An Overview of LIGO Length Sensing and Control

Peter Shawhan
(LIGO Lab / Caltech)

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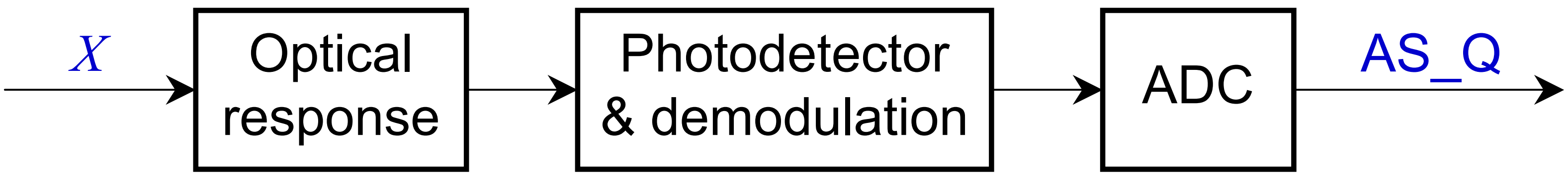
LIGO-G020064-00-D

Length Sensing

Interferometer response to an externally-induced mirror displacement X, neglecting servo feedback



Neglecting: anti-alias filter; whitening filter pair

Fabry-Perot cavity introduces frequency dependence

$$\frac{\text{AS\_Q}}{X} = \frac{C_o}{1 + i\,(f / f_c)} \equiv C(f)$$

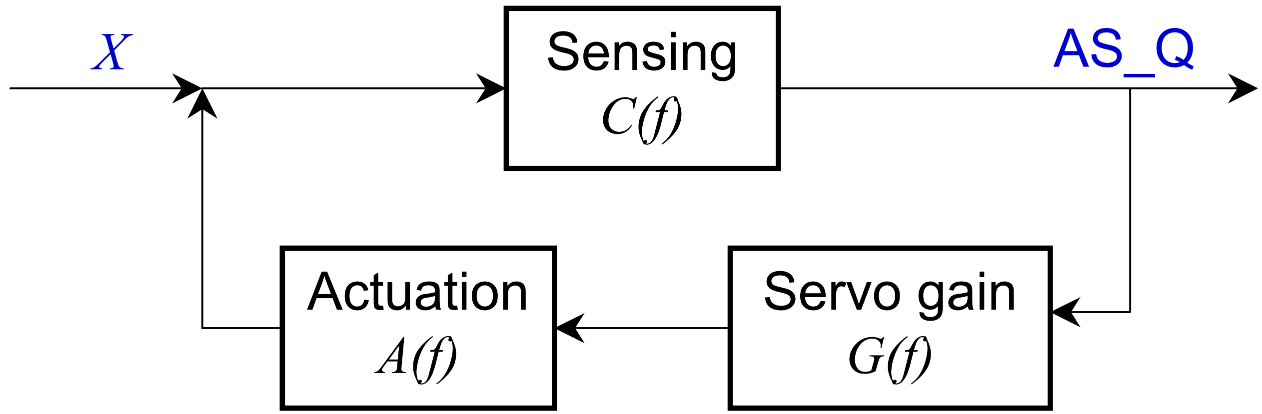
(units: AS_Q counts per meter)

Cavity pole frequency f_c depends on cavity length & finesse

Nominally 180 Hz for 2km, 90 Hz for 4km

Determine C_o by shaking mirror and measuring AS_Q signal

Effect of Servo Feedback



Loop gain modifies transfer function

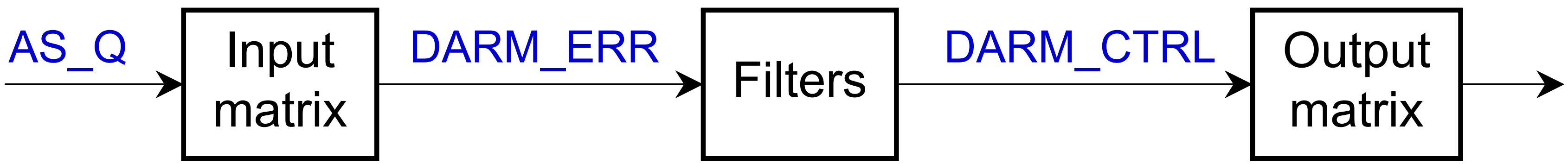
$$\frac{\text{AS\_Q}}{X} = \frac{C}{1 - G\,A\,C}$$

Loop gain is >>1 at low frequencies, <<1 at high frequencies

Unity gain frequency is within LIGO's sensitive band ➔
Servo has substantial effect on response function for astrophys. analyses

Servo Gain *G(f)*

Overall servo gain is the product of several operations



- Input and output matrices are frequency-independent
- Filters are digital ➔ completely deterministic

Aside: AS_Q and DARM_CTRL are coherent, except for small (?) “off-diagonal” terms in input matrix

***G(f)* has a rather complicated frequency dependence**

- Can be described as a set of poles and zeros

Can either model *G(f)* or measure it empirically *(see later)*

Actuation Transfer Function $A(f)$

Neglecting: dewhitening filter pair; analog filtering in coil driver

Relates an electronic signal to absolute mirror displacement

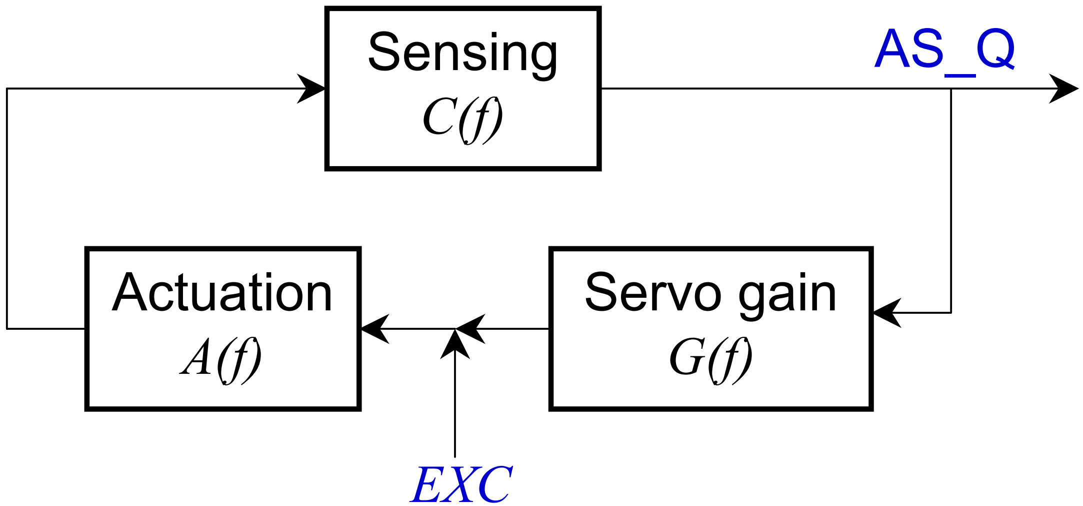
Pendulum response introduces frequency dependence

$$A(f) = \frac{A_o}{1-(f/f_p)^2 + i\,(f/f_p)/Q} \approx \frac{-A_o\, f_p^2}{f^2}$$

f_p ~ 0.75 Hz, Q ~ 10

Determine A_o by moving a mirror with the servo disabled and observing interference fringes

Response of AS_Q to a Calibration Excitation



$$\frac{\text{AS\_Q}}{\text{EXC}} = \frac{A\,C}{1 - G\,A\,C}$$

To get the response function to an externally-induced displacement, just divide by $A(f)$

A swept-sine excitation traces out the full transfer function

Notes

There are some other complications

Filters neglected in this discussion need to be characterized/modeled

Response may include an absolute time delay

Response function can be represented as poles & zeros

Mathematically true as long as G, A, & C have pole/zero representations

Some of these will be complex-valued, in general

A swept-sine calibration yields a frequency series; have to fit this (with some choice of functional form) to get a pole/zero representation

Summary

The amplitude and phase of the response to a gravitational wave have nontrivial frequency dependence

Even without the servo, the cavity pole introduces frequency dependence

The servo has a significant effect on the response function, but it can be modeled or simply measured with a swept-sine

The actuation transfer function has an absolute scale factor which needs to be measured

The calibration procedure has become better understood since the E7 run

Techniques for determining A_o

Understanding / modeling $G(f)$

Techniques for fitting swept-sine data