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BicoMon

Steve Penn, Hobart and William Smith Colleges

LSC Meeting — August 2002

LIGO Hanford Observatory

LIGO-G020390-00-Z

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Synopsis

- Introduction to Higher Order Statistics
  - 1D: Correlation, Coherence, Power Spectra
  - 2D: Bicorrelation, Bicoherence, Bispectrum
- Monitor Update
- Future work

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What are Higher Order Statistics?

- 1D Statistics:
  - » Correlation: $C_{xy}(t) = \int_{-\infty}^{\infty} x(\tau)\, y(t+\tau)\, d\tau \;\Leftrightarrow\; X(f)\, Y^{*}(f) = S_{xy}(f)$
  - » Power Spectral Density: $C_{2x}(t) \;\Leftrightarrow\; X(f)\, X^{*}(f) = S_{2x}(f)$
  - » Coherence: $C_{xy}(f) = \dfrac{S_{xy}(f)}{\sqrt{S_{2x}(f)\, S_{2y}(f)}}$
    - – Tells us power and phase coherence at a given frequency

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Second Order Statistics

- 2D Statistics:
  - » Bicumulant:

$$C_{xyz}(t,t') = \int_{-\infty}^{\infty} x(\tau)\, y(t+\tau)\, z(t'+\tau)\, d\tau \quad \Leftrightarrow \quad X(f_1)\, Y(f_2)\, Z^*(f_1+f_2) = S_{xyz}(f_1, f_2)$$

  - » Bispectral Density:

$$C_{3x}(t) \quad \Leftrightarrow \quad X(f_1)\, X(f_2)\, X^*(f_1+f_2) = S_{3x}(f_1, f_2)$$

  - » Bicoherence:

$$\mathrm{C}_{xyz}(f) = \frac{S_{xyz}(f_1, f_2)}{\sqrt{S_{2x}(f_1)\, S_{2y}(f_2)\, S_{2z}(f_1, f_2)}}$$

    - – Tells us power and phase coherence at a coupled frequency

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Zero-lag Cumulants

Mean	Variance	Skewness	Kurtosis
$C_x(0)$	$C_{2x}(0)$	$C_{3x}(0)$	$C_{4x}(0)$
		0 if Symmetric	0 if Gaussian

Useful statictical values, but…

Skewness = 0 does not prove symmetry

Kurtosis = 0 does not prove Gaussianity

Variations in skew and kurtosis not well quantified.

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Why Higher Order Statistics?

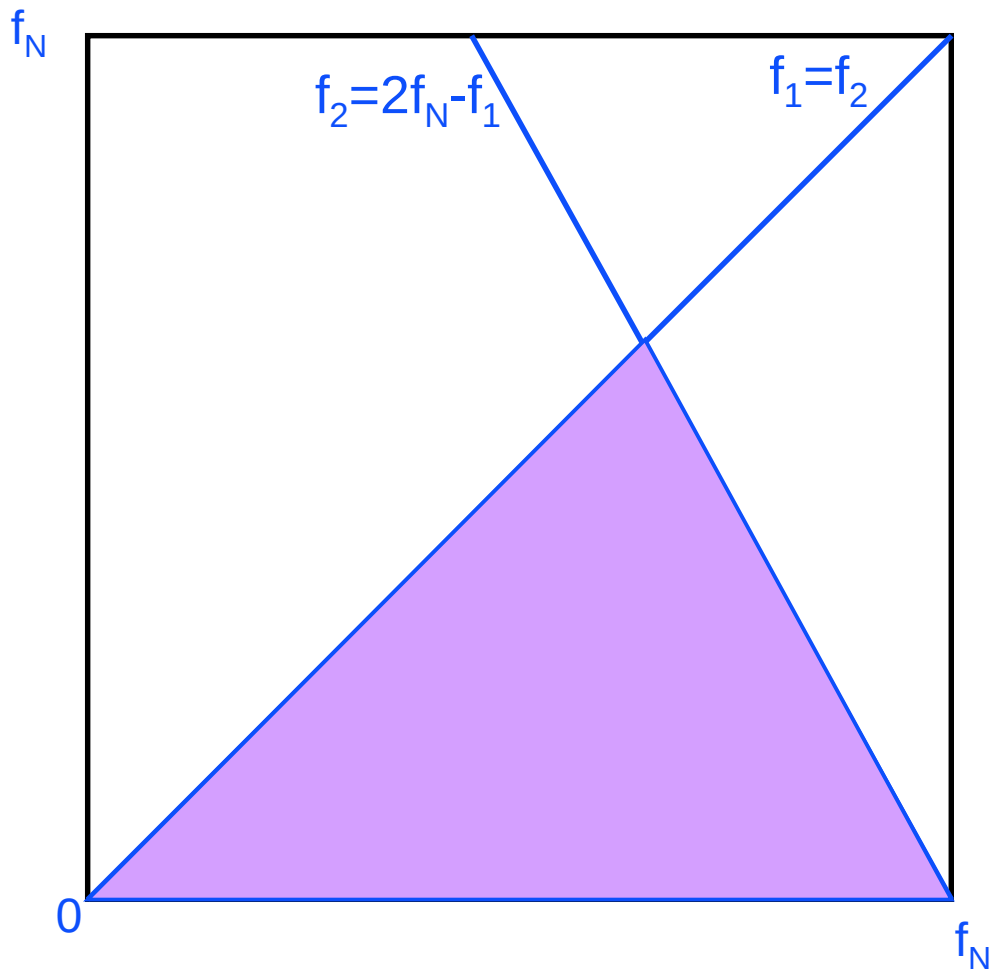
- For a Gaussian process: $C_{nx}(t) = 0, \text{ for } n > 2$

- For independent processes:

$$z(t) = x(t) + y(t), \quad C_{nz}(t) = C_{nx}(t) + C_{ny}(t) \xrightarrow{n>2} C_{ny}(t)$$

- Allows for separation of Gaussian process for n>2
  - » Visual check of frequency coupling and phase noise
  - » Statistical test for the probability of gaussianity and linearity
  - » Iterative process to reconstruct nongaussian signal from the higher order cumulants

Auto-Bispectrum Unique Area



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Monitor Features

- Plots (cross-)bicoherence, (cross-)bispectrum, & PSD's
  - » Operates on 1–3 channels
  - » User-specified Bispectrum method (Direct / InDirect)
  - » Automatically decimates to the lowest channel rate.
  - » User selected f_{max} and Δf Limited to factor 2^n
  - » User specifies the accuracy
  - » Monitor Outputs the Time span (gives user the update time)
  - » User-specified Bispectrum method (Direct / InDirect)
  - » Windowing: Optimized Rao-Gabr windowing
  - » Outputs GIF files of the plots
  - » Help facility

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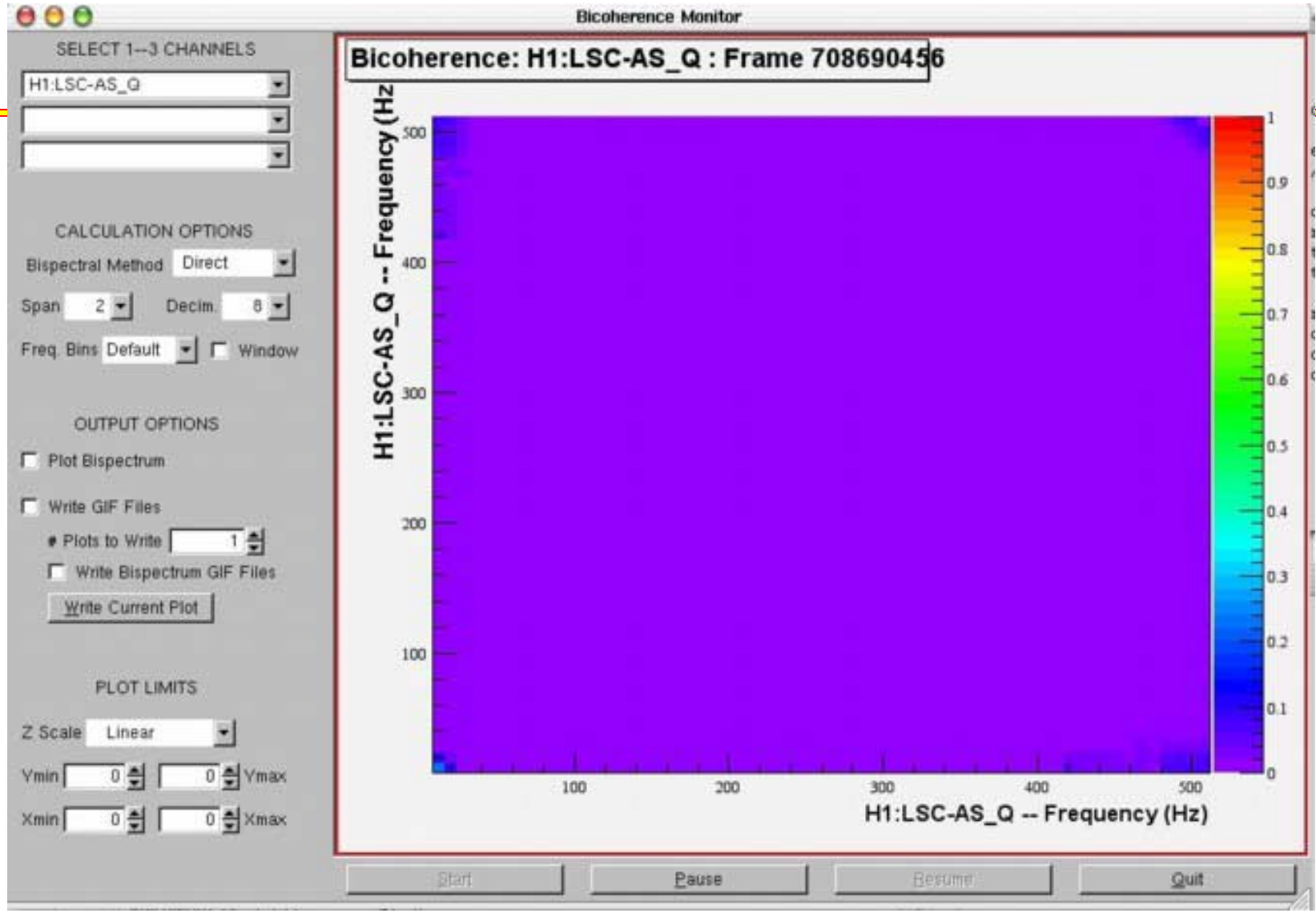
Bicoherence Monitor
SELECT 1--3 CHANNELS
H1:LSC-AS_Q
CALCULATION OPTIONS
Bispectral Method
Direct
Span
2
Decim.
8
Freq. Bins
Default
Window
OUTPUT OPTIONS
Plot Bispectrum
Write GIF Files
Plots to Write
1
Write Bispectrum GIF Files
Write Current Plot
PLOT LIMITS
Z Scale
Linear
Ymin
0
Ymax
Xmin
Xmax
Bicoherence: H1:LSC-AS_Q : Frame 708690456
H1:LSC-AS_Q -- Frequency (Hz)
H1:LSC-AS_Q -- Frequency (Hz)
Start
Pause
Resume
Quit

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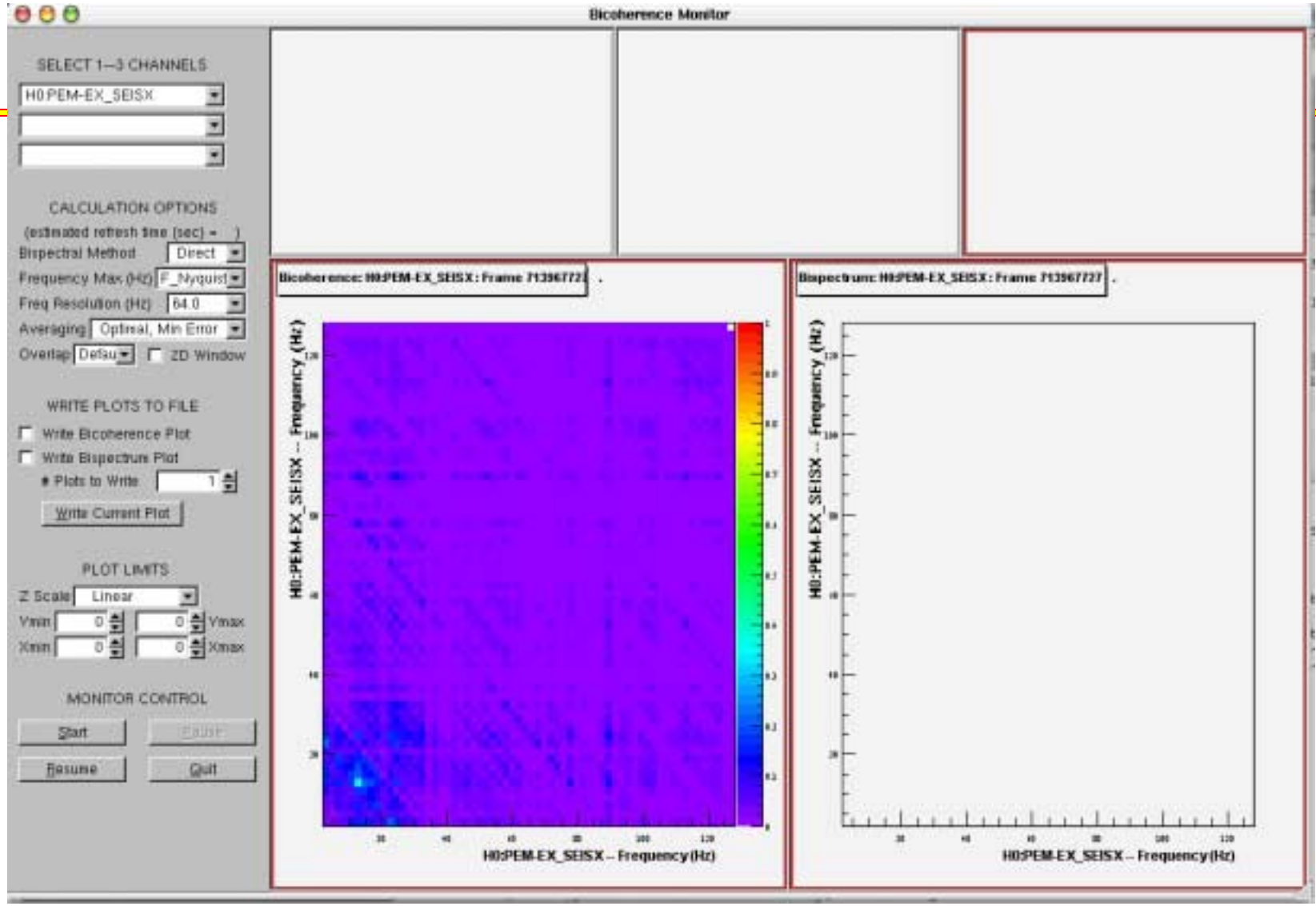
Bicoherence Monitor
SELECT 1–3 CHANNELS
H0:PEM-EX_SEISX
CALCULATION OPTIONS
(estimated refresh time (sec) =)
Bispectral Method
Direct
Frequency Max (Hz)
F_Nyquist
Freq Resolution (Hz)
64.0
Averaging
Optimal, Min Error
Overlap
Defau
ZD Window
WRITE PLOTS TO FILE
Write Bicoherence Plot
Write Bispectrum Plot
Plots to Write
1
Write Current Plot
PLOT LIMITS
Z Scale
Linear
Vmin
Vmax
Xmin
Xmax
MONITOR CONTROL
Start
Pause
Resume
Quit
Bicoherence: H0:PEM-EX_SEISX: Frame 713967727
H0:PEM-EX_SEISX -- Frequency (Hz)
H0:PEM-EX_SEISX -- Frequency (Hz)
Bispectrum: H0:PEM-EX_SEISX: Frame 713967727
H0:PEM-EX_SEISX -- Frequency (Hz)
H0:PEM-EX_SEISX -- Frequency (Hz)

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Improvements / Future Directions

- GUI, frequency-selection method, freq. resolution

- Vijay Chickarmane and I are working together to monitor the time variation of regions of high bicoherence. --> Background monitoring

  - we are devising a method to fit the bilinear features for tracking of coefficients and/or subtraction