $\times$ light travel time
  - most sources of $\Omega_{\rm inst}$ should still be there

  $\longrightarrow$ mean or std dev of time-shifted $\mathcal{Y}$ measurements
  can set bounds on $\Omega_{\rm inst}$ at zero time lag

- Small time shifts:
  - Equiv to careful look at freq domain behavior of CC integrand
  - Can use to set something like a χ^2 veto

$\longrightarrow$ Should calibrate this procedure with Monte Carlo simulations

Current Prescription (for S1)

1. Set straightforward two-sided limit on $\Omega_{GW}+\Omega_{inst}$

2. Decide if Ω_{inst} significant (time shifts, anomalous $\mathcal{Y}$)
   - If no, set straightforward upper limit on Ω_{GW}
   - If yes, set no upper limit on Ω_{GW}

Currently exploring estimates of significant Ω_{inst} for future searches