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Bicoherence Monitor

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Synopsis

- Introduction to Higher Order Statistics
  - 1D: Correlation, Coherence, Power Spectra
  - 2D: Bicorrelation, Bicoherence, Bispectrum
  - 3D …
- Bispectrum diagnostic
- Gaussianity Test
- Linearity Test

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What are Higher Order Statistics?

- 1D Statistics:
  - » Correlation: $C_{xy}(t) = \int_{-\infty}^{\infty} x(\tau)\, y(t+\tau)\, d\tau \quad \Leftrightarrow \quad X(f)\, Y^{*}(f) = S_{xy}(f)$
  - » Power Spectral Density: $C_{2x}(t) \quad \Leftrightarrow \quad X(f)\, X^{*}(f) = S_{2x}(f)$
  - » Coherence: $C_{xy}(f) = \dfrac{S_{xy}(f)}{\sqrt{S_{2x}(f)\, S_{2y}(f)}}$
    - Tells us power and phase coherence at a given frequency

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Second Order Statistics

- 2D Statistics:
  - » Bicumulant:

$$C_{xyz}(t,t') = \int_{-\infty}^{\infty} x(\tau)\, y(t+\tau)\, z(t'+\tau)\, d\tau \quad \Leftrightarrow \quad X(f_1)\, Y(f_2)\, Z^*(f_1+f_2) = S_{xyz}(f_1,f_2)$$

  - » Bispectral Density:

$$C_{3x}(t) \quad \Leftrightarrow \quad X(f_1)\, X(f_2)\, X^*(f_1+f_2) = S_{3x}(f_1,f_2)$$

  - » Bicoherence:

$$C_{xyz}(f) = \frac{S_{xyz}(f_1,f_2)}{\sqrt{S_{2x}(f_1)\, S_{2y}(f_2)\, S_{2z}(f_1,f_2)}}$$

    - Tells us power and phase coherence at a coupled frequency

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Zero-lag Cumulants

Mean	Variance	Skewness	Kurtosis
$C_x(0)$	$C_{2x}(0)$	$C_{3x}(0)$	$C_{4x}(0)$
		0 if Symmetric	0 if Gaussian

Useful statictical values, but…

- Skewness = 0 does not prove symmetry
- Kurtosis = 0 does not prove Gaussianity
- Variations in skew and kurtosis not well quantified.

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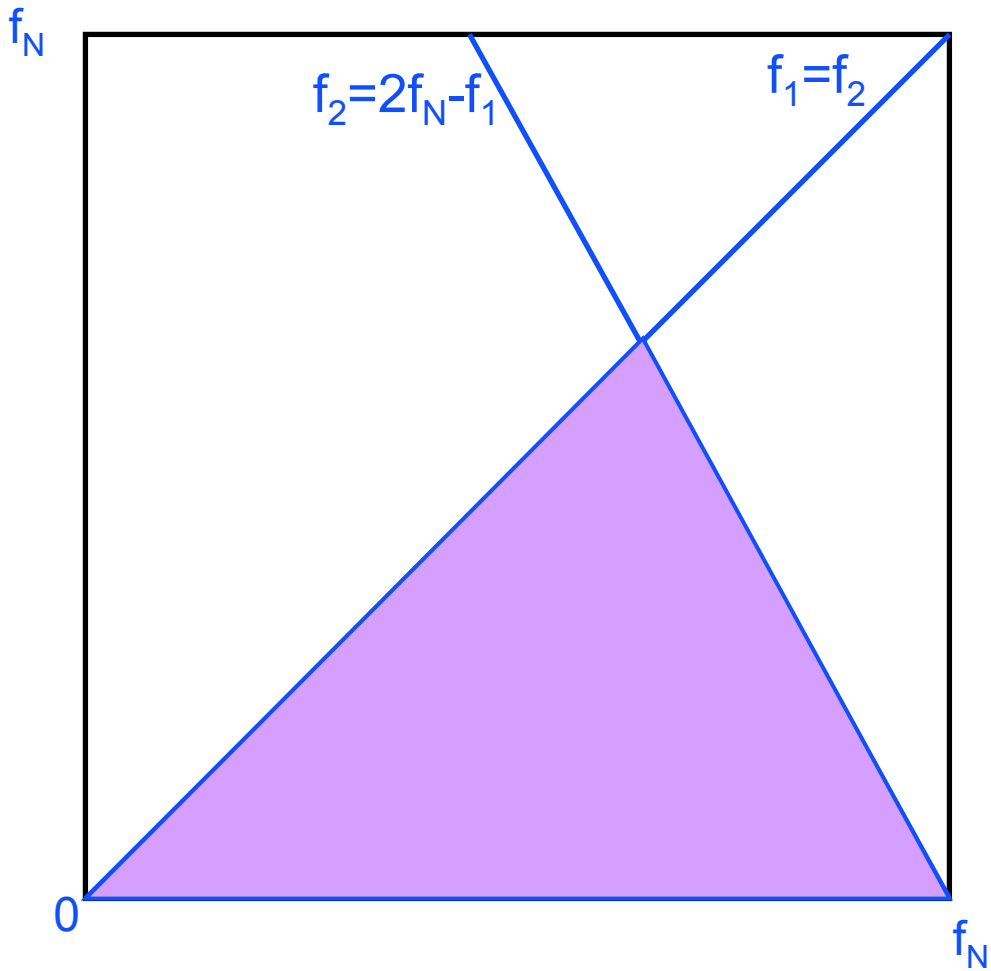
Why Higher Order Statistics?

- For a Gaussian process: $C_{nx}(t) = 0, \text{ for } n > 2$
- For independent processes:

$$z(t) = x(t) + y(t), \quad C_{nz}(t) = C_{nx}(t) + C_{ny}(t) \xrightarrow{n>2} C_{ny}(t)$$

- Allows for separation of Gaussian process for n>2
  - » Visual check of frequency coupling and phase noise
  - » Statistical test for the probability of gaussianity and linearity
  - » Iterative process to reconstruct nongaussian signal from the higher order cumulants

Bispectrum Unique Area



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Monitor Features

- Plots cross-bicoherence and cross-bispectrum (optional).
  - » Operates on 1–3 channels
  - » Automatically decimates to the lowest channel rate.
  - » Further decimation (by factor 2^n) user-specified (sets f_{max})
  - » Time span (by 2^n seconds) user-specified (sets Δf)
  - » User-specified Bispectrum method

Indirect $\qquad\qquad$ Direct

$$C_{xyz}(t,t') = \int_{-\infty}^{\infty} x(\tau)\, y(t+\tau)\, z(t'+\tau)\, d\tau \quad \Leftrightarrow \quad X(f_1)\, Y(f_2)\, Z^*(f_1+f_2) = S_{xyz}(f_1, f_2)$$

  - » Windowing: Optimized Rao-Gabr windowing
  - » Outputs EPS files of the plots
  - » New, stable Help facility

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Improvements since last LSC

- USERS! Thanks to Nelson Christiansen and Dennis Ugolini!
- User-selectable features listed on previous slide
- **SPEED!** Major code rewrite for speed.
  - » Old version: 2k, unwindowed channels were almost real-time.
  - » New version: 16k, windowed channels ARE real-time!
  - » ==> speed improvement factor of a few hundred!
- GUI – in progress but not quite finished yet. Sorry.
- Gaussianity Test: same as above.

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How to use XBic

Example:

- `cd /home/spenn`
- `./XBic --help`
- `./XBic L0:IOO-PSL1_MIC L2:LSC-AS_Q -direct -MaxFrame 15`
- `./XBic L2:LSC-AS_Q -direct -MaxFrame 15`