Current Status of PN Computations of Binary Inspiral

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Most Recent Review:
Luc Blanchet,
Living Reviews in Relativity,
gr-qc 0202016

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** FYI - This document will be updated/appended in the next week.*

GW from ICB

- Computations of GW from ICB requires control of 3 independent modules

  1. Motion

  2. Generation

  3. Radiation Reaction

MOTION

- Status of PN EOM *satisfactory*
  Agreement between different approaches and techniques

- 2.5PN

  Damour, Deruelle: Harmonic coords, Riesz regularisation

  Schafer : ADM, Hadamard partie finie

  Kopeijkin and Grischuk:
  Physical computation using self gravitating extended bodies

  Blanchet, Faye and Ponsot:
  Direct PN iteration, Matching

  Itoh, Futamase and Asada: Variant of surface integral approach of EIH

- 3PN

  Jaranowski, Schafer and Damour:
  ADM coords, Hadamard regularisation, EOM has an arbitrary parameter ω_{static}

  Blanchet, Faye and Andrade:
  Harmonic coords, (extended) Hadamard regularisation, EOM has an arbitrary parameter λ

$$\lambda = -\frac{3}{11}\omega_{static} - \frac{1987}{3080}$$

- The undetermined constant reflects the incompleteness of the Hadamard regularisation

- Hadamard regularisation does not satisfy distributivity of products $(FG)_1 \neq (F)_1(G)_1$. Violates Leibniz rule for differentiation of a product

- Dimensional Regularisation preserves the gauge symmetry of perturbative GR underlying the link between Bianchi identities and EOM and hence respects ALL basic properties of algebraic and differential calculus of ordinary functions

- Damour, Jaranowski and Schafer: Dimensional regularisation gives $\omega_{static} = 0$ so that $\lambda = -\frac{1987}{3080} = -.645..$

- 3PN EOM and ALL conserved quantities available for General Orbits

WORK IN PROGRESS

- Computation without regularisation: Calculate 3PN EOM for extended bodies taking into account internal structure (pressure, density..) and then take its limit as 'radius' goes to zero. Compare with the point mass regularised result

  2PN: Kopeijkin and Grischuk implemented this and showed *effacement* of internal structure

  3PN: Can one determine λ???
  Is this consistent with $\omega_{static} = 0$
  (Blanchet, Esposito-Farese, Poujade)

- Can one compute EOM in harmonic coords using dimensional regularisation and determine λ?

- Upto now one has been discussing the conservative motion of the binary

- The radiative part of the EOM is available only upto leading order (2.5PN)

- Deriving the full *relative* 3PN/3.5PN radiation reaction ie *absolute* 5.5PN/6PN contributions is impossible with present technology

- Thus we move to the second module

GENERATION

- Apply wave generation formula to compute the work done by radiation reaction force i.e. total energy flux at null infinity

  Computation of *Source* multipole moments I_L and J_L

  Determination and control of *Tails* and non-linear effects relating source moments to *Radiative* moments

- 2PN
  Blanchet, Damour, BRI, Will and Wiseman
  BDI - Multipolar Post Minkowskian method, Hadamard/Riesz self-field regularisation
  WW - Direct Integration of Relaxed Einstein (DIRE) ; Epstein-Wagoner-Thorne + *retarded integral*

- Mathematical Equivalence of Both approaches (Blanchet)

- 3PN Instantaneous part
  Blanchet, BRI, Joguet; Circular Orbits;
  Harmonic coords + Hadamard regularn of infinite self-field
  General orbits (In Progress: Blanchet, BRI)

- Hereditary part : Blanchet
  Tails : 1.5PN, 2.5PN, 3.5PN
  Tails of Tails, $(\text{Tail})^2$: 3PN

- 3 undetermined constants in the the Mass Quadrupole combine to a single undetermined constant θ in GW Luminosity in addition to the λ coming from EOM

DETERMINATION of θ??

- Can θ be computed by an Extended Body Computation?? (Blanchet, BRI..)

- Can we formulate the wave generation in ADM coords?? 2PN , Tails, ????

- Can self-fields in harmonic coordinates be controlled by the Dimensional Regularn in the generation problem??
  Need to first discuss EOM in Harmonic coordinates with Dimensional Regularn
  Setting up the *entire* MPM generation formalism in d dimensions seems non-trivial
  Rotation group in higher dimensions,
  Propagator in higher dimensions,
  Backscattering/Tails..
  Can one be smart enough to apply Dimensional regularisation *only where required* without setting up the whole edifice???

ECCENTRIC BINARIES

- Will and Wiseman, Gopakumar and BRI
  2PN Energy Flux, Waveform

- Gopakumar and BRI
  AM Flux, Evolution of orbital elements, GW polarisations without inspiral but 2PN accurate periastron precession

- GW polarisations with RR
  (In progress: Damour, Gopakumar, BRI)

RADIATION REACTION

- We assume a energy balance equation

$$\frac{dE}{dt} = \mathcal{L}$$

- Though *physically obvious*, no general proof from first principles of GR of the correctness of the above balance eqn beyond 1PN/1.5PN

- Blanchet, Faye, BRI, Joguet
  3.5PN GW Phasing

SPINNING BINARIES

- Spins affect the GWF in several ways

  Cause precession of Orbital plane of binary; changes its orientation wrt observer leading to modulation of shape

  Modifies Amplitude of GWF

  Affect GW Luminosity

  Contribute to Orbital decay and hence Accumulated phase of GWF

- Assume spins unaffected by GW Radn damping:

- Circular orbits with SS exist only if orbital plane and spins are constant over a orbit

- EOM

$$\mathbf{a} \equiv \ddot{\mathbf{x}} = \mathbf{a}_{SO}^{(1.5)} + \mathbf{a}_{SS}^{(2)} + \mathbf{a}_{SO}^{(2.5)} + \mathbf{a}_{RR-SO}^{(4)} \,,$$

  Papapetrou
  Kidder, Will and Wiseman
  Tagoshi, Ohashi and Owen
  Gopakumar and Will (In Progress)

- Spin Precession

$$\dot{\mathbf{S}_1} \sim \frac{1}{r^3}\{(\mathbf{L_N} \times S_1), (S_1 \times S_2)\}$$

  Barker, O'Connell
  Apostolatos, Kidder:
  Effect of Precession on WF

- Conserved Energy

$$E = E_{SO}^{1.5} + E_{SS}^{2} \,,$$

  Kidder, Will and Wiseman

- GW Luminosity

$$(\frac{dE}{dt})_{FZ} = \dot{E}^{1.5}_{SO} + \dot{E}^{2}_{SS} + \dot{E}^{2.5}_{BH} \,,$$

Kidder, Will and Wiseman
$S_1.S_2$ terms computed not S_i^2
$\frac{dJ}{dt}$ also computed ; SS term not complete
Alvi (Absorption/emission of energy by horizon for BH)

- GW Polarisations

$$(h_{+,\times})_{\text{inst}} = H^{(1)}_{SO} + H^{(3/2)}_{SS} + H^{(2)}_{SO} \,,$$

Kidder
Owen, Tagoshi and Ohashi
(non-precessional 2PN SO; No associated RR)

- Orbital Freq evoln and accumulated orbital phase with SO and SS
  Kidder, Will and Wiseman

- EM tensor: Fluid stress energy tensor:
  Kidder, Will and Wiseman; Gopakumar and Will
  δ-fn based on Dixon:
  Mino Shibata and Tanaka; Owen, Tagoshi and Ohashi
  δ-fn of Bailey and Israel:
  Cho; Tagoshi, Ohashi and Owen

- Complications of Spin supplementary conditions

TEST PARTICLE RESULTS

- Review on BH Perturbation: Mino, Sasaki, Shibata, Tagoshi and Tanaka

- In the Test particle limit GW are computed as Linear perturbations about curved backgrounds of Schwarzschild and Kerr

- Schwarzschild BH background

  Conserved energy is exactly known

  Exact GW luminosity : Numerically

  GW luminosity for Circular orbits,
  analytically to 5.5PN order
  Tanaka, Tagoshi and Sasaki

  Slightly eccentric orbits $\mathcal{O}(e^2)$ corrections;
  4PN Energy and AM fluxes

- Kerr BH background

  Circular orbits in Equatorial plane; 4PN
  Tagoshi, Shibata, Tanaka and Sasaki

  Slightly eccentric orbits on Eq. plane; 2.5PN; Tagoshi

  Circular orbits with small incination to Eq. plane; 2.5PN Energy and AM fluxes
  Shibata, Sasaki, Tagoshi and Tanaka

  Spinning particles around Kerr; 2.5PN

- BH absorption; $\mathcal{O}(v^8)$ relative to leading quadrupole for Schwarzschild $\mathcal{O}(v^5)$ relative to leading quadrupole for Kerr

- Provide the best tests for the PN computations

BEYOND

- The PN expansion is very slowly convergent

- The convergence may be improved by Resummation methods like Pade approximants

- Effective one body method is a very efficient way to investigate the conservative motion of the binary

- The early inspiral is well modelled by the adiabatic approximation

- A combination of Resummation methods and EOB is necessary to go beyond the adiabatic approximation and discuss late inspiral, plunge and subsequent merger since BBH are the most likely sources for LIGO

- See Talks of Buonanno and Grandclement

- GWDA using 3PN EOB and sensitivity of the overlaps to flexibility parameters and 3PN unknown parameters
  (Damour, BRI, Jaranowski and Sathyaprakash - In Progress)