Extended Hierarchical Search for Inspiraling Compact Binaries

A. Sengupta, S. Dhurandhar, A. Lazzarini, T. Prince*, P. Shawhan

Plan of the Talk

- The 1-step search with 2PN templates
- The 2-step search with hierarchy only in the masses
- The 2-step search with hierarchy in masses and decimation in time

* Participated in the early phases of the project

The restricted PN waveform in the SPA

$$\tilde{h}(f) = \mathcal{N} f^{-7/6} \exp i\psi(f;\lambda^\alpha) + 2\pi i t_c f + i\phi_0$$

$$\psi(f;\lambda^\alpha) = \Sigma\,\theta^i(\lambda^\alpha)\zeta_i(f)$$

2PN and spinless templates: $\zeta_1(f) = f^{-5/3}, \zeta_2(f) = f^{-1}, \zeta_3(f) = f^{-2/3}, \zeta_4(f) = f^{-1/3}$

$$\begin{aligned}
\theta_1 &= \frac{3}{128\eta}(\pi M)^{-5/3} \\
\theta_2 &= \frac{1}{384\eta}\left(\frac{3715}{84} + 55\eta\right)(\pi M)^{-1} \\
\theta_3 &= -\frac{3\pi}{8\eta}(\pi M)^{-2/3} \\
\theta_4 &= \frac{3}{128\eta}\left(\frac{15293365}{508032} + \frac{27145}{504}\eta + \frac{3085}{72}\eta^2\right)
\end{aligned}$$

The detection strategy

Maximum Likelihood Approach:

Parameters: Amplitude, t_c, ϕ_0, τ_0, τ_3

Strategy:

- Use normalised templates
- Two templates for $\phi_0 = 0, \pi/2$ and add in quadrature
- Use FFT for scanning over t_c
- For τ_0, τ_3 template bank required

The Parameter Space:

Parameters in which the ambiguity function is almost independent of location

$$\tau_0 = \frac{5}{256\mu M^{2/3}}(\pi f_a)^{-8/3}$$

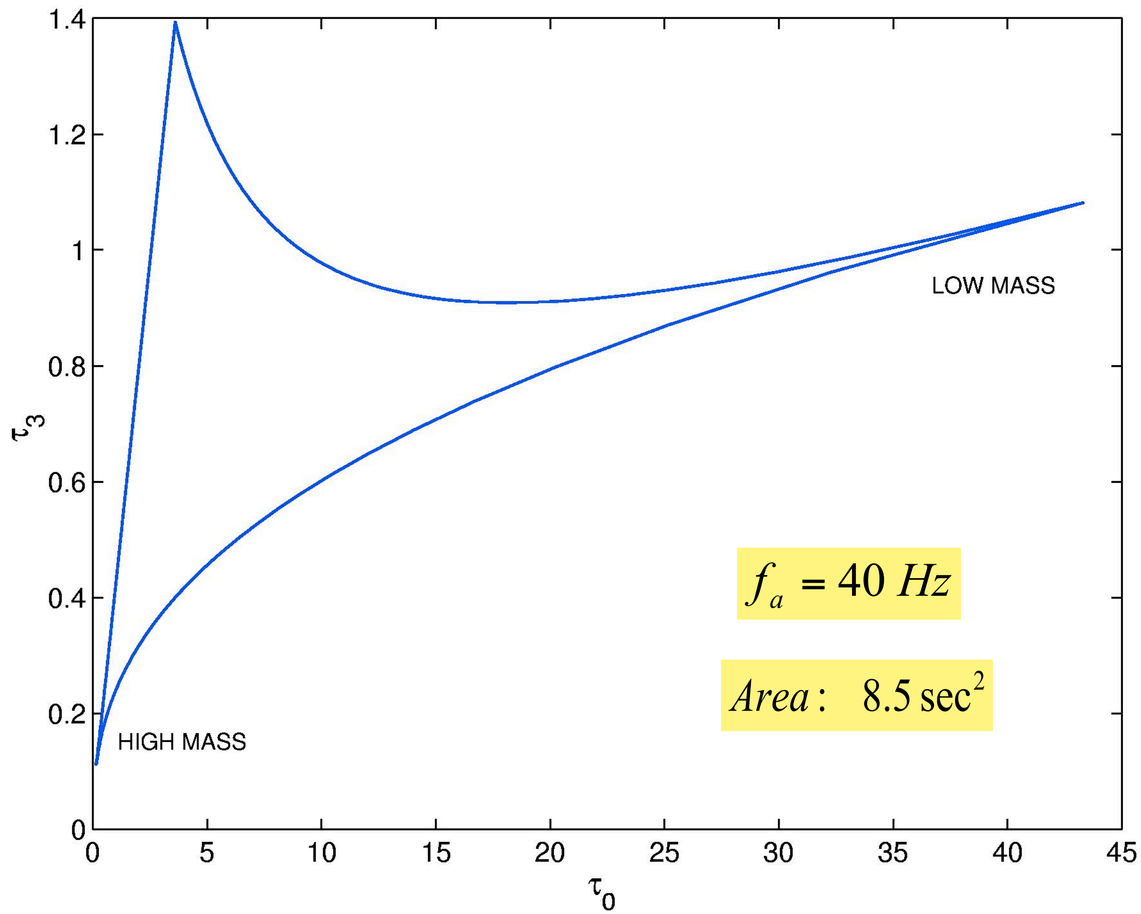
$$\tau_3 = \frac{1}{8\mu}\left(\frac{M}{\pi^2 f_a^5}\right)^{1/3}$$

$f_a = 40$ Hz.

Mass Range: $1\ M_\odot \le m_1, m_2 \le 30\ M_\odot$

Area: 8.5 sec^2

Parameter Space



Mismatch and Ambiguity Function

$$H(\lambda^\alpha, \Delta\lambda^\alpha) = \langle s(\lambda^\alpha), s(\lambda + \Delta\lambda^\alpha\rangle$$

Intrinsic Ambiguity Function:

$$\mathcal{H}(\tau_0, \tau_3; \Delta\tau_0, \Delta\tau_3) = \max_{\Delta t_c, \Delta\phi_0} \ H(\lambda^\alpha, \Delta\lambda^\alpha)$$

Minimal match $= 0.97$

$$n_t \sim 10^4$$

Online speed for the 1-step search (square lattice): ~ 3 GFlop

The hierarchical search

The principle:

- Two thresholds and two banks of templates:
  - Lower threshold: η_1 and a coarse grid of templates
  - Higher threshold: $\eta_2 = \eta$ and the 1-step fine grid of templates
- η_1 sufficiently large - few false alarms - minimise cost of the fine search.
- η_1 small enough - coarse grid - minimise cost in the trigger stage.

The Total Computational Cost

$$C_1 \sim n_t^{(1)} \times 6N \log_2 N$$

$$C_2 \sim n_c \times (n_t^{(2)}/n_t^{(1)}) \times 6N \log_2 N \times \alpha$$

α is a covering factor which we will take to be little over 2.

Total cost $C = C_1 + C_2$.

Only hierarchy in masses: gain factor 25-30

The extended hierarchical search

Include decimation in time in the previous hierarchical search

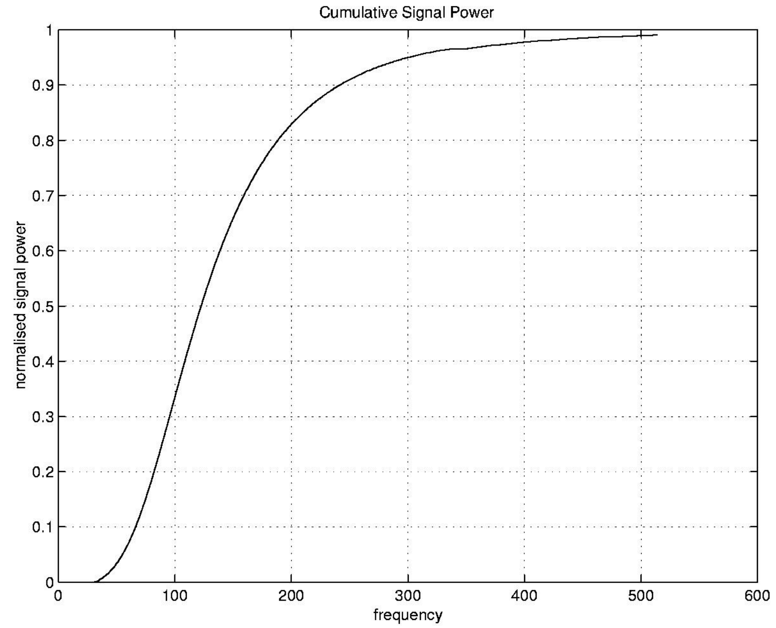
Cumulative Signal Power:

92% power at

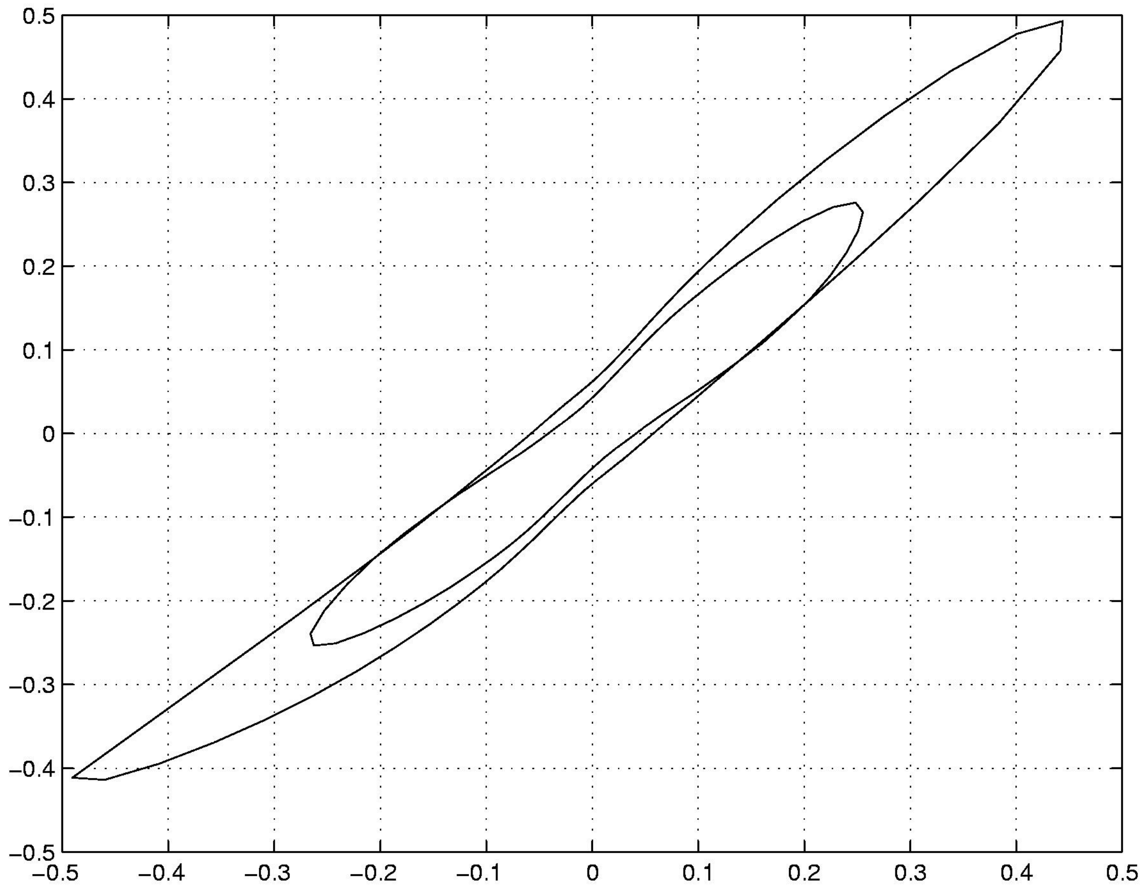
f_c = 256 Hz

Factor of 4

In FFT cost

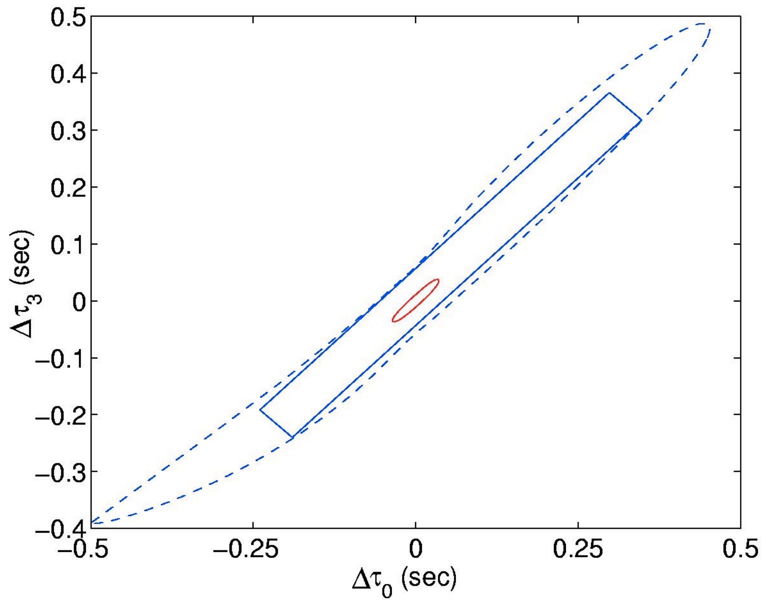


Comparison of contours (H = 0.8) with 256 Hz and 1024 Hz cut-off

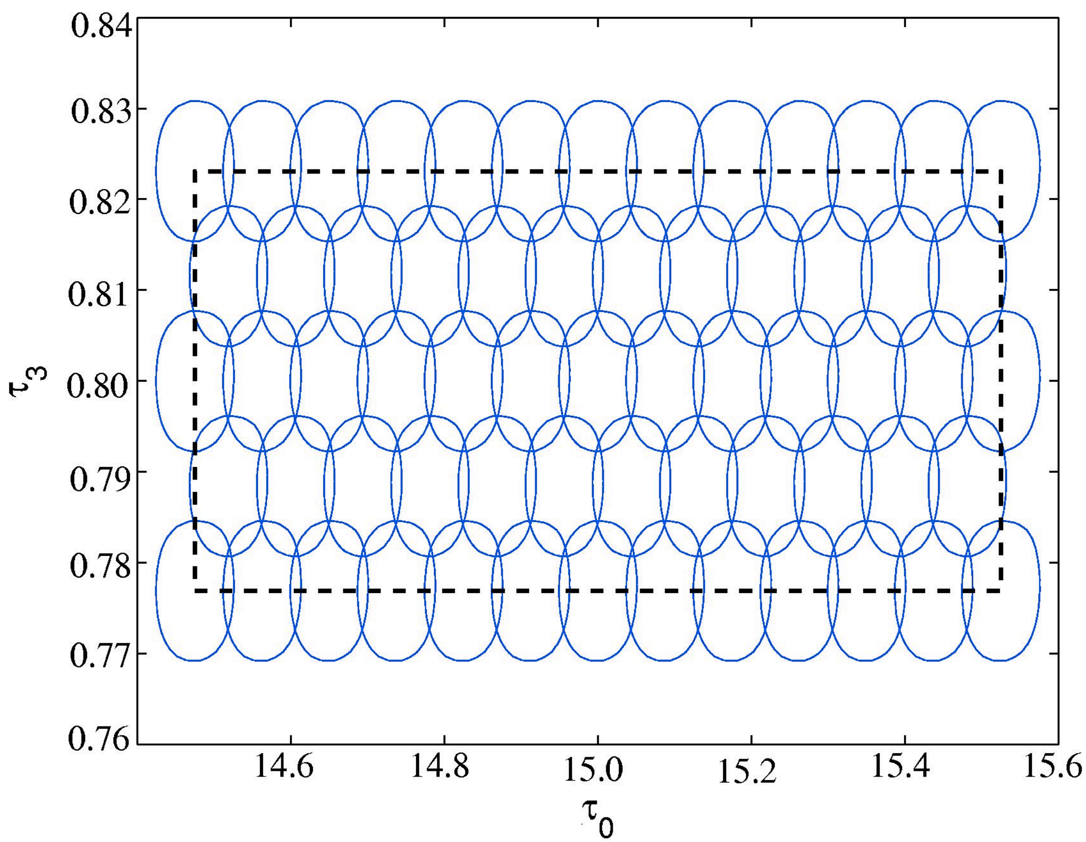


Chirp cut at 256 Hz

Contour level $= (\eta_1 + \Delta S)/(.92 \times S_{min}) \sim 0.8$



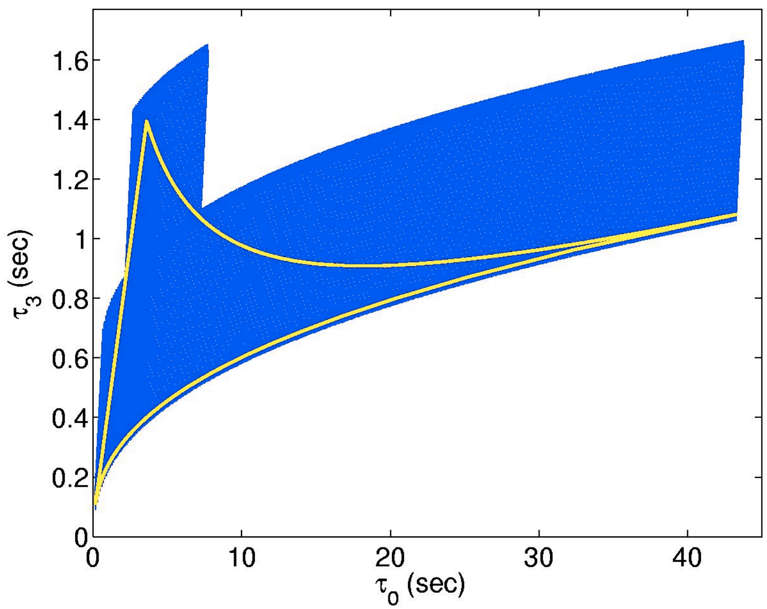
Relative size of boat/ellipse: .97 at 1 kHz and .8 at 256 Hz

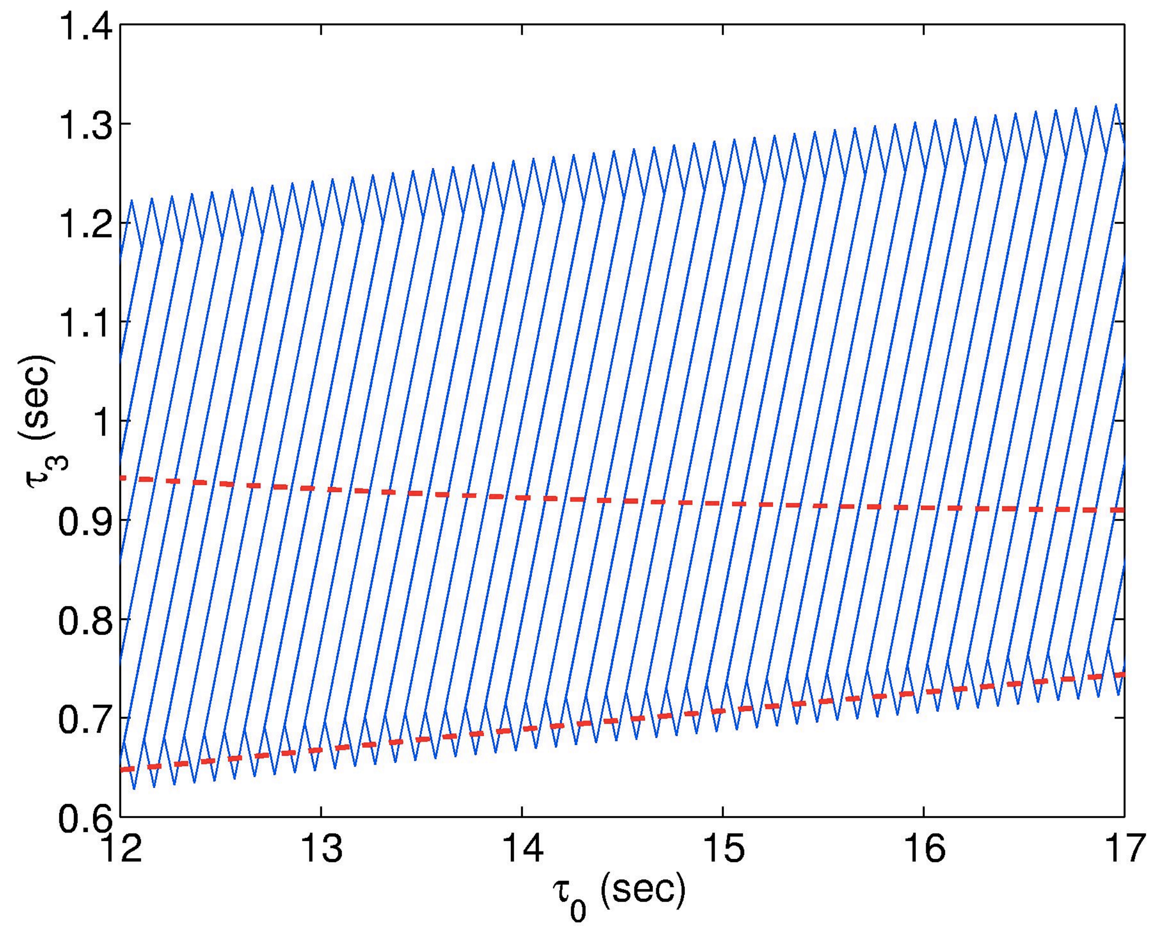

0.84
0.83
0.82
0.81
0.80
0.79
0.78
0.77
0.76
τ_3
14.6
14.8
15.0
15.2
15.4
15.6
τ_0

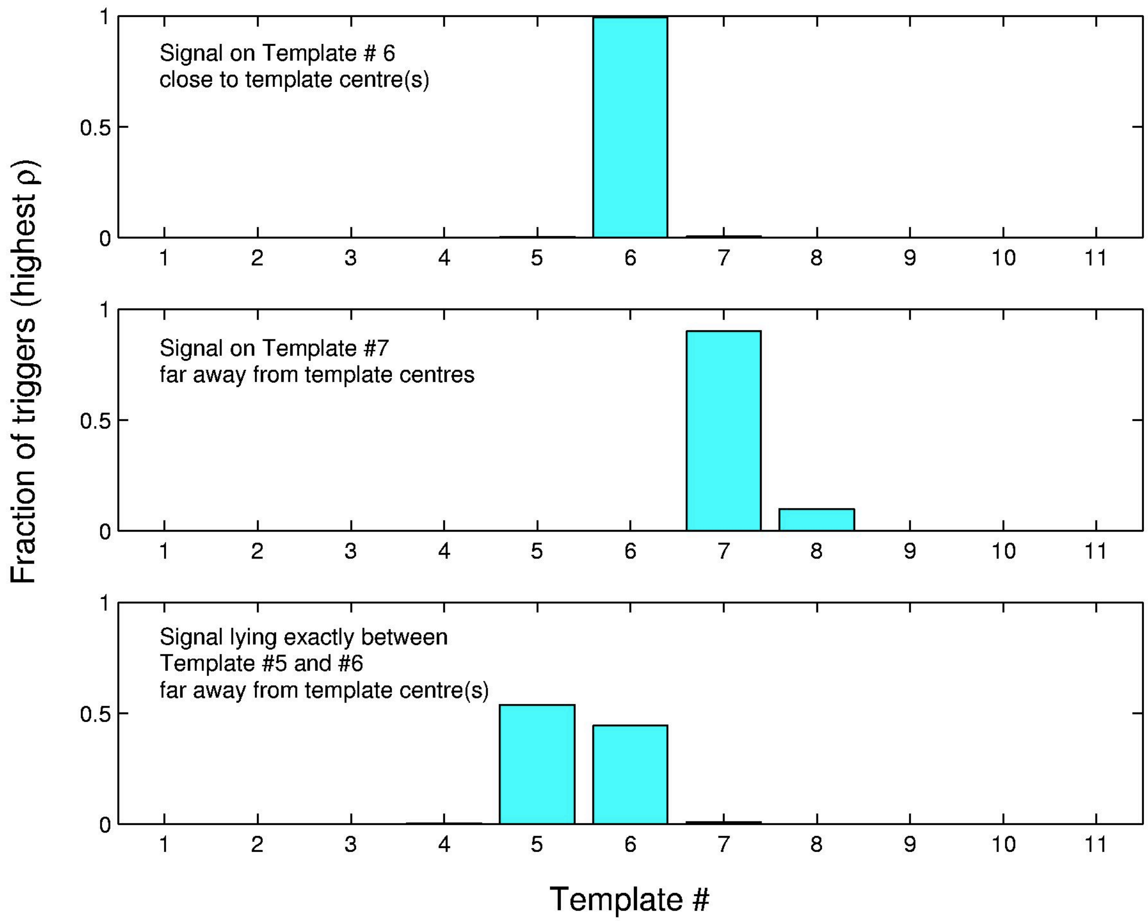
Boundary Effects

Inefficient tiling near the low mass end !!

For low mass cut-off of $1 M_\odot$: Need 535 templates




1.4
1.3
1.2
1.1
1
0.9
0.8
0.7
0.6
τ_3 (sec)
12
13
14
15
16
17
τ_0 (sec)


Signal on Template # 6
close to template centre(s)
Signal on Template #7
far away from template centres
Signal lying exactly between
Template #5 and #6
far away from template centre(s)
Fraction of triggers (highest ρ)
Template #
0
0.5
1
1
2
3
4
5
6
7
8
9
10
11

Computational Cost

Search parameter range	$1\ M_\odot < m_1, m_2 < 30\ M_\odot$,
Area of interest	8.5 $\sec^2$
Number of templates (first stage)	~ 535
Computational speed (first stage)	~ 33 MFlops
Computational speed (second stage)	~ 10 MFlops
Computational speed (total)	~ 43 MFlops

Flat search speed: 3 GFlops

Gain ~ 70

SUMMARY

• Results shown for Gaussian noise

- Need to look at real (E7, S1) data

- Performance with non-Gaussian noise is the next question

- event multiplicity needs to be condensed into a single event

• Reducing computational cost with hierarchical approach frees up CPU power for additional parameters

- spins

G020360-00-Z

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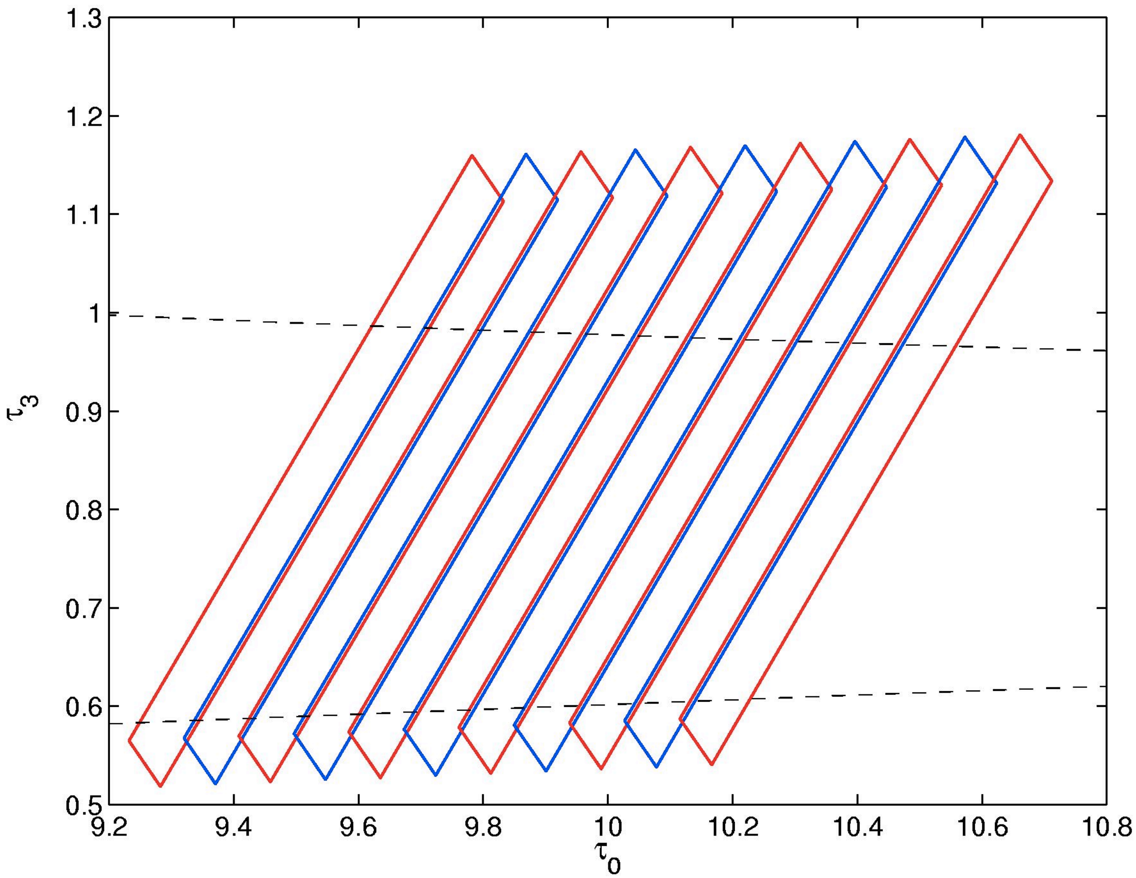


1.3
1.2
1.1
1
0.9
0.8
0.7
0.6
0.5
τ_3
9.2
9.4
9.6
9.8
10
10.2
10.4
10.6
10.8
τ_0

Detection probability Q_d

Defn: Signal Strength $S = c$ for perfect match of the parameters

c follows a Rician distribution

$\sim$ Gaussian with mean S, if $S >> 1$

With two adjacent templates:

For $Q_d = .95$ we get $S_{minmin} \sim \eta + 0.7$

$S_{min} \sim 8.2 + 0.7 + 3\% \sim 9.2$

Thresholding

In absence of the signal:

Each $c_0, c_{\pi/2}$ is Gaussian distributed with mean zero.

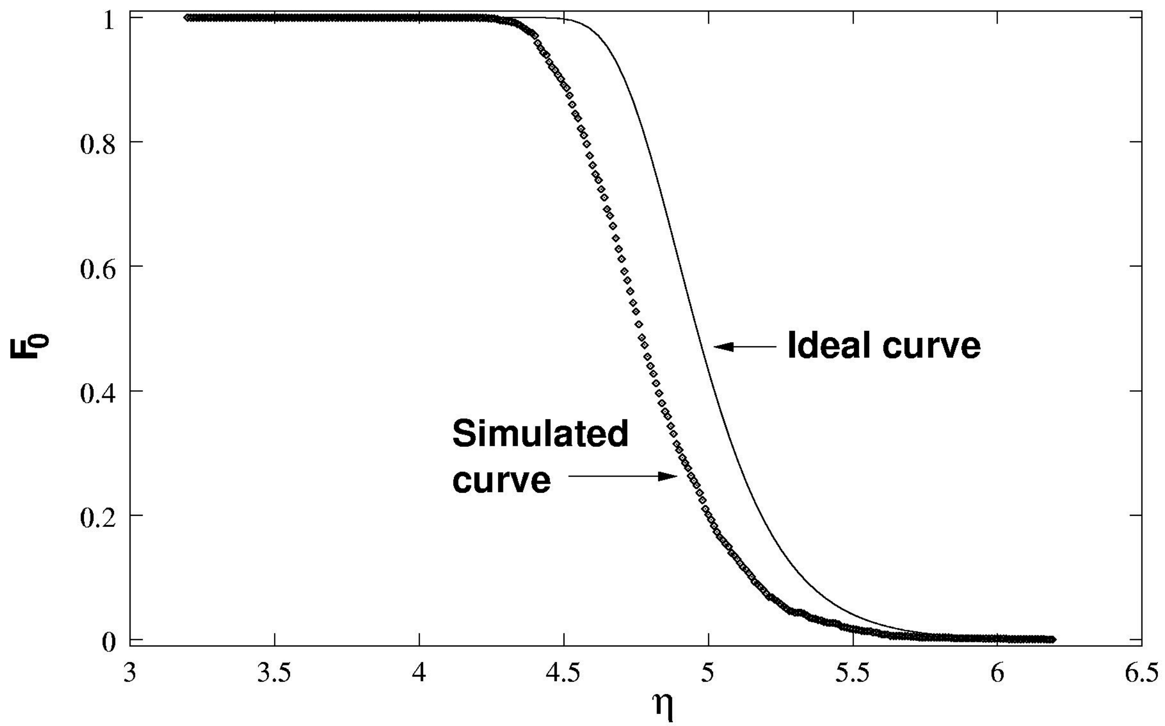
While $c = \sqrt{c_0^2 + c_{\pi/2}^2}$ is Raleigh distributed:

$R(c) = c \exp(-c^2/2)$

1 false alarm/yr, sample at 2 kHz, $n_t \sim 10^4$ at 3% mismatch

Range $1M_\odot \leq m_1, m_2 \leq 30M_\odot$ - LIGO I curve.

$\int_\eta^\infty R(c)dc = P_F$ gives $\eta \sim 8.2$



The average number of crossings: $n_c = n_t^{(1)} \times F_0(\eta)$

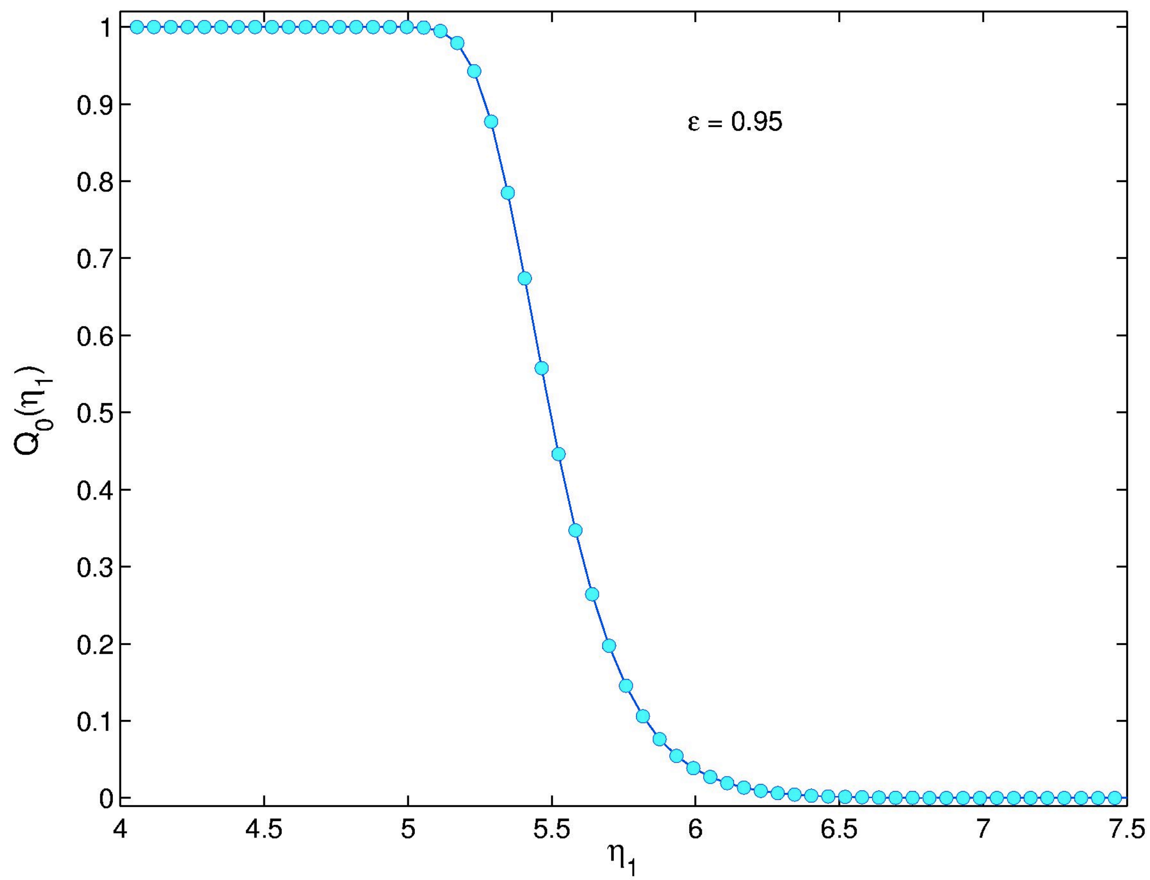
Decimation in time

Use the fact that there is a lot of power at low frequencies in the chirp

At 256 Hz there is 92 % of the signal power

Sample at lower frequencies but lose little in power

Gain in reducing FFT operations in the trigger stage



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