Detection template families for spinning high-mass binary black holes

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[preliminary results]

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Nonspinning high-mass BBHs

- **Nonspinning BH/BH binary with $M = 10 - 40 M_\odot$: possible to miss GW signal if PN templates are used *naively*.**
  - **Resummation techniques** [Damour, Sathyaprakash & Iyer 97, AB & Damour 99, 01]
  - **Importance of modeling signal amplitude with *cutoff frequency* and signal phase with *arbitrary coefficients*** [AB, Chen & Vallisneri 02]

Non-modulated detection template family in Fourier domain

$$h_{\rm DTF}(f) = \mathcal{A}(f)\, e^{i\psi(f)}$$

$$\mathcal{A}(f) = f^{-7/6}\, \left(1 - \alpha\, f^{2/3}\right)\, \theta(f_{\rm cut} - f)$$

$$\psi(f) = f^{-5/3}\, \left(\psi_0 + \psi_{1/2}\, f^{1/3} + \psi_1\, f^{2/3} + \psi_{3/2}\, f + \cdots\right)$$

α is an arbitrary *extrinsic* parameters and $f_{\rm cut}, \psi_0, \psi_{1/2}, \psi_1, ...$ are arbitrary *intrinsic* parameters on which the signal-to-noise ratio is maximized

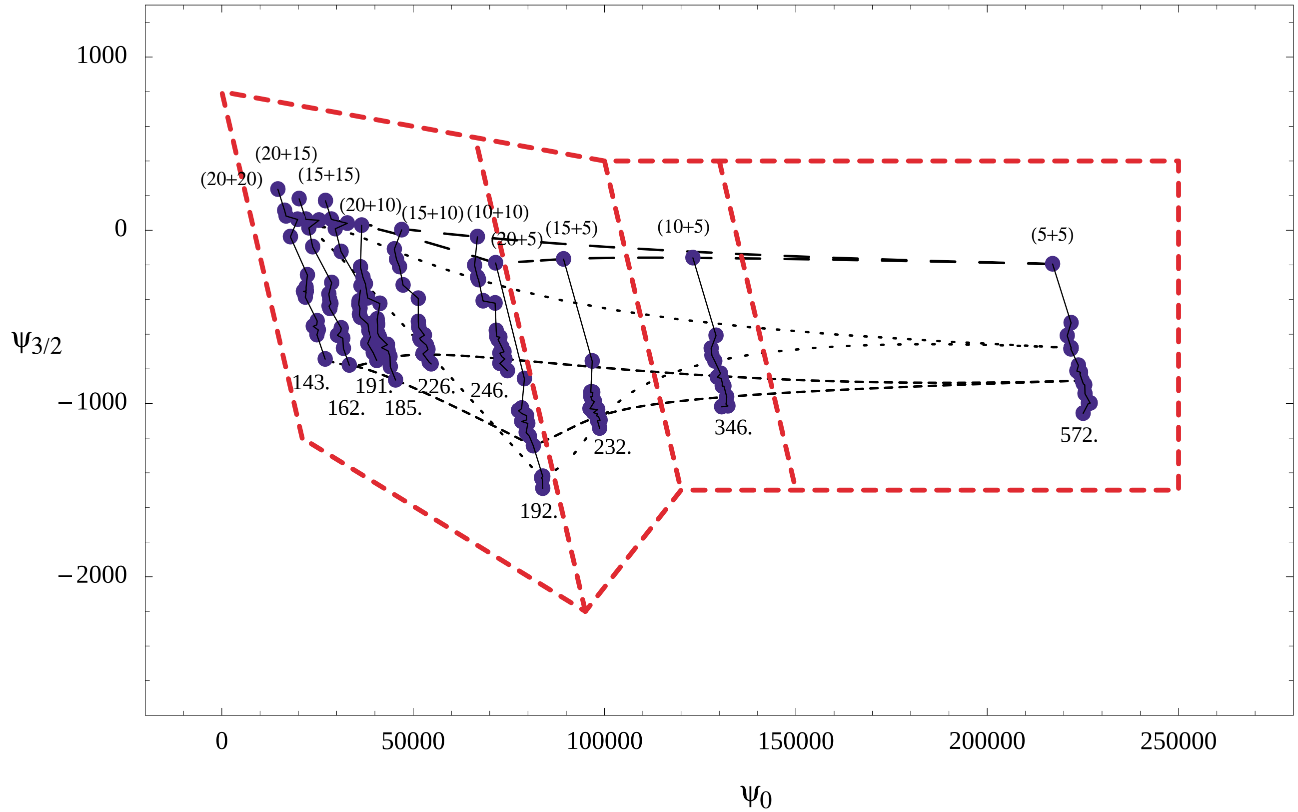
Nonspinning high-mass BBHs

$\mathcal{N}_{\text{templates}} \sim 4 \times 10^4$

$\mathcal{L}_{\text{event rates}} \lesssim 17\%$

with FF $= 0.96$

and MM $= 0.98$



Estimation of chirp mass: $\sim 3\% - 40\%$

Including spin effects

- **Do black holes in binaries carry spin? How big is the spin? We do not know!**

- **The theoretical waveforms depend on** $many$ **parameters:** m_1, m_2 $\vec{S}_1$, $\vec{S}_2$, **orientation of the binary with respect to the detector, etc.**

- **Analytical solutions in special cases. Apostolatos' ansatz**

  [Apostolatos, Cutler, Sussman & Thorne 94, Apostolatos 95, 96]

- **Results for NS/BH binary with** $M \leq 10 M_\odot$ **and Newtonian dynamics**

  [Grandclément, Kalogera & Vecchio 02]

- **DTF for spinning high-mass BBHs but also NS/BH**

  [AB, Chen & Vallisneri, in preparation]

How do we generate the GW signal?

Two-body dynamics in the adiabatic limit at 2PN and 3PN order including spin-orbit and spin-spin effects

$$\dot{\omega} = F_\omega(\omega, \widehat{\boldsymbol{L}}_N \cdot \widehat{\boldsymbol{S}}_1, \widehat{\boldsymbol{L}}_N \cdot \widehat{\boldsymbol{S}}_2, \widehat{\boldsymbol{S}}_1 \cdot \widehat{\boldsymbol{S}}_2)$$

Precession equations including spin-orbit and spin-spin effects

$$\dot{\boldsymbol{S}}_1 = F_{\boldsymbol{S}_1}(\omega, \boldsymbol{S}_1, \boldsymbol{S}_2, \widehat{\boldsymbol{L}}_N)\,, \quad \dot{\boldsymbol{S}}_2 = F_{\boldsymbol{S}_2}(\omega, \boldsymbol{S}_1, \boldsymbol{S}_2, \widehat{\boldsymbol{L}}_N)$$

$$\dot{\widehat{\boldsymbol{L}}}_N = F_{\widehat{\boldsymbol{L}}_N}(\omega, \boldsymbol{S}_1, \boldsymbol{S}_2, \widehat{\boldsymbol{L}}_N)$$

GW signal is extracted at quadrupole order (with Finn-Chernoff convention)

The motivation for building modulated detection-template family relies on the dynamics

$$h_{\rm GW}(t) = -\frac{2\mu}{D}\,\frac{M}{r(t)}\left[\boldsymbol{e}_{+}^{ij}(t)\ \cos 2\Phi(t) + \boldsymbol{e}_{\times}^{ij}(t)\ \sin 2\Phi(t)\right]\times$$

$$[T_{+\,ij}(\Theta,\varphi)\,F_{+}(\theta,\phi,\psi) + T_{\times\,ij}(\Theta,\varphi)\,F_{\times}(\theta,\phi,\psi)]$$

$$\mathbf{e}_{+}(t) \equiv \mathbf{e}_1(t)\,\mathbf{e}_1(t) - \mathbf{e}_2(t)\,\mathbf{e}_2(t)\,, \quad \mathbf{e}_{\times}(t) \equiv \mathbf{e}_1(t)\,\mathbf{e}_2(t) + \mathbf{e}_2(t)\,\mathbf{e}_1(t)$$

$$\widehat{\boldsymbol{n}}(t) = \boldsymbol{e}_1(t)\ \cos\Phi(t) + e_2(t)\ \sin\Phi(t)\,, \quad \mathbf{T}_{+} \equiv \boldsymbol{e}_x^R\,\boldsymbol{e}_x^R - \boldsymbol{e}_y^R\,\boldsymbol{e}_y^R\,, \quad \widehat{\boldsymbol{N}}(\Theta,\,\varphi)$$

New convention (frame independent):

$e_{1,2}(t)$ are an orthonormal basis of the instantaneous orbital plane which are evolved such that the condition $\dot{\Phi} = \omega$ is preserved

$\Rightarrow$ Φ *does not* depend on directional parameters and is almost non-modulated

Crucial to apply Apostolatos' ansatz to both phase and amplitude!

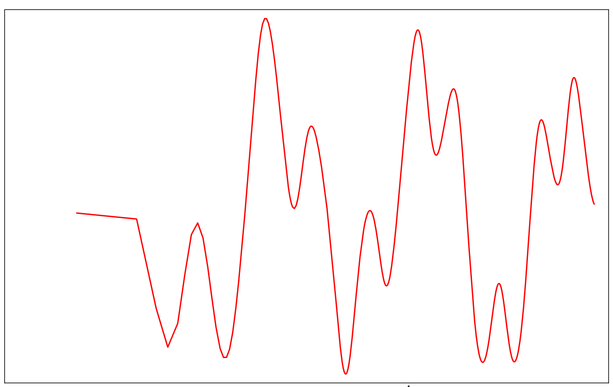
At leading order in stationary-phase approximation:

$$h_{\mathrm{GW}}(f) = -h_C(f)\left[e_+^{jk}(t_f) + i e_\times^{jk}(t_f)\right](T_{+\,jk}\,F_+ + T_{\times\,jk}\,F_\times)$$

- $h_C(f)$ is Fourier transform of carrier $h_C(t) = \frac{2\mu}{D}\frac{M}{r(t)}\cos 2\Phi(t)$ (almost non-modulated!)
- t_f is the time at which the carrier has instantaneous frequency f

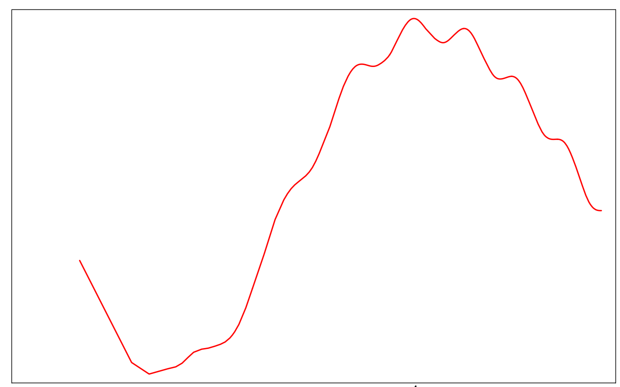
Ansatz motivated by Apostolatos: $e^{ij}_{+,\,\times}(t_f) \propto \mathcal{C}^{ij}_{+,\,\times}\cos\left(\mathcal{B}\,f^{-2/3} + \delta^{ij}_{+,\,\times}\right)$

$M = (20+5)M_\odot$



$\omega^{-2/3}$

$M = (10+10)M_\odot$



$\omega^{-2/3}$

Modulated detection template family in Fourier domain

$$h^{\rm mod}_{\rm DTF}(f) = \mathcal{A}^{\rm mod}(f)\, e^{i\psi(f)}$$

$$\mathcal{A}^{\rm mod}(f) = f^{-7/6}\,\theta(f_{\rm cut} - f)\times$$

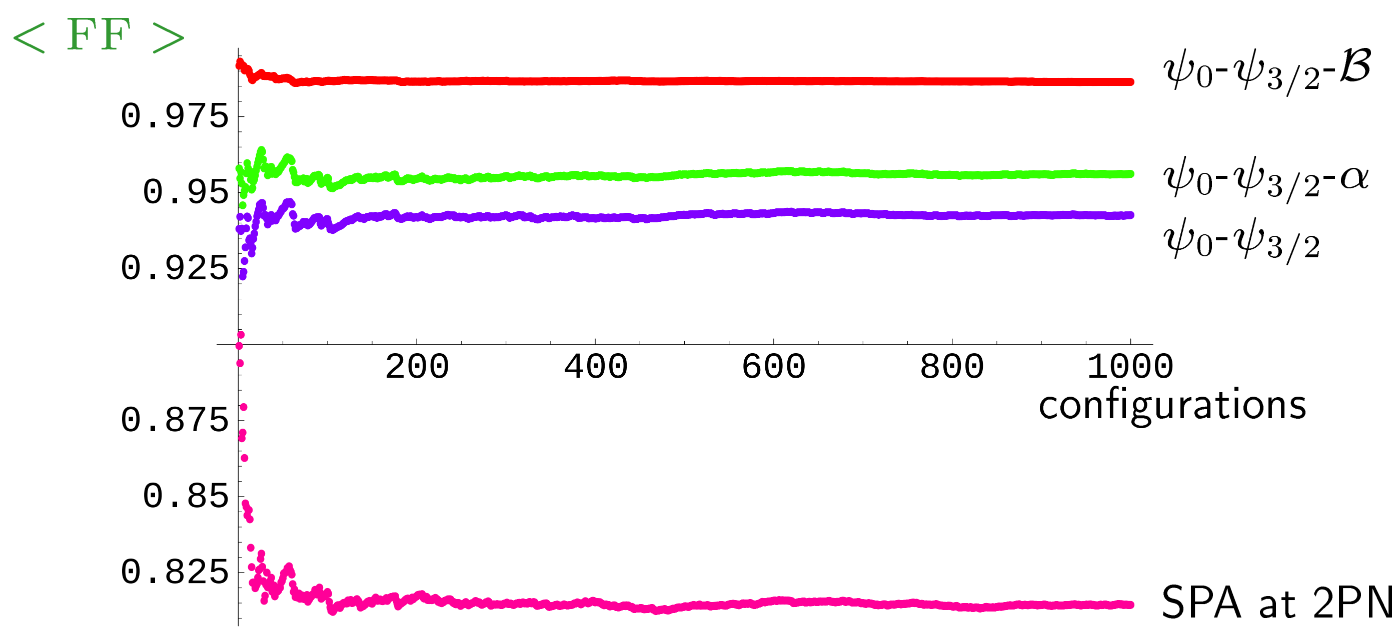
$$\times \left[(\gamma_1 + i\gamma_2) + (\gamma_3 + i\gamma_4)\ \cos\left(\mathcal{B} f^{-2/3}\right) + (\gamma_5 + i\gamma_6)\ \sin\left(\mathcal{B} f^{-2/3}\right)\right]$$

$$\psi(f) = f^{-5/3}\ \left(\psi_0 + \psi_{1/2}\, f^{1/3} + \psi_1\, f^{2/3} + \psi_{3/2}\, f + \cdots\right)$$

$f_{\rm cut}, \mathcal{B}$, $\psi_0, \psi_{1/2}, \psi_1, \ldots$ are arbitrary *intrinsic* parameters and $\alpha, \gamma_1, \gamma_2, \ldots$ are arbitrary *extrinsic* parameters on which the signal-to-noise ratio is maximized

Convergence of the average FF for BBH with $M = (15 + 15)\, M_\odot$

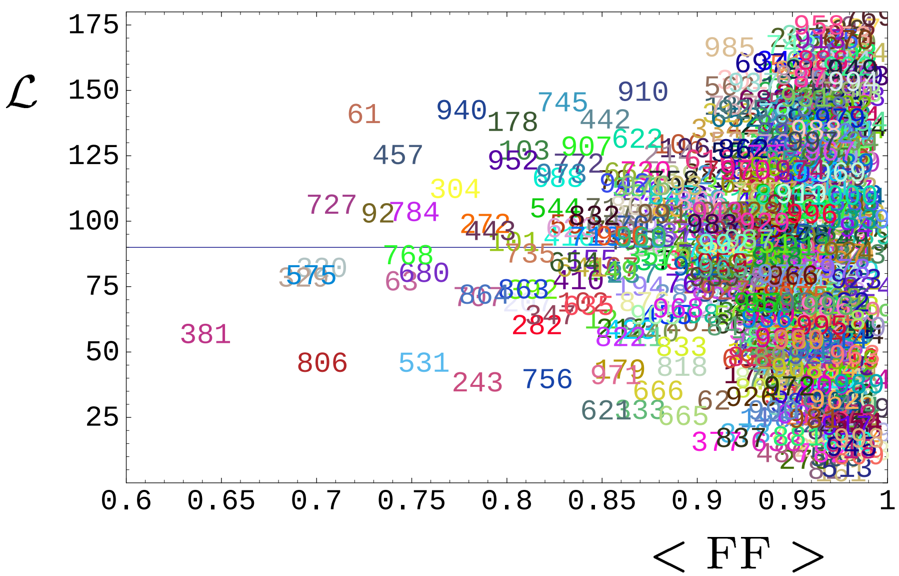
$$< \mathrm{FF} >= \left(\frac{\sum_n \mathrm{overlap}^2(n)}{\sum_n 1} \right)^{1/2}$$



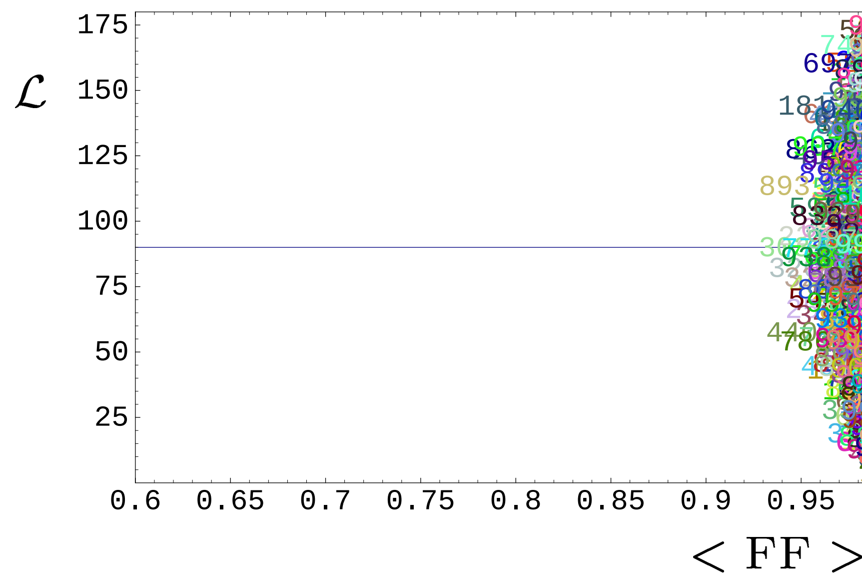
We chose 1000 initial configurations for the spins

Distribution of $< \mathrm{FF} >$ for BBH with $M = (15 + 15)\, M_\odot$

DTF: ψ_0-$\psi_{3/2}$



DTF: ψ_0-$\psi_{3/2}$-$\mathcal{B}$



$\mathcal{L} \rightarrow$ angle between initial Newtonian angular-momentum direction and direction of observation

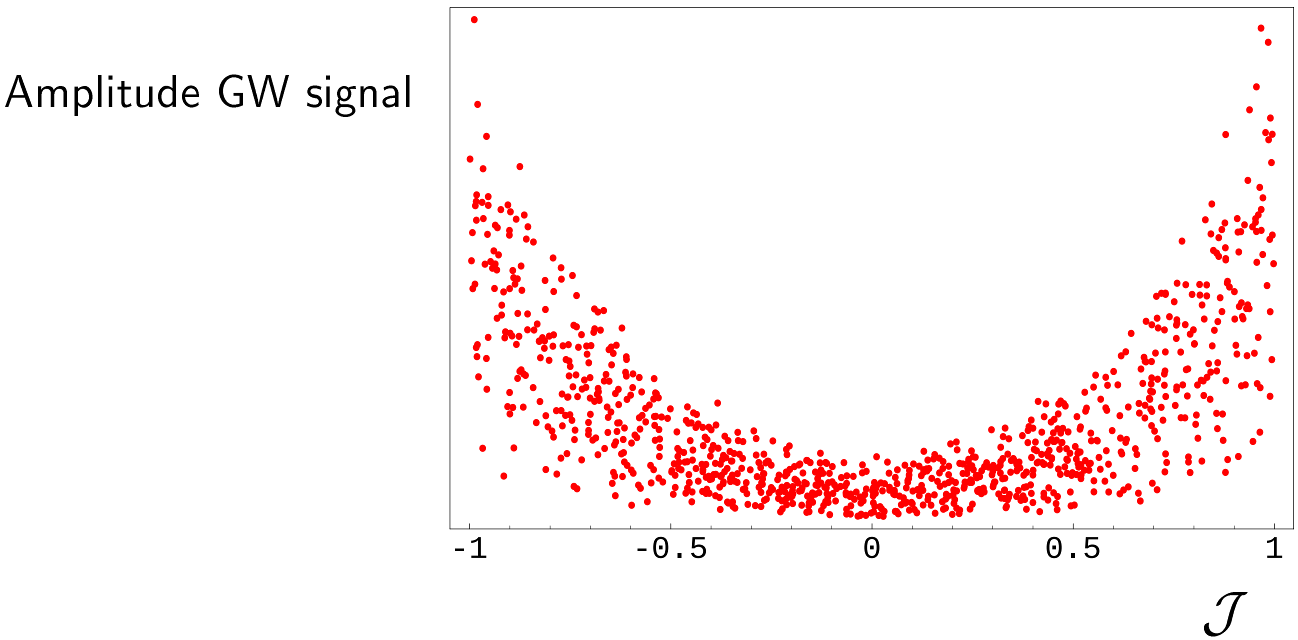
Performances for high-mass BBHs

	$(7+5)M_\odot$		$(10+10)M_\odot$		$(15+15)M_\odot$		$(20+5)M_\odot$	
	$\overline{\mathrm{FF}}$	$\overline{\mathrm{FF}}_{\mathrm{ampl}}$	$\overline{\mathrm{FF}}$	$\overline{\mathrm{FF}}_{\mathrm{ampl}}$	$\overline{\mathrm{FF}}$	$\overline{\mathrm{FF}}_{\mathrm{ampl}}$	$\overline{\mathrm{FF}}$	$\overline{\mathrm{FF}}_{\mathrm{ampl}}$
SPA at 2PN	0.909	0.939	0.899	0.920	0.818	0.828	0.864	0.884
ψ_0-$\psi_{3/2}$	0.931	0.959	0.945	0.966	0.944	0.962	0.897	0.918
ψ_0-$\psi_{3/2}$-α	0.934	0.962	0.951	0.970	0.957	0.973	0.903	0.921
ψ_0-$\psi_{3/2}$-$\mathcal{B}$	0.976	0.983	0.986	0.989	0.986	0.989	0.975	0.979

$$\overline{\mathrm{FF}} = \left(\frac{\sum_n \mathrm{overlap}^3(n)}{\sum_n 1} \right)^{1/3}$$

$$\overline{\mathrm{FF}}_a = \left(\frac{\sum_n \mathrm{overlap}^3(n) \sqrt{\langle s_n, s_n \rangle}^3}{\sum_n \sqrt{\langle s_n, s_n \rangle}^3} \right)^{1/3}$$

Amplitude distribution for equal-mass BBH of $(15 + 15)M_\odot$



$\mathcal{J} \rightarrow$ cosine of angle between initial total angular-momentum direction and direction of observation

Summary of results for high-mass BBHs

- **By introducing arbitrary coefficients in the phase, $\overline{\mathrm{FF}}_a$ increases by $\sim 2.1\%$–15% with respect to SPA**

- **By changing the shape of the amplitude with α, $\overline{\mathrm{FF}}_a$ increases by $\sim 0.3\%$–1.1% with respect to non-modulated DTF**

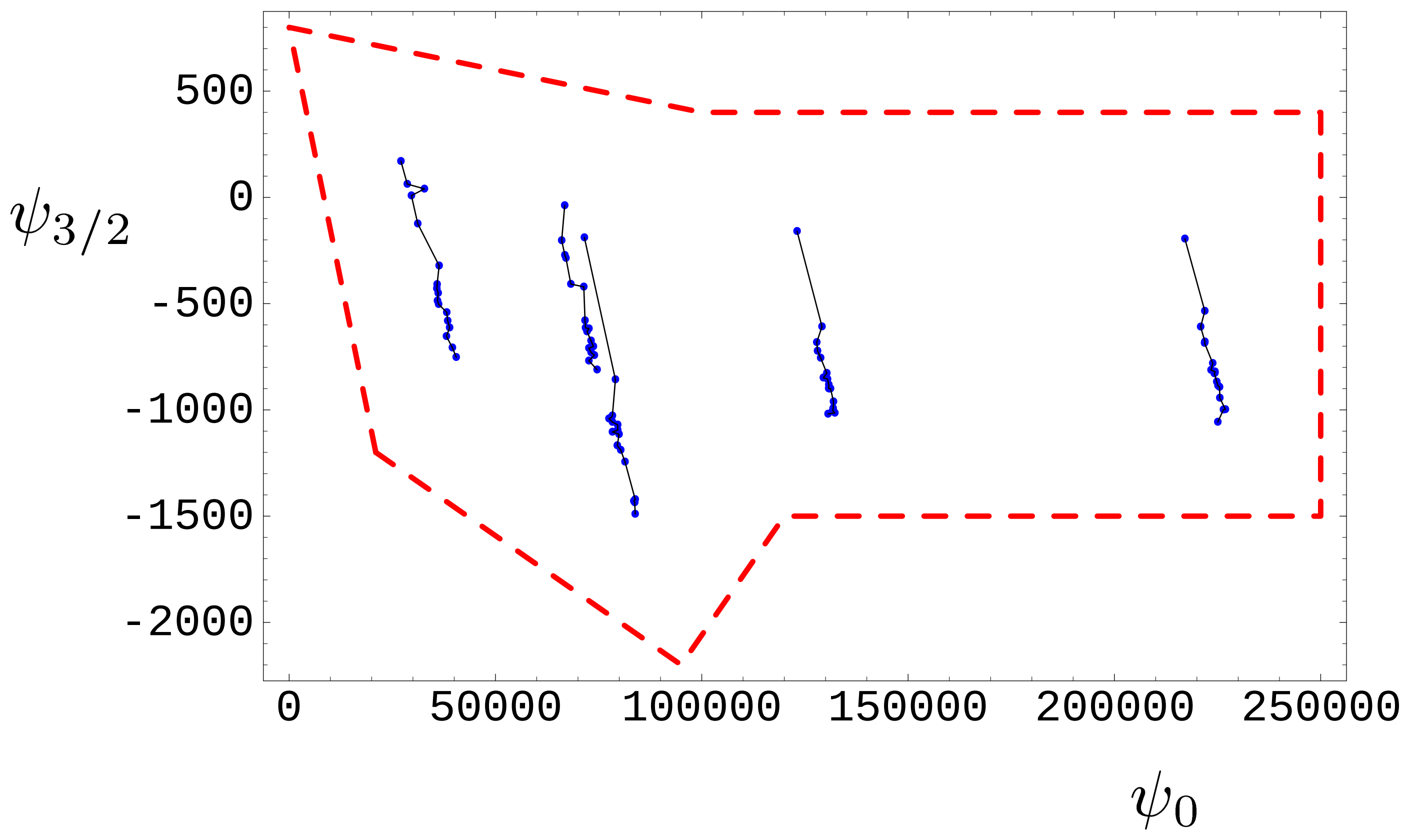
- **By modulating both amplitude and phase with γ_i, $\overline{\mathrm{FF}}_a$ increases by by $\sim 3\%$–7% with respect to non-modulated DTF**

Warning:

Keeping the same false alarm probability, we are justified in using more coefficients γ_i only if the gain in overlap is larger than the increase in detection threshold

High-mass BBHs: projection onto ψ_0-$\psi_{3/2}$–plane of non-modulated DTF

Zero-spin and aligned-spin configurations



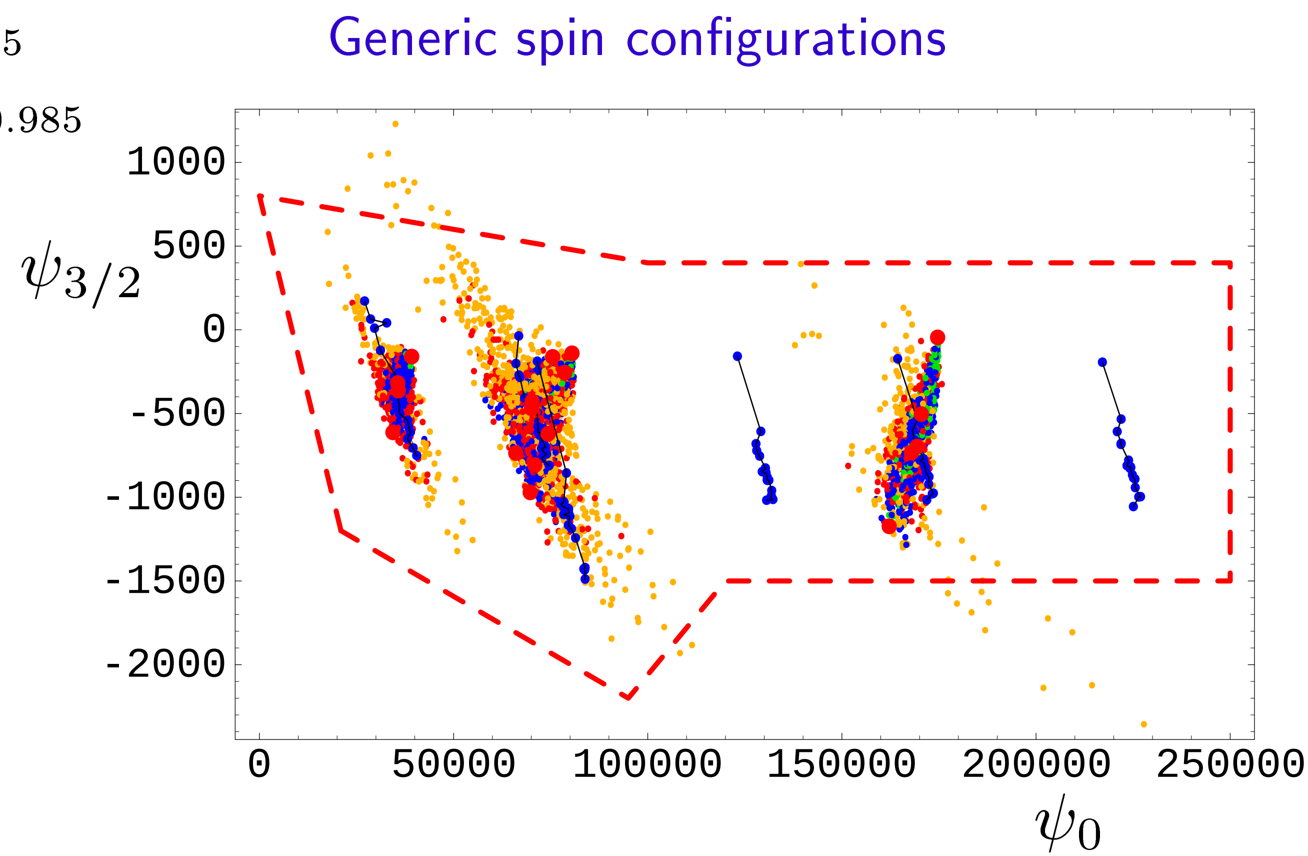
High-mass BBHs: projection onto ψ_0-$\psi_{3/2}$–plane of non-modulated DTF

yellow dots $\rightarrow \overline{\mathrm{FF}} < 0.9$

red dots $\rightarrow 0.9 < \overline{\mathrm{FF}} < 0.95$

blue dots $\rightarrow 0.95 < \overline{\mathrm{FF}} < 0.985$

green dots $\rightarrow \overline{\mathrm{FF}} > 0.985$



High-mass BBHs: projection onto ψ_0-$\psi_{3/2}$–plane of modulated DTF

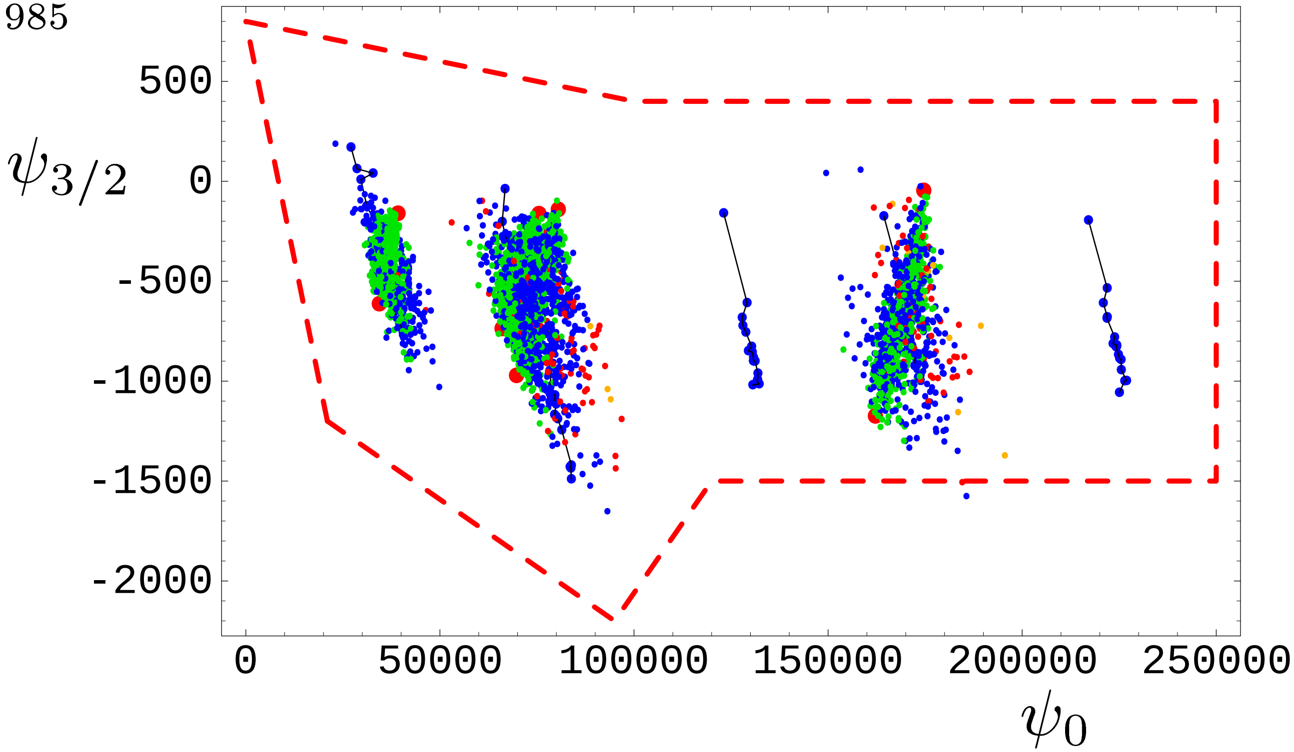
yellow dots $\rightarrow \overline{\mathrm{FF}} < 0.9$

red dots $\rightarrow 0.9 < \overline{\mathrm{FF}} < 0.95$

blue dots $\rightarrow 0.95 < \overline{\mathrm{FF}} < 0.985$

green dots $\rightarrow \overline{\mathrm{FF}} > 0.985$

Generic spin configurations



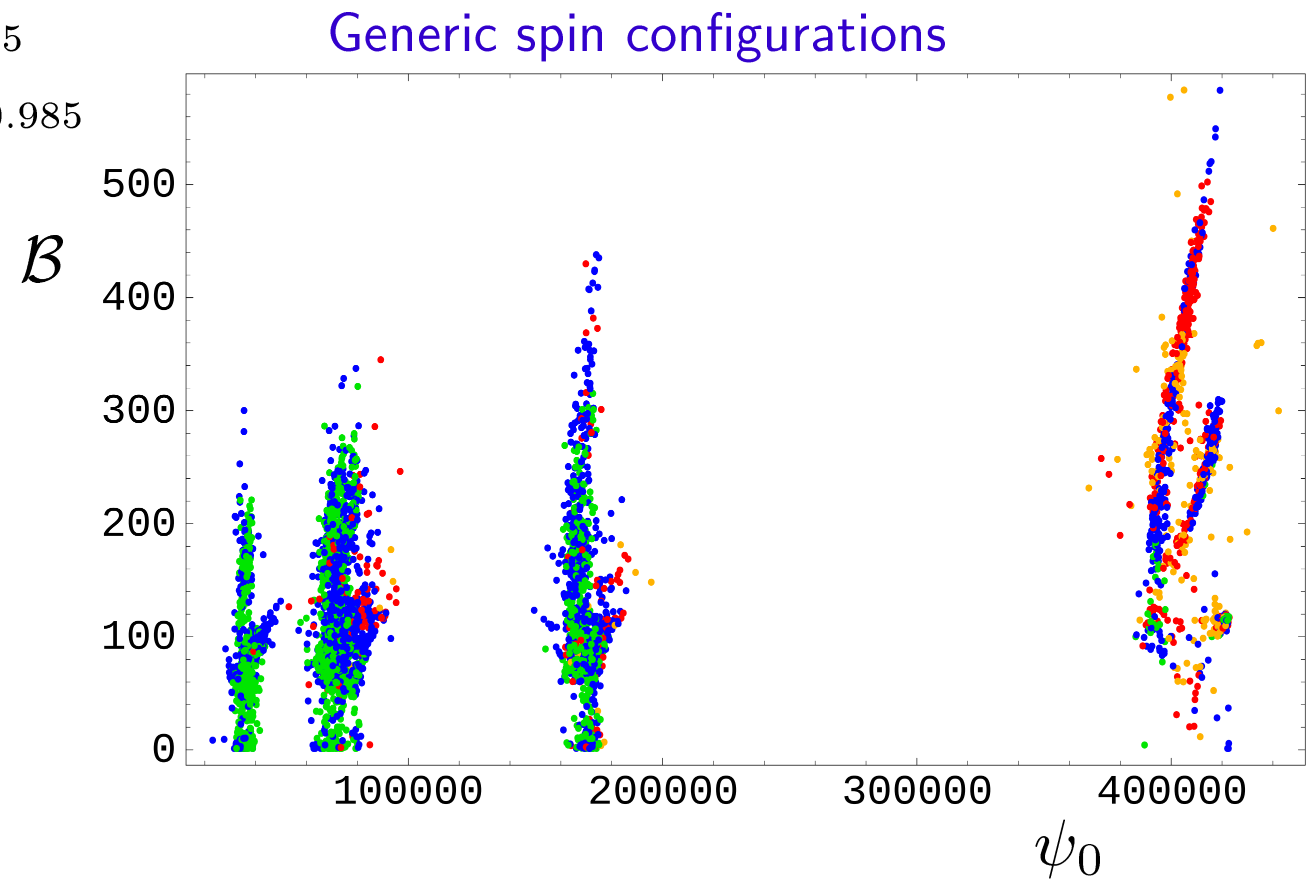
Compact binaries: projection onto ψ_0-$\mathcal{B}$–plane of modulated DTF

yellow dots $\rightarrow \overline{\mathrm{FF}} < 0.9$

red dots $\rightarrow 0.9 < \overline{\mathrm{FF}} < 0.95$

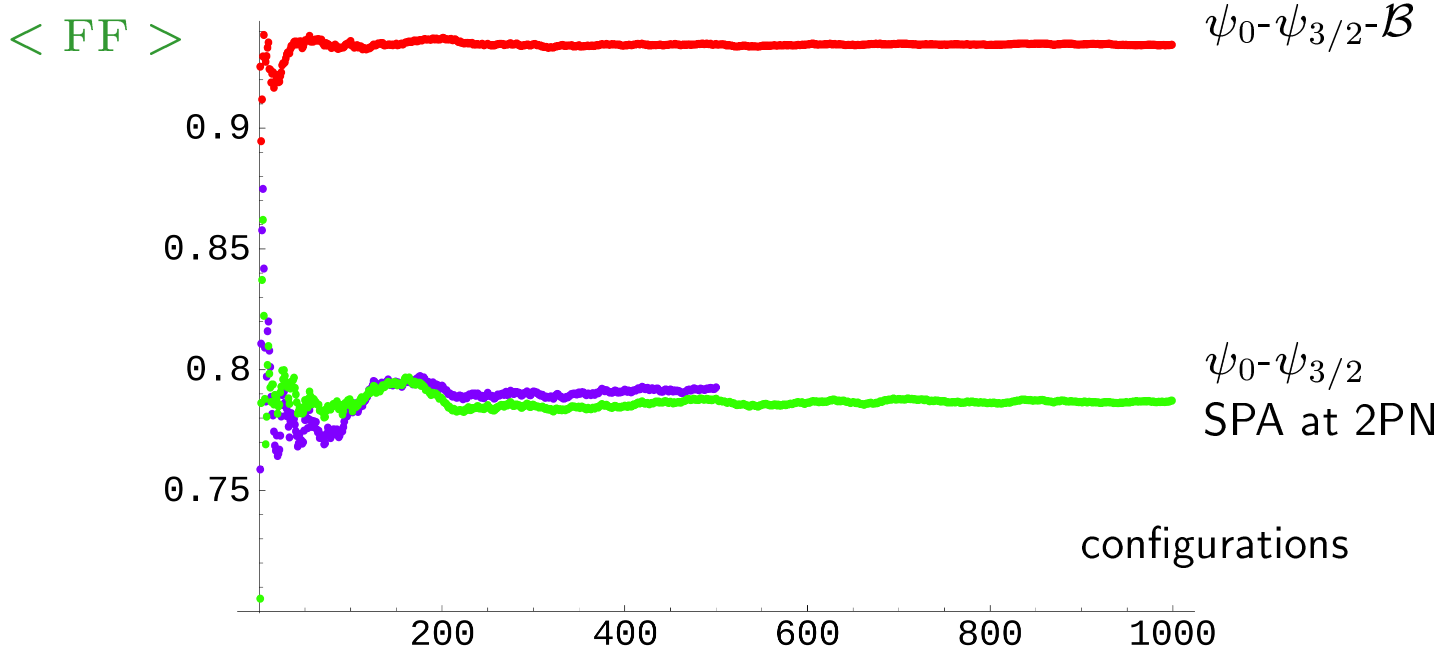
blue dots $\rightarrow 0.95 < \overline{\mathrm{FF}} < 0.985$

green dots $\rightarrow \overline{\mathrm{FF}} > 0.985$



Convergence of the average FF for BH/NS with $M = (10 + 1.4)\, M_\odot$

$$< \mathrm{FF} >= \left(\frac{\sum_n \mathrm{overlap}^2(n)}{\sum_n 1} \right)^{1/2}$$



We chose 1000 initial configurations for the spin

Performances for BH/NS binary

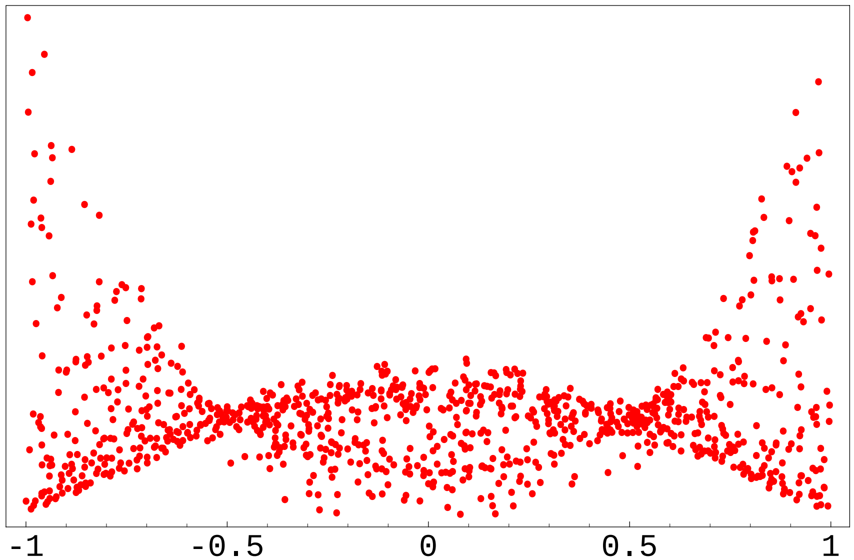
	$(10+1.4)M_\odot$	
	$\overline{\mathrm{FF}}$	$\overline{\mathrm{FF}}_{\mathrm{ampl}}$
SPA at 2PN	0.794	0.817
ψ_0-$\psi_{3/2}$	0.802	0.834
ψ_0-$\psi_{3/2}$-$\mathcal{B}$	0.936	0.945

$$\overline{\mathrm{FF}} = \left(\frac{\sum_n \mathrm{overlap}^3(n)}{\sum_n 1} \right)^{1/3}$$

$$\overline{\mathrm{FF}}_a = \left(\frac{\sum_n \mathrm{overlap}^3(n) \sqrt{\langle s_n, s_n \rangle^3}}{\sum_n \sqrt{\langle s_n, s_n \rangle^3}} \right)^{1/3}$$

Amplitude distribution for BH/NS of $(10+1.4)\,M_\odot$

Amplitude GW signal



$\mathcal{J}$

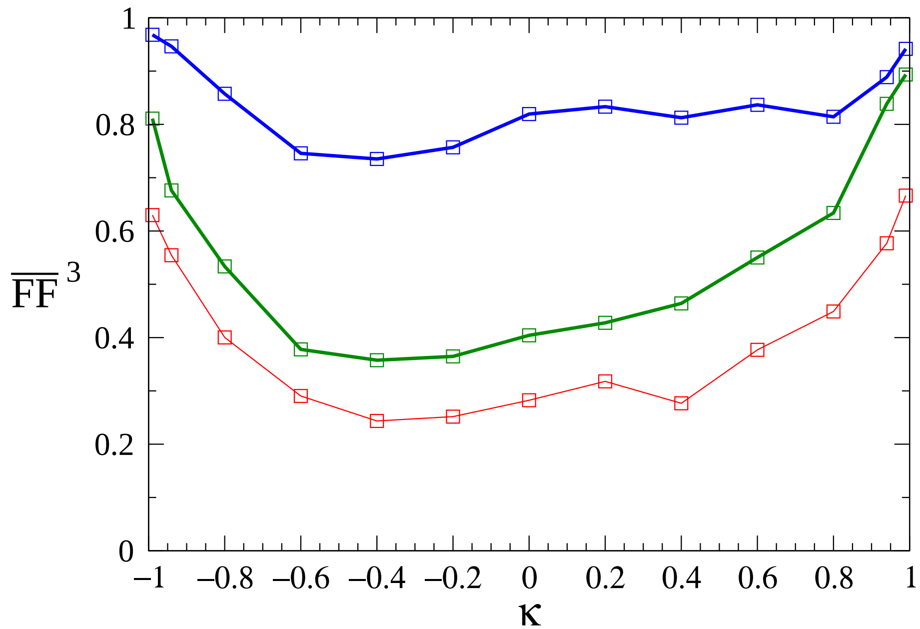
$\mathcal{J}$ $\rightarrow$ cosine of angle between initial total angular-momentum direction and direction of observation

Performances for BH/NS binary

blue line $\longrightarrow$ target: adiab. at 2PN; template: ϕ_0-$\psi_{3/2}$-$\mathcal{B}$

red line $\longrightarrow$ target: adiab. at Newt. order; template: SPA at 2PN

green line $\longrightarrow$ target: adiab. at 2PN; template: ϕ_0-$\psi_{3/2}$



$\kappa \longrightarrow$ cosine of the angle between Newtonian angular-momentum direction and BH's spin

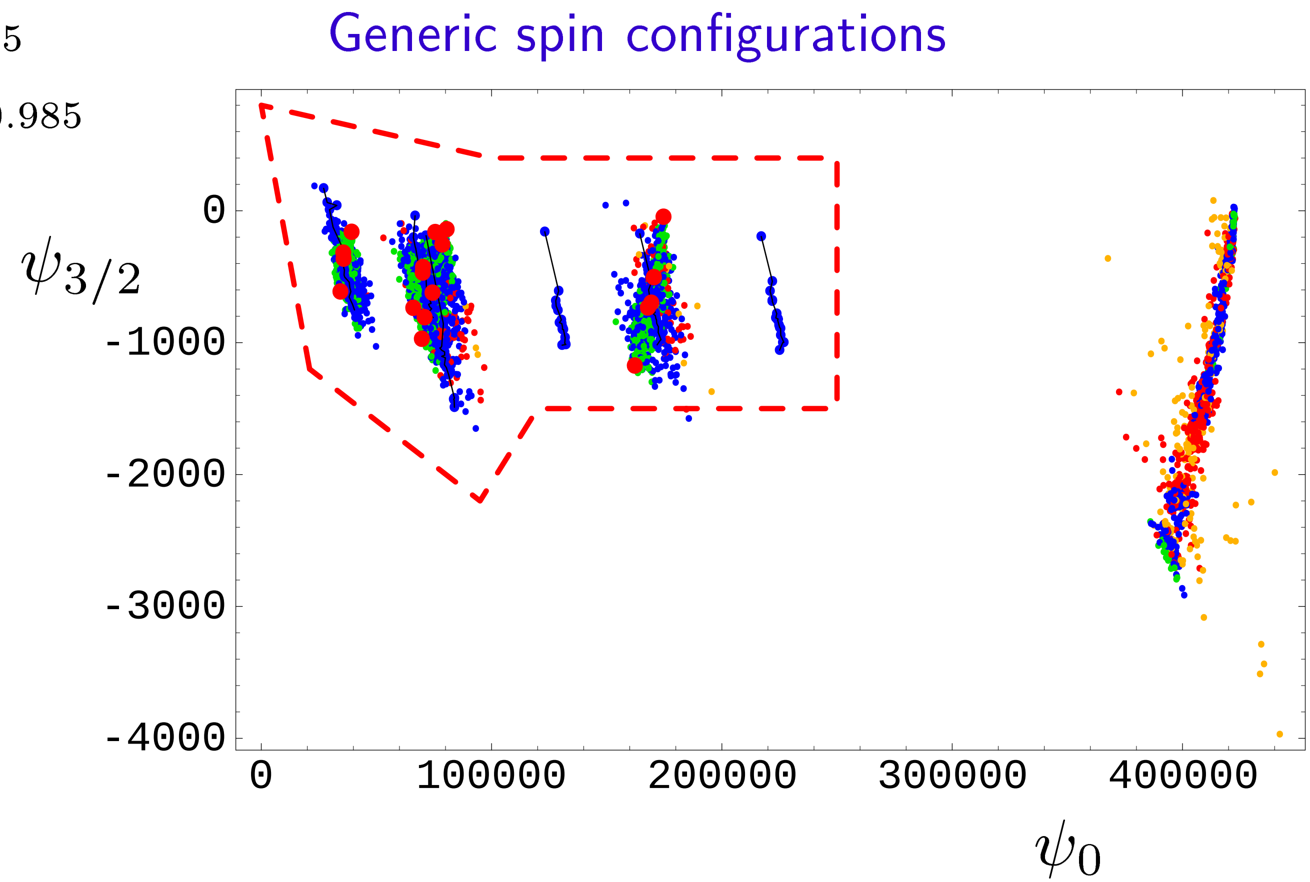
Compact binaries: projection onto ψ_0-$\psi_{3/2}$–plane of modulated DTF

yellow dots $\rightarrow \overline{\text{FF}} < 0.9$

red dots $\rightarrow 0.9 < \overline{\text{FF}} < 0.95$

blue dots $\rightarrow 0.95 < \overline{\text{FF}} < 0.985$

green dots $\rightarrow \overline{\text{FF}} > 0.985$



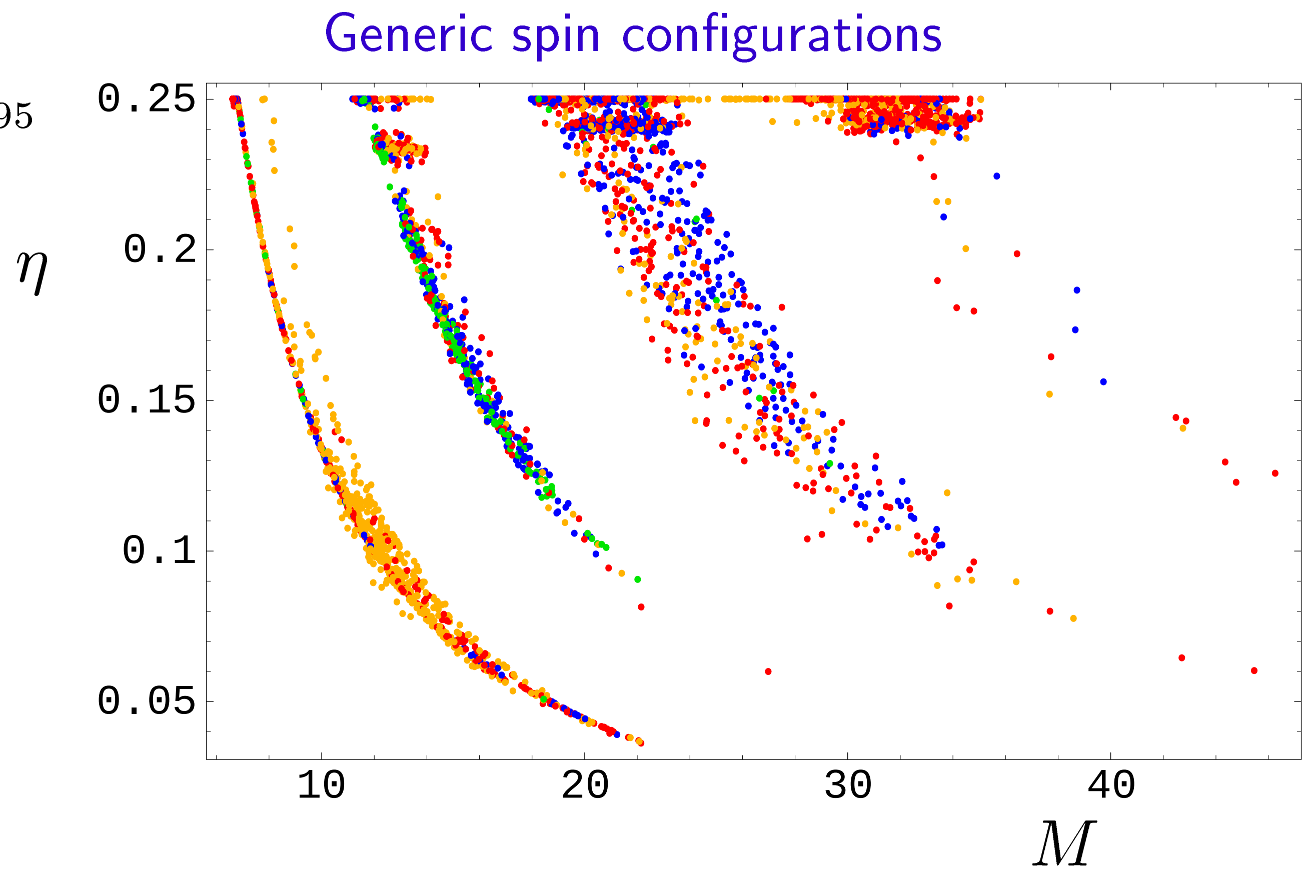
Projection on M-η–plane of SPA at 2PN order

yellow dots $\rightarrow \overline{\mathrm{FF}} < 0.8$

red dots $\rightarrow 0.8 < \overline{\mathrm{FF}} < 0.9$

blue dots $\rightarrow 0.9 < \overline{\mathrm{FF}} < 0.95$

green dots $\rightarrow \overline{\mathrm{FF}} > 0.95$



Summary and remarks

- **Non-modulated DTF quite promising for high-mass BBHs**

- **New convention for expressing the quadrupole GW signal**

- **Apostolatos' ansatz works if applied both to phase and amplitude. Indeed, the modulated DTF mimicks quite well the GW signal** $\overline{\mathrm{FF}}_a > 0.96$

- **The issue of** $real$ **gain in overlap keeping the same false alarm probability**

- **In the new convention and for NS/BH, the non-modulated GW signal depends only on two initial-configuration parameters: manageable!**

  [AB, Chen & Vallisneri, in preparation]

Realistic and manageable template model for NS/BH?

$$h_{\rm GW}(t) = -\frac{2\mu}{D}\frac{M}{r(t)}\left[\boldsymbol{e}_+^{ij}(t)\,\cos 2\Phi(t) + \boldsymbol{e}_\times^{ij}(t)\,\sin 2\Phi(t)\right]\times$$

$$[T_{+\,ij}(\Theta,\varphi)\,F_+(\theta,\phi,\psi) + T_{\times\,ij}(\Theta,\varphi)\,F_\times(\theta,\phi,\psi)]$$

$\boldsymbol{e}_+^{ij}$, $\boldsymbol{e}_\times^{ij}$ and Φ depend only on two parameters: S_1 and $\widehat{\boldsymbol{L}}_N \cdot \boldsymbol{S}_1$

$T_{\times,+ij}$ and $F_{+,\times}$ can be optimized automatically