Calibration in Stochastic Source Searches

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Idealized Definition of Method

Cross-correlation statistic:

$$Y = \int_{-\infty}^{\infty} \tilde{h}_1(f)^* \tilde{Q}(f) \tilde{h}_2(f)\, df$$

Optimal filter:

$$\tilde{Q}(f) = \mathcal{N} \frac{\gamma(f)}{|f|^3 P_1(f) P_2(f)}$$

Normalization:

$$\mathcal{N} = \frac{20\pi^2}{3H_0^2} \left(\int_{-\infty}^{\infty} \frac{\gamma(f)^2}{|f|^6 P_1(f) P_2(f)} df \right)^{-1}$$

chosen so that

$$\langle Y \rangle = \Omega_0 T$$

Role of Calibration

Putting it all together:

$$Y = \frac{20\pi^2}{3H_0^2} \frac{\int_{-\infty}^{\infty} \frac{\gamma}{|f|^3} \frac{\widetilde{h}_1{}^* \widetilde{h}_2}{P_1 P_2} \, df}{\int_{-\infty}^{\infty} \frac{\gamma^2}{|f|^6} \frac{1}{P_1 P_2} \, df}$$

Actually work w/uncalibrated signal $\widetilde{h}^{\mathsf{U}}(f)$ and PSD $P^{\mathsf{U}}(f)$

$$\widetilde{h}(f) = \widetilde{R}(f)\widetilde{h}^{\mathsf{U}}(f) \qquad P(f) = \left|\widetilde{R}_1(f)\right|^2 \left|\widetilde{R}_2(f)\right|^2 P^{\mathsf{U}}(f)$$

So we measure:

$$Y = \frac{20\pi^2}{3H_0^2} \frac{\int_{-\infty}^{\infty} \frac{\gamma}{|f|^3} \frac{\widetilde{h}_1^{\mathsf{U}*} \widetilde{h}_2^{\mathsf{U}}}{P_1^{\mathsf{U}} P_2^{\mathsf{U}}} \frac{1}{\widetilde{R}_1 \widetilde{R}_2^*} \, df}{\int_{-\infty}^{\infty} \frac{\gamma^2}{|f|^6} \frac{1}{P_1^{\mathsf{U}} P_2^{\mathsf{U}}} \frac{1}{\left|\widetilde{R}_1\right|^2 \left|\widetilde{R}_2\right|^2} \, df}$$

Effects of Calibration Errors

Writing $R(f) = \rho(f)e^{i\theta(f)}$, err in measurements of Y (hence both point estimate & error bars) linear in err $\delta\rho$ and $\delta\theta$

$$\frac{\delta Y}{Y} = \frac{\int_{-\infty}^{\infty} a(f)\left(-\frac{\delta\rho_1}{\rho_1} - \frac{\delta\rho_2}{\rho_2} - i\delta\theta_1 + i\delta\theta_2\right) df}{\int_{-\infty}^{\infty} a(f)\, df} - \frac{\int_{-\infty}^{\infty} b(f)\left(-2\frac{\delta\rho_1}{\rho_1} - 2\frac{\delta\rho_2}{\rho_2}\right) df}{\int_{-\infty}^{\infty} b(f)\, df}$$

where

$$a(f) = \frac{\gamma}{|f|^3} \frac{\widetilde{h}_1^{\mathsf{U}*}\widetilde{h}_2^{\mathsf{U}}}{P_1^{\mathsf{U}} P_2^{\mathsf{U}}} \frac{1}{\widetilde{R}_1 \widetilde{R}_2^*} = \frac{\gamma}{|f|^3} \frac{\widetilde{h}_1{}^*\widetilde{h}_2}{P_1 P_2}$$

and

$$b(f) = \frac{\gamma^2}{|f|^6} \frac{1}{P_1^{\mathsf{U}} P_2^{\mathsf{U}}} \frac{1}{\left|\widetilde{R}_1\right|^2 \left|\widetilde{R}_2\right|^2} = \frac{\gamma^2}{|f|^6} \frac{1}{P_1 P_2}$$

Impact of Calibration on Weighting Factor $1/\sigma^2_{\text{theor}}$

With time-varying noise, we combine jobs weighted by

$$\lambda = \frac{T}{\sigma^2_{\text{theor}}} = \frac{9H_0^2}{100\pi^2}\left(\int_{-\infty}^{\infty}\frac{\gamma^2}{|f|^6}\frac{1}{P_1^{\text{U}}P_2^{\text{U}}}\frac{1}{\left|\widetilde{R}_1\right|^2\left|\widetilde{R}_2\right|^2}df\right)^{-1}$$

so

$$\frac{\delta\lambda}{\lambda} = \frac{\int_{-\infty}^{\infty} b(f)\left(-2\frac{\delta\rho_1}{\rho_1} - 2\frac{\delta\rho_2}{\rho_2}\right)df}{\int_{-\infty}^{\infty} b(f)\,df}$$

linear in calibration amplitude errors; insensitive to phase errors