LIGO-G020227-00-D

Sidebands Imbalance: causes and effects

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Ligo Seminar

1. **Estimation of the sidebands imbalance noise** which limits the sensitivity of gravitational wave interferometers

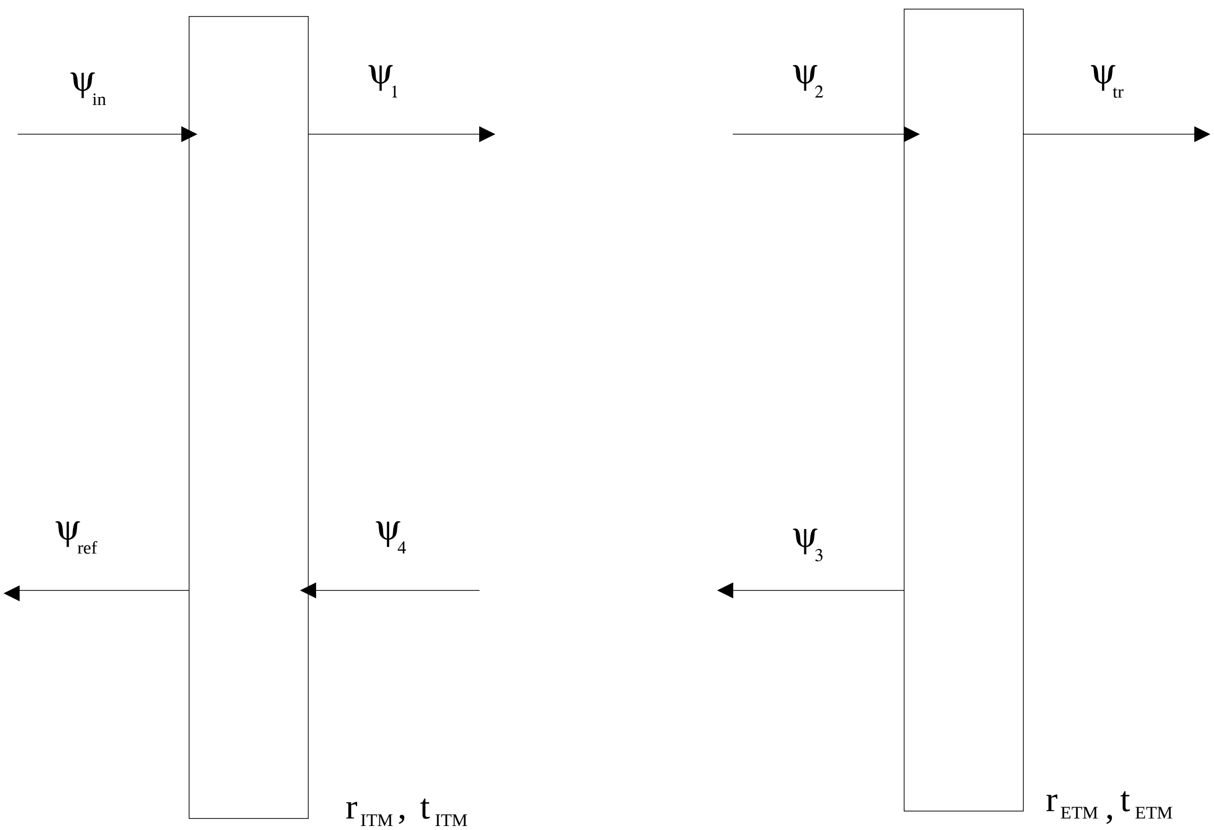
2. Investigation of the **physical mechanisms** which induce sideband imbalance

3. Introduction of a 2X2 optical model constructed with a view to take into account symmetries and unitarity

A gravitational wave is measurable as a phase change in the electromagnetic field that is reflected out of an **optical resonator driven by monochromatic light** (carrier field)

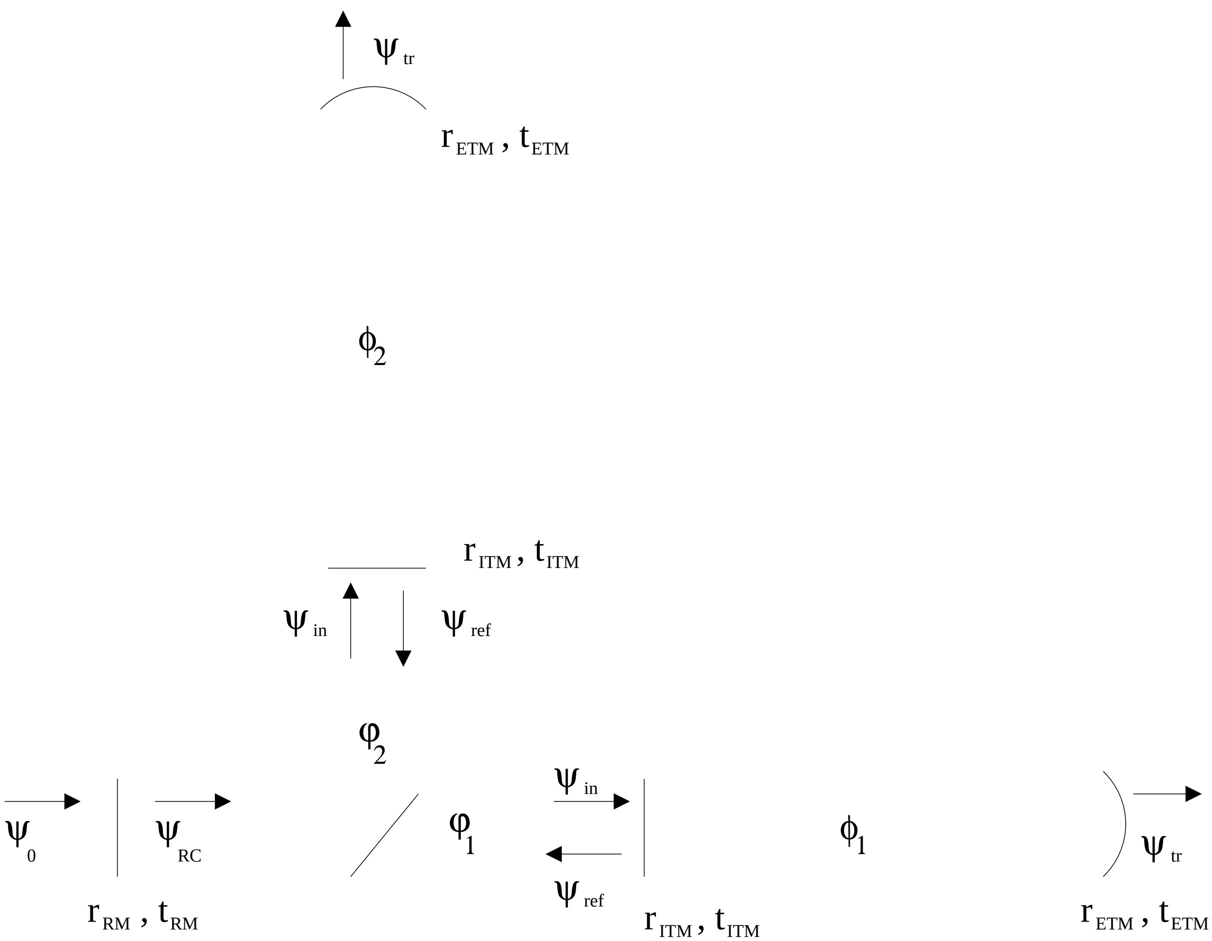
$$r_{eff}(\phi) = \frac{r_{ITM} - r_{ETM}e^{i\phi}}{1 - r_{ITM}r_{ETM}e^{i\phi}}$$

$$r_{eff}(\phi_{gw}) \overset{r_{ETM}\to 1}{\longrightarrow} \exp \frac{2i\mathcal{F}\phi_{gw}}{\pi}$$



$$\mathcal{F} \equiv \frac{\pi\sqrt{r_{ITM}r_{ETM}}}{1 - r_{ITM}r_{ETM}}$$

The detector can be designed in order to make the application of the basic idea work easier and a more complex scheme is used for this purpose



$$t_1 = \frac{1}{\sqrt{2}} \frac{t_{RM} e^{\frac{i\varphi_1}{2}}}{[1 - r_{RM}(r_{eff}(\phi_1)e^{i\varphi_1} + r_{eff}(\phi_2)e^{i\varphi_2})]}$$

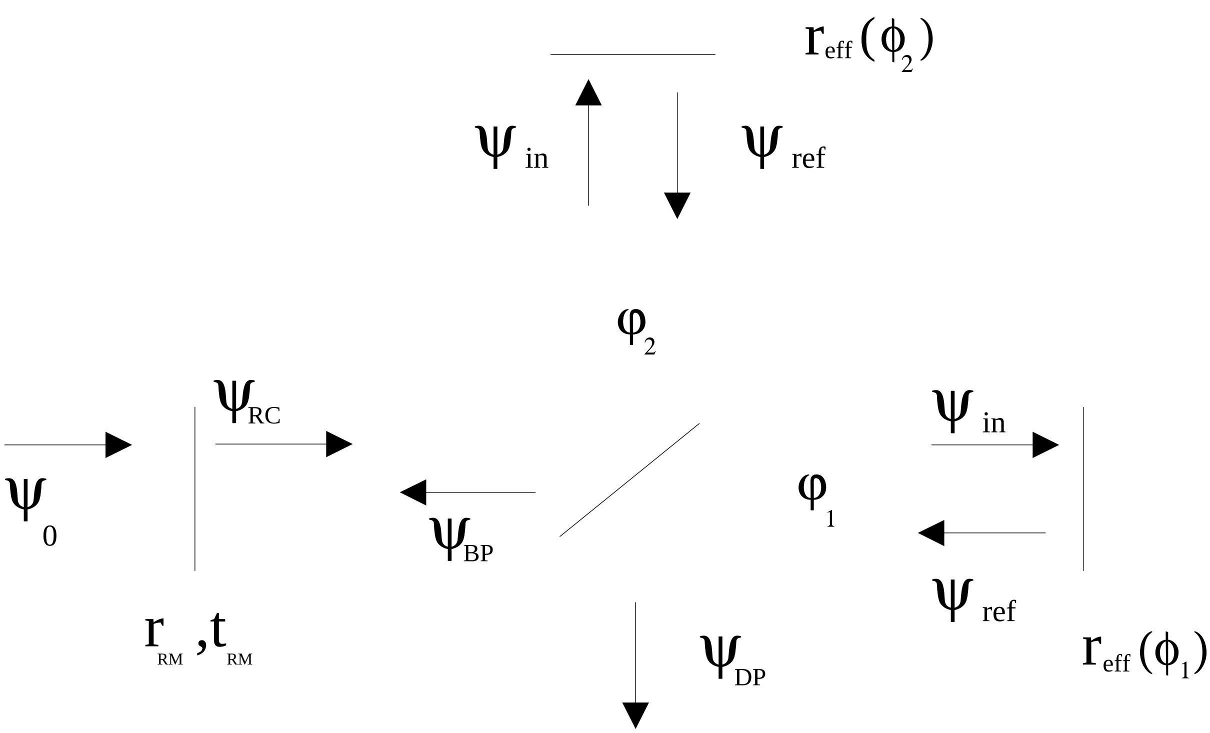
$$t_2 = \frac{1}{\sqrt{2}} \frac{t_{RM} e^{\frac{i\varphi_2}{2}}}{[1 - r_{RM}(r_{eff}(\phi_1)e^{i\varphi_1} + r_{eff}(\phi_2)e^{i\varphi_2})]}$$

In an ideal interferometer the amplitude of the field at the dark port is proportional to the phase variation induced by the gravitational wave

$$\Psi_{DP} = \frac{1}{2}[r_{eff}(\phi_1)e^{i\varphi_1} - r_{eff}(\phi_2)e^{i\varphi_2}]\Psi_{RC}$$

at the working point

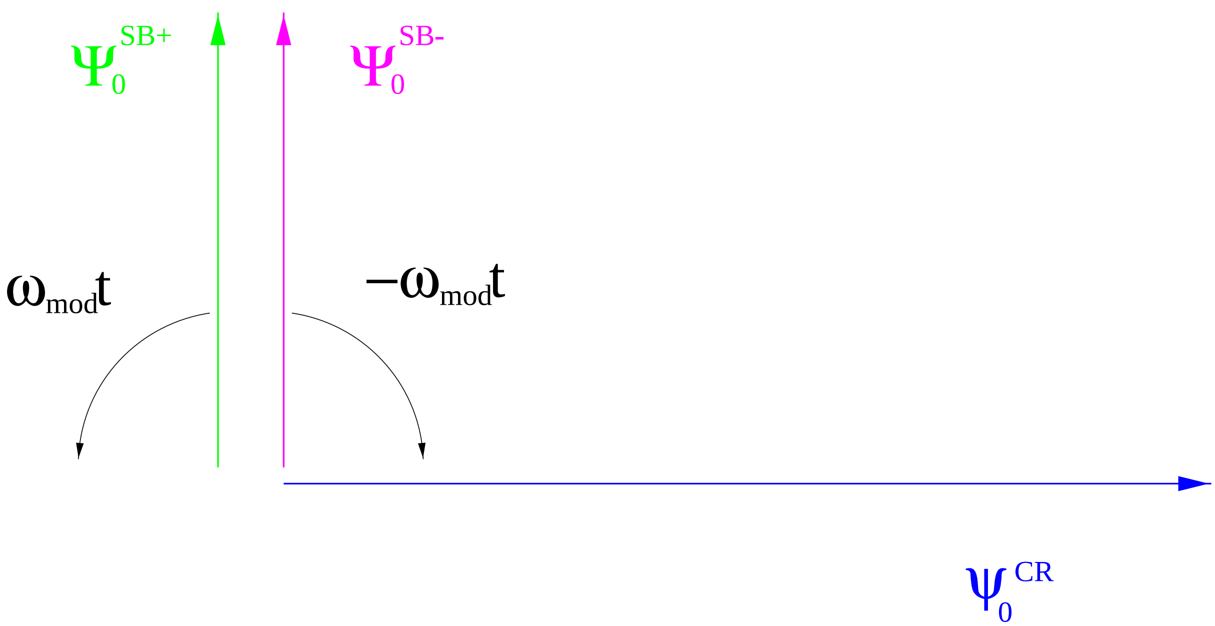
$$\phi_1 = \phi_2 = 0 \qquad \varphi_1 = \varphi_2 = 0$$



$$\phi_1 = -\phi_2 = \phi_{gw}$$

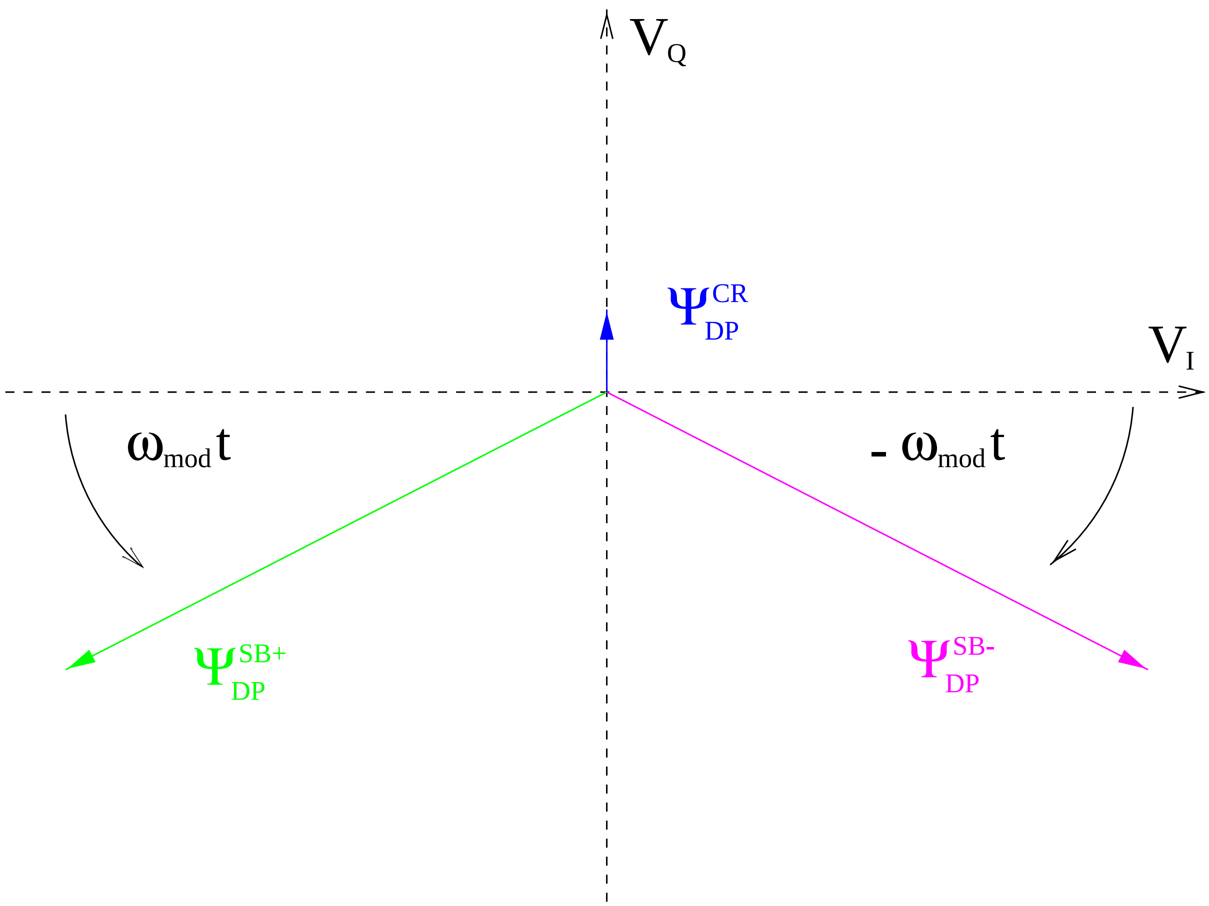
A reference frame for detecting the variation of the phase

$$
\begin{aligned}
\Psi &= \Psi_l e^{i\Gamma \cos \omega_{mod} t} \\
&\simeq J_0(\Gamma)\Psi_l + iJ_1(\Gamma)\Psi_l e^{i\omega_{mod} t} + iJ_1(\Gamma)\Psi_l e^{-i\omega_{mod} t} \\
&= \Psi_0^{CR} + \Psi_0^{SB+} e^{i\omega_{mod} t} + \Psi_0^{SB-} e^{-i\omega_{mod} t}
\end{aligned}
$$



$$
\begin{aligned}
P_{DP} &= |\Psi_{DP}|^2 = |\Psi_{DP}^{CR}|^2 + |\Psi_{DP}^{SB+}|^2 + |\Psi_{DP}^{SB-}|^2 + \\
&\quad 2\Re[(\Psi_{DP}^{CR}\Psi_{DP}^{SB-\,*} + \Psi_{DP}^{SB+}\Psi_{DP}^{CR\,*}) \exp(i\omega_{mod} t)] \\
&\quad + 2\Re[\Psi_{DP}^{SB+}\Psi_{DP}^{SB-\,*} \exp(2i\omega_{mod} t)]
\end{aligned}
$$

For the "ideal" configuration

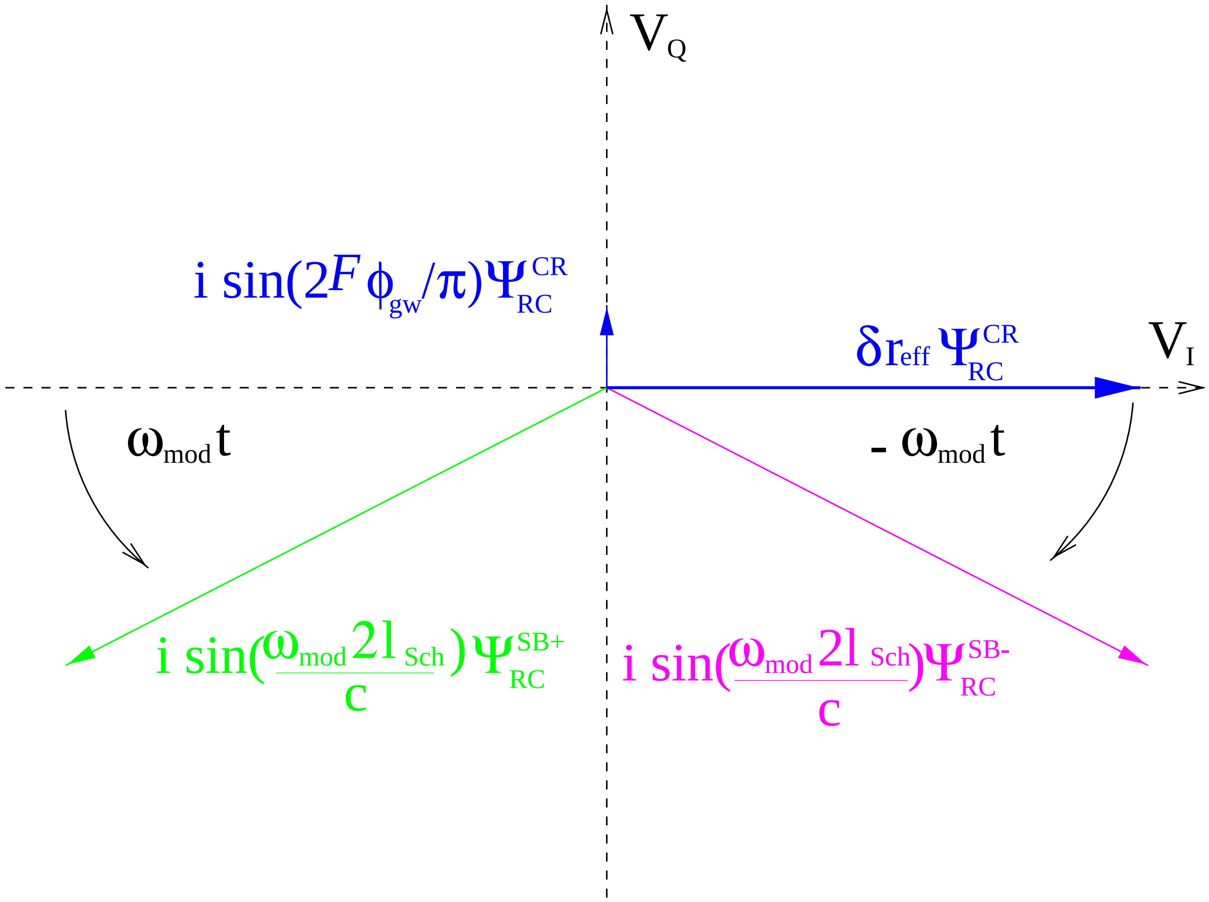


$$V_I = \int_0^T \frac{P_{DP} \cos \omega_{mod} t}{T} dt = \Re[\Psi_{DP}^{CR} \Psi_{DP}^{SB-\,*} + \Psi_{DP}^{SB+} \Psi_{DP}^{CR\,*}]$$

$$V_Q = \int_0^T \frac{P_{DP} \sin \omega_{mod} t}{T} dt = -\Im[\Psi_{DP}^{CR} \Psi_{DP}^{SB-\,*} + \Psi_{DP}^{SB+} \Psi_{DP}^{CR\,*}]$$

$$D.C. = \int_0^T \frac{P_{DP}}{T} dt = |\Psi_{DP}^{CR}|^2 + |\Psi_{DP}^{SB+}|^2 + |\Psi_{DP}^{SB-}|^2$$

In a "realistic" interferometer



- Requirements for limiting any variation equivalent to a phase change due to some gravitational signal

  $\tilde{x}(10Hz) \leq 10^{-19} m/\sqrt{Hz}$ TMs displacement noise

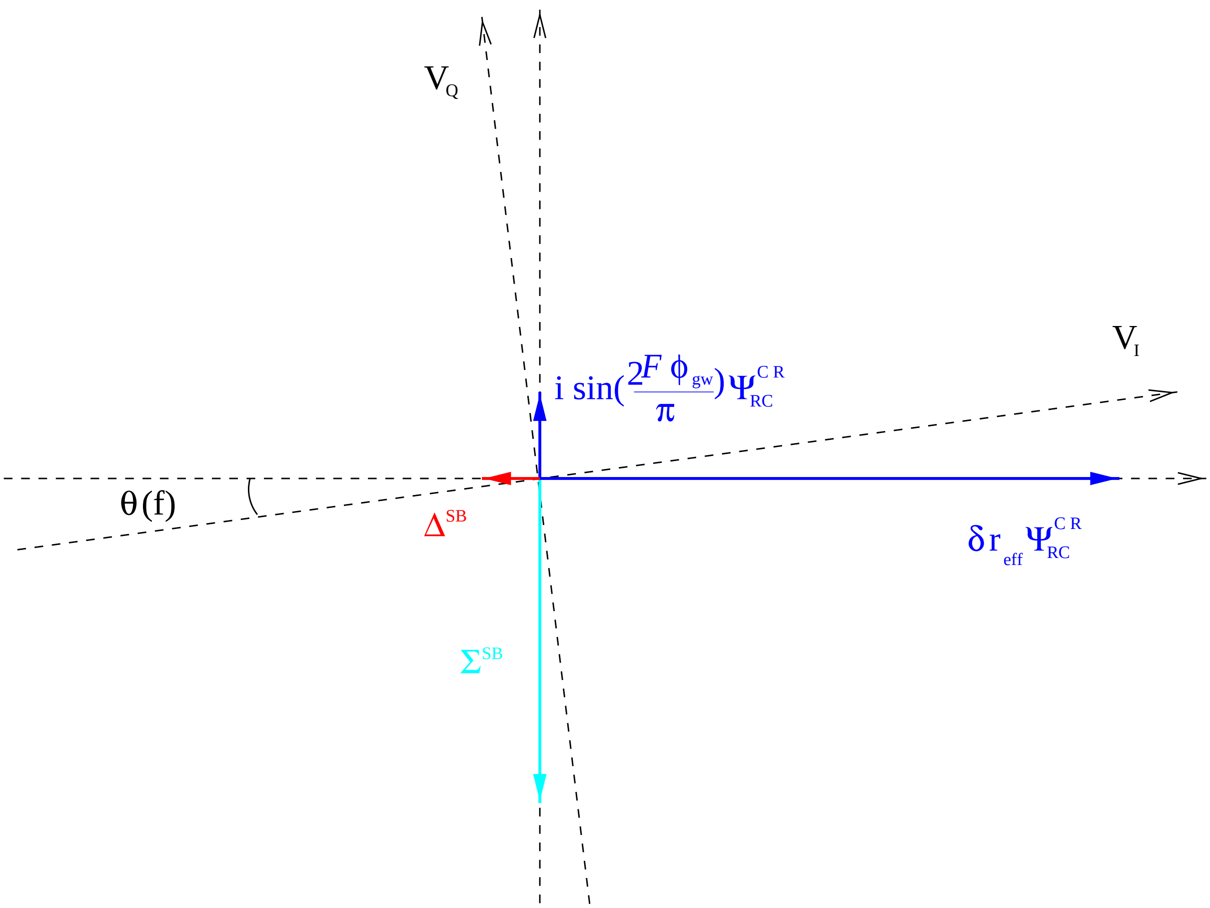
- laser frequency and amplitude must be stabilized to reduce fluctuations

  relative intensity noise $\leq 10^{-9} - 10^{-8}/\sqrt{Hz}$

  laser frequency noise $\delta\nu_0 \leq 10^{-7} Hz/\sqrt{Hz}$

First estimation of the sidebands imbalance noise in V_Q

$$V_Q = \sin(\frac{2\mathcal{F}\phi_{gw}}{\pi})\Psi_{RC}^{CR}\Sigma^{SB}\cos\theta + \delta r_{eff}\Psi_{RC}^{CR}\Delta^{SB}\sin\theta$$



$$\phi_{gw} = 2k\frac{Lh(f)}{2} \qquad \delta r_{eff}(ls_1 - ls_2) = \frac{1}{2}[r_{eff}(ls_1) - r_{eff}(ls_2)]$$

$$h(f) = 1.07\,10^{-19} \times (\frac{\theta(f)}{10^{-4}}) \times (\frac{\delta r_{eff}}{0.33\%}) \times \frac{\Delta^{SB}}{\Sigma^{SB}}$$

Can the sidebands imbalance noise affect the sensitivity of Ligo I and II?

using Ligo I parameters

$$t^2_{RM} = 0.0244 \qquad t^2_{ITM} = 0.03 \qquad t^2_{ETM} = 15\,10^{-6}$$

$$|ls_1 - ls_2| = 10^{-4} \quad \lambda = 1.064\,10^{-6}m \quad L = 4\,10^3 m$$

$$h(f) = 1.07\,10^{-19} \times (\frac{\theta(f)}{10^{-4}}) \times (\frac{\delta r_{eff}}{0.33\%}) \times \frac{\Delta^{SB}}{\Sigma^{SB}}$$

using Ligo II parameters

$$h(f) = 1.06\,10^{-19} \times (\frac{\theta(f)}{10^{-4}}) \times (\frac{|ls_1 - ls_2|}{10^{-4}}) \times \frac{\Delta^{SB}}{\Sigma^{SB}}$$

because of the dependance

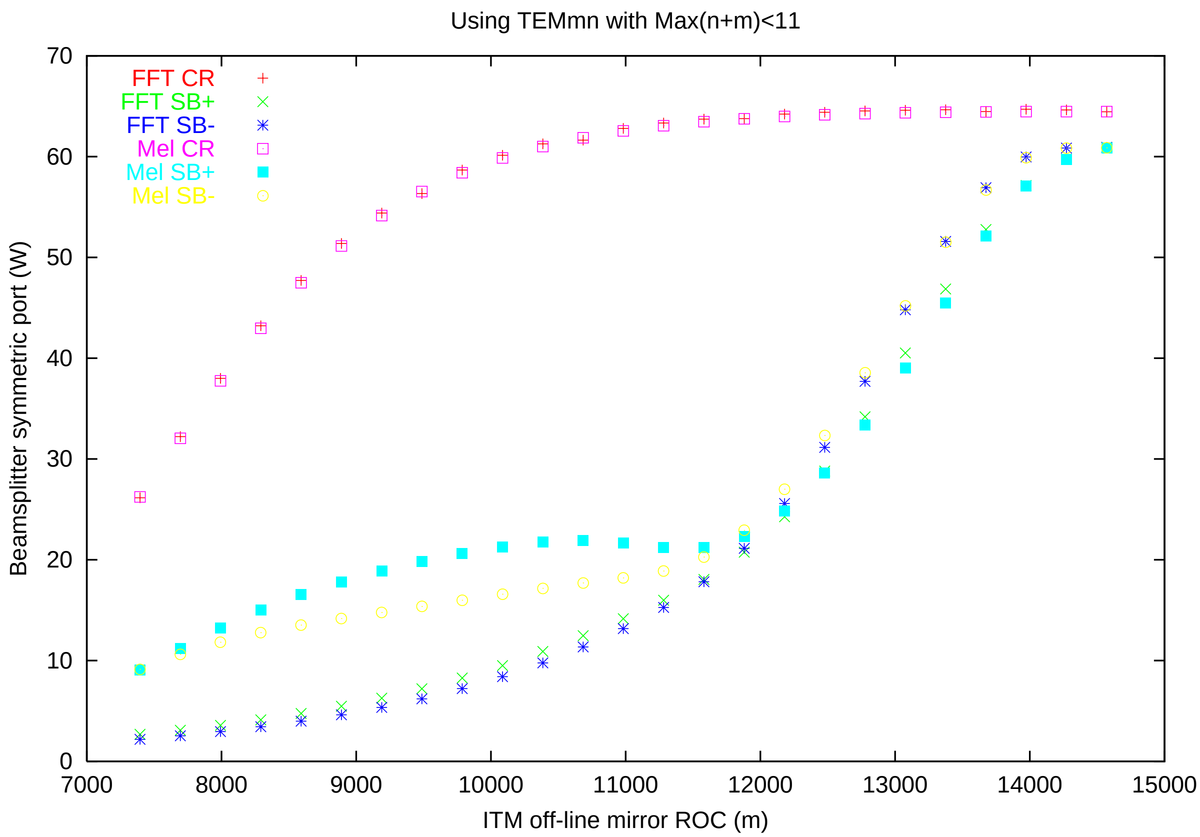
$$\delta r_{eff}(ls_1 - ls_2) = \frac{1}{2}[r_{eff}(ls_1) - r_{eff}(ls_2)] \simeq \frac{\mathcal{F}}{2\pi}(ls_1 - ls_2)$$

What causes sidebands imbalance?

- **Geometrical asymmetries that make the two branches of the interferometer different**, even if everything else is perfectly designed including the lengths of the cavities that are supposed to match the macroscopic conditions for the sidebands to be anti-resonating in the Fabry-Perot cavities and resonating in the recycling cavity ($L_i = (2n_i + 1)\frac{\lambda_{mod}}{4}$).

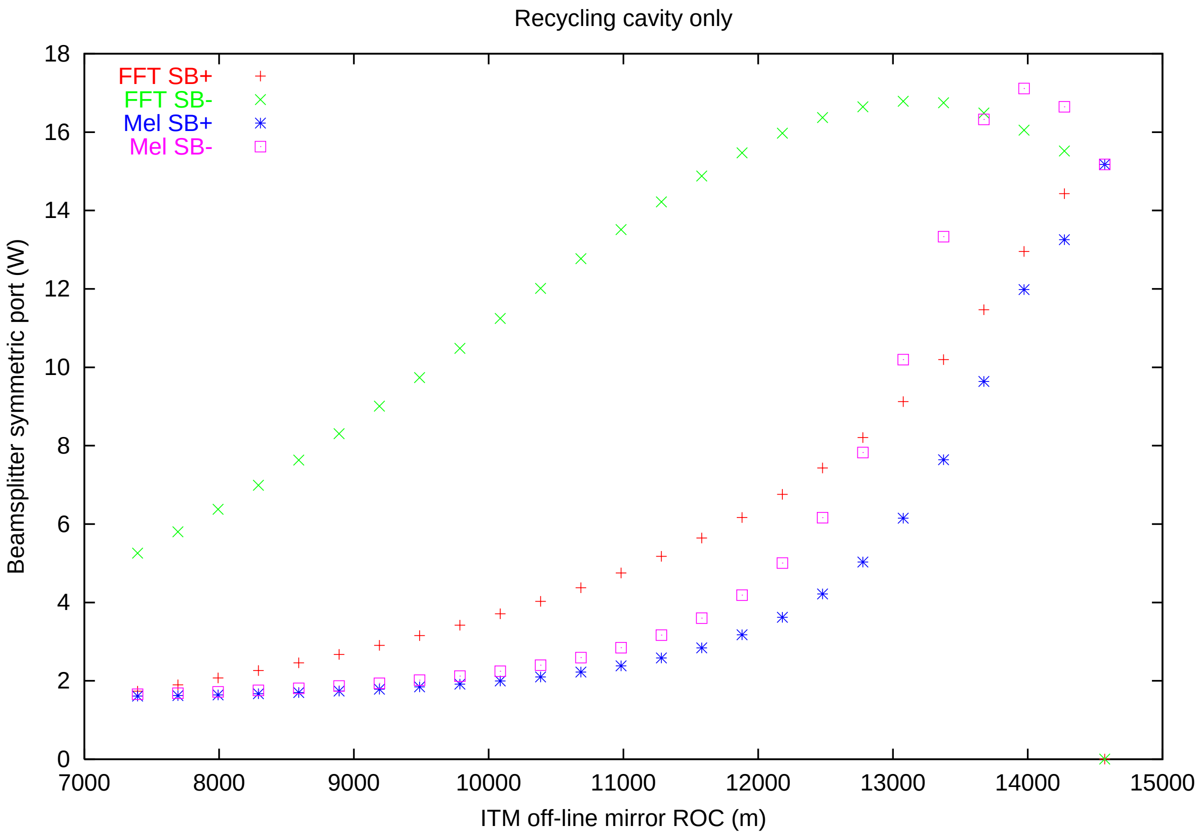
- **The two branches of the interferometer are identical but** the lengths of the Fabry-Perot and recycling cavities do not match the macroscopic condition so that **the phase difference between the sidebands is no longer a multiple of** 2π as it is when all the lengths are a multiple of $\frac{\lambda_{mod}}{4}$

One arm is perturbed and sidebands imbalance is produced because of the geometrical asymmetry



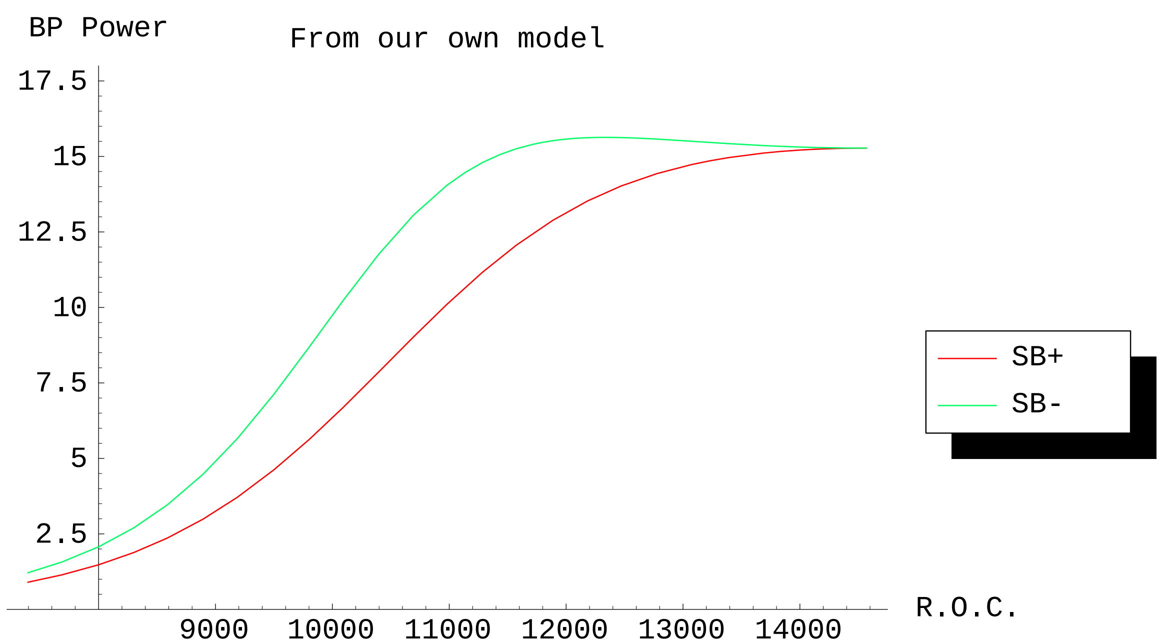
The results obtained by an FFT code and a modal model are compared: even using 66 modes the agreement between them is not good for perturbations larger than $\sim 2.5km$ in the radius of curvature of the ITM off-line mirror. However the agreement for the carrier is very good even for large distortions $\sim 7.5km$.

Both the imbalance and the disagreement between the FFT-code and the modal model become worse when the recycling cavity is simulated switching off the arm cavities



If the sidebands are sensitive to a geometrical asymmetry in the two branches let's construct an analytical model that explicitly implies those features in a two dimensional model.

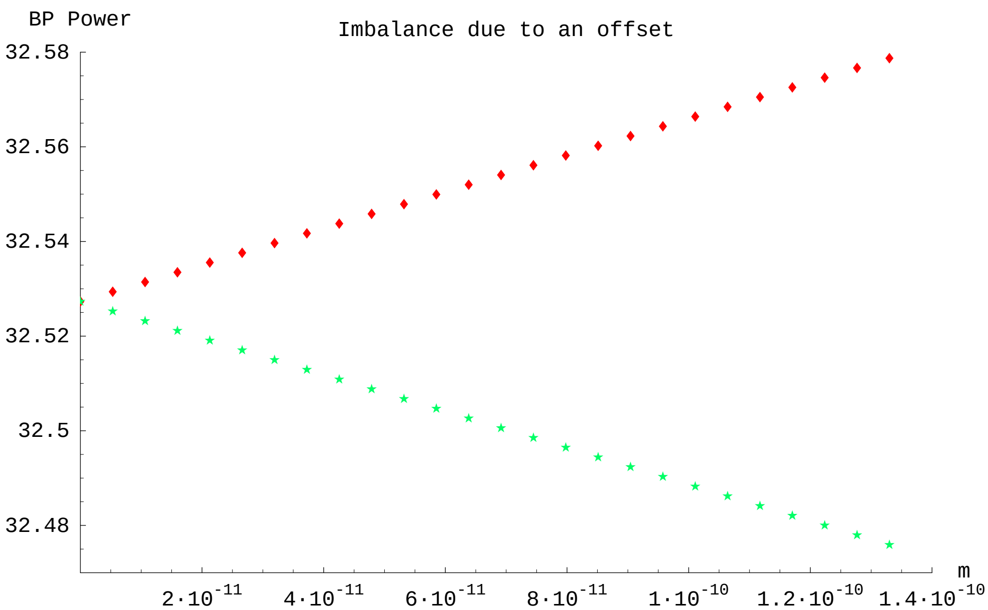
William P. Kells and I developped a simple model with a view to do analytical calculations. The aim was to understand the underlying physical mechanisms and we explicitly started from the basic principles we wanted to be included such as unitarity and symmetries.

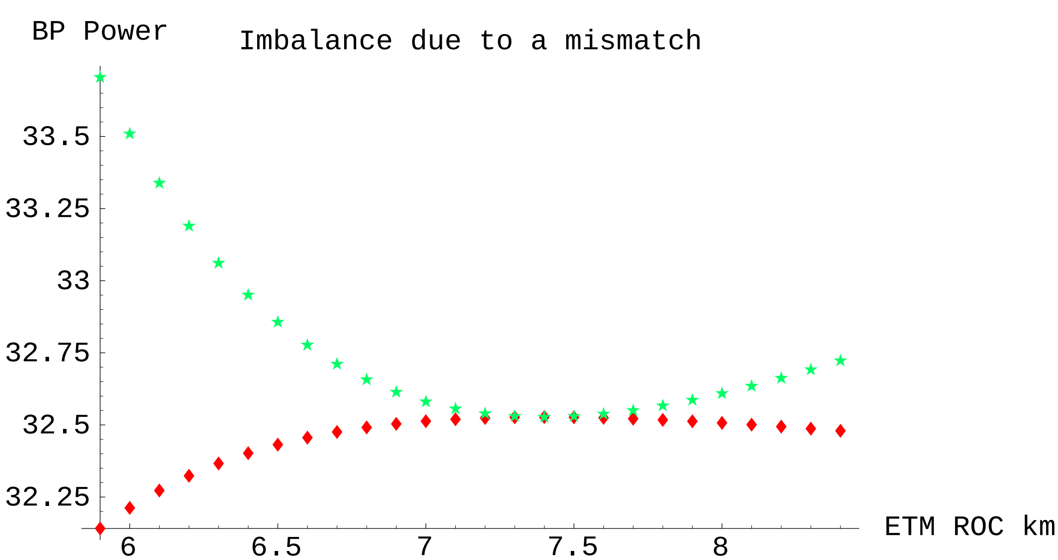


More details (maybe) after...

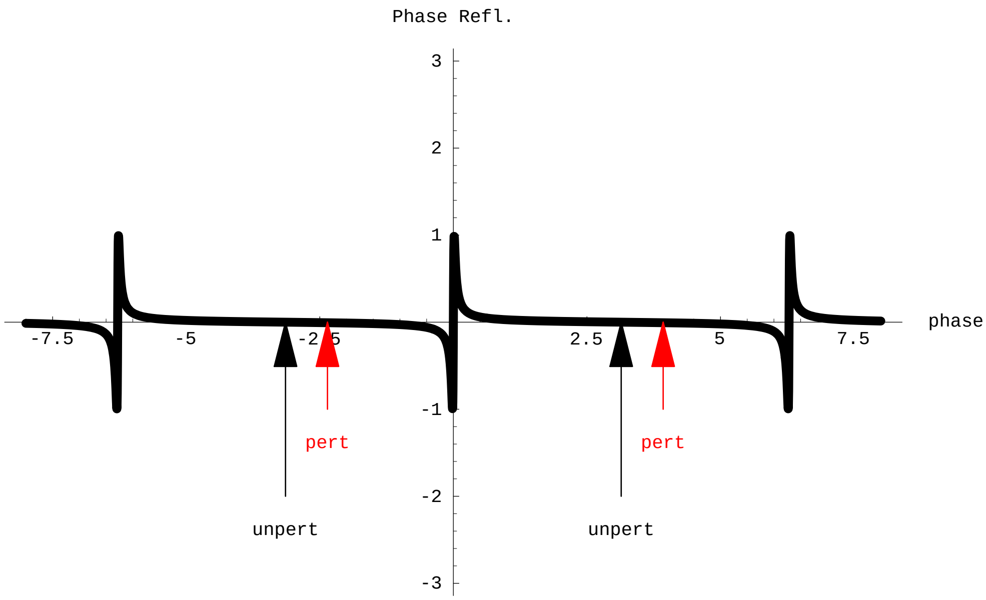
Sidebands imbalance can occur simply because the macroscopic condition on the lengths is not satisfied

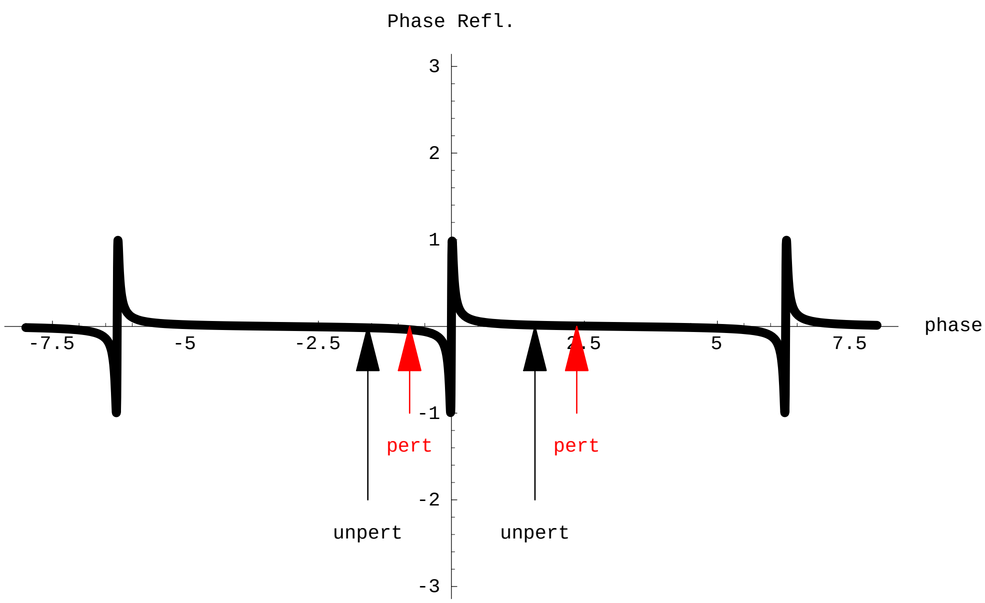
$$L_{arm} \neq (2n+1)\lambda_{mod}$$



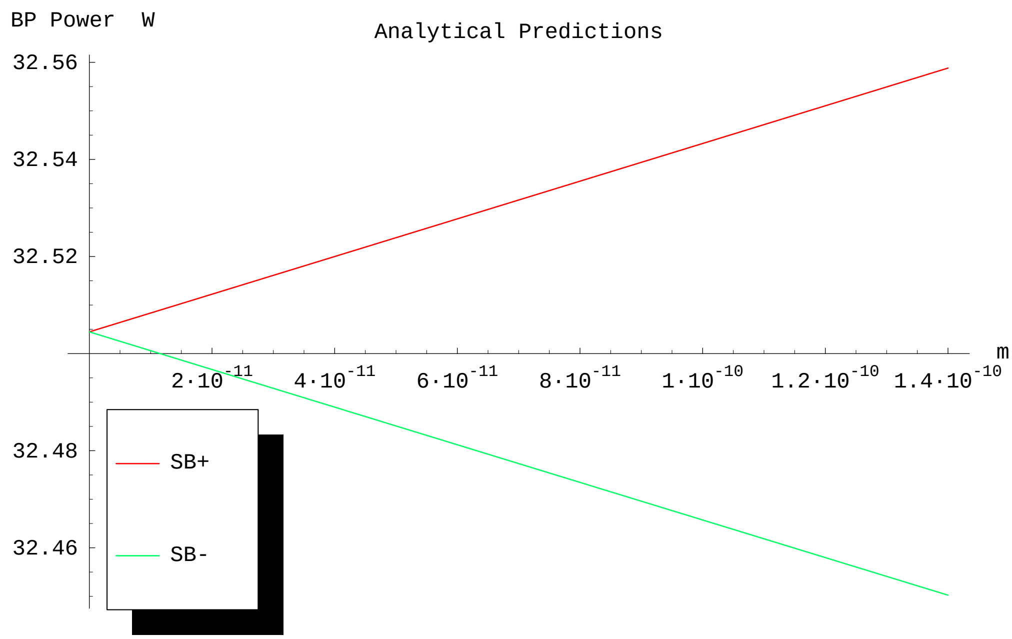


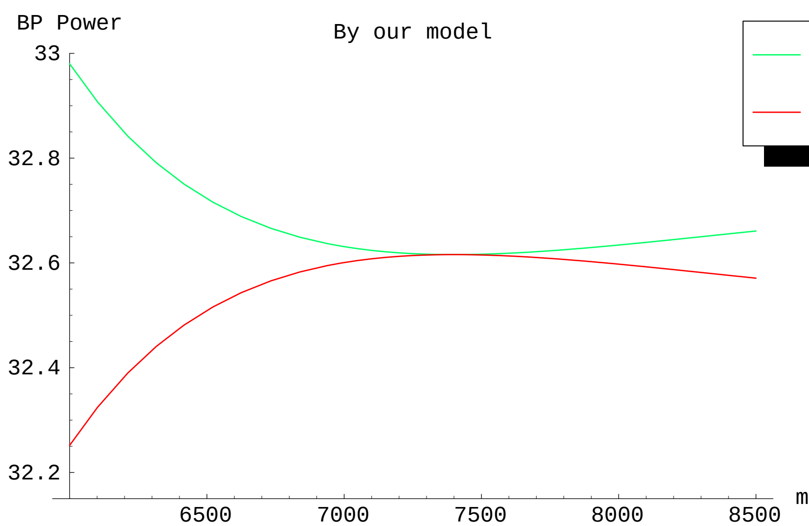
Where does the imbalance come from in a completely symmetrical interferometer?



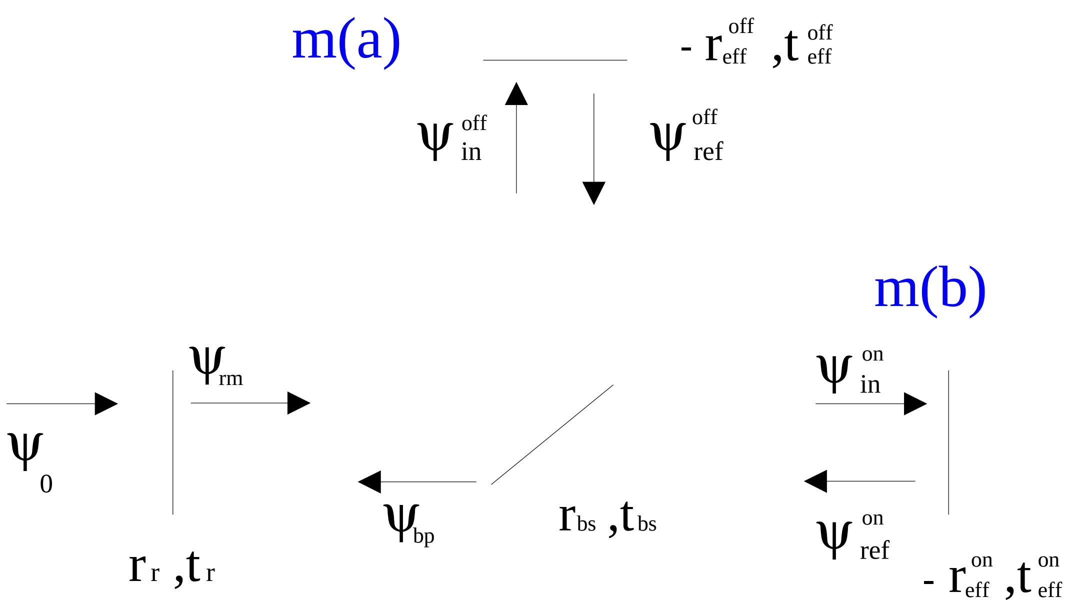


We understand the underlying mechanisms that can generate sidebands imbalance. They are related with a **broken symmetry**: the left-right symmetry in the system or the left-right symmetry around the value for resonance in the phase domain.





Prescriptions in our model



1.

$$m(a)\,,\,m(b) \qquad \text{unitary}$$

2.

$$m(a) = m(\frac{a-b}{2})m(\frac{a+b}{2}) \qquad \text{and similarly for } m(b)$$

$$\Downarrow$$

$$m(a) = \exp[ia\vec{n}\cdot\vec{\sigma}]$$

At the second order our request overrides the rules obtained by other methods.

$$e^{i\alpha}\begin{pmatrix} e^{-i\beta}\cos\gamma & i\sin\gamma \\ i\sin\gamma & e^{i\beta}\cos\gamma \end{pmatrix}$$

The quantity γ represents the scattering from one mode to another.

When the Guoy phase is changed β stands for that variation.

There is a factor in front of the matrix when the optimal position of the mirror must be varied.

tilt

$$k\vec{u}_z \longrightarrow k\sin\vartheta\vec{u}_r + k\cos\vartheta\vec{u}_z$$

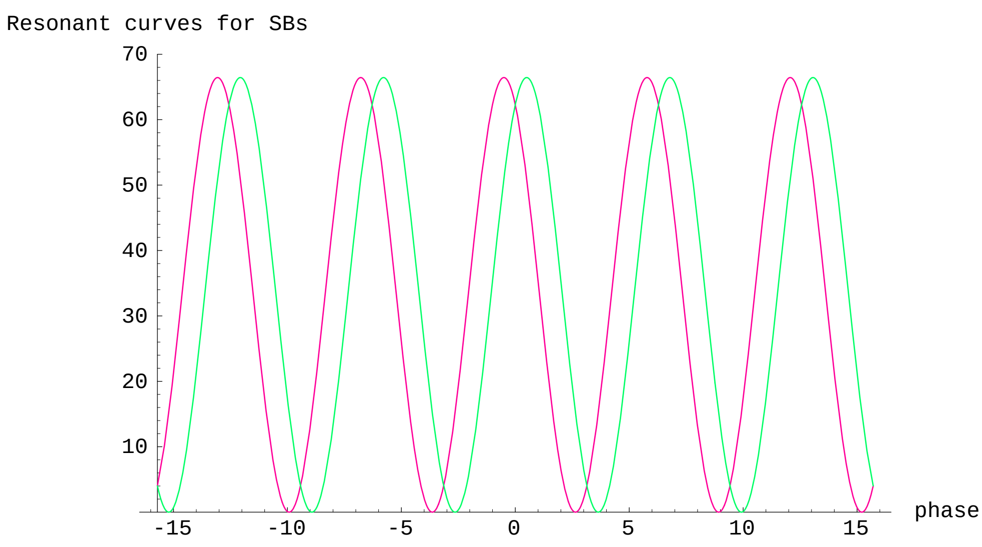
radius of curvature

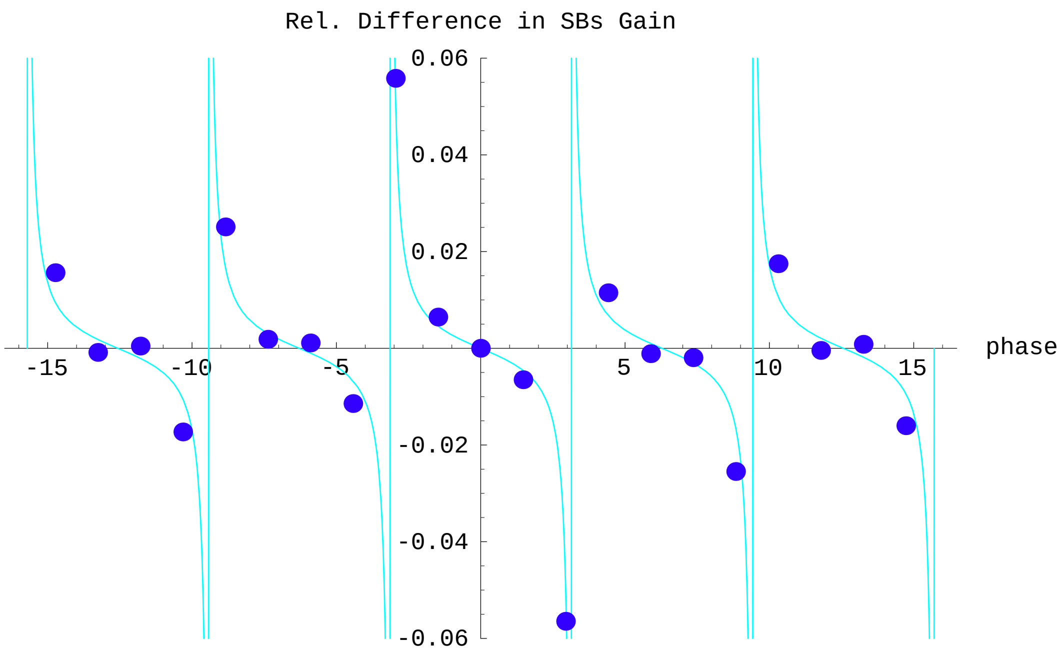
$$\frac{\mathrm{d}\theta_g}{\mathrm{d}R}\Delta R \longrightarrow \beta$$

The sidebands imbalance can be induced by several different technical reasons (for instance any asymmetry in the interferometer or inappropriate macroscopic lengths of the cavities).

Balance can be restored by tuning the microscopic lengths although this is an additional tuning according to the definition of all distances defined on the carrier resonant condition.

Since the sidebands are not resonating in the arms the **realistic configuration of the interferometer** makes different the optimal tuning for the carrier from the **optimal tuning for the sidebands balance**. This simple phenomenon induces sidebands noise which can be reduced by *applying an offset*. An useful test is to alter the sidebands phase although the carrier is always resonating. This can be done by adjusting the macroscopic length of the cavities (*a conceptual test*).





Apparently balance can always be restored by tuning the frequency and the beam mode at the input.

$$\Psi^{SB}_{input}(k_x, k_y, k_z) \qquad \text{for res./anti-res. conditions}$$

The sidebands imbalance arises because of imperfections in the interferometer.

When correctly parametrized in terms of the perturbation it is a second order correction.

It is clear that sidebands balance is not equivalent to **no carrier light through the antisymmetric port**. Exact **balance** would imply both same **gain** and **mode composition**. The general issue is that an imbalance can cause a noise term but we also had in mind other motivations for this problem to be tackled.