The Analysis of LIGO Data

Physics 237b Guest Lecture

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Outline of this lecture

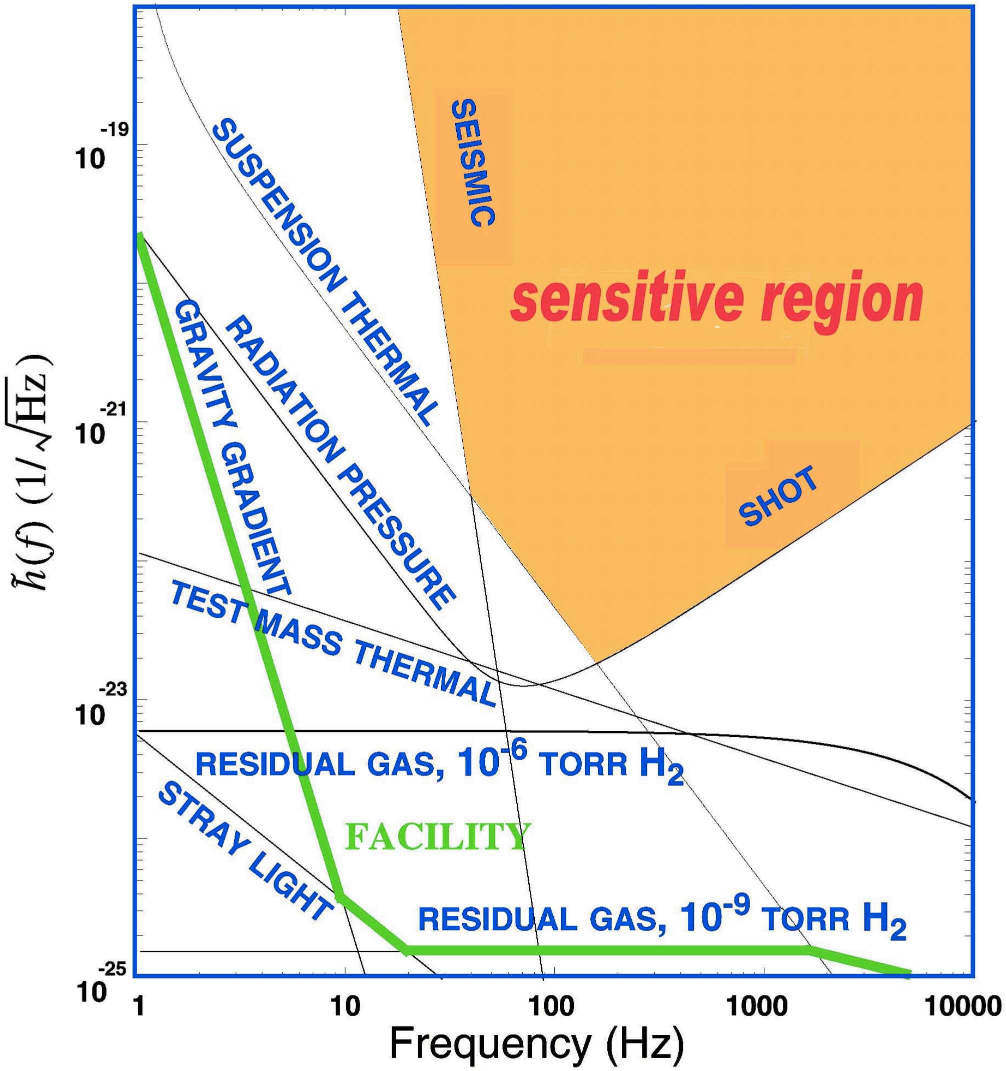
- LIGO data attributes
- Noise processes
  - *Time and frequency domain properties*
- Time-frequency classification of GW signal properties
- Search tecniques for specific classes of signals
  - *Computational implementation*

LIGO First Generation Detector

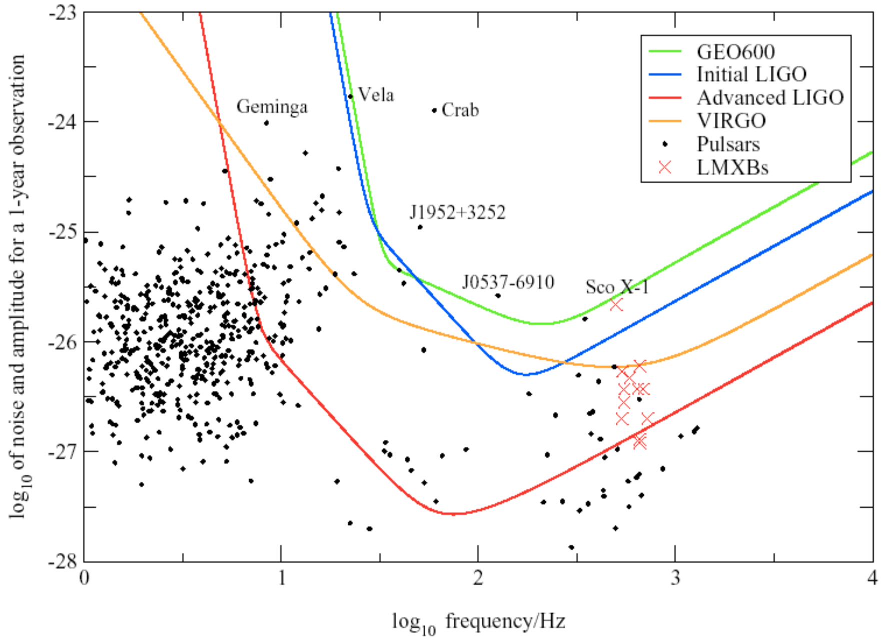
Limiting noise floor

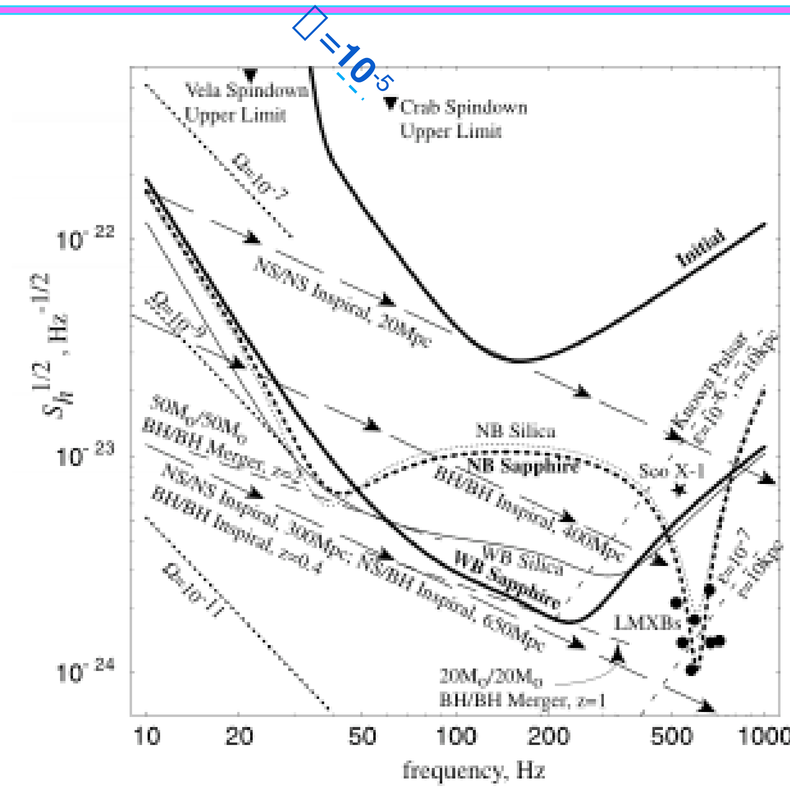
- **Interferometry is limited by three fundamental noise sources**
  - <u>seismic noise</u> at the lowest frequencies
  - <u>thermal noise</u> (Brownian motion of mirror materials, suspensions) at intermediate frequencies
  - <u>shot noise</u> at high frequencies

- **Many other noise sources lie beneath and must be controlled as the instrument is improved**



Detection is expected to be at the limits of LIGO I sensitivity

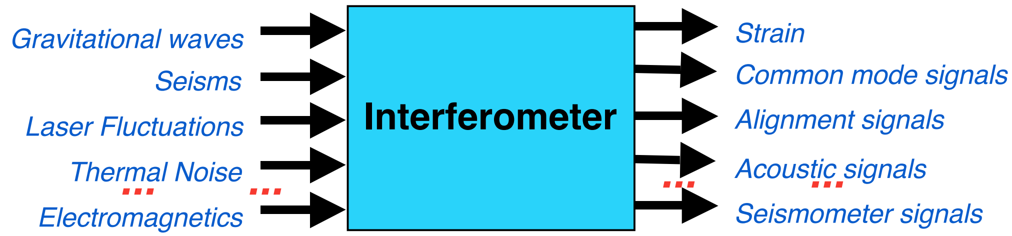




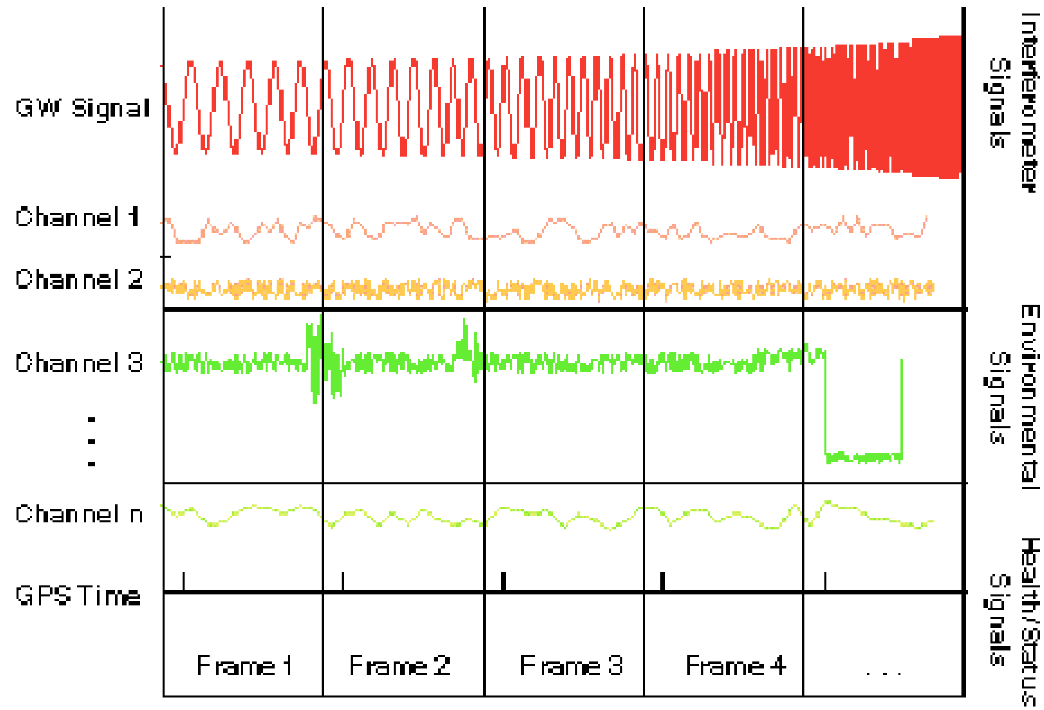
Jones, gr-qc/0111007

Provided by K. Thorne and others for the "LIGO II Conceptual Project Book", September 1999, Document # LIGO-M990288

Interferometer Data Channels



- **All interferometric detector projects have agreed on a standard data format**
- **Anticipates joint data analysis**
- **LIGO Frames for 1 interferometer are ~3MB/s**
  - **32 kB/s strain**
  - **~2 MB/s other interferometer signals**
  - **~1MB/s environmental sensors**
  - ***Strain is ~1% of all data***



Collection of Multiple Data Channels

- 1% of data is the GW channel, “h[t]”
- 99% are auxiliary channels (over 5000 channels!):
  - *Monitor interferometer servos (>50 loops)*
    Laser f, laser I, mirror □,□, …
  - *Monitor instrument state vector*
    Gains, offsets, …
  - *Monitor environment*
    Power line voltage stability, $v_{n\cdot 60\ Hz}[t]$ -- look for transient pick up
    Accelerometers -- high f shocks, vibrations, f>100Hz)
    Magnetic fields, B[f] -- electromagnetic interferences
    Seismometers -- low f ground motion, f< 100Hz)
    Acoustics near test mass vacuum chambers
    Vacuum monitors -- pressure outgassing bursts
    Muon shower detectors -- site-wide cosmic ray showers

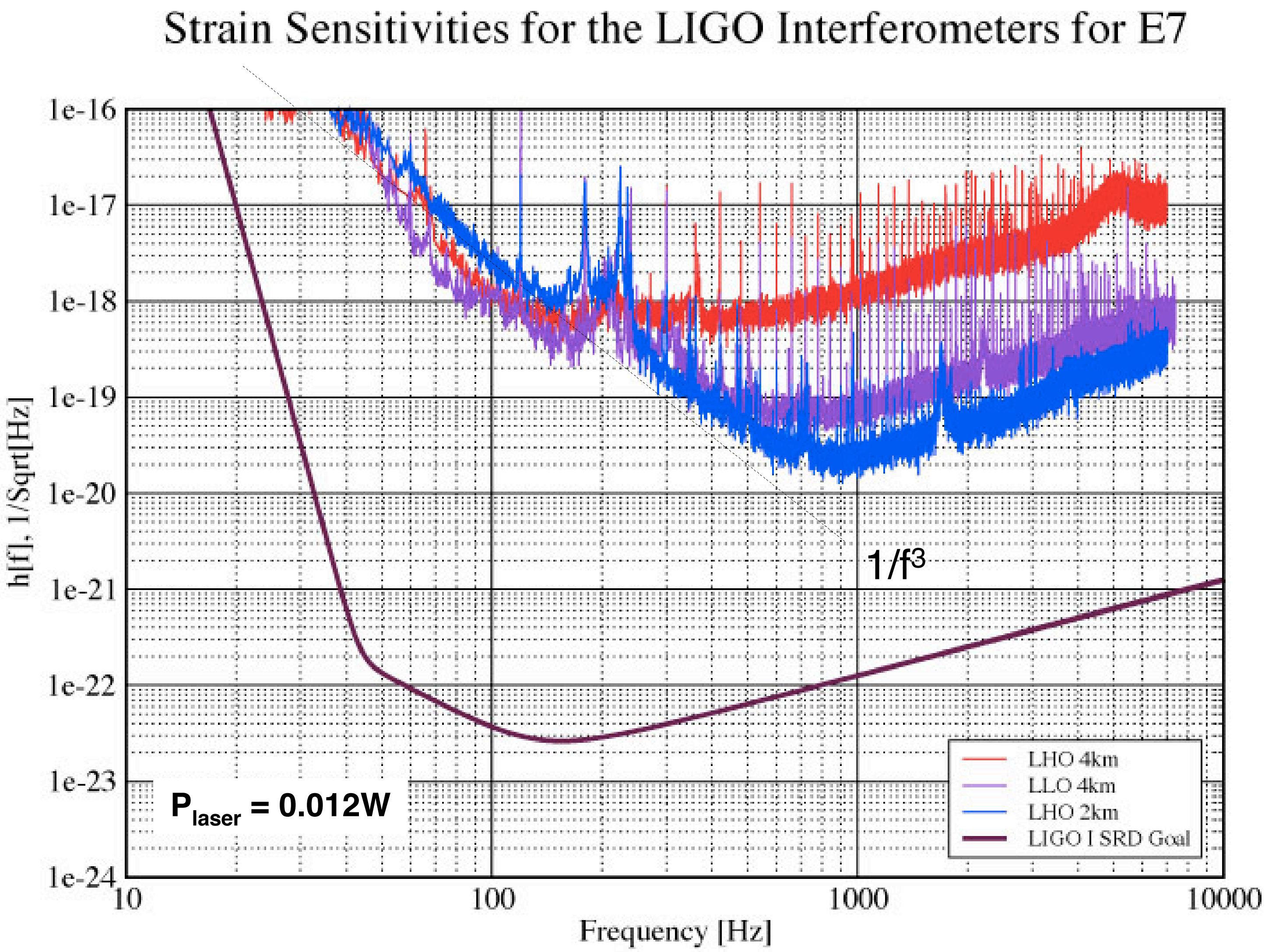
Collection of Multiple Data Channels

- How will these be used?
  - *Cross-correlation and regression analysis in order to reduce RMS of h[t] channel*
    - *Useful for high SNR signals with known transfer function to h[t]*
  - *Validation of "nominal" instrument behavior*
    - *Continuously monitor interferometer, environment for "nominal behavior", within, say +/-3σ for stable channels…*
    - *Develop an archive of "typical channel behavior", power spectra, etc.*
    - *Empirically derived template bank of instrumental "glitches" for detection & vetoing*
    - *Generate vetoes using auxiliary channels when these pick up transients in the environment*
      - ***Traffic***
      - ***Logging (in Livingston)***
      - ***…***

E7 sensitivities for LIGO Interferometers

28 December 2001 - 14 January 2002

- Used 10^{-3} of ultimate laser power
  - "Automatic" 30x improvement at high f
- Investigating "f^{-3}" noise at low f
- Electronics upgrade for sensitive servos
- Damping of structural resonances near 200 Hz on optics benches
- Upgrade seismic system at Livingston to provide greater availability



The Battle: Noise vs Signal

- So…
  - *Unless Nature is very serendipitous …*
- We will have to work very hard to sieve through the interferometer data to look for putative events with the initial inferferometers
  - *Integrated signal-to-noise ratios of O[10]*
  - *Instantaneous signal-to-noise ratios of O[10^{-4}] or less …*
- Focus will be on understanding, reducing noise
  - *Instrumental improvements prior to digitization & acquisition*
  - *Signal processing techniques after acquisition*
- Hypothesis testing:
  - *Signal is present vs. signal is not present*

(Very) Brief Summary of Random Processes & Signal Noise

- ***n(t) is a randomly varying signal***
  - ***Gaussian process:***

$$P(n) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{\frac{-(n-\mu)^2}{2\sigma^2}}$$

$$< n >= \mu = \int_{-\infty}^{\infty} n P(n) dn$$

$$< n^2 >= \sigma^2 + \mu^2 = \int_{-\infty}^{\infty} n^2 P(n) dn$$

  - ***Can assume μ=0 without loss of generality***
- ***Ensemble averages, time averages***
  - ***Time average:***

$$< n(t) >_t = \lim_{T \to \infty} \frac{1}{T} \int_{-T/2}^{T/2} n(t) dt$$

    Ensemble average:

$$< n >_P = \int_{-\infty}^{\infty} n P(n) dn$$

  - ***Stationarity: μ, σ , etc. do not vary with time***
  - ***Ergodicity: Probability distribution of n(t) over a long period T is the same as P(n) at one instant, t'***

    crucial assumption because we do not have N>>1 interferomters taking data at the same time!

Fourier Transforms of Time Dependent Signals

- Continuous-time (infinite time duration)

$$\hat{x}(f) = \int_{-\infty}^{\infty} x(t) e^{-2\pi i f t} dt \equiv \mathcal{F}[x(t)]$$

$$x(t) = \int_{-\infty}^{\infty} \hat{x}(f) e^{2\pi i f t} df \equiv \mathcal{F}^{-1}[\hat{x}(f)]$$

$$\mathcal{F}^{-1}\mathcal{F}[x(t)] = x(t) \Rightarrow \delta(t - t') = \int_{-\infty}^{\infty} e^{-2\pi i f (t - t')} df$$

$$\int_{-\infty}^{\infty} \delta(t - t') dt = 1$$

- Discrete-time (finite time stretch T)

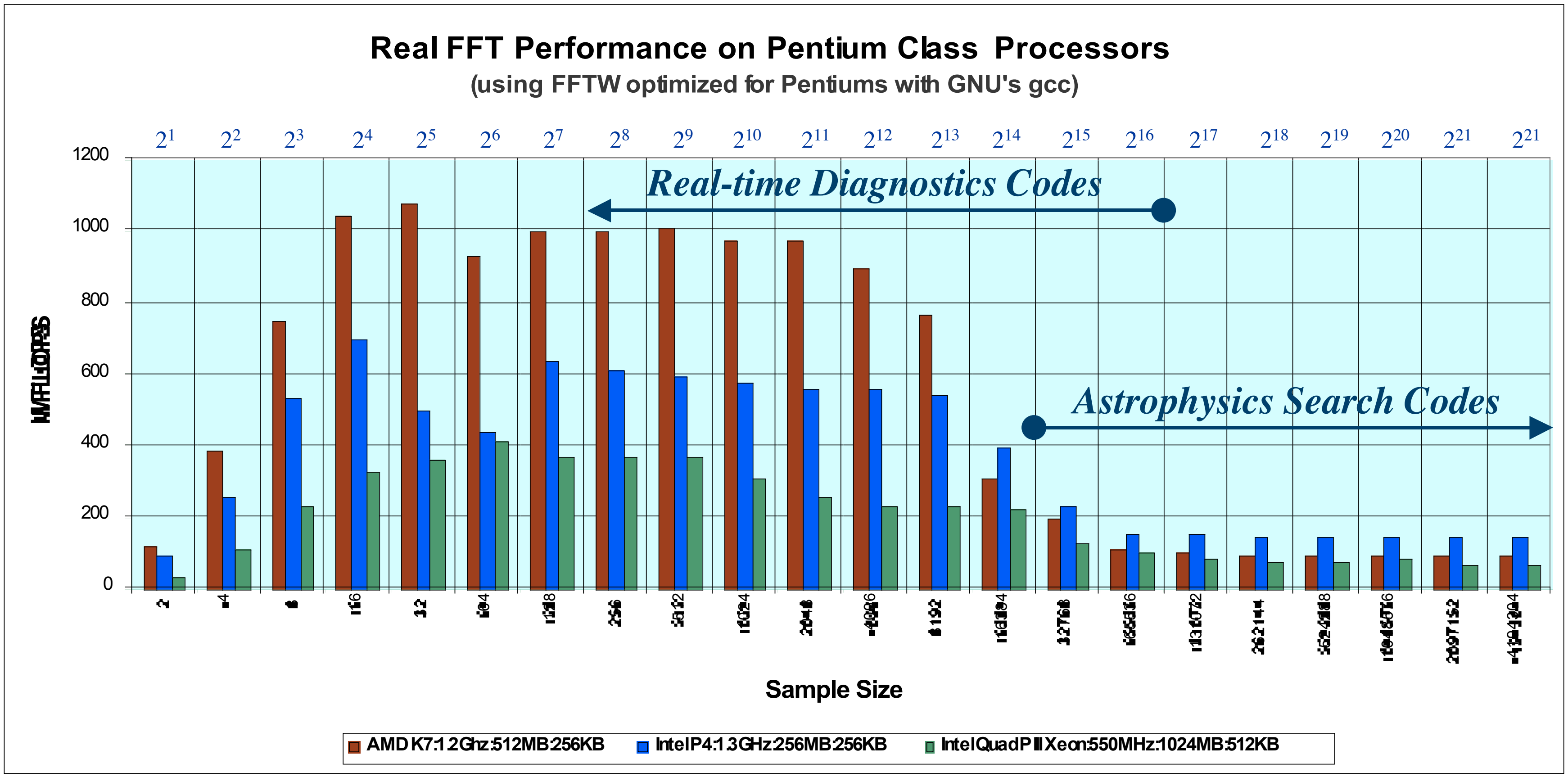
$$\hat{x}(f_i) = \sum_{n=0}^{N-1} x(t_n) e^{-2\pi i f_i t_n} \Delta t \equiv \mathcal{F}[x(t_n)] \; ; \; t_n = n\Delta t = \frac{nT}{N}$$

$$x(t_n) = \sum_{i=0}^{N-1} \hat{x}(f_k) e^{-2\pi i f_k t_n} \Delta f \equiv \mathcal{F}^{-1}[\hat{x}(f_k)] \; ; \; f_k = k\Delta f = \frac{k}{T}$$

$$\mathcal{F}^{-1}\mathcal{F}[x(t_j)] = x(t_i) \Rightarrow \delta_{ij}$$

- Computational cost to perform transform (FFT): *$5 N \log_2 [N]$*
  - ***1000 s of 16384 Sample/s data: 2×10^9 floating point operations (FLOP)***
  - ***To keep up with data: 2 MFLOP/s (MFLOPS)***
  - ***Perform 20,000 at same time: 40 GFLOPS -> clusters of CPUs***
  - ***90+% of CPU time involved in f<->t transformations***

Recent CPU Node Performance



- **Pipeline analysis of LIGO data computationally dominated by cost of Fast Fourier Transforms (FFT).**
  - **Non-Hierarchical Binary Inspiral Search spends an average of ~90% of CPU cycles performing FFT.**
- **Most practical/efficient data segment size as much as 2^{20} points for Binary Inspiral Search.**

Useful Formulae and Identities

- Translation:

$$\mathcal{F}[x(t+t')] = \hat{x}(f)e^{-2\pi i f t'}$$

- Useful identity:

$$\hat{n}(f) = \hat{n}^*(-f) \quad for \quad real \quad n(t)$$

- Convolution

$$c(\tau) = \int_{-\infty}^{\infty} a(t)\,b(\tau - t)\,dt$$

$$\hat{c}(f) = \int_{-\infty}^{\infty} d\tau \int_{-\infty}^{\infty} dt \;\; a(t)\,b(\tau - t)\,e^{-2\pi i f \tau}; \;\; t' = \tau - t$$

$$\hat{c}(f) = \int_{-\infty}^{\infty} dt'\, b(t')\,e^{-2\pi i f t'} \int_{-\infty}^{\infty} dt \;\; a(t)\,e^{-2\pi i f t}$$

$$\hat{c}(f) = \hat{a}(f)\,\hat{b}(f)$$

•Parseval's Theorem

$$\int_{-\infty}^{\infty} x(t)^2\,dt = \text{"Integrated Signal Energy"}$$

$$= \int_{-\infty}^{\infty} \Big[\int_{-\infty}^{\infty} \hat{x}(f)e^{2\pi i f t}\,df\Big]\Big[\int_{-\infty}^{\infty} \hat{x}(f'')e^{2\pi i f'' t}\,df''\Big]dt$$

$$= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} df''\,df\,\{\hat{x}(f)\hat{x}(f'')\}\int_{-\infty}^{\infty} e^{2\pi i (f+f'')t}\,dt$$

$$= \int_{-\infty}^{\infty} |\hat{x}(f)|^2\,df$$

- Correlation

$$R_{xy}(\tau) = \int_{-\infty}^{\infty} x(t)\,y(\tau + t)\,dt$$

$$\frac{1}{2}S_{xy}(f) \equiv \int_{-\infty}^{\infty} R_{xy}(\tau)\,e^{-2\pi i f \tau}\,d\tau$$

$$= \int_{-\infty}^{\infty} d\tau \int_{-\infty}^{\infty} dt \;\; x(t)\,y(\tau + t)\,e^{-2\pi i f \tau}; t' = t + \tau$$

$$= \int_{-\infty}^{\infty} dt'\, y(t')\,e^{-2\pi i f t'} \int_{-\infty}^{\infty} dt \;\; x(t)\,e^{2\pi i f t}$$

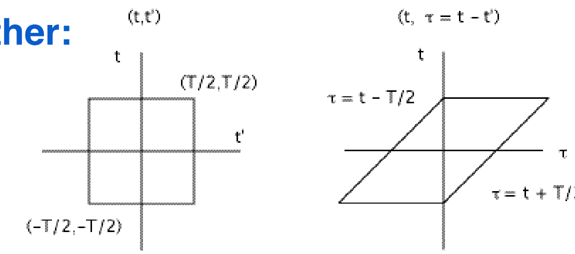
$$= \hat{x}(f)\,\hat{y}^*(f) \Leftarrow \text{Cross-correlation}$$

$$\frac{1}{2}S_{xx}(f) = |\hat{x}(f)|^2 \Leftarrow \text{Autocorrelation}$$

S_x is a one-sided func tion

Important property of an Ergodic Gaussian Process

- **Different Fourier components of n(t) are independent of each other:**



$$\langle n(f)n^{*}(f')\rangle_P = \lim_{T\to\infty} \left\langle \int_{-T/2}^{T/2} dt\ \ n(t)e^{-2\pi ift} \int_{-T/2}^{T/2} dt'\ \ n(t')e^{2\pi if't'} \right\rangle_T$$

$$= \lim_{T\to\infty} \int_{-T/2}^{T/2} dt \int_{-T/2}^{T/2} dt'\ \langle n(t')n(t)\rangle_T\, e^{-2\pi i(ft - f't')}; t' = t + \tau$$

$$= \lim_{T\to\infty} \frac{1}{T}\left(\int_{-T}^{0} \int_{-T/2}^{\tau+T/2} e^{-2\pi it(f-f')} R_n(\tau) e^{-2\pi if\tau} d\tau dt + \int_{0}^{T} \int_{\tau-T/2}^{T/2} e^{-2\pi it(f-f')} R_n(\tau) e^{-2\pi if\tau} d\tau dt \right)$$

$$= \lim_{T\to\infty} \int_{-T}^{T} d\tau \left[e^{-\pi i(f-f')\tau} \frac{\sin\left[\pi(T-|\tau|)(f-f')\right]}{\pi(T-|\tau|)(f-f')} \left(1-\frac{|\tau|}{T}\right) \right] R_n(\tau) e^{-2\pi if\tau}$$

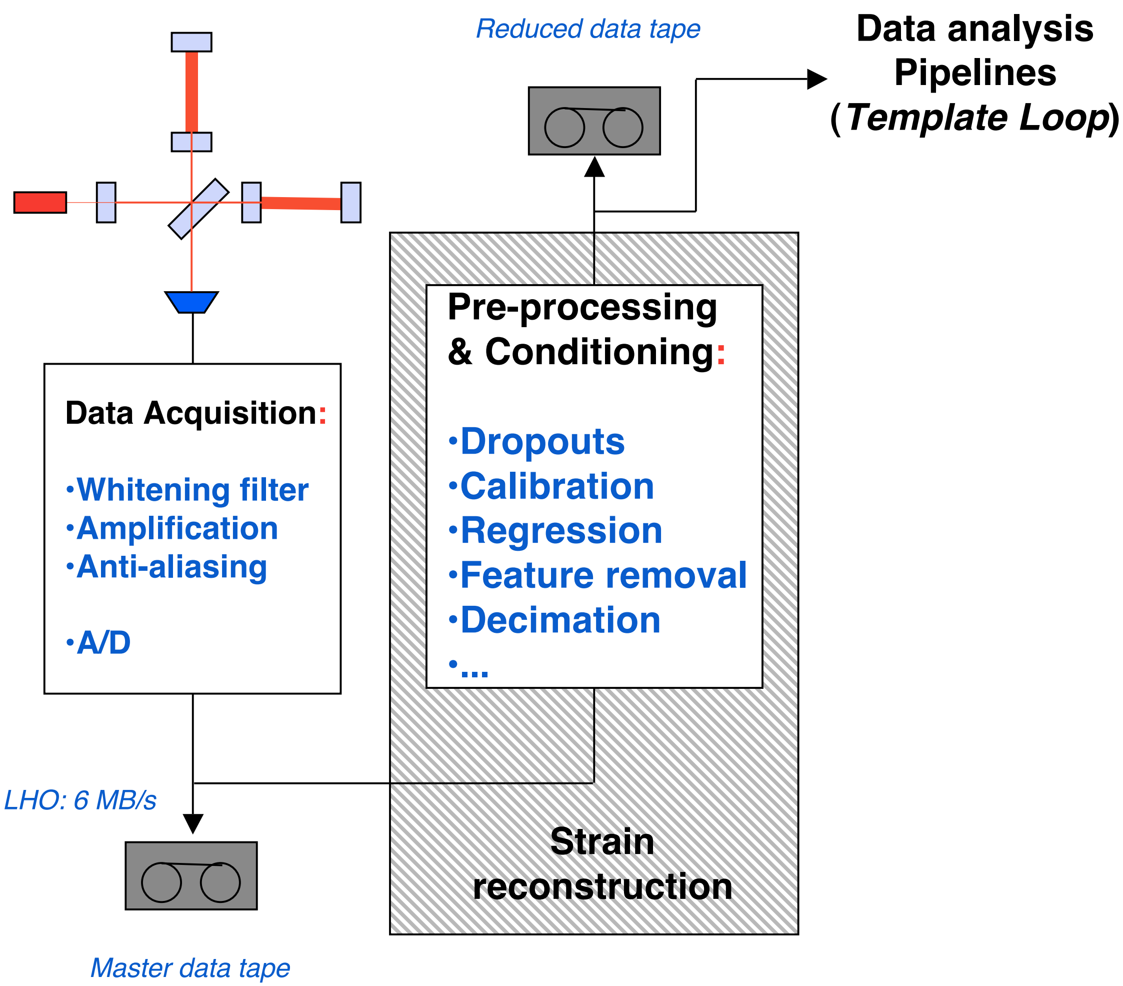
$$= \frac{1}{2} S_n(f)\delta(f-f')$$

δ(f-f') for sufficiently large T

- **Independence of frequency bins allows one to estimate statistics of signal, noise using properties of Gaussian noise (central limit theorem) and statistical tests, such as Neiman-Pearson, χ^2 , …**

As soon as noise properties exhibit transient behavior (on the time scale of the analysis), this introduces frequency correlations in the spectra

Data Flow: Pre-processing



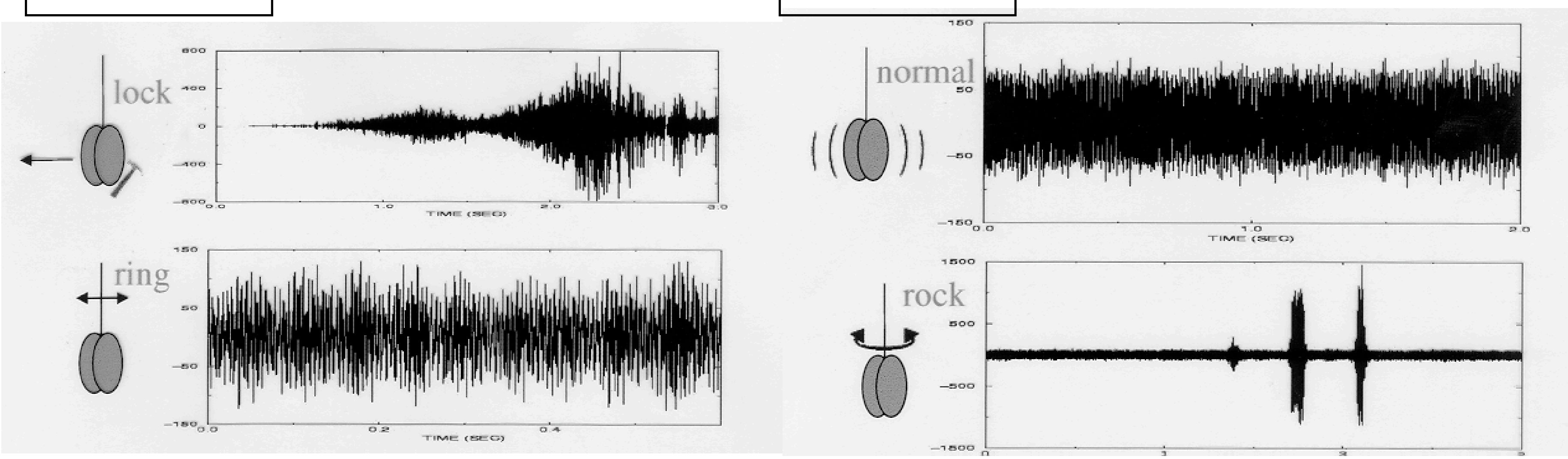
Interferometer Data

Instrumental transients from Caltech 40 m prototype

Real interferometer data are UGLY!!!

(Gliches - known and unknown)

LOCKING **NORMAL**



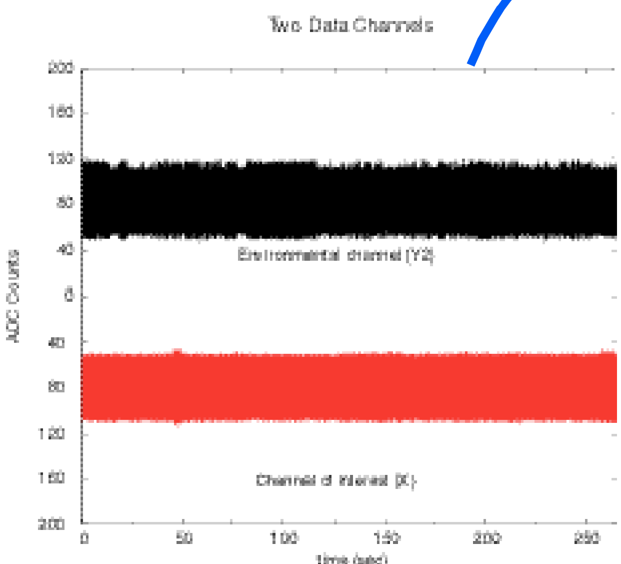
RINGING **ROCKING**

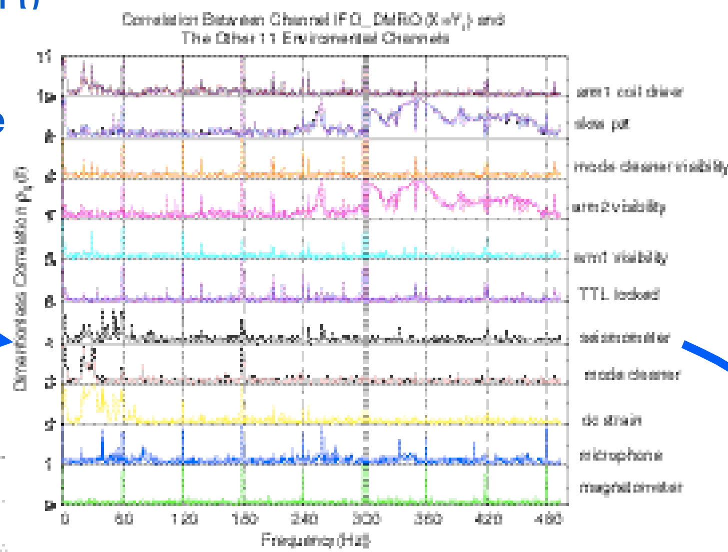
Data Pre-processing: removing instrumental effects

- Cross channel regression will be used to improve signal to noise ratios when possible (need adequate SNR)

Raw channel data (40m prototype

$$s_a(t) \Rightarrow \hat{s}_a(f)$$
$$s_b(t) \Rightarrow \hat{s}_b(f)$$
$$s_a(t)s_b(t+\tau) \Rightarrow R_{ab}(\tau) \Rightarrow \hat{S}_{ab}(f)$$



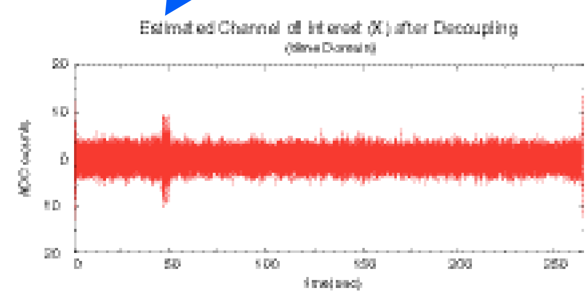


Cross channel spectral correlation

$$\hat{s}'_a(f) = \hat{s}_a(f) - \hat{M}_{aj}(f)\hat{s}_j(f);\; j \neq a$$
$$\Rightarrow \int \hat{s}'_a(f)\hat{s}_j{}^*(f)df = 0;$$
$$\hat{M}_{aj}(f) = \left[\hat{S}(f)\right]^{-1}_{jk}\hat{S}_{ka}(f);$$

$\Rightarrow$ *generalization of 2-channel coherence matrix*

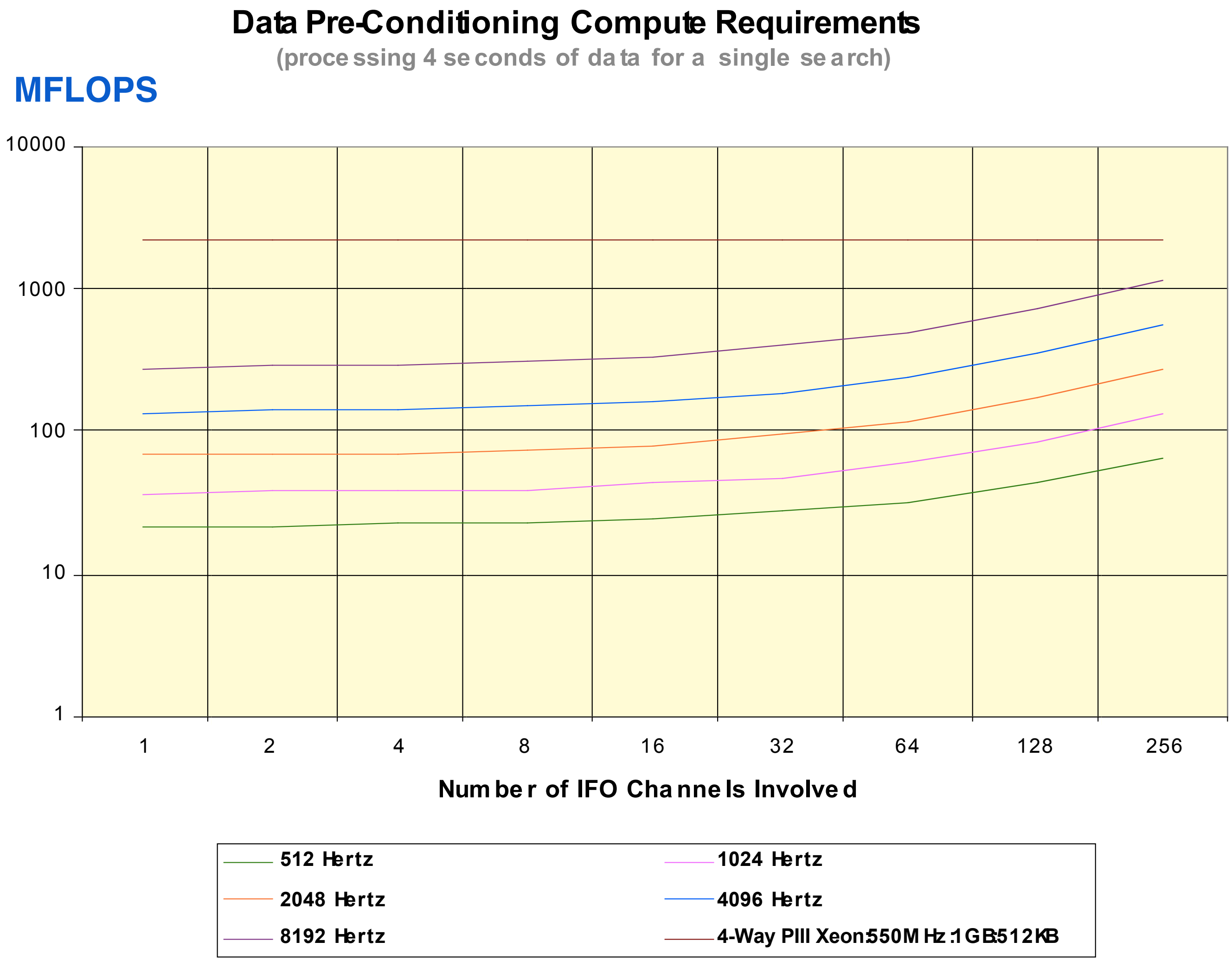
$$\gamma_{ab}(f) \equiv \frac{\hat{S}_{ab}(f)}{\sqrt{\hat{S}_a(f)\;\hat{S}_b(f)}};$$



Reduced data channel

ref: Allen, Hua, Ottewill (gr-qc/9909083)

LIGO Data Conditioning Computational Costs

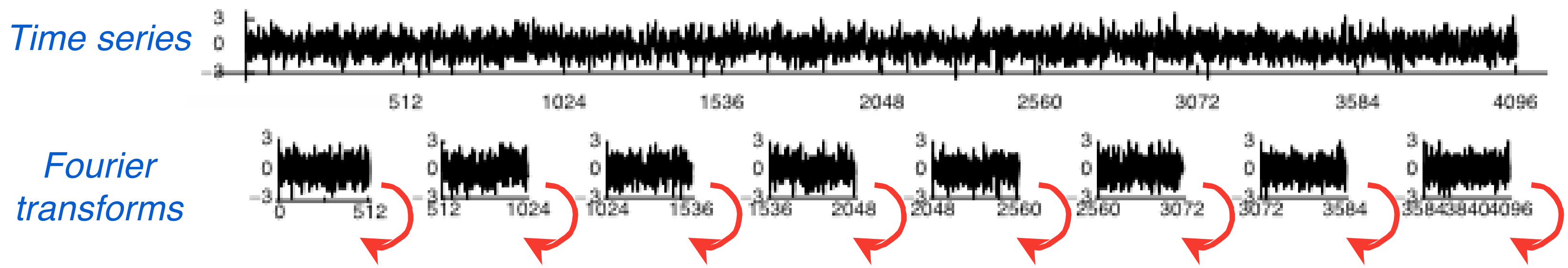


- **Pre-Conditioning steps involved:**
  - *64K samples of input data per channel*
  - *Data Drop-Out Correction on 10% of data.*
  - *Line Removal of 64 lines.*
  - *Calibration of GW channel.*
  - *Resampling (see legend).*
  - *Linear Regression using all input signals.*
- **Roughly 4-8 unique searches expected to be active in LDAS.**

Analyzing discretely sampled time series data

Time-Frequency Analysis of Data

Spectrograms



- Dynamically changing time series contain information not well represented in a single power spectrum or FFT of the data
- Time-frequency analysis allows “visualization” of the dynamics in the data -- time dependent of individual frequency components - produce “image” of a process
- Many different ways to create the time-frequency image -- all derive from a generalized f-t transform

Time-frequency analysis

Generalized t-f transform

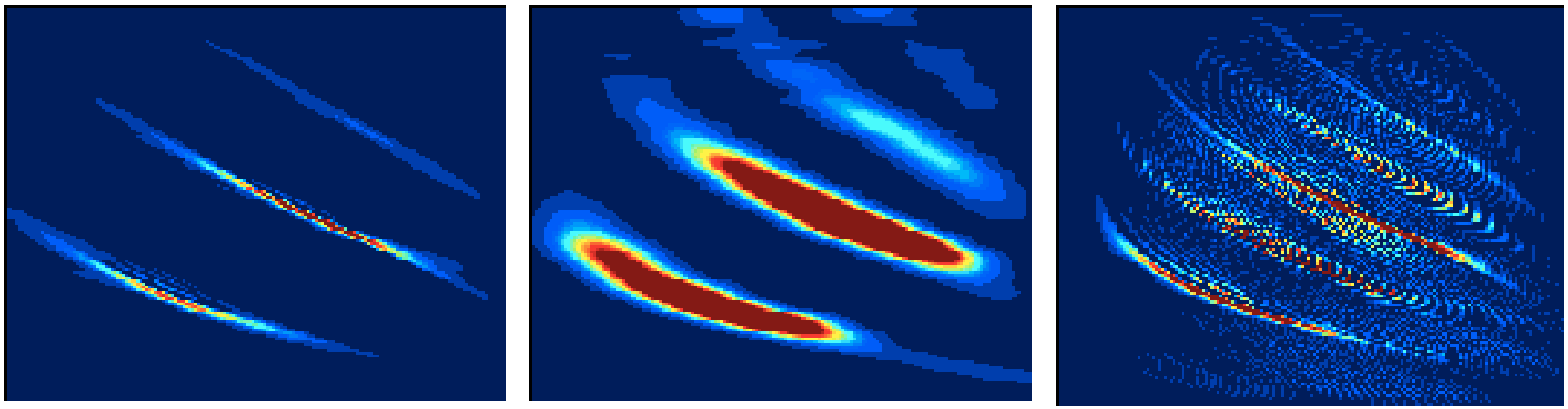
$$P(t,\omega)=\frac{1}{4\pi^2}\iiint e^{-2\pi i(\theta t+\tau\omega-\tau u)}\phi(\theta,\tau)\hat{S}^*[u+\theta/2]\hat{S}[u-\theta/2]\,du\,d\tau\,d\theta$$

Reference	Kernel $\phi(\theta,\tau)$	Distribution $P(t,\omega)$
Wigner[1], Ville[2]	1	$\frac{1}{2\pi}\int e^{-i\tau\omega}s^*[t-\tau/2]s[t+\tau/2]d\tau$
Margenau & Hill[3]	$\cos(\theta\tau/2)$	$\mathrm{Re}\left\{\frac{1}{\sqrt{2\pi}}s(t)e^{-it\omega}\hat{S}^*(\omega)\right\}$
Kirkwood[4], Rihaczek[5]	$e^{i\theta\tau/2}$	$\frac{1}{\sqrt{2\pi}}s(t)e^{-it\omega}\hat{S}^*(\omega)$
sinc[6]	$\frac{\sin(a\theta\tau)}{a\theta\tau}$	$\frac{1}{4\pi a}\int\frac{e^{-i\tau\omega}}{\tau}\int_{t-a\tau}^{t+a\tau}s^*[x-\tau/2]s[x+\tau/2]dx\,d\tau$
Page[7]	$e^{i\theta\vert\tau\vert/2}$	$\frac{\partial}{\partial t}\left\vert\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{t}e^{-i\omega x}s[x]dx\right\vert^2$
Choi & Williams[8]	$e^{-(\theta\tau)^2/\sigma}$	$\frac{1}{4\pi^{3/2}}\iint\sqrt{\frac{\sigma}{\tau^2}}e^{-\sigma(x-t)^2/(4\tau^2)-i\tau\omega}s^*[x-\tau/2]s[x+\tau/2]dx\,d\tau$
Spectrogram	$\int dx\,e^{-i\theta x}h^*[2\pi(x-\tau/2)]h[2\pi(x+\tau/2)]$	$\left\vert\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{t}e^{-i\omega x}s[x]h[x-t]dx\right\vert^2$

1) E.P. Wigner, Phys. Rev., Vol 40 (1932) p749.
2) J. Ville, Cables et Transmissions, Vol. 2A (1948) p61.
3) H. Margenau & R. N. Hill, Prog. Theor. Phys., Vol. 26 (1961) p722.
4) J.G. Kirkwood, Phys. Rev., Vol. 44 (1933) p31.
5) W. Rihaczek, IEEE Trans. Informat. Theory, Vol. IT-14 (1968) p369.
6) L. Cohen, J. Math. Phys., Vol 7 (1966) p781.
7) C.H. Page, J. Appl. Phys., Vol 23 (1952) p103.
8) H.I. Choi & W. J. Williams, IEEE Trans. Acoust., Speech, Signal Processing, Vol. ASSP-37 (1989)

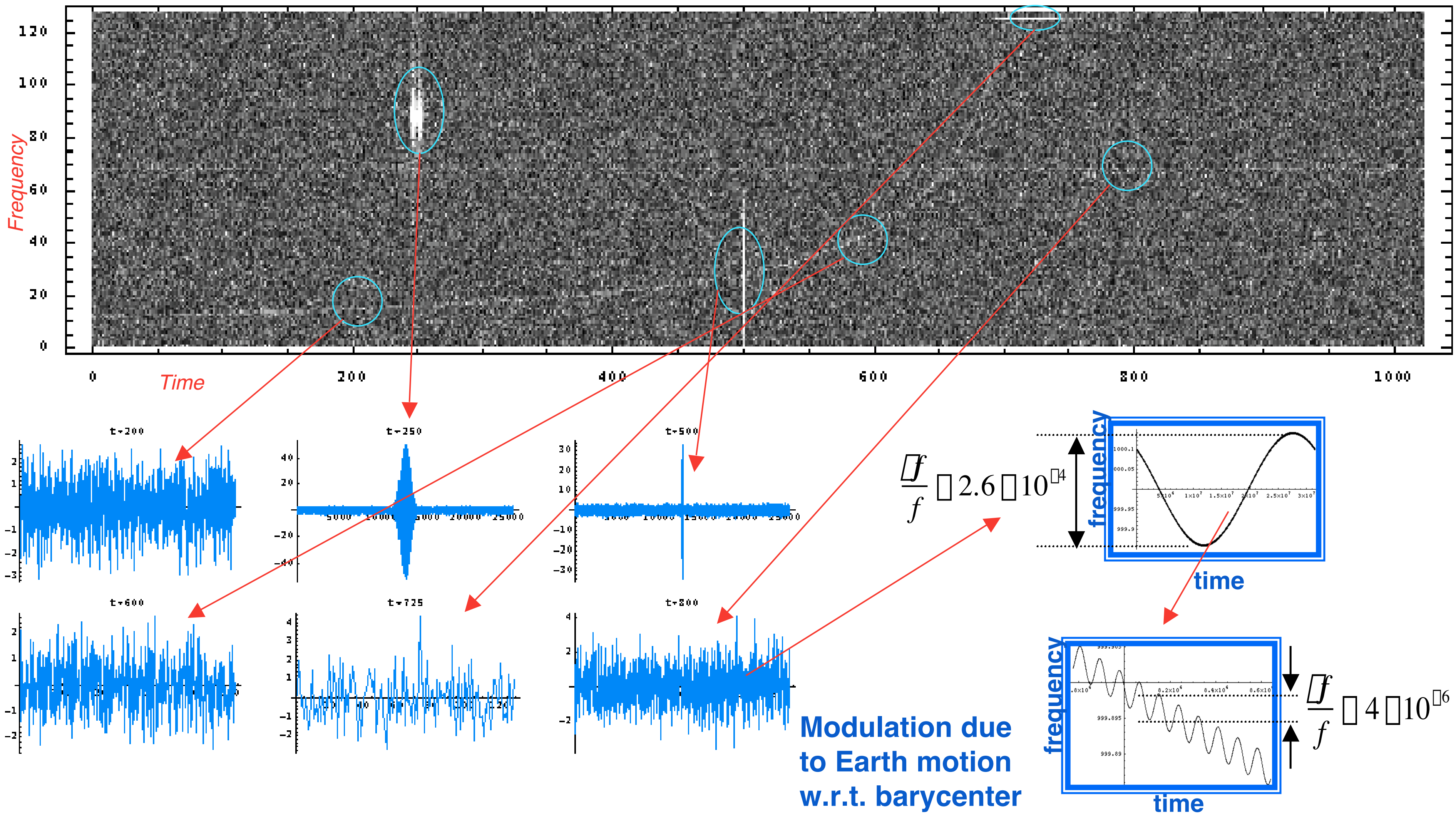
Time-frequency spectrograms

Example from acoustics & zoology: Bat chirps

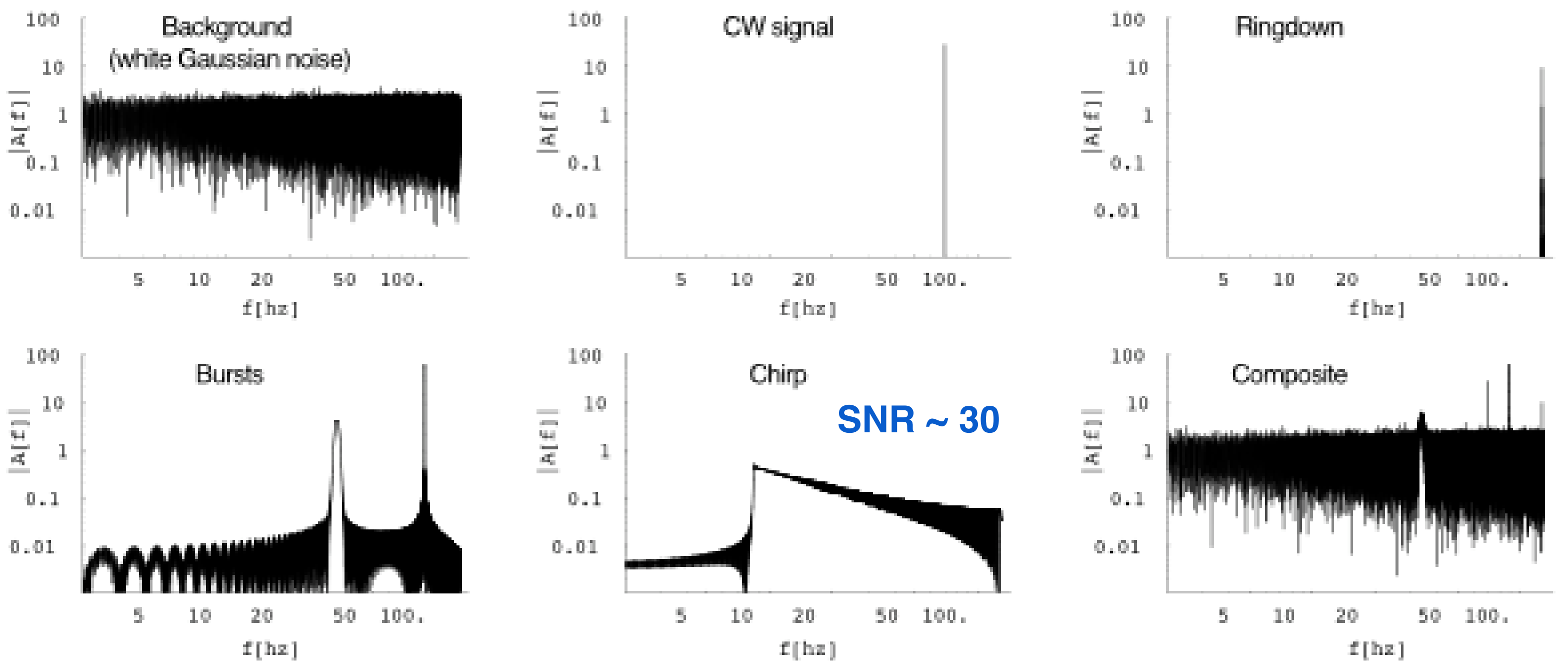


- Good sources of information:
  - *http://www-dsp.rice.edu/software/TFA*
  - *L. Cohen, Proc. of the IEEE, Vol 77, No. 7, July 1989*
  - *W. Anderson & R. Balasubramanian, PRD D60 102001*
    Applied to GW detection& references therein.

LIGO *Time-Frequency Characteristics of GW Sources*



LIGO *Time-Frequency Characteristics of GW Sources*

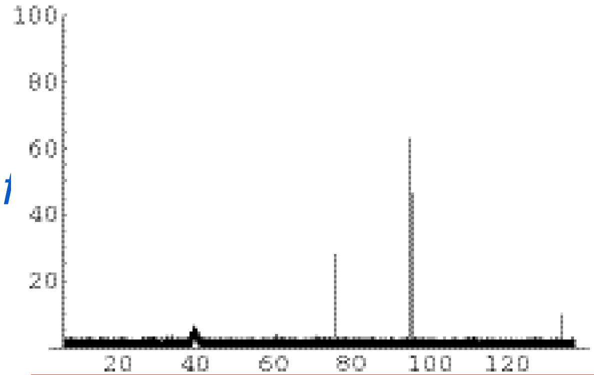


- Long time Fourier transforms of time series h[t] for different components of previous f-t map.
  - *250k points in series*

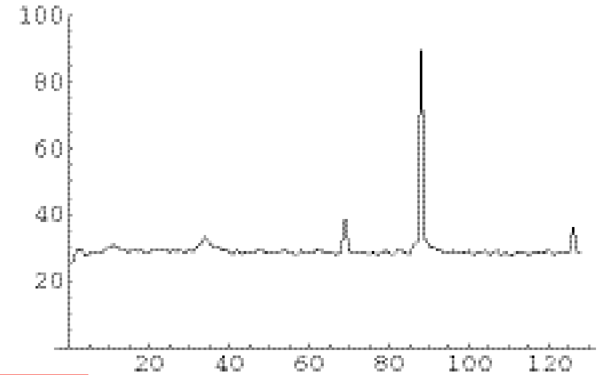
Time-Frequency Characteristics of GW Sources
Loss of SNR due to short time FTs

- For CW signal:
  - *SNR (high res.) ~ 30;*
  - *SNR(low res.) ~ 1.4*
  - *For unresolved (line) signal: signal in single f bin remains constant, but background power grows as Δf increases amplitude SNR~1/Sqrt[Δf]*

Fourier transform of full time series (high res.)



Stack of Fourier transforms of f-t columns (low res.)



$$SNR_{power} = \frac{A^2/2}{S_n(f)\cdot \Delta f}$$
$$s(t) = A\sin(2\pi f t);$$
$$S_n(f) = \text{power spectral density of noise}$$

- Stacking is more computationally efficient, for N data points and m stacks:

$$C_1 \approx 5N\cdot\log_2 N$$
$$C_m \approx m\cdot\left(5\frac{N}{m}\cdot\log_2\frac{N}{m}\right) = 5N\cdot\left[\log_2 N - \log_2 m\right]$$
$$R_m \equiv \frac{C_m}{C_1} = \left(1 - \frac{[\log_2 m]}{[\log_2 N]}\right)$$
$$N = 10^9; m = 10^6$$
$$R_m = 0.3333$$

Time-Frequency Characteristics of GW Sources

- **Bursts are short duration, broadband events**
- **Chirps explore the greatest time-frequency area**
  - **BH ringdowns will be associated with chirps**
- **CW sources have FM characteristics which depend on position on the sky (*<u>and source parameters</u>*)**
- **Stochastic background is stationary and broadband**

- **For deterministic sources, the optimal signal to noise ratio is obtained by integrating signal *<u>along</u>* the trajectory**
  - **If SNR >> 1, kernel μ |*signal*|^2** *power or square-law detector*
  - **If SNR $\leq$ 1, kernel μ**
    - **|*template* signal*|** *matched filter or*
    - **|*signal*$_j$* *signal*$_k$|** *cross-correlation for stochastic singals*
  - **Optimal filter:**
    - **kernel μ 1/(*noise power*)** *weight integral by inverse of noise --- look where detector is most quiet*

Science in LIGO I
Data analysis strategies

- Compact binary inspiral: *"chirps"*
  - *NS-NS waveforms are well described*
  - *BH-BH need better waveforms*
  - *search technique: matched templates, fast-chirp transform*
- Pulsars in our galaxy: *"periodic"*
  - *search for observed neutron stars (freq., doppler shift) - matched filters*
  - *all sky search (computing challenge)*
  - *r-modes*
- Cosmological Signals *"stochastic background"*
  - *Search technique: optimal Wiener filter for different models*

Modeled Sources

- Supernovae / GRBs: *"bursts"*
  - *burst search algorithms – excess power; time-freq patterns*
  - *burst signals - coincidence with signals in E&M radiation*
  - *prompt alarm (~ 1 hr) with ν detectors [SNEWS]*

Unmodeled Sources

Optimal Filters

- For a given source of gravitational waves
  - *Optimal filter may be derived based on a model of:*
    - *Noise properties of the instrument*
      - ***Gaussian is assumed because it makes the analysis tractable***
      - ***Ultimately need to rely on instrument performance improvements in order to approach Gaussian characteristics of the data***
      - ***Requiring coincidence among multiple similar devices further reduces non Gaussian noise***
    - *Interaction of the antenna (GW detector) with the putative gravitational waves from the source*
      - ***Requires model of waveform, statistics, time-frequency domain of signal***
      - ***The more information, the more robust the filter***
      - ***Any information is useful and can be used to formulate a filter***

Optimal Filters - cont.

- Compact coalescences
  - *Best studied sources - probably best understood*
    - *Phase, amplitude dependence can be modeled*
    - *Provides family of parametrized templates for matching against the data*
- Stochastic gravitational wave background
  - *Assumption of white (at least over the few decades LIGO will observe), Gaussian, isotropic background makes analysis tractable*
  - *Requires cross-correlation between (at least) two detectors*
  - *Use of bar detector to modulate signal by physically rotating bar can be exploited to develop an optimal filter in the presence of terrestrial correlated noise*

Optimal Filters - cont.

- Spinning neutron stars (CW sources, GW pulsars)
  - *Long lived signal - not a transient*
  - *Daily and yearly modulations of frequency due to Earth's motion relative to the Solar System barycenter provides a unique signature with which to develop an optimal filter*
  - *Source may exhibit intrinsic variaitons of period (spindown) - constitutes a large parameterizable space over which to search*
  - *Frequency modulation swamps frequency resolution of analysis*
    - *Many different templates required, depending on source sky location, intrinsic parameters*
  - *The one problem for LIGO that could potentially use infinite CPU power to fully exploit the data*
    - *"Unbiased all-sky search"*

$$f \to f' = f\frac{v}{c}\cos\theta$$

$$\Delta f = \left|f' - f\right| \approx 10^{-4} f \sin\theta\,\delta\theta \approx \delta f\big|_{\text{Fourier trasform}} = \frac{1}{T_{obs}}$$

$$\delta\Omega \approx (\delta\theta)^2 = 10^8\left(\frac{1}{fT_{obs}}\right)^2;\ \ fT_{obs} = N_{cycles}$$

$$N_{\text{sky pixels}} \geq \frac{4\pi}{\delta\Omega} = 4\pi \times 10^{-8}\left(N_{cycles}\right)^2;$$

$$1 \text{ month @ } 500Hz \Rightarrow N_{cycles} \approx 10^9$$

$$N_{\text{sky pixels}} \geq 10^{11}\left(\frac{T}{1\ month}\right)^2$$

Optimal Filters - cont.

- Transient (burst) phenomena
  - *Waveforms are less well known*
    - *Model in terms of time-frequency volume and shape of volume occupied by any source type*
  - *Most challenging experimentally because many instrumental, geophyiscal phenomena can manifest themselves as burst noise in the interferometer data*
  - *Most like the cryogenic resonant bar detectors*
    - *Adopt/adapt experiences and techniques from this community for analysis*
  - *Rely on coincidence analysis among multiple detectors (of all types) to reduce backgrounds*
  - *Expect useful collaboration with e.g., astroparticle physics detectors for □,□ event coincidences*

Signals with parametrizable waveforms

Deterministic

CW: http://www.lsc-group.phys.uwm.edu/pulgroup/

Inspiral: http://www.lsc-group.phys.uwm.edu/iulgroup/

Statistical

Stochastic background: http://feynman.utb.edu/~joe/research/stochastic/upperlimits/ /

- **Measurement s, (may) contain signal h, contains noise n:**

$$s(t) = h(t) + n(t);\ \ n >> h$$
$$\langle n(t)\rangle = 0\ \ ;\ \ h(t) = \alpha T_i(t);$$
$$\langle n(t)h(t)\rangle = 0;$$

- **$\alpha T_i(t)$ is one of a family of fiducial (expected) waveforms or *templates* of type *i* with unknown time origin t_0 and amplitude α; e.g.**
  - ***i={+ or x;m_i;ϕ_0; …}***

- **Design optimal (linear) correlation filter,Q, that maximizes chance of detecting *T* in s(t).**
  - ***General expression for a correlation filter:***

$$C(t_0) = \int_{-\infty}^{\infty} s(t - t_0)Q(t)dt;$$

$$\langle C\rangle = \int_{-\infty}^{\infty} \alpha T(t - t_0)Q(t)dt$$
$$= \alpha \int_{-\infty}^{\infty} \hat{T}(f)e^{-2\pi i f t_0}\hat{Q}^*(f)df$$

$$N = C - \langle C\rangle;\ \langle N\rangle \equiv 0$$
$$\langle N^2\rangle = \langle C^2\rangle - \langle C\rangle^2$$
$$= \left\langle \int \hat{n}(f)\hat{Q}^*(f)df \int \hat{n}(f')\hat{Q}^*(f')df' \right\rangle$$
$$= \int df \int df' \hat{Q}^*(f)\hat{Q}^*(f')\langle \hat{n}(f)\hat{n}(f')\rangle$$
$$= \frac{1}{2}\int S_n(f)|Q(f)|^2 df$$

Optimal Filtering and Signal Processing

- **Maximize P(detection) => Maximize signal-to-noise ratio (SNR):**

$$Max[SNR = \frac{\langle C \rangle}{\langle N^2 \rangle^{1/2}}] \Rightarrow \quad Max[(SNR)^2 = \frac{\langle C \rangle^2}{\langle N^2 \rangle}]$$

$$\delta(SNR)^2 = \frac{\delta\langle C \rangle^2}{\langle N^2 \rangle} - \frac{\langle C \rangle^2 \delta\langle N^2 \rangle}{\langle N^2 \rangle^2} = 0$$

$$= \frac{2\langle C \rangle}{\langle N^2 \rangle^2}\left(\langle \delta C \rangle \langle N^2 \rangle - \langle C \rangle \langle N \delta N \rangle\right) = 0$$

$$\langle \delta C \rangle \langle N^2 \rangle = \langle C \rangle \langle N \delta N \rangle$$

- **Variation and maximization is with respect to optimal filter, δQ:**

$$\langle C \rangle = \int_{-\infty}^{\infty} \hat{T}(f) e^{-2\pi i f t_0} \hat{Q}^*(f) df$$

$$\langle \delta C \rangle = \int_{-\infty}^{\infty} \hat{T}(f) e^{-2\pi i f t_0} \delta\hat{Q}^*(f) df$$

$$\langle N^2 \rangle = \frac{1}{2} \int S_n(f) |Q(f)|^2 df$$

$$\langle N \delta N \rangle = \frac{1}{2} \int S_n(f) Q(f) \delta Q^*(f) df$$

Optimal Filtering and Signal Processing

- **Equate LHS, RHS:**

$$\langle \delta C \, N^2 \rangle = \langle C \, N \delta N \rangle$$

$$\hat{T}(f) e^{-2\pi i f t_0} \left[\int S_n(f') |Q(f')|^2 df' \right] = S_n(f) Q(f) \left[\int \hat{T}(f') e^{-2\pi i f t_0} \hat{Q}^*(f') df' \right]$$

- **Require <u>frequency dependent</u> coefficients of [...] to be <u>equal for all f:</u>**

$$\hat{T}(f) e^{-2\pi i f t_0} = S_n(f) Q(f) \Rightarrow \boxed{Q(f) = \frac{\hat{T}(f) e^{-2\pi i f t_0}}{S_n(f)}}$$

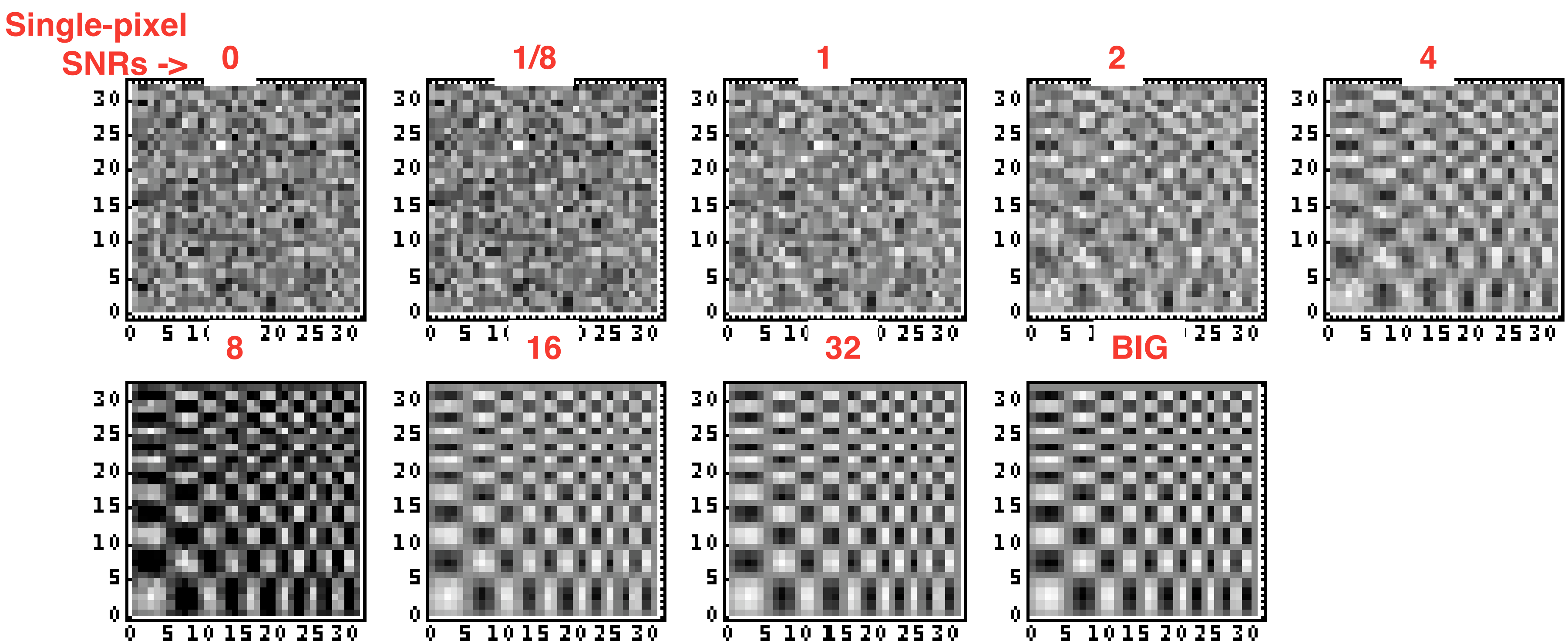
<u>Optimal filter for this problem</u>

- **Check that [...] == [...] with Q(f) from above:**

$$\left[\int S_n(f') |Q(f')|^2 df' \right] ?=? \left[\int \hat{T}(f') e^{-2\pi i f t_0} \hat{Q}^*(f') df' \right] \quad !$$

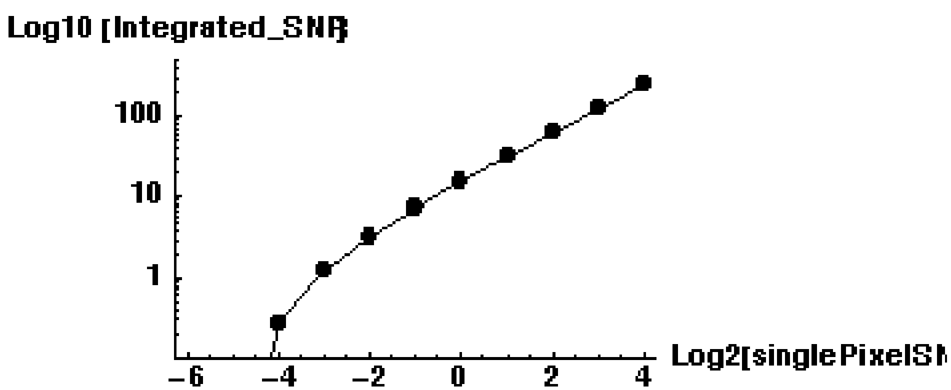
Detection vs. Parameter Estimation

- Detection is possible at lower SNR than parameter extraction
  - *Can detect before you can "see" the pattern*



Detections vs. Parameter Estimation

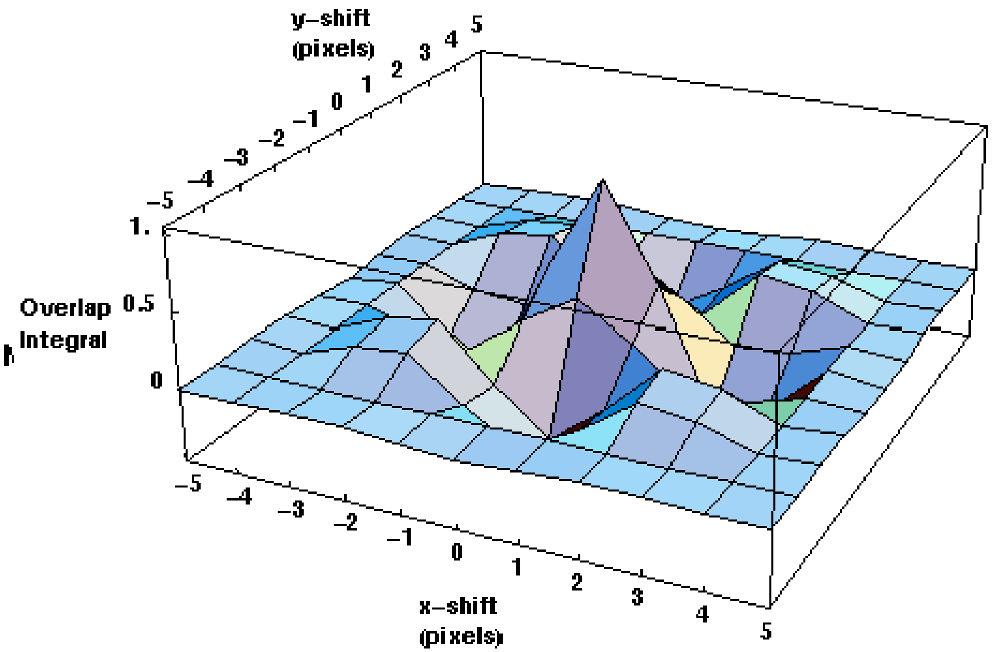
•…what if you don't know the orientation, or pattern exactly?



$$n = noise; \quad d = noisy\ \ data; \quad T = noiseless\ \ template$$

$$SNR = \frac{\sum_{i,j} d(i,j)T(i,j)}{\sqrt{\sum_{i,j} n^2(i,j)T^2(i,j)}}$$

- Integrated SNR == Sum over entire image after it is multiplied by noiseless signal (template)



$$A(\delta i, \delta j) = \sum_{i,j} T(i,j)T(i+\delta i, j+\delta j)$$

- Effect on Integrated SNR of (unknown) shift between ***noiseless*** signal and template
- Shift +/- 1 pixel reduces correlations => need 32^2 templates to cover a 32x32 pixel image...

Matched Filtering with Templates

$$C(t_0:\{p\}) = \int \frac{\hat{s}(f)\hat{T}^{*}_{\{p\}}(f)}{S_n(f)} e^{2\pi i f t_0} df$$

- $C(t_0{:}\{p\})$ is a *family* of derived correlation time series
  - *{p} is a set of parameters used to characterize the templates T*

    Intrinsic parameters: masses, other GR parameters

    Extrinsic parameters: distance, orbital inclination, phase, position in the sky, …

    Dimensions of {p} can be HUGE (e.g., 10^4 - 10^5) if one wants a reasonable certainty of detecting a weak signal with unknown parameters

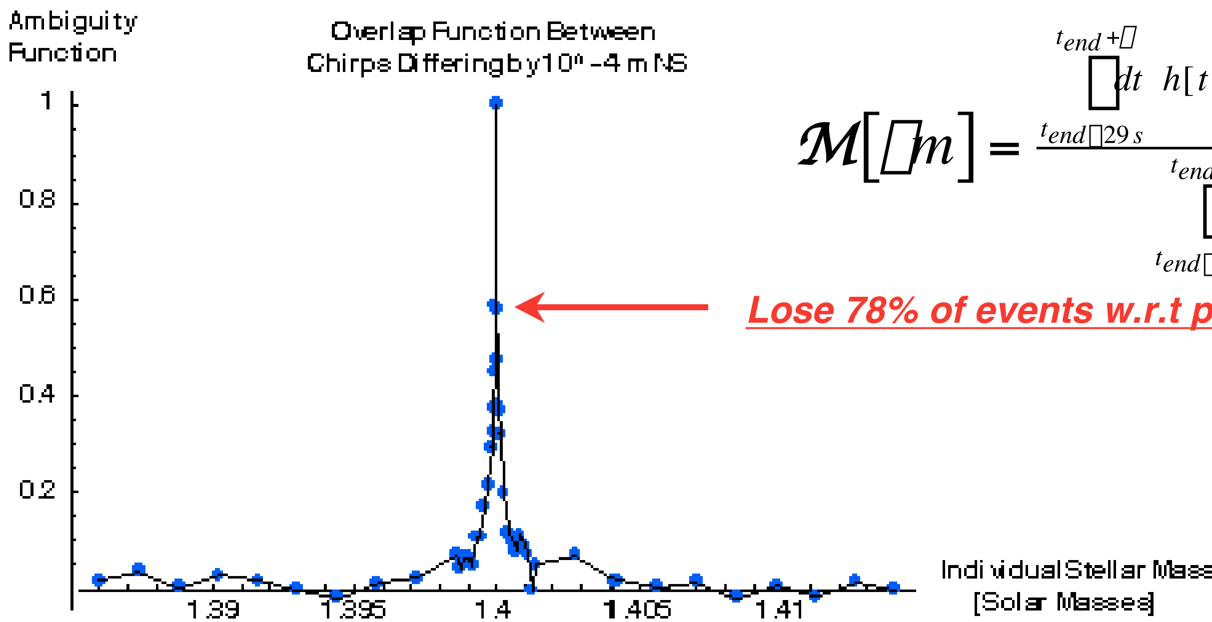
    Example: Search for a sinusoid with unknown frequency between 10 Hz < f < 500Hz in a data stream lasting T = 1000s;

    To see signal, need to acquire data at S > 1000 s^{-1} (several factors greater)

    Number of frequency bins in Fourier transform of data = ST ~ few x 10^6

    ! ***Each bin is orthogonal to all others -> need to test every bin since sinusoid will be contained within only one bin***

Matched Filtering with Templates

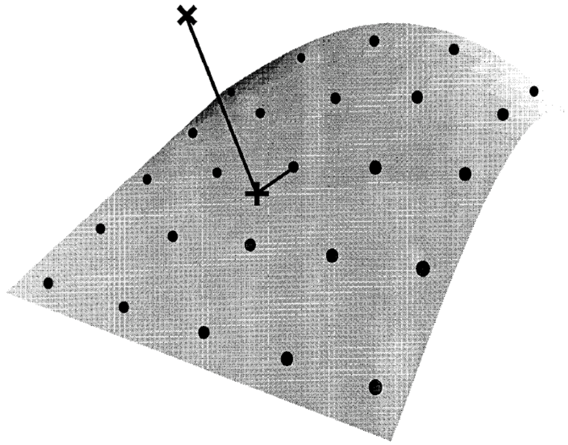


$$\mathcal{M}[\Delta m] = \frac{\int_{t_{end}-29\,s}^{t_{end}+\infty} dt \;\; h[t, M_{NS}, M_{NS}, R_0] \cdot h[t, M_{NS}, M_{NS}+\Delta m, R_0]}{\int_{t_{end}-29\,s}^{t_{end}+\infty} dt \;\; h[t, M_{NS}, R_0] \cdot h[t, M_{NS}, R_0]}$$

Lose 78% of events w.r.t peak!

- For $1.4M_{sun} + 1.4M_{sun}$ binary, starting at R = 300km;
  - *Chirp duration ~ 30s from f = 40 Hz -> `Infinity'*
  - *To search range $1M_{sun} < M_{NS} < 3M_{sun};\ \Delta M = 2\ M_{sun}$*
  - *With resolution $\delta M = 1.4 \times 10^{-4}\ M_{sun}$*
  - Requires $N_{templates} = \Delta M / \delta M \sim 1.5 \times 10^4$ chirp templates
    - *1PN simple calculation*
    - *Ignores variation in density of templates over mass range*
    - *White noise background*

Mass parameter space for inspiraling binary coalescences



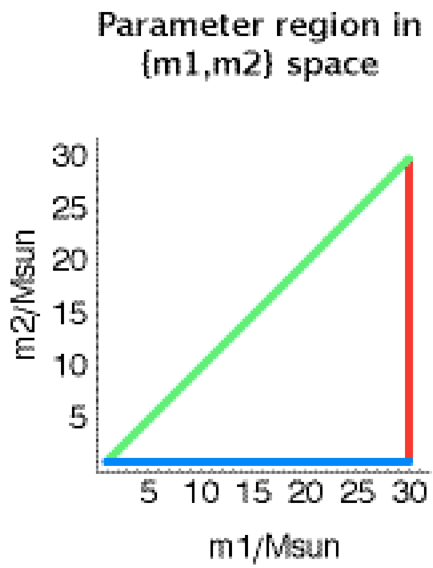
$$\mathcal{M}[\lambda,\Delta\lambda] = \frac{\int_{t_0}^{t_{end}} dt\;\; h[t:\lambda]\cdot h[t:\lambda+\Delta\lambda]}{\int_{t_0}^{t_{end}} dt\;\; h[t:\lambda]\cdot h[t:\lambda]};\;\; \lambda = \{m_1, m_2, S_1, S_2 \ldots\}$$

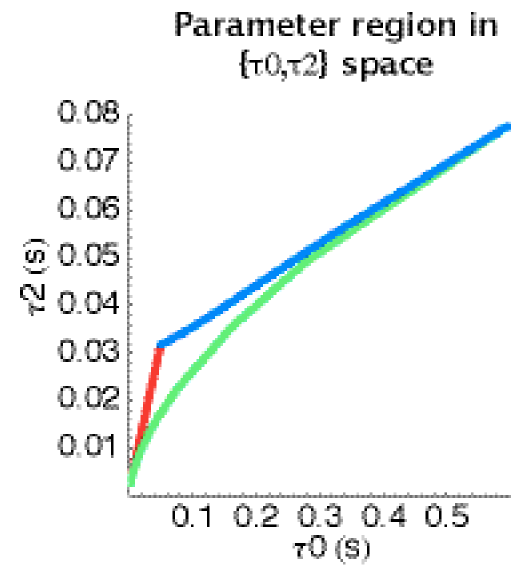
$$\left.\frac{\partial \mathcal{M}}{\partial \lambda_i}\right|_{\Delta\lambda=0} = 0$$

$$1 - \mathcal{M}[\lambda,\Delta\lambda] = -\frac{1}{2}\left(\left.\frac{\partial^2 \mathcal{M}}{\partial\Delta\lambda_i \partial\Delta\lambda_j}\right|_{\Delta\lambda=0}\right)\Delta\lambda_i\Delta\lambda_j + \ldots$$

$$g_{ij}(\lambda) \equiv -\frac{1}{2}\left(\left.\frac{\partial^2 \mathcal{M}}{\partial\Delta\lambda_i \partial\Delta\lambda_j}\right|_{\Delta\lambda=0}\right)\Delta\lambda_i\Delta\lambda_j \quad \text{metric on template manifold}$$

References: B. J. Owen, PRD Vol. 53, No. 12 (1996) 6749
B.S.. Sathyaprakash, Vol. 50, No. 12 (1994) R7111





- Optimal placement of templates in parameter space is determined by requiring the *metric* 1-$\mathcal{M}$ to be no smaller than a minimum value, e.g.,0.97 (10% loss in event rate)
- "Best" coordinates to use: corrections to coalescence time τ coming from successive PN corrections:

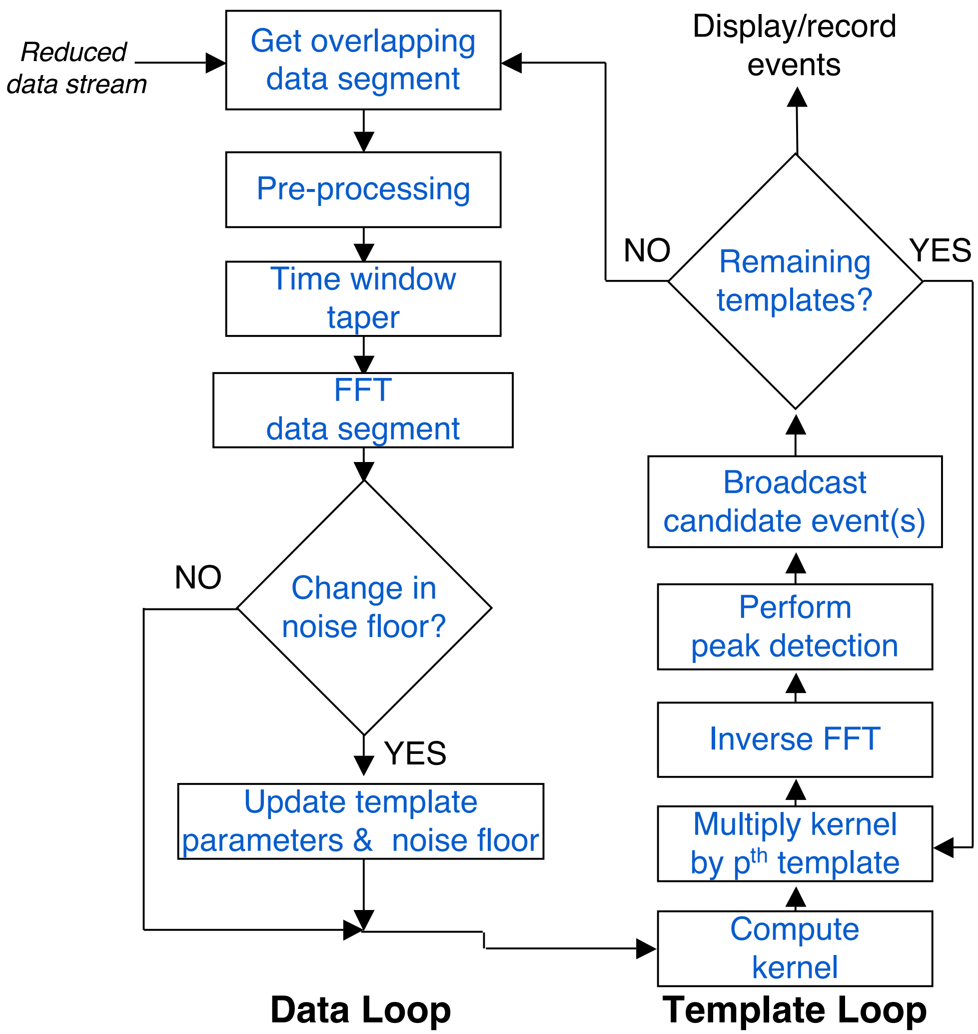
$$\tau_0 = \frac{5}{256}\eta^{-1} M^{-5/3}(\pi f_0)^{-8/3}$$

$$\tau_2 \approx \frac{3715}{64512}(\eta M)^{-1}(\pi f_0)^{-2}$$

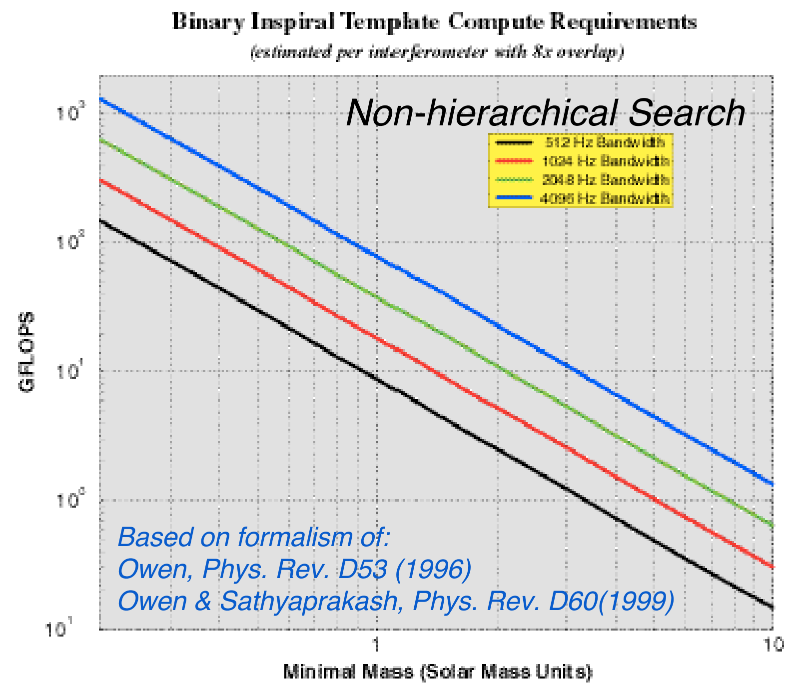
$$M = total\ mass;\ \eta = \frac{\mu}{M};\ \mu = reduced\ mass$$

$$f_0 = reference\ \ frequency\ \ at\ \ start\ \ of\ \ epoch$$

Compact Binary Inspirals Data Analysis Flow



$$\rho_p[t_c] = 2\int \frac{\hat{T}_p^*(f)\;\hat{s}(f)}{\hat{S}_n(|f|)} e^{-2\pi i f t_c}\, df$$

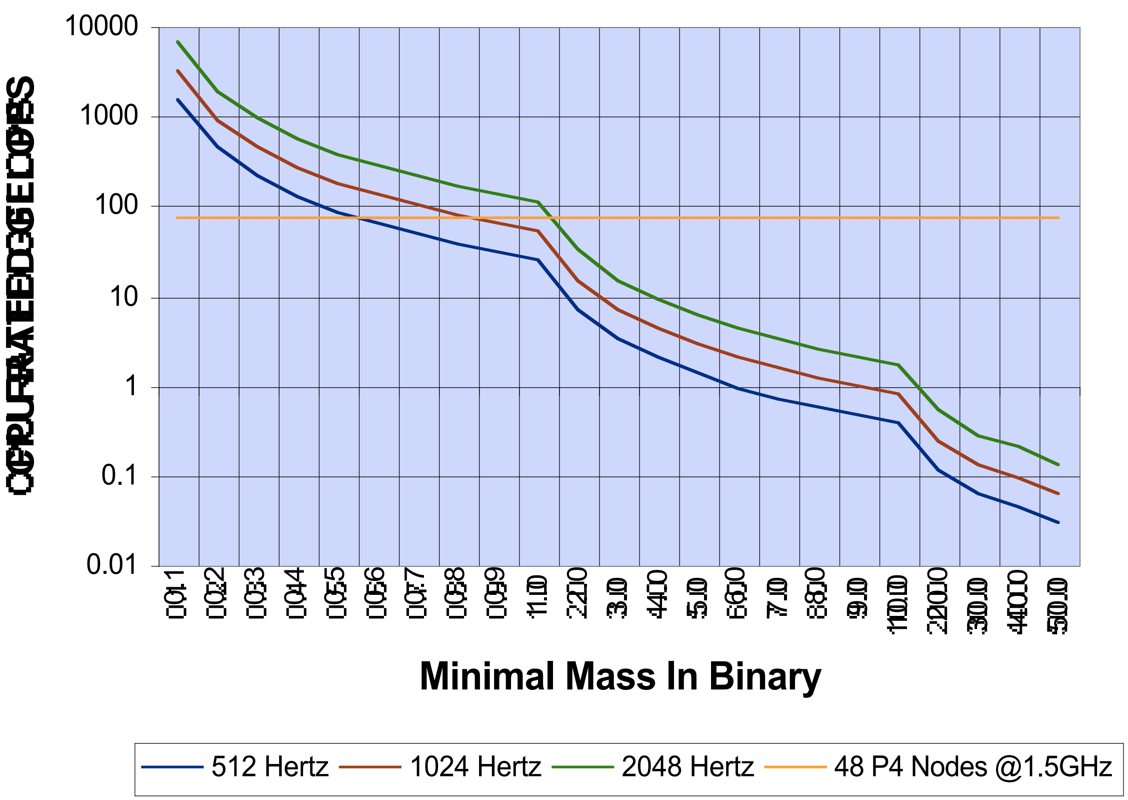


- **Process data at real time rate**
- **Improvements:**
  - ***Hierarchical searches developed***
  - ***Phase coherent analysis of multiple detectors (Finn, in progress)***

Computation of Templated Binary Inspiral Search

Computation for Binary Inspiral Search Using 1.5GHz P4's

(8x overlap, no hierarchy, 10% detection loss, 10% efficiency for FFTs)



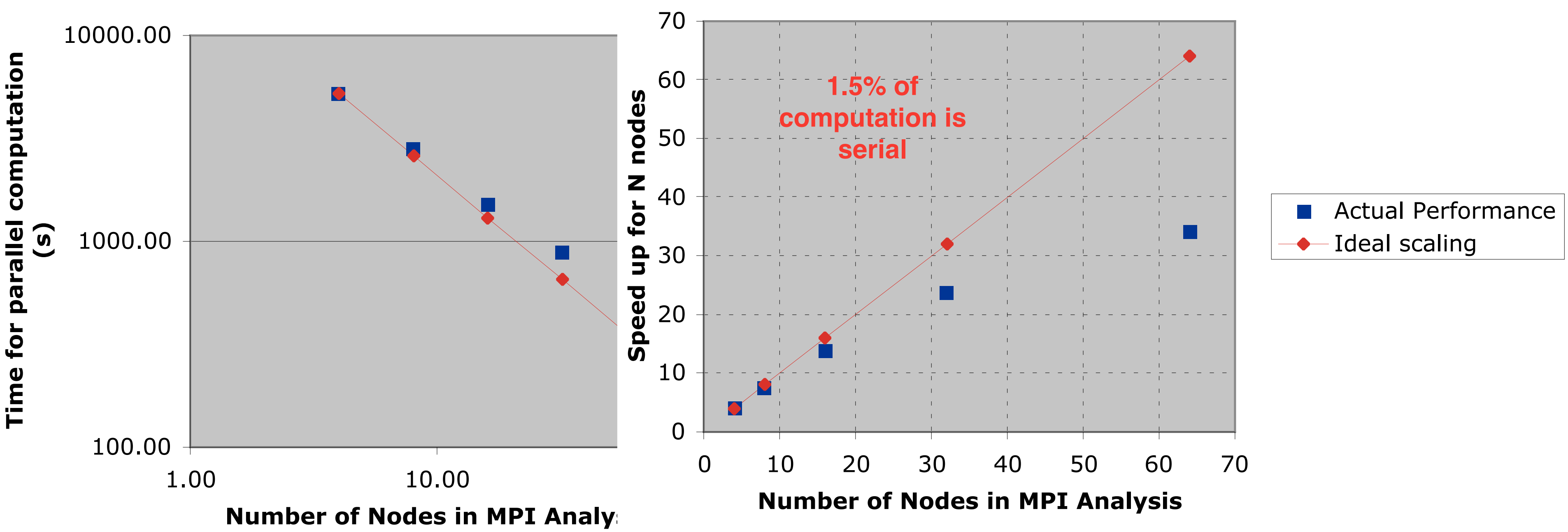
- **Non-Hierarchical Search for NS-NS Inspiral ($1.4M_S$, $1.4M_S$) ~ {15, 32, 67} GFLOPS.**
- **512 Hertz band is adequate for detections (blue curve).**
- **Hierarchical strategies expected to decrease cost by 5x to 30x.**
- **Ringdown Search estimated to be roughly 10% as costly.**
- **Other searches (excluding all-sky pulsar search) are single node compute problems.**

Inspiral E7 tests - Template Search

- Preliminary results
  - *Code performance in a parallel computer is degraded by serial (bottleneck) components of code -- usually interprocessor I/O*
  - *Governed by Amdahl's Law:*

$$\frac{T_1}{T_N} = \frac{s+p}{s+\frac{p}{N}} = \frac{N}{1+s(N-1)} \to \begin{cases} N, & s \to 0 \\ 1, & s \to 1 \end{cases}$$

MPI computation performance with increasing number of nodes



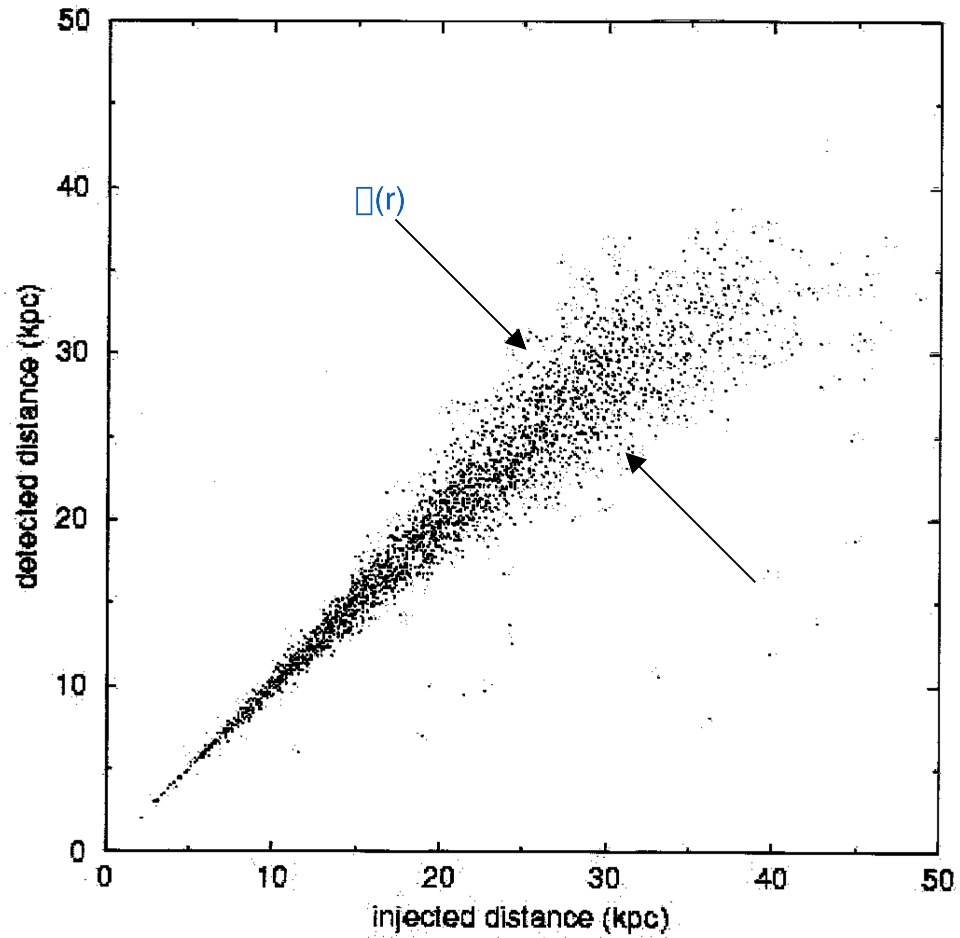
Reference: Amdahl, G.M. Validity of the single-processor approach to achieving large scale computing capabilities. In AFIPS Conference Proceedings vol. 30 (Atlantic City, N.J., Apr. 18-20). AFIPS Press, Reston, Va., 1967, pp. 483-485.

Determining Detection Efficiency

Monte Carlo (Statistical) techniques are needed to characterize complex detection probabilities

- **Inject a large number of simulated signals varying, e.g., distance, (m_1, m_2).**
  - **Retrieve events by same algorithm used for data**
  - **Confirm detected parameters**
  - **Determine efficiency of search (model dependent)**

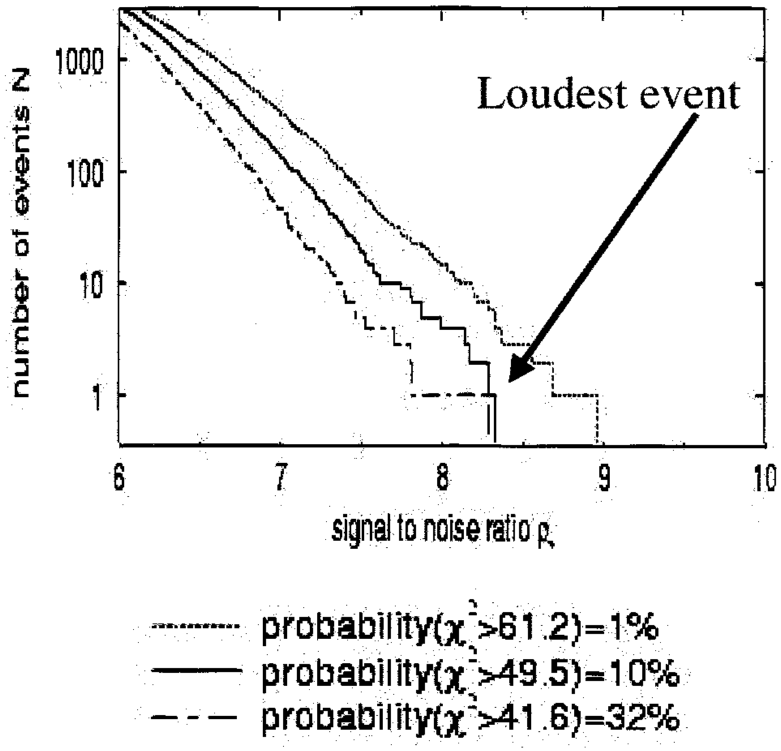
- **Errors in distance measurements from presence of noise are consistent with SNR fluctuations**



Setting a limit on inspiral coalescence rate within the galaxy

(1994 40m prototype data)

Quantitative Science: making a probabilistic statement about the likelihood of an observation (or lack thereof)



Upper limit on event rate can be determined from SNR of 'loudest' event

Limit on rate:

R < 0.5/hour with 90% CL

□= 0.33 = detection efficiency

An ideal detector would set a limit:

R < 0.16/hour

B. Allen et al., gr-qc/9903108

Stochastic GW Background Detection

- Cross-correlate the output of two (*independent*) detectors with a suitable filter kernel:

$$C(T) = \int_{-T/2}^{T/2} dt \int_{-\tau/2}^{\tau/2} d\tau' \; s_1(t)s_2(t-\tau')Q(\tau')$$

- Requires:
  - *(i) Two detectors must have overlapping frequency response functions i.e.,* $s_1(f)s_2(f) \neq 0, \; \{f\} \neq \varnothing$
  - *(ii) Detectors sensitive to same polarization state (+, x) of radiation field,* h_{GW}.
  - *(iii) Baseline separation must be suitably "short":*

$$L < \lambda_{GW}(f) \Rightarrow \frac{fL}{c} < 1$$

Stochastic Background Correlation

- Ideally, the stochastic background correlation increases with integration time as:

$$SNR \approx \frac{3H_0^2}{10\pi^2}\sqrt{T_{\mathrm{int}}}\left[\int df\,\frac{\gamma^2(f_0)\,\Omega_{GW}^2(f)}{f^6 S_{1,n}(|f|)S_{2,n}(|f|)}\right]^{\frac{1}{2}}$$

  - *Assumes no additional sources of correlated noise*

    cannot discriminate with a single measurement

  - *Mutual orientation dependence of GW background signal may be exploited to discriminate among possible correlated sources*

•References:

»*P.F. Michelson, Mon. Not. Roy. Astron. Soc.* ***227****, 933 (1987).*

»*N. Christensen, Phys. Rev.* ***D46****, 5250 (1992)*

»*E. Flanagan, Phys. Rev.* ***D48****, 2389 (1993), astro-ph9305029*

»*B. Allen and J. Romano, Phys. Rev.* ***D59****, 102001 (1999), gr-qc9710117*

»*M. Maggiore, Trieste, June 2000: Gravitational Waves: A Challenge to Theoretical Astrophysics, gr-qc-0008027*

»*L.S. Finn and A. Lazzarini, Phys. Rev. D, 15 (2001)*

Optimal filtering in the presence of background correlation

$$C(T) = \int_{-T/2}^{T/2} dt \int_{-T/2}^{T/2} dt' \; s_1(t) s_2(t') Q(t - t') \; ; \; s_i(t) = h_i(t) + n_i(t)$$

$$C(T) = \int_{0} df \int_{0} df' \; \delta_T(f - f') \tilde{s}^*{}_1(f) \tilde{s}_2(f') Q(f') \; ; \; \delta_T(f) \equiv T\left(\frac{\sin(\pi f T)}{\pi f T}\right)$$

$$\left\langle \tilde{h}_1^*(f) \tilde{h}_2(f') \right\rangle = \delta(f - f') \frac{3H_0^2}{20\pi^2 |f|^3} \Omega_{GW}(|f|) \gamma(|f|, \vec{\Omega}_1, \vec{\Omega}_2)$$

<=Template for this problem

$$\left\langle \tilde{n}_i^*(f) \tilde{n}_j(f') \right\rangle = \frac{1}{2} \delta(f - f') S_{ij}(|f|)$$

$$\left\langle \tilde{n}_i^*(f) \tilde{n}_j(f') \tilde{n}_i^*(f'') \tilde{n}_j(f''') \right\rangle = \frac{1}{4}\Big(S_{ii}(|f|) S_{jj}(|f'|) \delta(f + f'') \delta(f' + f''') + S_{ij}(|f|) S_{ij}(|f''|) \delta(f + f'') \delta(f' + f''') + S_{ii}(|f|) S_{jj}(|f'|) \delta(f + f'') \delta(f' + f''') \Big)$$

h_i=GW signal in detector i

n_i= noise in detector i

$\Omega_{GW}(f) = 1/\rho_0 \, d\rho_{GW}/d(\ln[f])$

$\gamma(f, \Omega_1, \Omega_2)$ = geometric overlap reduction factor depends on antenna orientations

Optimal filtering in the presence of background correlation

$$\left\langle C(T,\vec{\Omega}_1,\vec{\Omega}_2)\right\rangle = T\int_0^{\infty} df \left[\pm\frac{3H_0^2}{20\pi^2|f|^3}\Omega_{GW}(|f|)\gamma\left(|f|,\vec{\Omega}_1,\vec{\Omega}_2\right) + S_{12}(|f|)\right]\tilde{Q}(f)\ ;$$

Choose two orientations of one detector $\{\Omega_1, \Omega_1'\}$, for which $\gamma(f, \Omega_1, \Omega_2) = -\gamma(f, \Omega_1', \Omega_2)$, denote C_+, C_- values of integrated correlation in these two orientations:

$$\langle C(T)\rangle = \langle C_+(T/2) - C_-(T/2)\rangle$$

$$\langle C(T)\rangle = T\int_0^{\infty} df \left[\frac{3H_0^2}{20\pi^2|f|^3}\Omega_{GW}(|f|)\gamma\left(|f|,\vec{\Omega}_1,\vec{\Omega}_2\right)\right]\tilde{Q}(f)$$

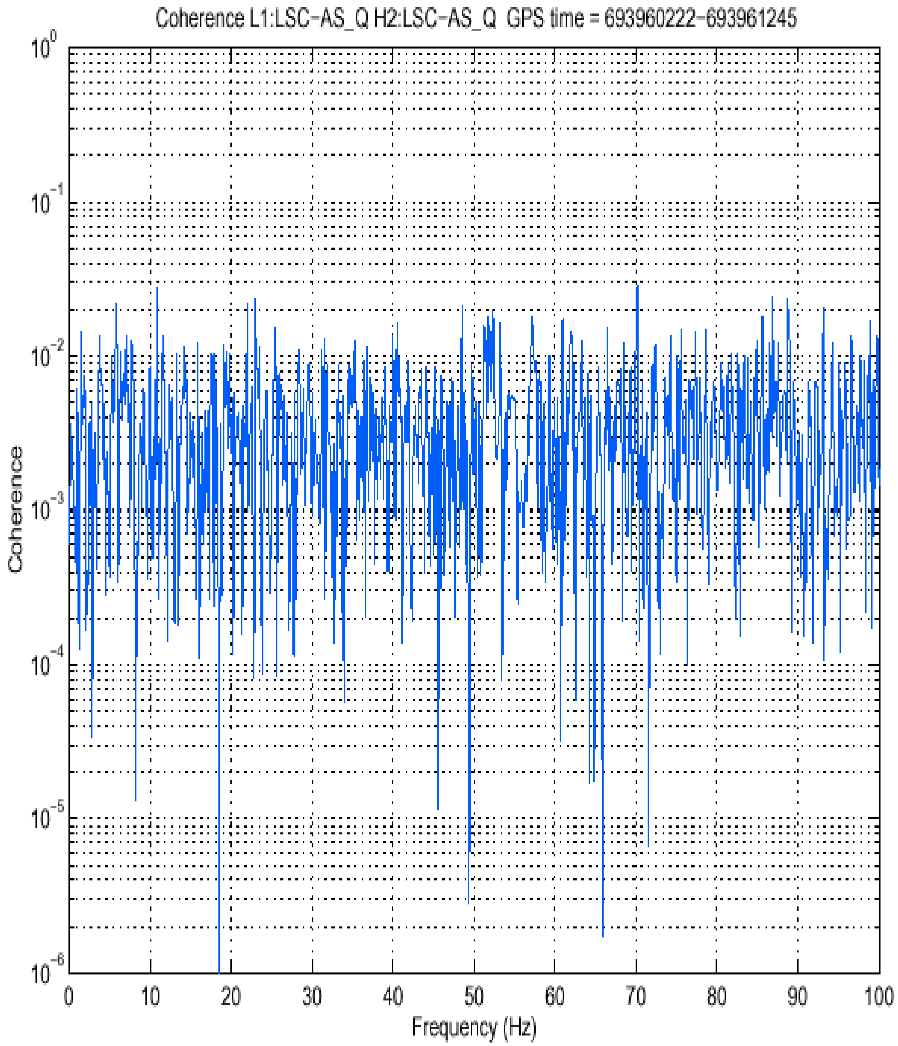
$$\sigma_C^2 = \left\langle C^2\right\rangle - \langle C\rangle^2 = 2\sigma_{C+,-}^2$$

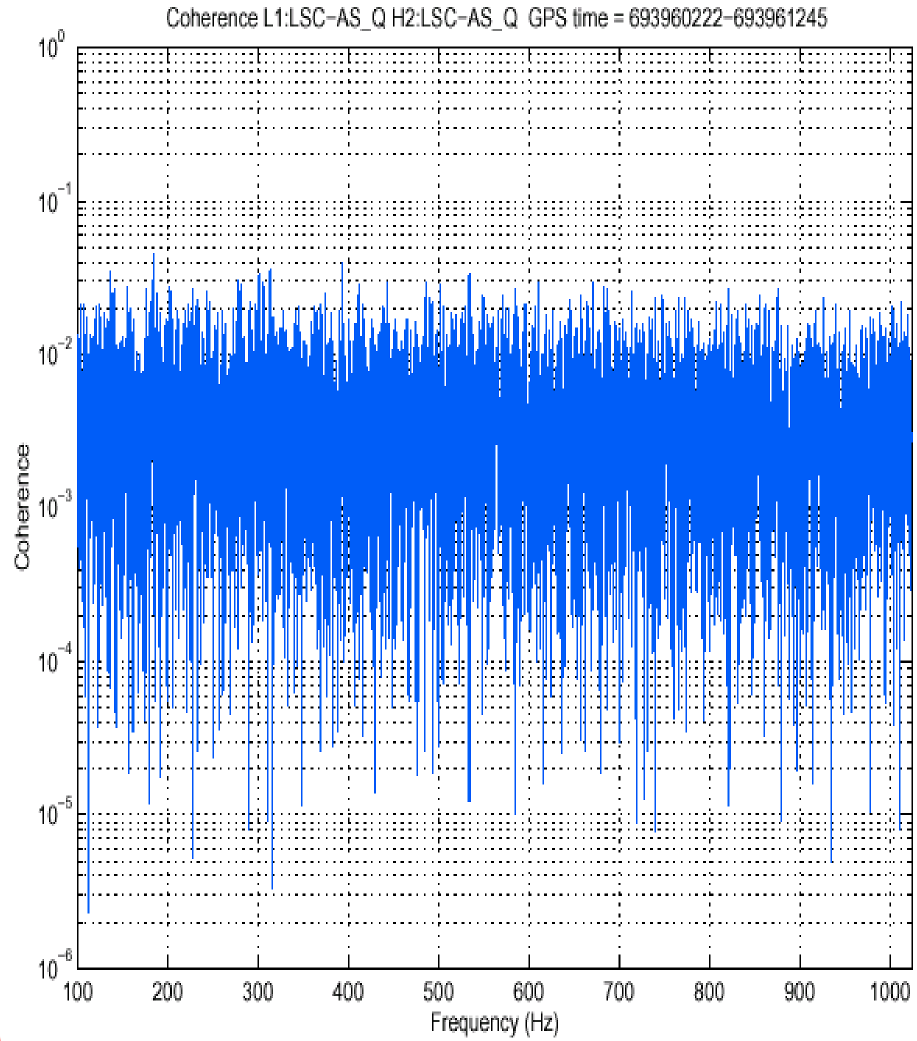
$$\sigma_C^2 = \frac{T}{2}\int_0^{\infty} df\ \left(S_1(|f|)S_2(|f|) + S_{12}^2(|f|)\right)\left[\tilde{Q}(f)\right]^2$$

$$SNR = \frac{\langle C\rangle}{\sigma_C}\ \xrightarrow{max}\ \frac{\delta[SNR]}{\delta[\tilde{Q}]} = 0\ \Rightarrow\ \tilde{Q}(f) = \left[\frac{\gamma\left(|f|,\vec{\Omega}_1,\vec{\Omega}_2\right)\Omega_{GW,model}(|f|)}{|f|^3\left(S_1(|f|)S_2(|f|) + S_{12}^2(|f|)\right)}\right]$$

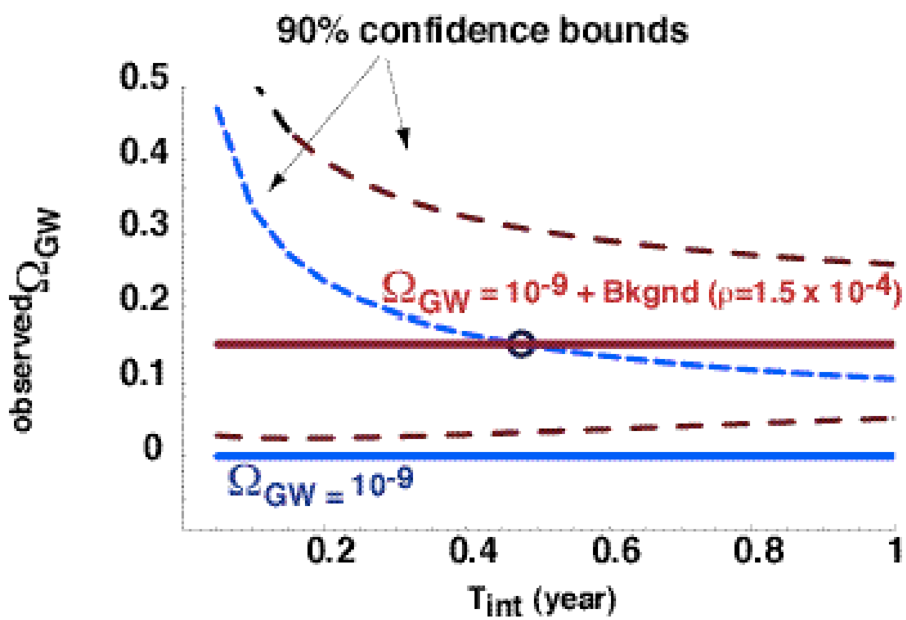
Optimal filter for this problem

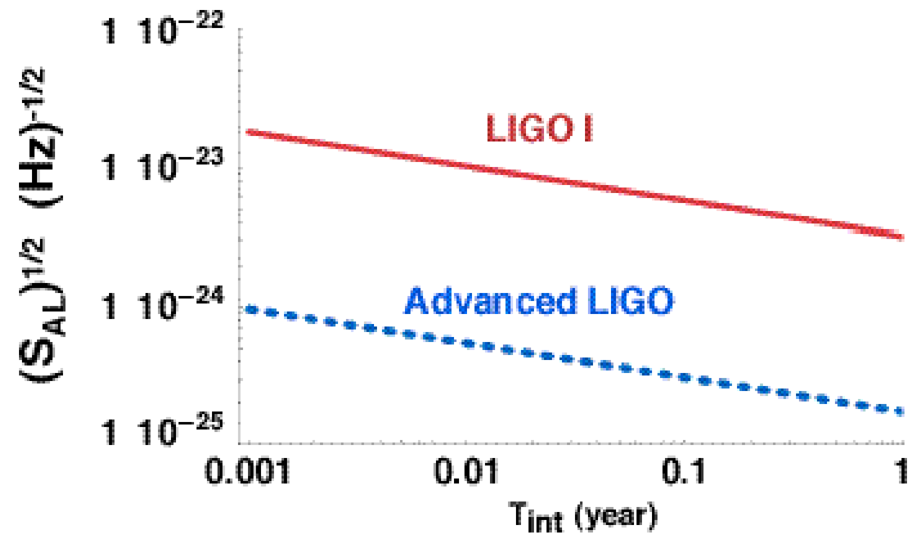
Coherence plots (LLO-LHO 2k)





Effect of correlated background on observable upper limits for Ω_{GW}





$$\rho \equiv \frac{S_{12}}{\sqrt{S_1 S_2}}$$

Signal detection

- Detection: choosing between two hypotheses:

  H0: y = n vs. H1: y = s + n

- Two types of error:
  - *False alarm:*

    □ = P(H1 l H0)

  - *False dismissal:*

    □(s) = P(H0 l H1)

LIGO

Data series

$\bar{\mathbf{x}} = \bar{\mathbf{n}} + \bar{\mathbf{s}}$ - measured series of data points

$\bar{\mathbf{x}} = \{x_1, x_2, \cdots, x_N\} \in R^N$ - - a vector of length N

$\bar{\mathbf{n}} = \{n_1, n_2, \cdots, n_N\}$ □ noise vector

$\bar{\mathbf{s}} = \{s_1, s_2, \cdots, s_N\}$ □ signal vector (may or may not be present)

$< \bar{\mathbf{n}}^{\mathbf{T}} \cdot \bar{\mathbf{n}} >= \tilde{\mathbf{C}}$ □ noise correlation matrix (diagonal if noise is white, Gaussian)

*W*hat is probability of observing the measured data if $\bar{\mathbf{x}} = \bar{\mathbf{n}} + \bar{\mathbf{s}}$ vs. $\bar{\mathbf{x}} = \bar{\mathbf{n}}$?

$$P[n_i] = \frac{1}{\sqrt{2\pi \tilde{C}_i^2}} e^{-n_i^2/2C_i^2}; P[\bar{\mathbf{n}}] = \frac{1}{\sqrt{2\pi \det\left\|\tilde{C}\right\|}} e^{-\frac{1}{2}\bar{\mathbf{n}}^{\mathbf{T}} \cdot \tilde{\mathbf{C}}^{-1} \cdot \bar{\mathbf{n}}}$$

$$P[\bar{\mathbf{x}}|\mathbf{0}] = P[\bar{\mathbf{n}}] = \frac{1}{\sqrt{2\pi \det\left\|\tilde{C}\right\|}} e^{-\frac{1}{2}\bar{\mathbf{x}}^{\mathbf{T}} \cdot \tilde{\mathbf{C}}^{-1} \cdot \bar{\mathbf{x}}}$$

$$P[\bar{\mathbf{x}}|\bar{\mathbf{s}}] = P[\bar{\mathbf{x}} - \bar{\mathbf{s}}|\mathbf{0}] = \frac{1}{\sqrt{2\pi \det\left\|\tilde{C}\right\|}} e^{-\frac{1}{2}(\bar{\mathbf{x}} - \bar{\mathbf{s}})^{\mathbf{T}} \cdot \tilde{\mathbf{C}}^{-1} \cdot (\bar{\mathbf{x}} - \bar{\mathbf{s}})};$$

•Also applies on resultant of a linear filtering of **x**, e.g., □= $\mathbf{x}^T \cdot Q \cdot T$

Likelihood

Baye'sLaw:

$$P[\bar{\mathbf{s}}_i|\bar{\mathbf{x}}] = \frac{P[\bar{\mathbf{x}}|\bar{\mathbf{s}}_i|]P[\bar{\mathbf{s}}_i|]}{P[\bar{\mathbf{x}}]}; i \in \{1,2,3,\cdots,N\}\ \textit{family of signals}$$

$P[\bar{\mathbf{s}}_i|\bar{\mathbf{x}}]$: Probability$[signal_i|data]$

$P[\bar{\mathbf{x}}|\bar{\mathbf{s}}_i|]$: Probability$[data|signal_i]$

$P[\bar{\mathbf{s}}_i]$: A priori probability$[signal]$

$P[\bar{\mathbf{x}}]$: A priori probability$[data] = P[\bar{\mathbf{x}}|\bar{\mathbf{s}} = 0]P[0] + P[\bar{\mathbf{x}}|\bar{\mathbf{s}} \neq 0]P[\bar{\mathbf{s}} \neq 0]$

$P[\bar{\mathbf{x}}|\bar{\mathbf{s}} \neq 0] = \sum_i P[\bar{\mathbf{x}}|\bar{\mathbf{s}}_i|]$: sum (or integral) over all parameters characterizing family of signals

$$P[\bar{\mathbf{s}}_i|\bar{\mathbf{x}}] = \frac{P[\bar{\mathbf{x}}|\bar{\mathbf{s}}_i|]P[\bar{\mathbf{s}}_i|]}{P[\bar{\mathbf{x}}|0]P[0] + P[\bar{\mathbf{x}}|\bar{\mathbf{s}} \neq 0]P[\bar{\mathbf{s}} \neq \mathbf{0}]}$$

Ratio of probablities: likelihood

$$\Lambda[\bar{\mathbf{s}}_i] \equiv \frac{P[\bar{\mathbf{x}}|\bar{\mathbf{s}}_i]}{P[\bar{\mathbf{x}}|\mathbf{0}]}P[\bar{\mathbf{s}}_i]; \quad \Lambda[all\ \bar{\mathbf{s}}] = \sum_i \Lambda[\bar{\mathbf{s}}_i]$$

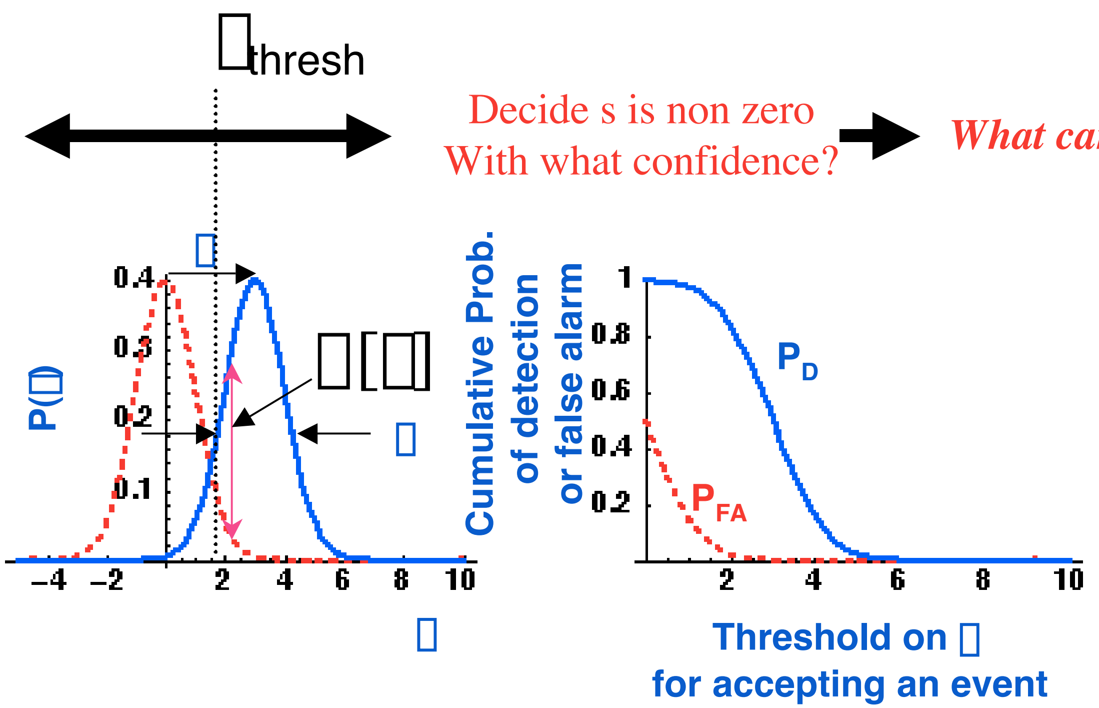
$$P[\bar{\mathbf{s}}_i|\bar{\mathbf{x}}] = \frac{P[\bar{\mathbf{x}}|\bar{\mathbf{s}}_i]P[\bar{\mathbf{s}}_i]}{P[\bar{\mathbf{x}}|\mathbf{0}]P[\mathbf{0}] + P[\bar{\mathbf{x}}|all\ \bar{\mathbf{s}}]P[all\ \bar{\mathbf{s}}]} = \frac{\Lambda[\bar{\mathbf{s}}_i]}{\Lambda[all\ \bar{\mathbf{s}}] + \dfrac{P[\mathbf{0}]}{P[\bar{\mathbf{s}}]}}$$

$$\frac{P[\bar{\mathbf{x}}|\bar{\mathbf{s}}_i]}{P[\bar{\mathbf{x}}|\mathbf{0}]} = e^{-\frac{1}{2}\left[(\bar{\mathbf{x}}-\bar{\mathbf{s}}_i)^{\mathbf{T}}\cdot\tilde{\mathbf{C}}^{-1}\cdot(\bar{\mathbf{x}}-\bar{\mathbf{s}}_i) - \bar{\mathbf{x}}^{\mathbf{T}}\cdot\tilde{\mathbf{C}}^{-1}\cdot\bar{\mathbf{x}}\right]} = e^{\left[\left(\bar{\mathbf{x}}^{\mathbf{T}}\cdot\tilde{\mathbf{C}}^{-1}\cdot\bar{\mathbf{s}}_i\right) - \frac{1}{2}\left(\bar{\mathbf{s}}_i^{\mathbf{T}}\cdot\tilde{\mathbf{C}}^{-1}\cdot\bar{\mathbf{s}}_i\right)\right]}$$

Detection:
False Alarm (P_{FA}) vs. Detection (P_D)

- x=n+s; <s>=□ ; □$_{□}$ =0; <n>=0 ; □$_n$ =□
- x=n; <n>=0 ; □$_n$ =□

Decide s = 0
With what confidence?



Decide s is non zero
With what confidence?

What can be said about <s>?

Searching for transient (burst) events

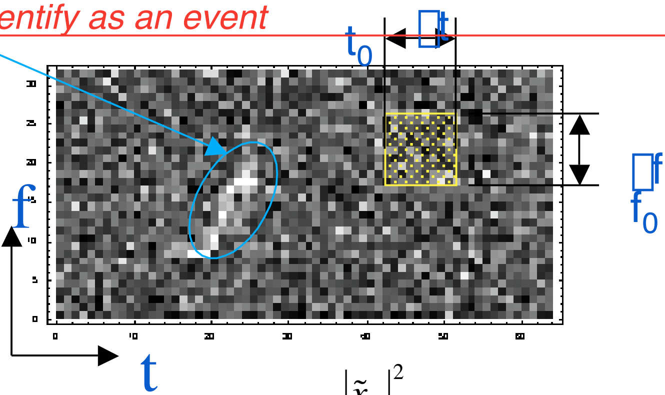
Burst group: http://www.ligo.caltech.edu/~ajw/bursts/bursts.html

Transient Signal Detection: optimization

- When ***s*** is a single, known waveform or parametrizable family:
  - *Neyman-Pearson lemma:threshold on likelihood ratio minimizes P(false dismissal) for any constraint on P(false alarm).*
    - *Inspiral templates, CW modulated signals, …*
- Optimality not well defined when ***s*** can take values in a subspace W (i.e. when H1 is a composite hypothesis):
  - *Bayesian: need to assume prior p(s), integrate likelihood over W, obtain Neyman-Pearson:*
    - Excess power (Anderson et al., gr-qc/0008066)
    - *Excess power for arbitrarily colored noise-- (Vicere,* LIGO-P010019*)*
  - *Average: minimize mean of P(false dismissal, s) over W, for a constraint on P(false alarm).*
    - Time domain filters -- slope detection (Virgo Orsay group, gr-qc/0010037)
  - *Minimax: minimize maximum of P(false dismissal, s) over W, for a constraint on P(false alarm).*
    - Tfclusters (J. Sylvestre, MIT, *http://www.ligo.caltech.edu/~ajw/bursts/bursts.html*)

Excess Noise Statistic

What if feature is not rectanglular? =>
do not fix shape, rather look for clusters of neighboring pixels all exceeding some statistic -> identify as an event



- Select a volume $\delta V_{ft} = \delta f \times \delta t$ in f-t plane and formulate a statistic to determine if the patch contains a signal, s.
- Background noise (Gaussian, locally white) has zero mean and variance σ_n.
  - **Frequency bins are independent <- ASSUMPTION!**
  - **Time bins are indpendent <- ASSUMPTION!**
- Sum of $N_V = 2\delta V_{ft}/(\delta f \delta t)$ random variables
  - *$\delta t = T/N_s$ of original data; $\delta f = 1/T$*
- Power statistic:

$$\mathcal{P} = \sum_{i,j} \frac{|\tilde{x}_{ij}|^2}{S_{ij}}; S_{ij} = \text{Noise power in pixel (i,j)}$$

$$P[\mathcal{P}|s=0,N_V] = e^{-\mathcal{P}/2}\left(\frac{\mathcal{P}}{2}\right)^{\frac{N_V}{2}-1};$$

$$P[\mathcal{P} > \mathcal{P}_{th}|s=0,N_V] = \frac{\Gamma\left(\frac{N_V}{2},\frac{\mathcal{P}_{th}}{2}\right)}{\Gamma\left(\frac{N_V}{2}\right)}; \Gamma(a,x) \equiv \int_x^\infty t^{a-1}e^{-t}dt$$

$$P[\mathcal{P}|s,N_V] = \frac{1}{2}e^{-(\mathcal{P}+s^2)/2}\left(\frac{\mathcal{P}}{s^2}\right)^{\frac{N_V-1}{2}} I_{N_V-1}\left(s\sqrt{\mathcal{P}}\right)$$

$$P[\mathcal{P} > \mathcal{P}_{th}|s,N_V] =; \int_{\mathcal{P}_{th}}^{\infty} P[\mathcal{P}|s,N_V]d\mathcal{P}$$

References:
Excess power statistic:
-W. Anderson et al., gr-qc/0001044
-W. Anderson et al., gr-qc/0008066

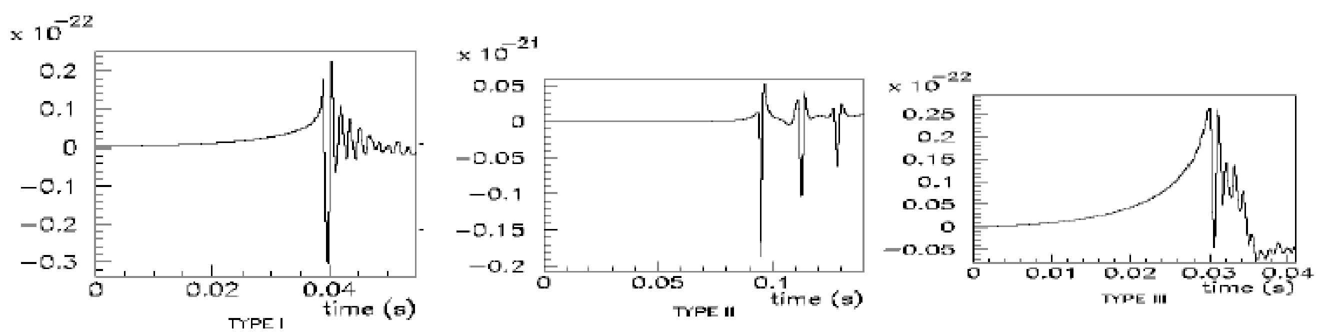
References:
t-f cluster analysis:
J. Sylvestre, MIT, thesis project LIGO-GG020028, LIGO -G010344, paper in process

Burst Searches

Excess Power Statistic (W. Anderson et al.)

- Search strategy is useful for signals where only general characteristics are known -- e.g. □t □ □f (bandwidth-time product)
  - *If one knows more, probably better to use some other method*
- Search assumes that all signals (of same □t □ □f volume) are equally likely
  - *Not true, since psd in signal space is not white*
  - *Need generalization to over-whitened data*

    Divide by psd



Burst Searches

Excess Power Statistic (W. Anderson et al.)

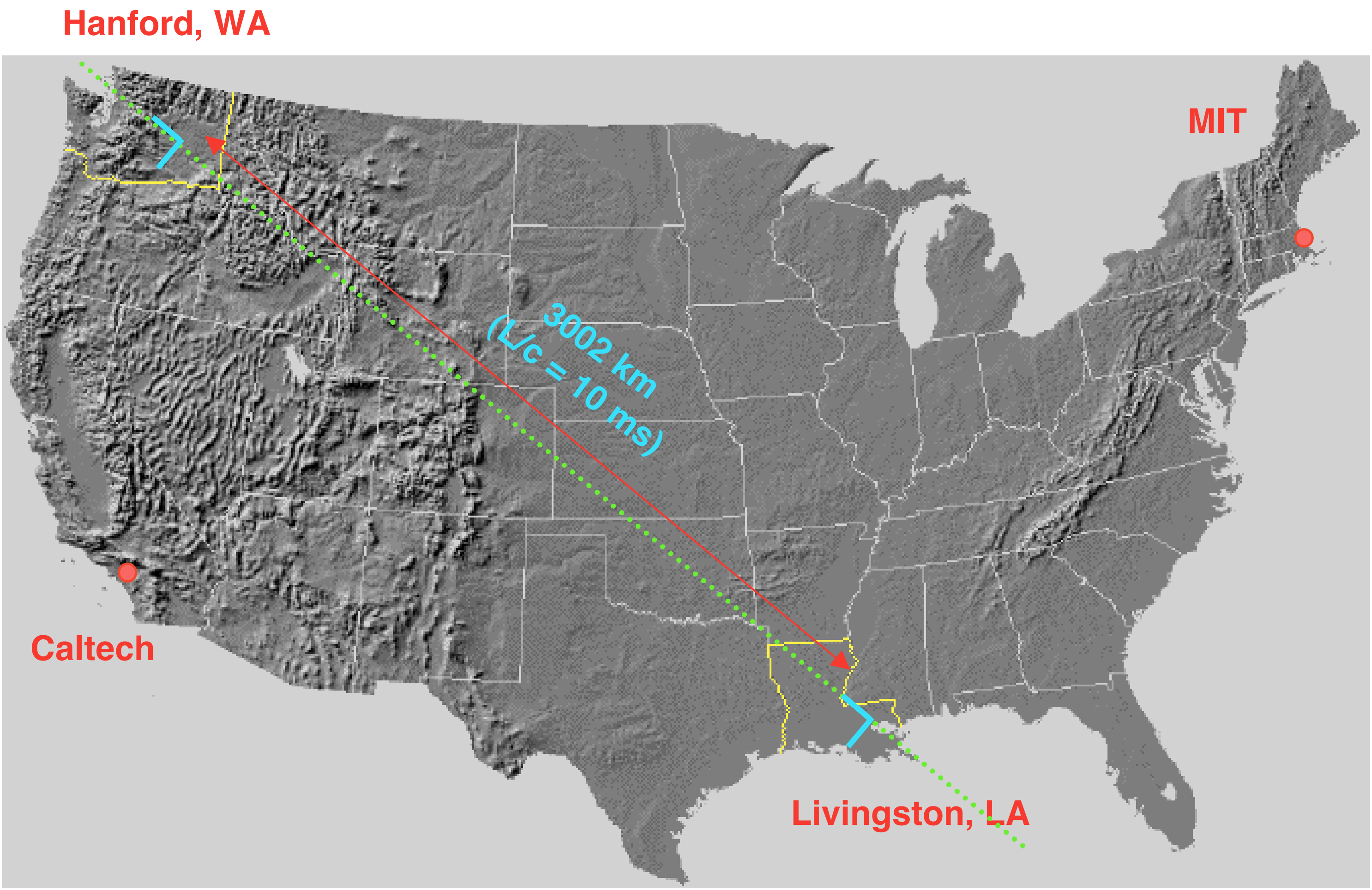
- The algorithm [1]:
  - *Pick a start time t_s, a time duration dt (containing N data samples), and a frequency band $[f_s; f_s + \Delta f]$.*
  - *Fast Fourier transform (FFT) the block of (time domain) detector data for the chosen duration and start time.*
  - *Sum the power in the frequency band $[f_s; f_s + \Delta f]$.*
  - *Calculate the probability of having obtained the summed power from Gaussian noise alone using a χ^2 distribution with $2 \times \Delta t \times \Delta f$ degrees of freedom.*
  - *If the probability is significantly small for noise alone, record a detection.*
  - *Repeat the process for all desired choices of start times t_s, durations Δt, starting frequencies f_s and bandwidths Δf.*

[1] A power filter for the detection of burst sources of gravitational radiation in interferometric detectors. Authors: Warren G. Anderson, Patrick R. Brady, Jolien D. E. Creighton, Eanna E. Flanagan. gr-qc/0001044

Use of multiple detectors in analysis

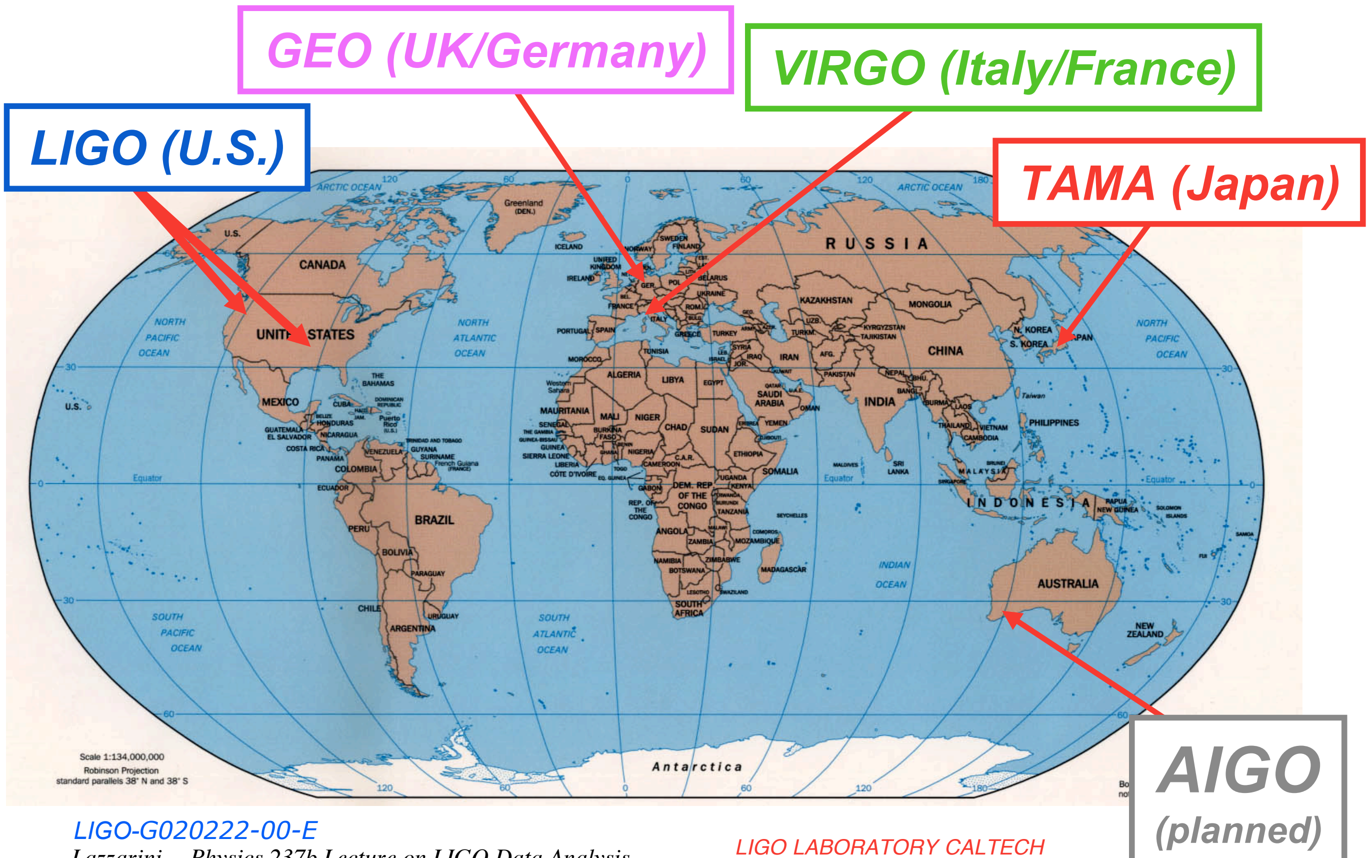
The LIGO Laboratory Sites

Interferometers are aligned along the great circle connecting the sites



International Network of Detectors

- A number of projects are bringing detectors on line during the next few years
- Operated as a phased array, they will augment the chances for detection by excluding backgrounds and localizing sources
- *True coincidences will be within milliseconds of each other*



- **detection confidence**
- **locate the sources**
- **decompose the polarization of gravitational waves**

Coincidence windows among detectors

- Rejection of statistically <u>uncorrelated</u> random events

Coincidence window duration determined by baselines, always less than 2*13000km/(300000km/s) = 0.086s

$$P_{n\neq 0}(\lambda_i;T_i) = 1 - e^{-\lambda_i T_i} \approx \text{ probability of at least one event in time } T_i$$

For $\lambda_i T_i << 1$, $P_{n\neq 0} \approx \lambda_i T_i$; if N detectors are statistically independent:

$$\lambda_{12...N} \approx \lambda_1 \prod_{i=2}^{N} \lambda_i T_{i1};$$

For N comparable detectors with comparable time windows

$$\frac{\lambda_{12...N}}{\lambda_1} \equiv \text{reduction of nonGaussian event noise by coincidence reqmnt.}$$

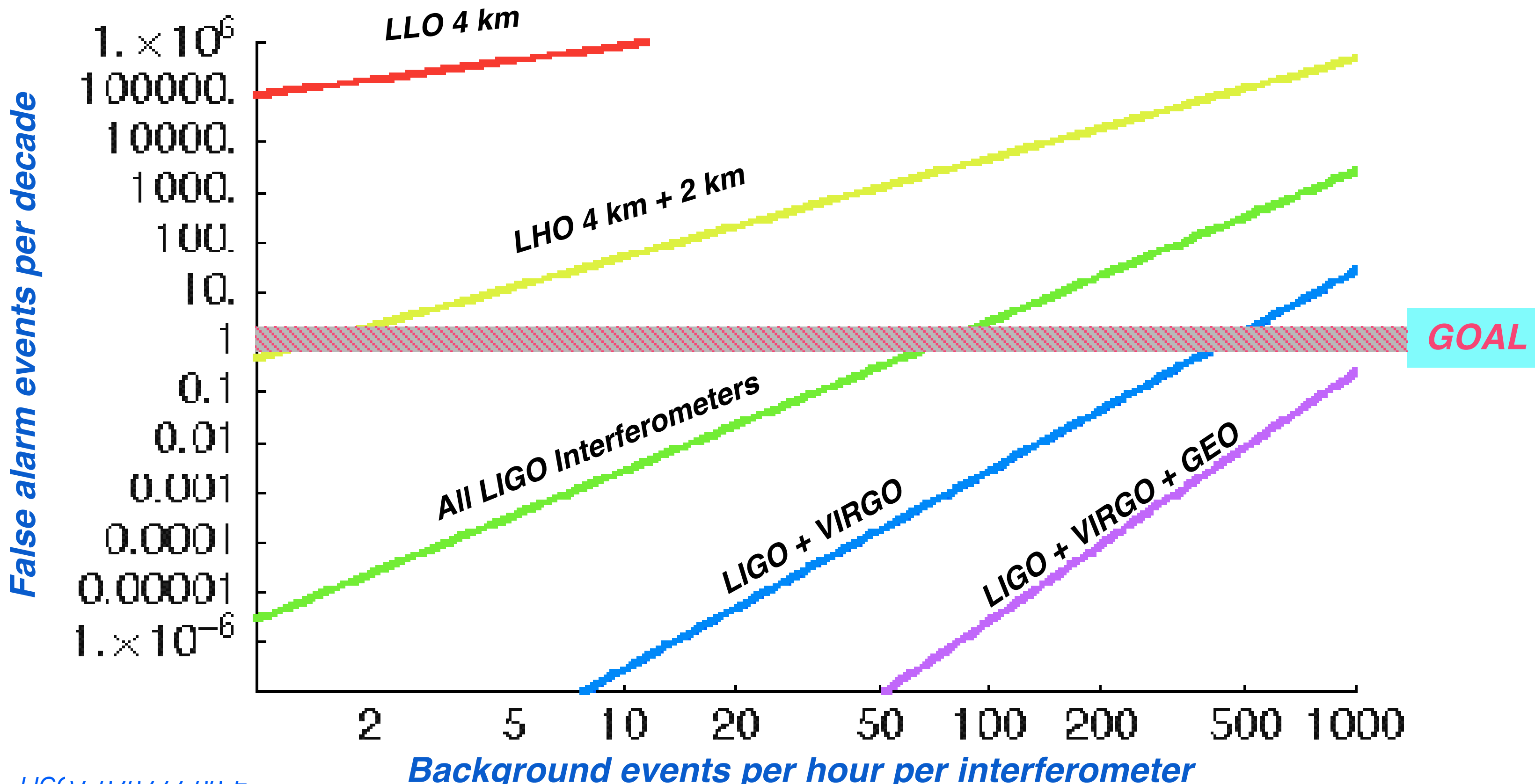
$$\approx \left[\lambda_1 T_{1j}\right]^{N-1} \text{and } \lambda_i T_i << 1$$

For λ= 1/min ,N=3 and T_{LIGO} =0.02s: rate reduction is 10^{-7}

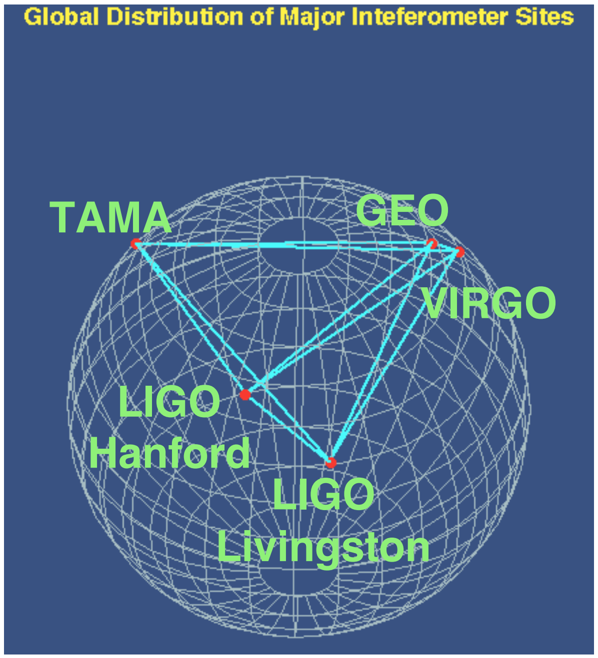
For λ= 1/min, N=4 and T_{12}=T_{23}= T_{LIGO} =0.02s and T_{34}=T_{max}=0.086s: rate reduction is 1.6×10^{-10}

Coincidence windows among detectors

- Rejection of statistically <u>uncorrelated</u> random events
- Two Sites - Three Interferometers
  - Single Interferometer - limited by non-gaussian noise ~70/hr
  - Hanford -- 2x coincidence requirement (**x1000 reduction**) ~1/day
  - Hanford + Livingston -- 3x coincidence (another **x5000 reduction**) **<0.1/yr**

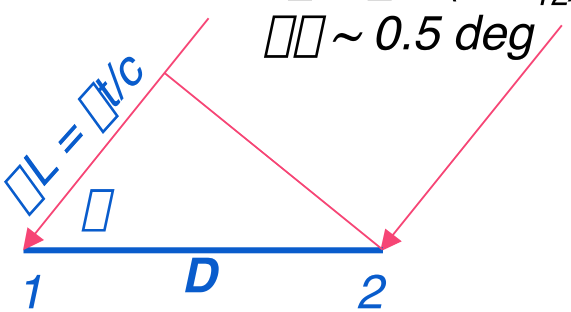


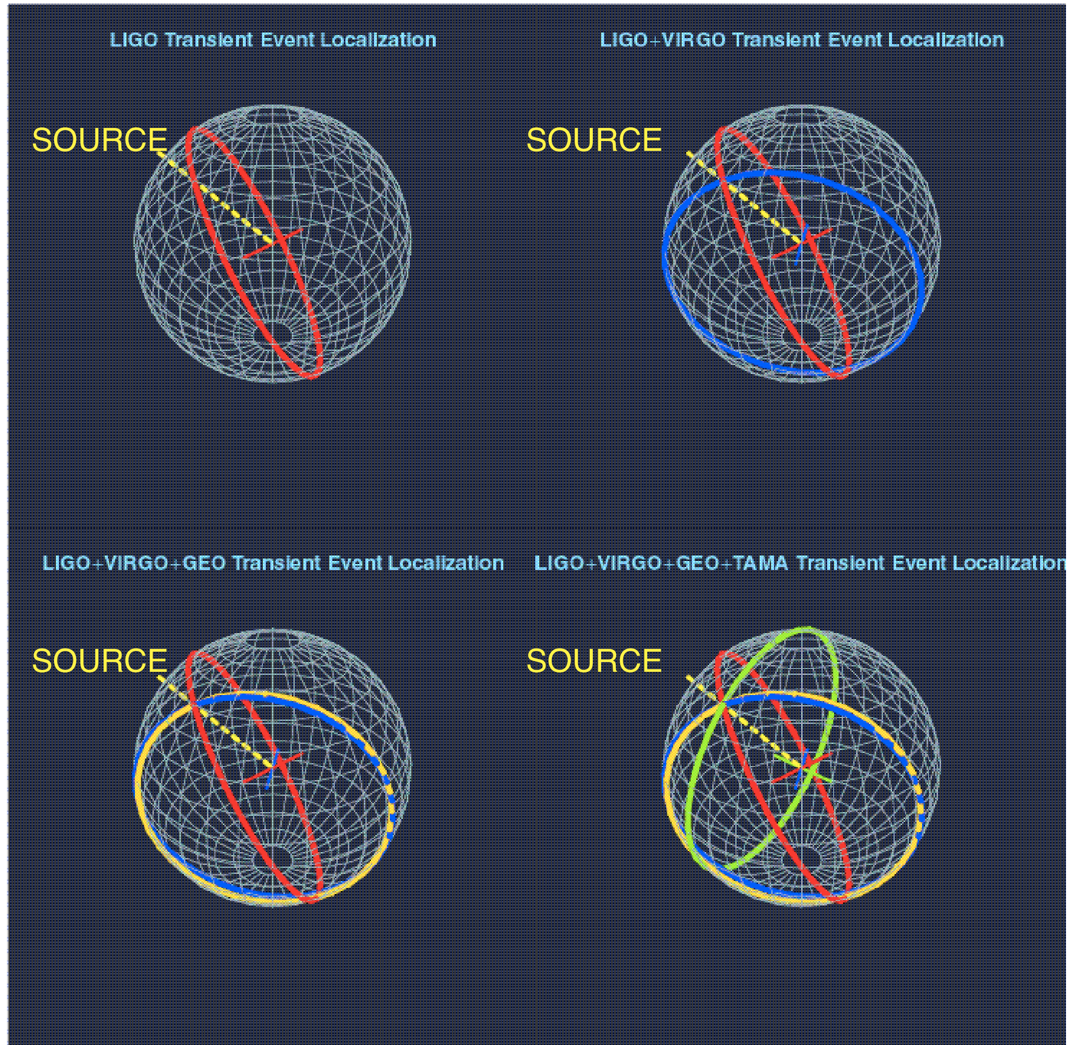
Event Localization With An Array of GW Interferometers



$\cos\square = \square t / (c\, D_{12})$

$\square\square \sim 0.5$ deg





Joint Data Analysis Among GW projects
From detection to validation

- For a *putative* detection:
  - Environmental, instrumental vetoes?
  - (Δt_i, $\Delta\Omega_i$): Seen by all detectors within consistent (time, position) windows?
  - δh_i: Is the amplitude of the signal consistent among detectors*?
  - $\Delta\lambda_i$: Are the deduced model parameters consistent?
- Follow up analyses
  - Independent
  - Coherent multi-detector analysis - maximum likelihood over all detectors: {t,Ω,h,λ}

$$h_i \to \vec{h}$$
$$\ln\Lambda(h_i,\lambda_i) \to \ln\Lambda(\vec{h},\lambda)$$
$$\sigma_i^2 \to C_{kl} \equiv \langle \vec{n}_k \; \vec{n}_l \rangle$$
$$\Lambda \propto \int \vec{h}^T \cdot C^{-1} \cdot \vec{T}\left(f; t_0 + \Delta t(\vec{\Omega})\right) df$$

- Discrepancies should be explainable, e.g.:
  - Not on line
  - Below noise floor
  - *Different polarization sensitivity, *etc.*

References:
L. S. Finn, gr-qc/0010033
S. Bose, gr-qc/0110041

FINIS