Multi-detector searches of inspirals: Current strategies & proposed improvements

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I. Single detector (1D) statistic:

$$\Lambda = |C| = \left[|C_0|^2 + |C_{\pi/2}|^2\right]^{1/2} \quad ,$$

where $C := \langle S, x\rangle = \langle S_0, x\rangle + i\langle S_{\pi/2}, x\rangle \quad ;$

$S =$template & $x =$ (noisy) data.

In the stationary-phase approximation,

$$\tilde{S}(f) \propto \left(\frac{f}{f_s}\right)^{-7/6} \exp\left[-i\Psi(f)\right] \quad ;$$

$\Psi(f;\vartheta^{\nu}) = 2\pi\varphi_{\mu}(f;\vartheta^{\nu})\vartheta^{\mu}$, $\mu, \nu = 0, 1, ...$

II. Multiple coaligned detectors:

$$\Lambda_{\mathrm{Net}} = \left|\sum_{I=1}^{M} C^I(\tau_I)\right| \quad ,$$

where τ_I is an offset time and

$$C^I := \langle S^I, x^I\rangle_{(I)} = \langle S_0^I, x^I\rangle_{(I)} + i\langle S_{\pi/2}^I, x^I\rangle_{(I)} \quad .$$

Two arbitrary interferometers

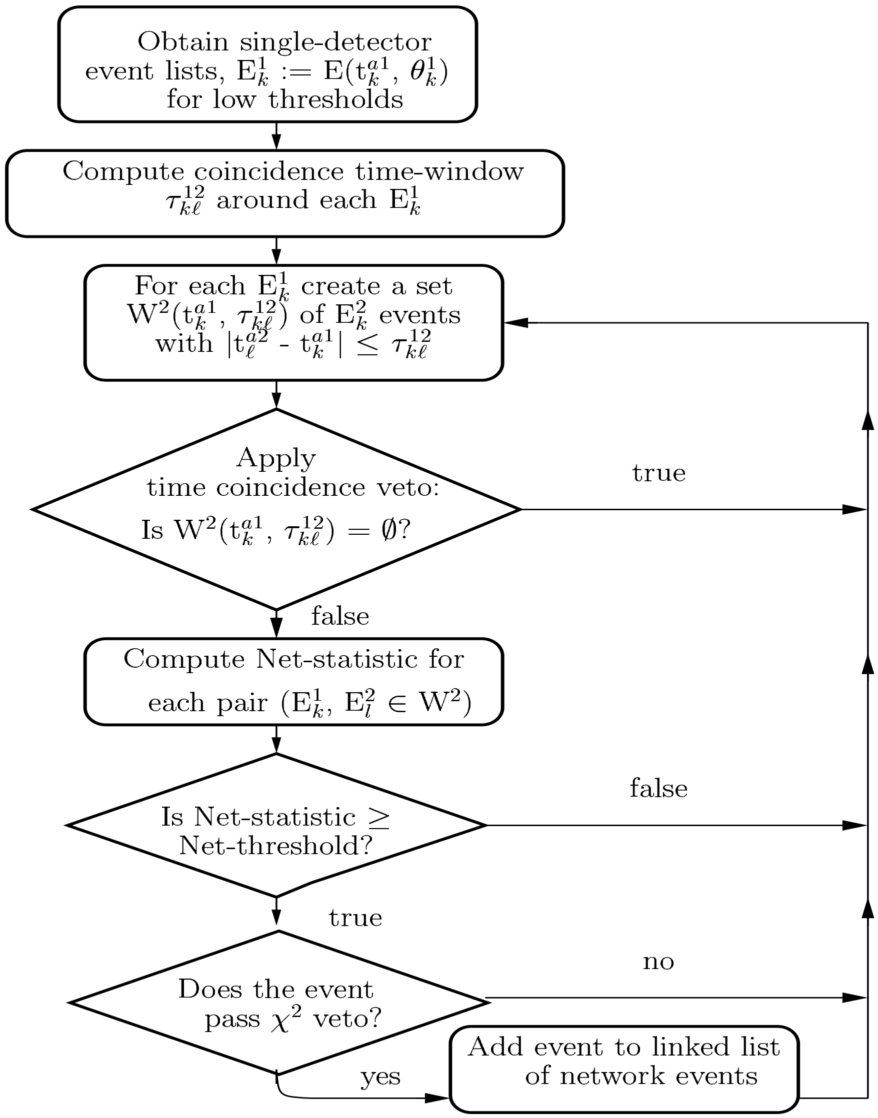
Statistic:

$$\Lambda(\tau)|_{\widehat{\epsilon},\widehat{\psi}} = \| \mathsf{C}(\tau) \|$$

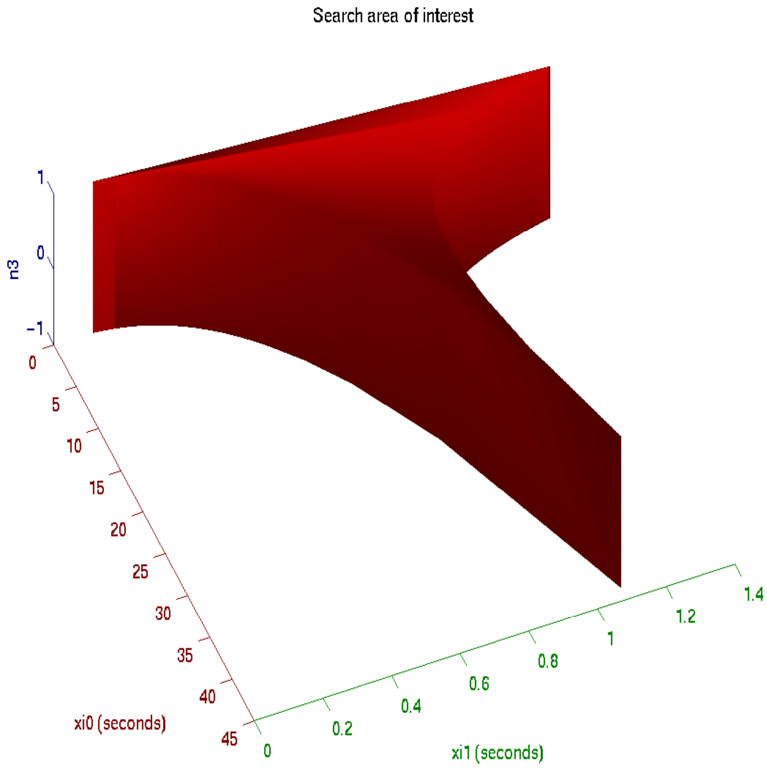
$$= \left[\ |C_1(\tau)|^2 + \left|C_2(\tau;\tau_{(2)})\right|^2 \ \right]^{1/2} \quad ,$$

which does *not* depend on $\{\psi, \epsilon\}$.
$\Lambda(\tau)|_{\widehat{\epsilon},\widehat{\psi}}$ depends only on the sum of the "ρ^2" statistics for the two detectors evaluated at the offset times τ and $\tau + \tau_{(2)}$, respectively.

2D search algorithm



2-detector parameter space



Above, ξ_0 and ξ_1 are the Newtonian and 1PN contributions, resp., to a chirp's dwell time.

Example. The 2D network of H1 and L1: One-step search on a 3-d parameter space, $\{f_s\xi_0,\ f_s\xi_1,\ n_3\}$, with metric:

$$g_{ij} = \begin{pmatrix} 0.0817 & 0.122 & 0 \\ \cdot & 0.185 & 0 \\ \cdot & \cdot & 3.615 \end{pmatrix} \quad ,$$

where $f_s = 40$ Hz (Seader, SB, *in prep.*). This extends the work in Owen, PRD 1996, & Balasubramanian *et al.*, PRD 1996.

Template spacing is determined by finding the eigencoordinates, $\{x_1,\ x_2,\ n_3\}$, in which the above metric is diagonalized:

$$(dx_1,\ dx_2,\ dn_3) = (\ 0.71,\ 9.25,\ 0.19) \left(\frac{1-MM}{0.03}\right)^{1/2} \quad .$$

Work nearing completion

Multi-detector (MD) statistic:

$$\Lambda|_{\hat{\epsilon},\hat{\psi}} = \|\mathbf{C}_{\mathcal{H}}\| = \left(|C^{+}|^2 + |C^{-}|^2\right)^{1/2} \quad ,$$

where

$$C^{\pm} := \mathsf{C} \cdot \hat{v}^{\pm} .$$

Here, $(\hat{v}^{+}, \hat{v}^{-})$ are two orthonormal real vectors. They form a basis in which any complex vector in $\mathcal{H}$ can be expanded. The vectors $\hat{v}^{\pm}$ depend on the detector orientations and the source-direction angles, i.e.,

$$\hat{v}^{\pm} = \hat{v}^{\pm}(\theta, \phi; \boldsymbol{\alpha}_{(1)}, ..., \boldsymbol{\alpha}_{(M)}) \quad .$$

Pai, Dhurandhar, SB, PRD 2001

3-detector networks

Example. Three coaligned LIGO-I detectors at the locations of LHO, LLO, and VIRGO: One-step search on a 4-d parameter space, $\{f_s\xi_0,\ f_s\xi_1,\ n_3,\ n_1\}$, with metric:

$$g_{ij} = \begin{pmatrix} 0.0817 & 0.122 & 0 & 0 \\ \cdot & 0.185 & 0 & 0 \\ \cdot & \cdot & 11.97 & 8.823 \\ \cdot & \cdot & \cdot & 102.98 \end{pmatrix} \quad .$$

Template spacing is determined by finding the eigencoordinates, $\{x_1,\ x_2,\ \Omega_1,\ \Omega_2\}$, in which the above metric is diagonalized:

$$(dx_1,\ dx_2,\ d\Omega_1,\ d\Omega_2) = (\ 0.95,\ 12.3,\ 0.05,\ 0.15)\left(\frac{1-MM}{0.03}\right)^{1/2} \quad ,$$

Hierarchical network searches

Example. L1-H1 (with LIGO-I noise):
Template spacings given by

$$(dx_1,\ dx_2,\ dn_3) = (\ 0.71,\ 9.25,\ 0.19)\left(\frac{1-MM}{0.03}\right)^{1/2} \quad .$$

Take $Q_{0\ \text{max}} = 1/\text{yr}$, $Q_{d\ \text{min}} = 0.95$, $S_{\text{min}} = 10$, $\xi_{\text{max}} \sim 140$ sec. Then by increasing, e.g., the $x_{1,2}$ spacing 3 times and the n_3 spacing 4 times, we get

$Q_d(\eta = 7.4) \simeq 0.96 > Q_{d\ \text{min}}$
and $Q_0(\eta = 7.4) \simeq 0.1/\text{yr} < Q_{0\ \text{max}}$.
Computational gain is $\sim 3^2 \times 4 = 36$

Computational gain in 2 steps

Network	n_{tot} (in 10^6)	Speed (Tflops)	Gain
LIGO-I	0.2	0.04	25
L1-H1	8	0.5	100
L1-H1-LV	1500	10^3	400

Assumed: 2PN chirp, $m_{\rm min} = 0.5 M_\odot$, minimal match of 97%, and data sampled at $\sim$ 2 kHz. "L1-H1-LV" denotes three co-aligned LIGO-I type detectors placed in Hanford, Livingston, and Pisa.

Time decimation increases the gain by a factor of ~ 4.

Future Work

1. Test LAL code for multiple-interferometer searches of inspirals

2. Develop LAL code for hierarchical multi-detector searches of inspirals

3. A FCT-based multi-detector strategy

4. Estimation of parameters based on multi-detector search results