LIGO: The Laser Interferometer Gravitational-wave Observatory

General Relativity...

Einstein...

A new window...

Astrophysics...

Gravitational Waves and the R-modes

Gregory Mendell

LIGO Hanford Observatory

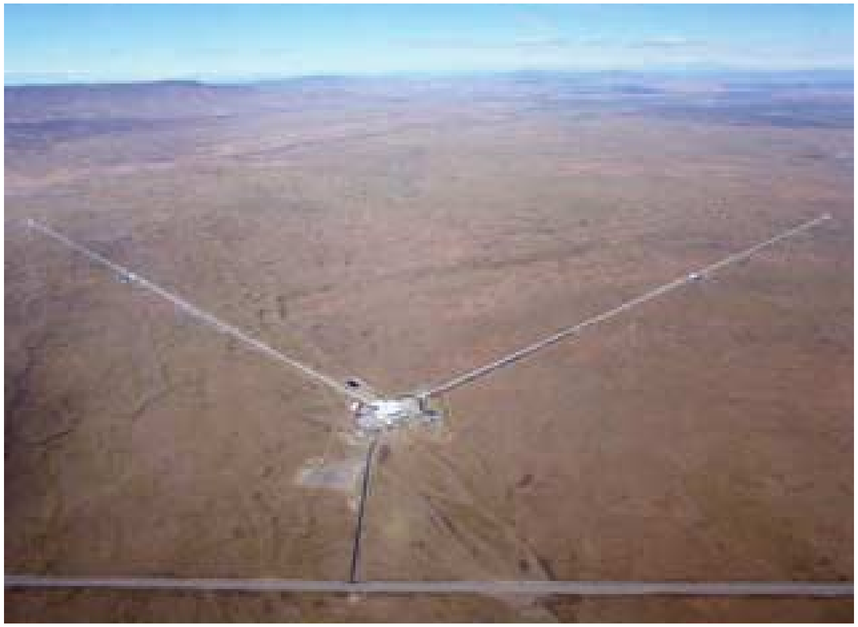
Who’s Involved?

Caltech, MIT, and the LIGO Science Collaboration

Institution	Heads	FTE	Heads	FTE	Heads	FTE
ACIGA	21	13.5	0	0.0	21	13.5
Caltech - CACR	3	0.7	3	0.7	0	0.0
Caltech - CaRT	6	3.1	1	0.4	1	2.7
Caltech - CEGG	2	1.6	1	0.3	2	1.3
Cal. State Dominguez Hills	5	4.6	5	4.6	0	0.0
Carleton University	1	0.8	1	0.8	0	0.0
Cornell	3	2.6	3	2.6	0	0.0
GEO 600	58	47.0	49	30.4	32	16.6
Harvard-Smithsonian	2	1.3	2	1.3	0	0.0
Inst. of Applied Physics - Russia	11	7.0	0	0.0	11	7.0
Inter-University Centre for Astronomy and Astrophysics (India)	5	2.2	5	2.2	0	0.0
Iowa State University	1	0.5	0	0.0	1	0.5
JILA (Univ. of Colorado)	5	1.5	0	0.0	5	1.5
Louisiana Tech	4	1.2	4	1.2	0	0.0
LSU	10	5.5	9	4.0	6	1.5
Moscow State University	9	9.0	0	0.0	9	9.0
NAOJ - TAMA	5	2.0	0	0.0	5	2.0
Oregon University	7	4.1	7	4.1	0	0.0
Penn State	14	13.3	10	8.6	6	4.7
Southern Univ/A&M Colledge	4	1.5	0	0.0	4	1.5
Stanford University	18	11.2	0	0.0	18	11.2
Syracuse University	5	5.0	2	1.0	5	4.0
University of Florida	16	14.0	16	11.6	6	2.4
University of Michigan	4	2.8	4	2.8	0	0.0
University of Texas - Brownsville	4	2.5	4	2.5	0	0.0
University of Wisconsin-Milwaukee	8	5.3	8	5.3	0	0.0
Total: Non-LIGO Laboratory	229	163.8	134	84.4	132	79.4

Sponsored by the National Science Foundation

The Observatories



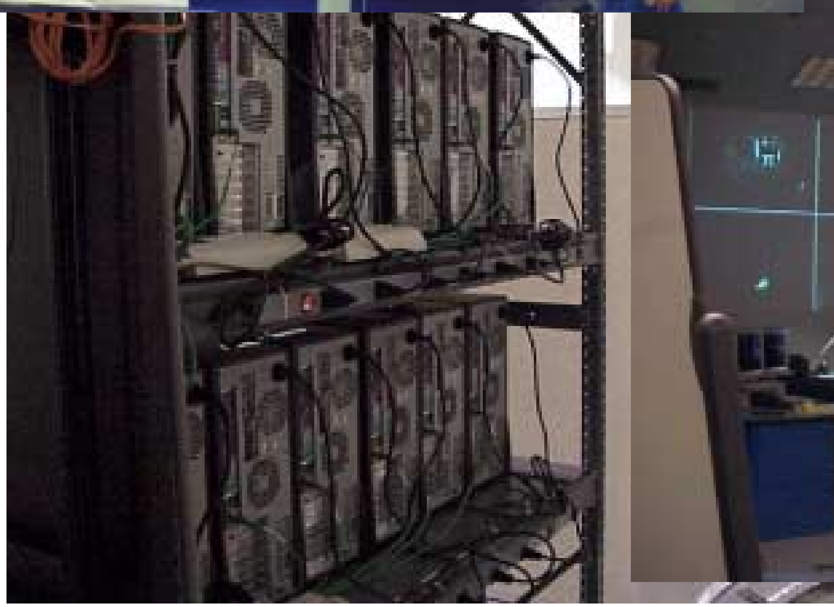
LIGO Hanford

LIGO Livingston

Photos: http:// www.ligo.caltech.edu; http://www.ligo-la.caltech.edu

LIGO

Inside



Gravitational Waves

- Gravitation = spacetime curvature described by the metric tensor: $ds^2 = g_{\mu\nu} dx^\mu dx^\nu$
- Weak Field Limit: $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$

$$\left(\nabla^2 - \frac{1}{c^2} \frac{\partial^2}{\partial t^2} \right) \bar{h}^{\mu\nu} = 0$$

- TT Gauge:

$$h_{\mu\nu}^{TT} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & h_+ & h_\times & 0 \\ 0 & h_\times & -h_+ & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} e^{2\pi i f t - i k z}$$

Gravitational-wave Strain

$$c\Delta t = 2\int_0^L \sqrt{1+h_+}\,dx \cong 2L + h_+ L = 2L + \Delta L$$

$$h = \Delta L / L$$

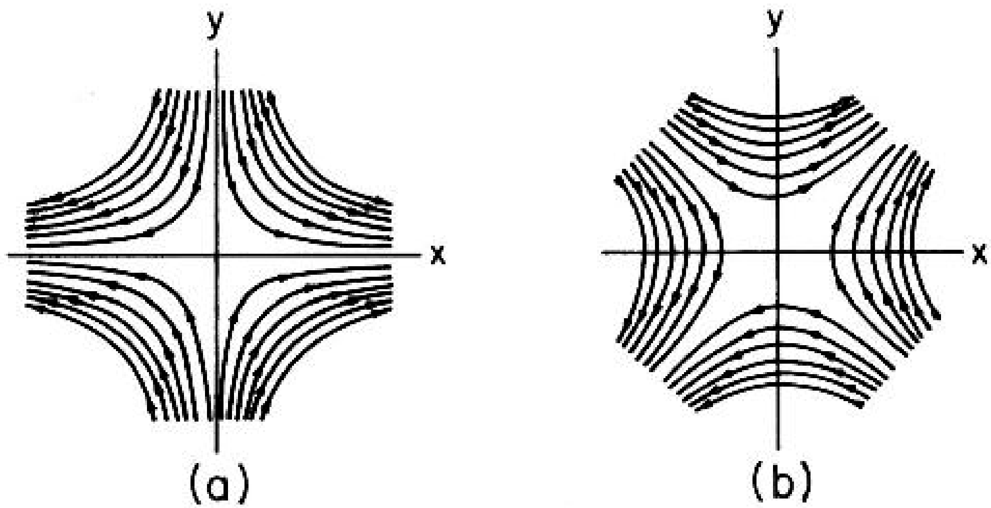
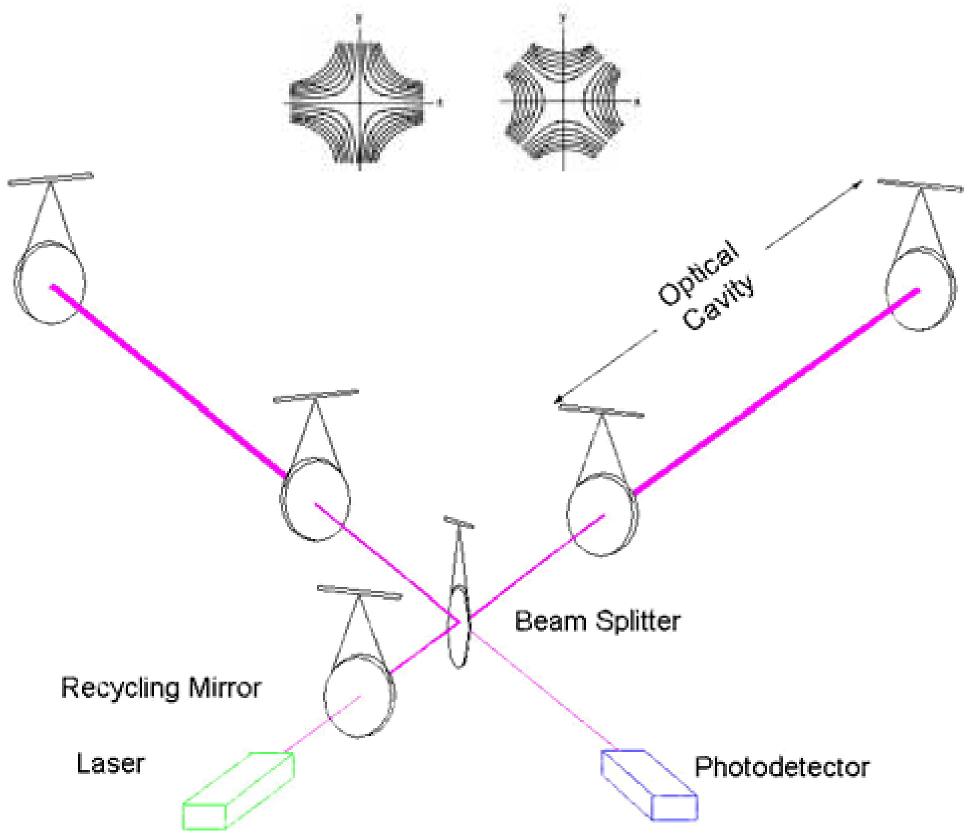


Figure 1. Direction of space deformation for a gravitational wave propagating along the z-axis, $+$ polarization (a) and $\times$ polarization (b).

D. Sigg LIGO-P980007-00-D

How Does LIGO Work?

Gravitational-wave Strain: $h = \Delta L / L$



LIGO is a lab looking for GW's.

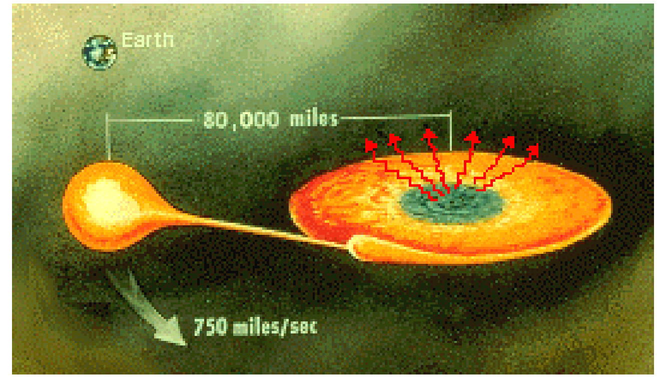
LIGO is an "ear" on the universe, listening for cosmic spacetime vibrations.

Figures: K. S. Thorne gr-qc/9704042; D. Sigg LIGO-P980007-00-D

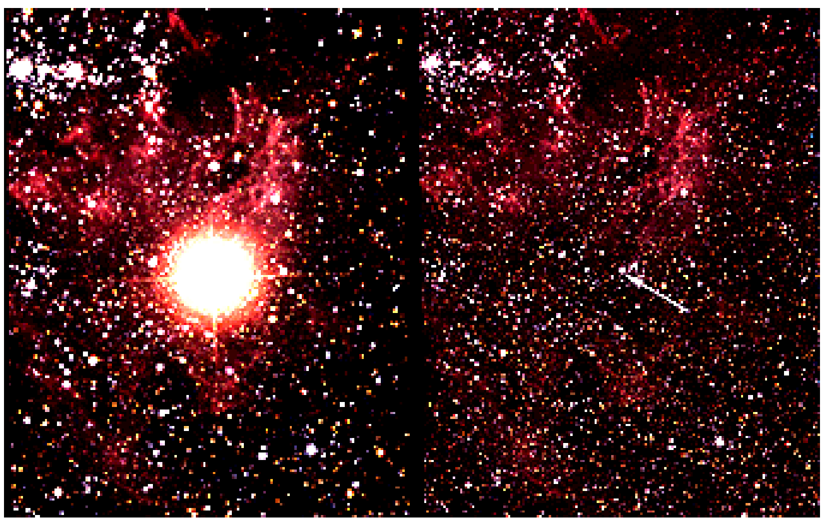
Black Holes

Pulsars

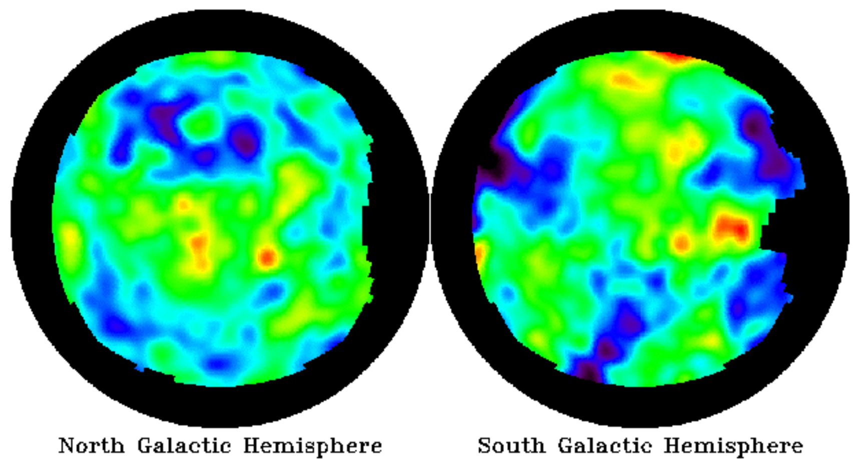
LMXBs



Astrophysical Sources



Supernovae



Stochastic Background

Photos: http://antwrp.gsfc.nasa.gov; http://imagine.gsfc.nasa.gov

$$h = \Delta L / L \approx (G / c^4)(\ddot{Q} / r)$$ "Newtonian" quadrupole formula.

- **Stochastic** (limit Ω_{GW}; cosmic strings; BH from massive pop III stars: $h = 10^{-23} - 10^{-21}$)

- **Burst** (SN at distance of Virgo Cluster: $h = 10^{-23} - 10^{-21}$; rate = 1/yr)

- **Inspiral** ($h_{max} = 10^{-22}$ for NS-NS@ 200 Mpc; rate = 3/yr; NS-BH; BH-BH)

- **Periodic** ($h = 10^{-25}$ for 10 ms pulsar with maximum ellipticity at 1 Kpc; $h = 10^{-27}$ for 2 ms LMXB in equilibrium at 1 Kpc)

Reviews: K. S. Thorne 100 Yrs of Gravitation; P. R. Saulson, Fund. of Interferometric GW Detectors

Noise Curves

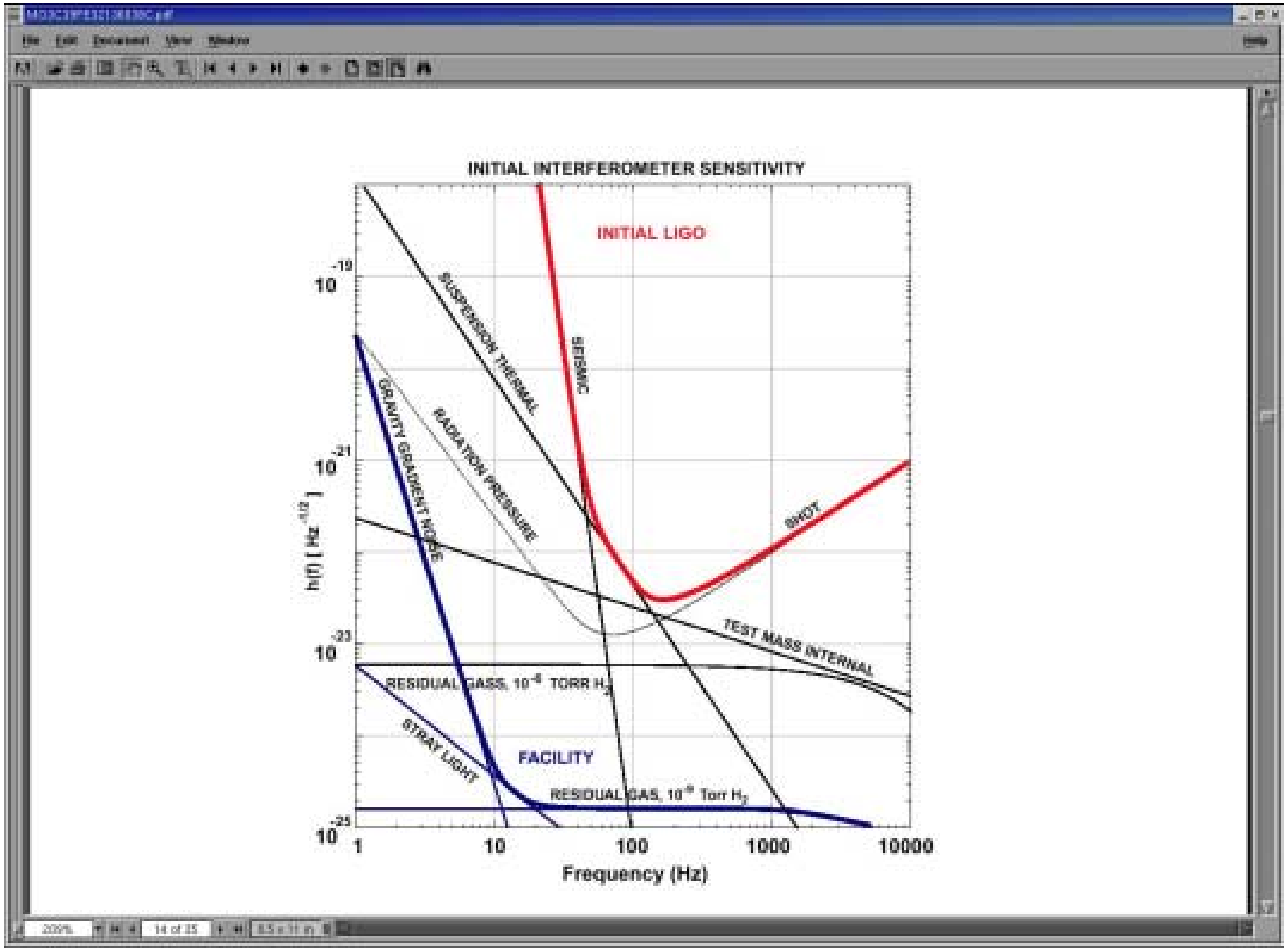


Figure: D. Sigg LIGO-P980007-00-D

Signal to Noise Ratio

$$\frac{S}{N} \cong \frac{h\sqrt{T}}{\sqrt{< n^2(f_c) >}}$$

- **h = signal amplitude**
- **T = observation time or duration of signal or period of the characteristic frequency of the signal.**
- **n^2 = power spectrum of the noise**

Sensitivity Curves

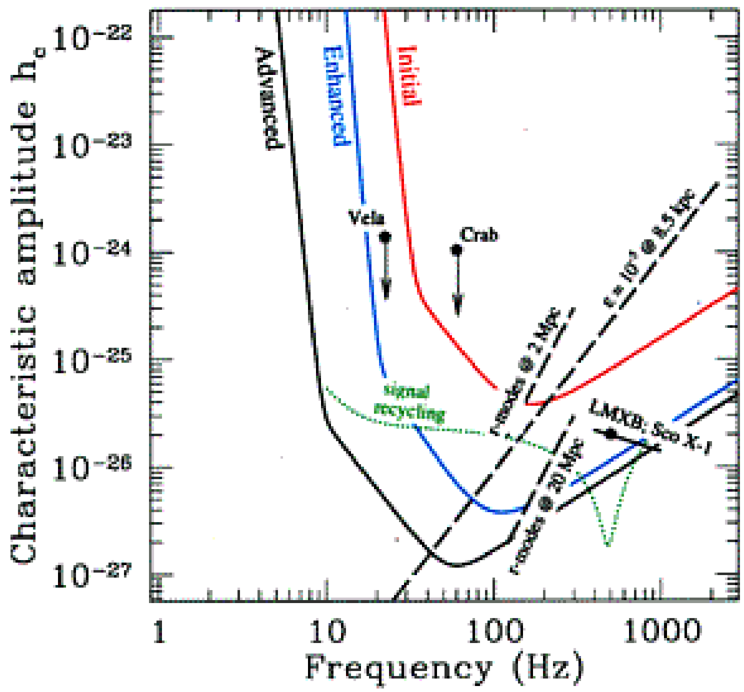
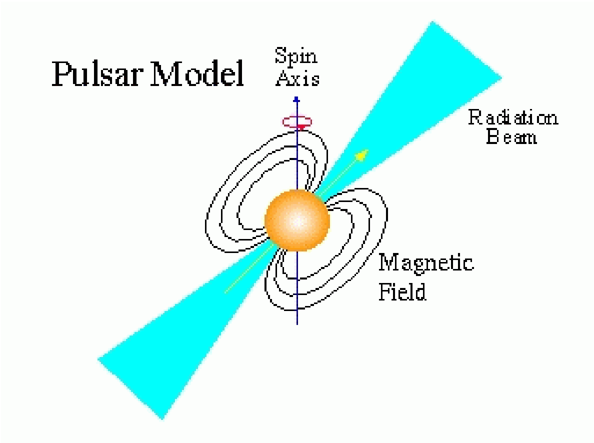
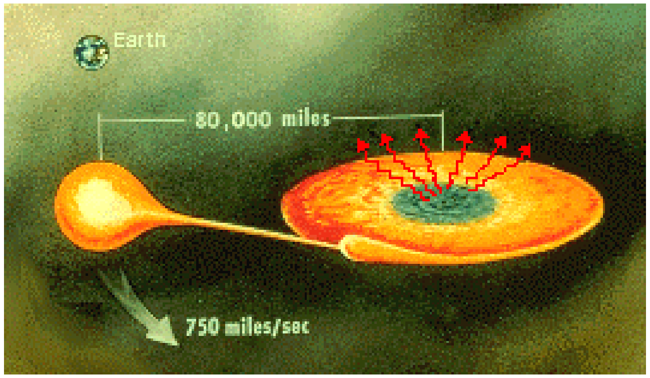


Figure: Brady ITP seminar summer 2000

Known Possible Periodic Sources

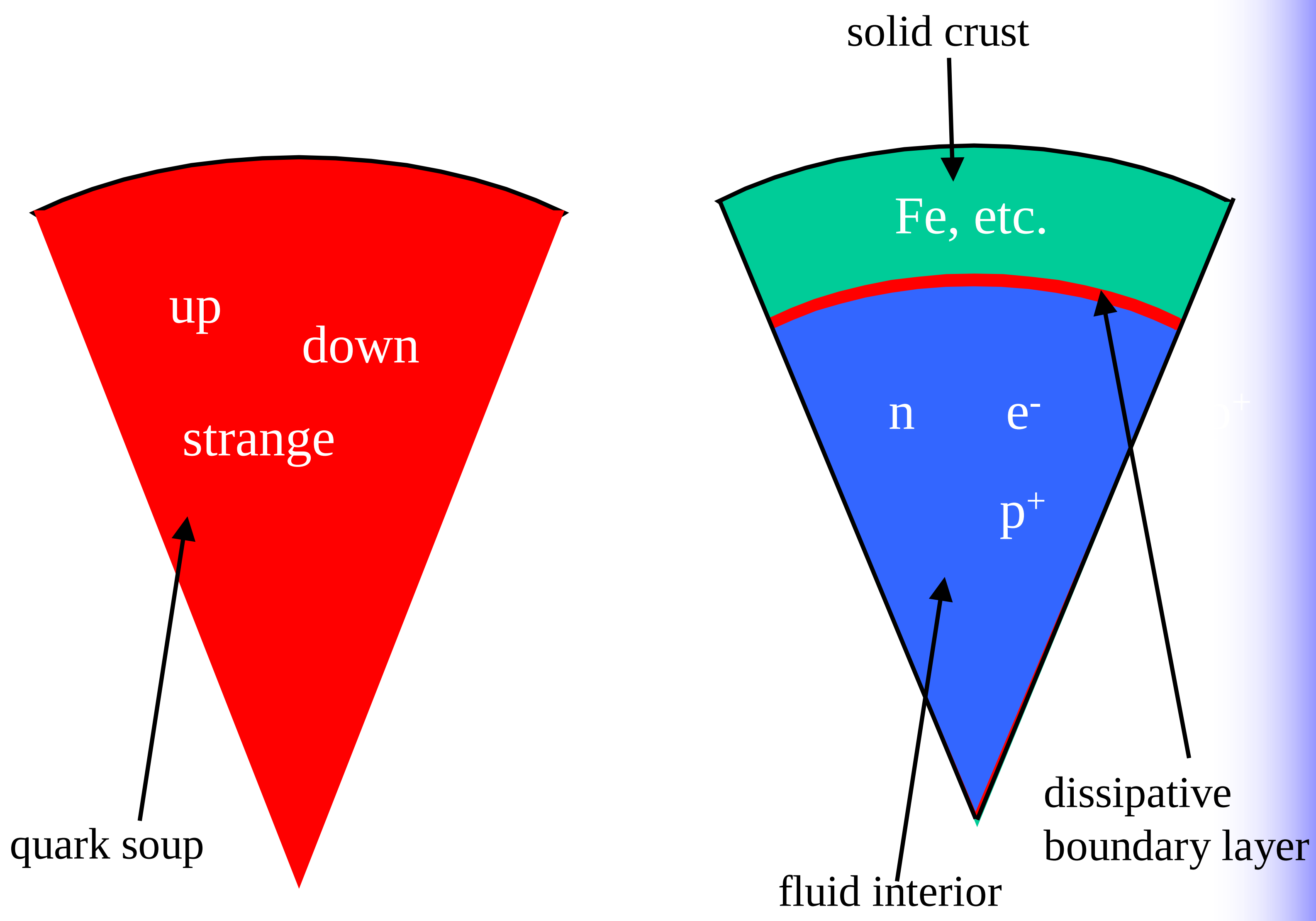




LMXBs

- Are neutron stars: the sun compress to size of city. Compact ($2GM/Rc^2 \sim .2$) and ultra dense (10^{14} g/cm^3).
- Are composed of (superfluid) neutrons, (superconducting) protons, electrons, + exotic particles (e.g., hyperons) or strange stars composed of an even more exotic up, down, and strange quark soup.
- Spin Rapidly (~ .1 Hz to 642 Hz i.e., within the LIGO band.)

Neutron (Strange) Star Models



Courtesy Justin Kinney

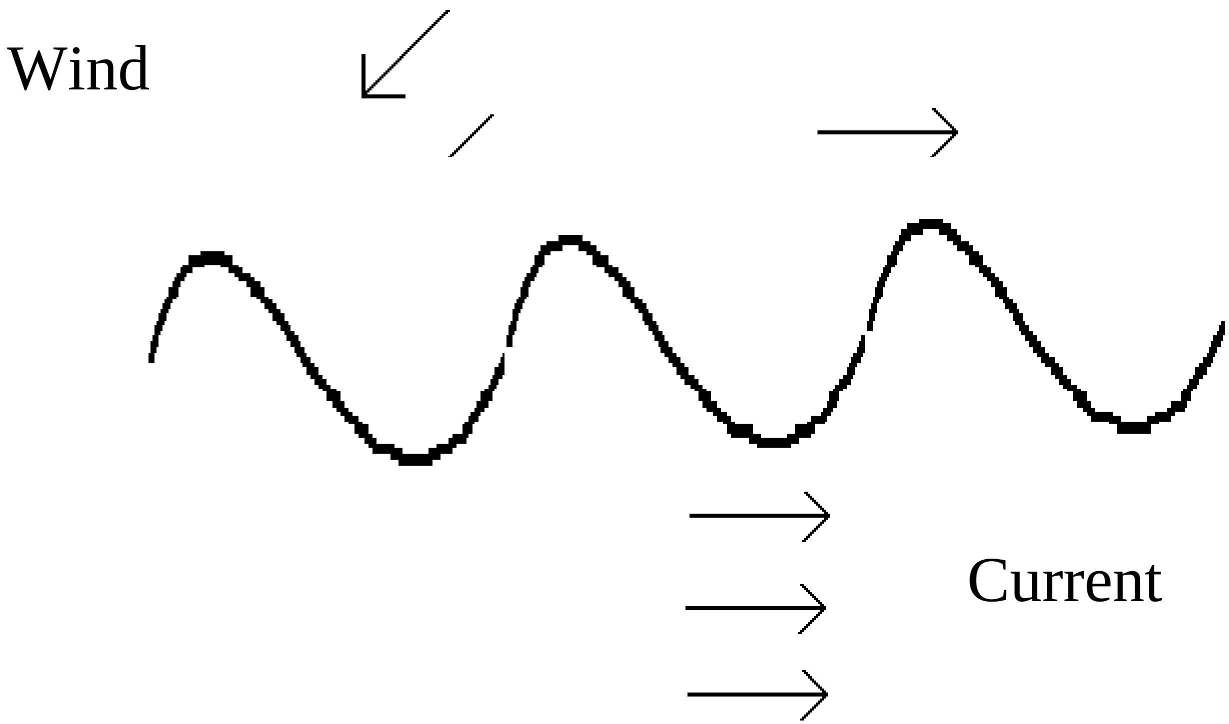
Periodic sources emit GWs due to…

- Rotation about nonsymmetry axis
- Strain induced asymmetry: $\varepsilon = \frac{I_1 - I_2}{I}$
- Accretion induced emission
- Unstable oscillation modes

Gravitational-radiation Driven Instability of Rotating Stars

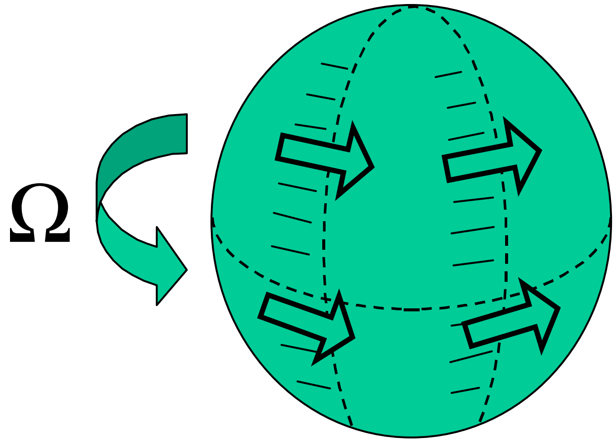
- GR tends to drive all rotating stars unstable!
- Internal dissipation suppresses the instability in all but very compact stars.

Ocean Wave Instability

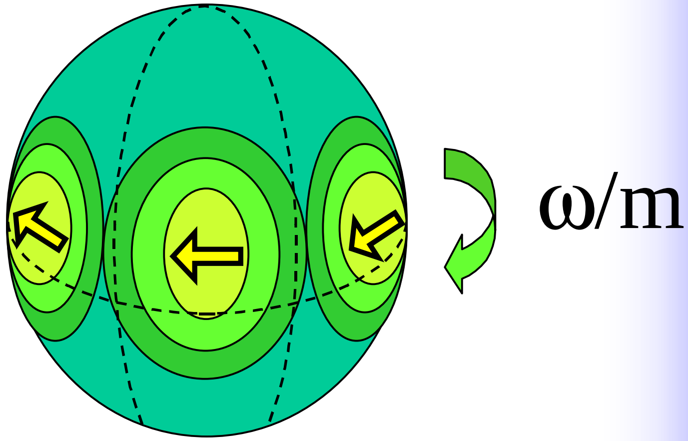


Perturbations in Rotating Neutron Stars

- Neutron star rotates with angular velocity $\Omega > 0$.
- Some type of "wave" perturbation flows in opposite direction with phase velocity ω/m, as seen in rotating frame of star.
- Perturbations create rotating mass and momentum multipoles, which emit GR.



Rotating neutron star.

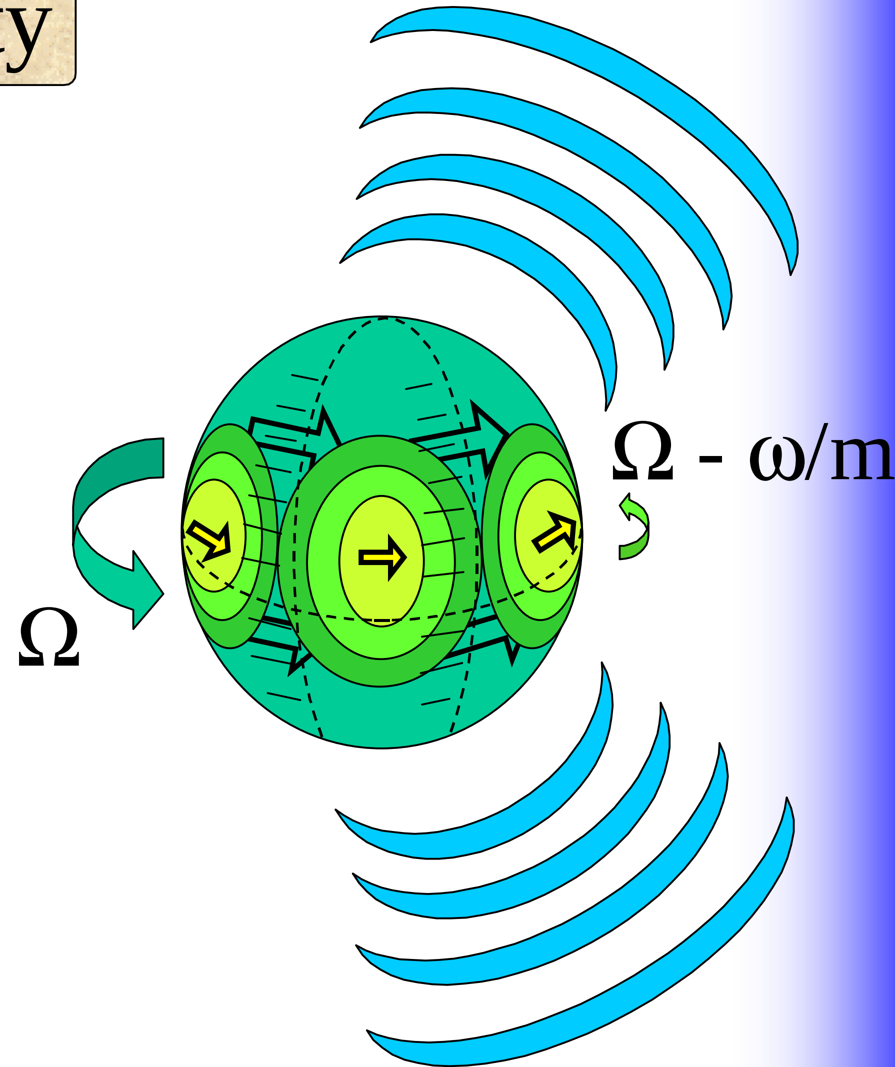


Perturbations in rotating frame.

Courtesy Justin Kinney

GR Causes Instability

- If $\Omega - \omega/m > 0$, star "drags" perturbations in opposite direction.
- GR caries away positive angular momentum.
- This adds negative angular momentum to the perturbations.
- This *increases* their amplitude!



Star drags perturbations in opposite direction. GR drives mode instability.

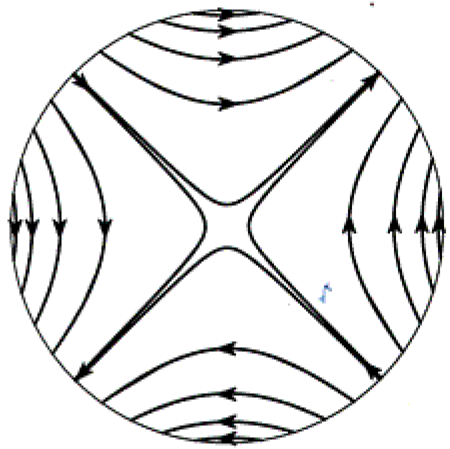
Courtesy Justin Kinney

The R-modes

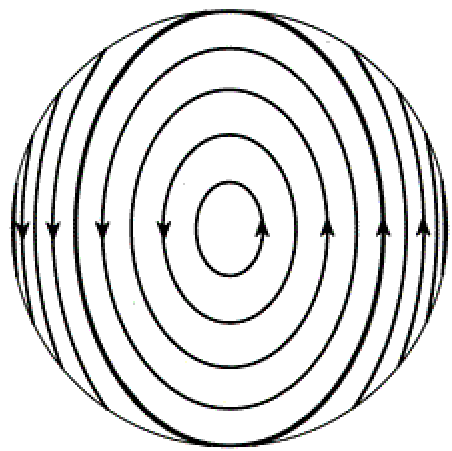
- The r-modes corresponds to oscillating flows of material (currents) in the star that arise due to the Coriolis effect. The r-mode frequency is proportional to the angular velocity, Ω .
- The current pattern travels in the azimuthal direction around the star as exp$(i\omega t + im\varphi)$
- For the $m = 2$ r-mode:
  - Phase velocity in the corotating frame: $-1/3\ \Omega$
  - Phase velocity in the inertial frame: $+2/3\ \Omega$

Flow Pattern for the $m = 2$ r-mode

Polar View

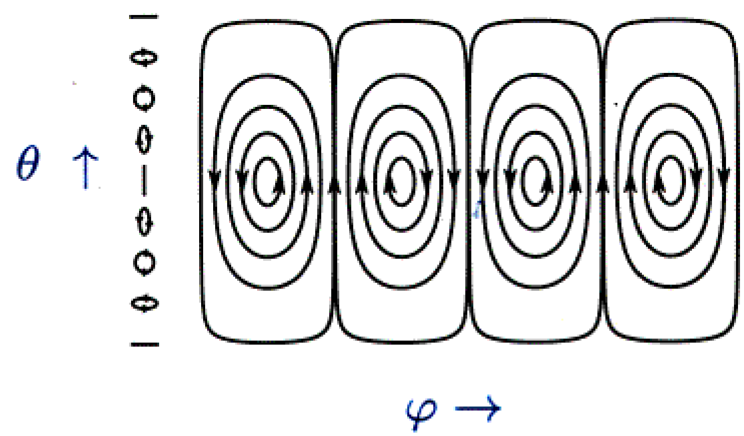


Equatorial View



Courtesy Lee Lindblom

Fluid Motion in the $m = 2$ r-mode



- The flow pattern is shown along with the small elliptical paths (on the left) of individual fluid elements. The flow pattern moves (to the left) past the fluid particles as the mode evolves.

Courtesy Lee Lindblom

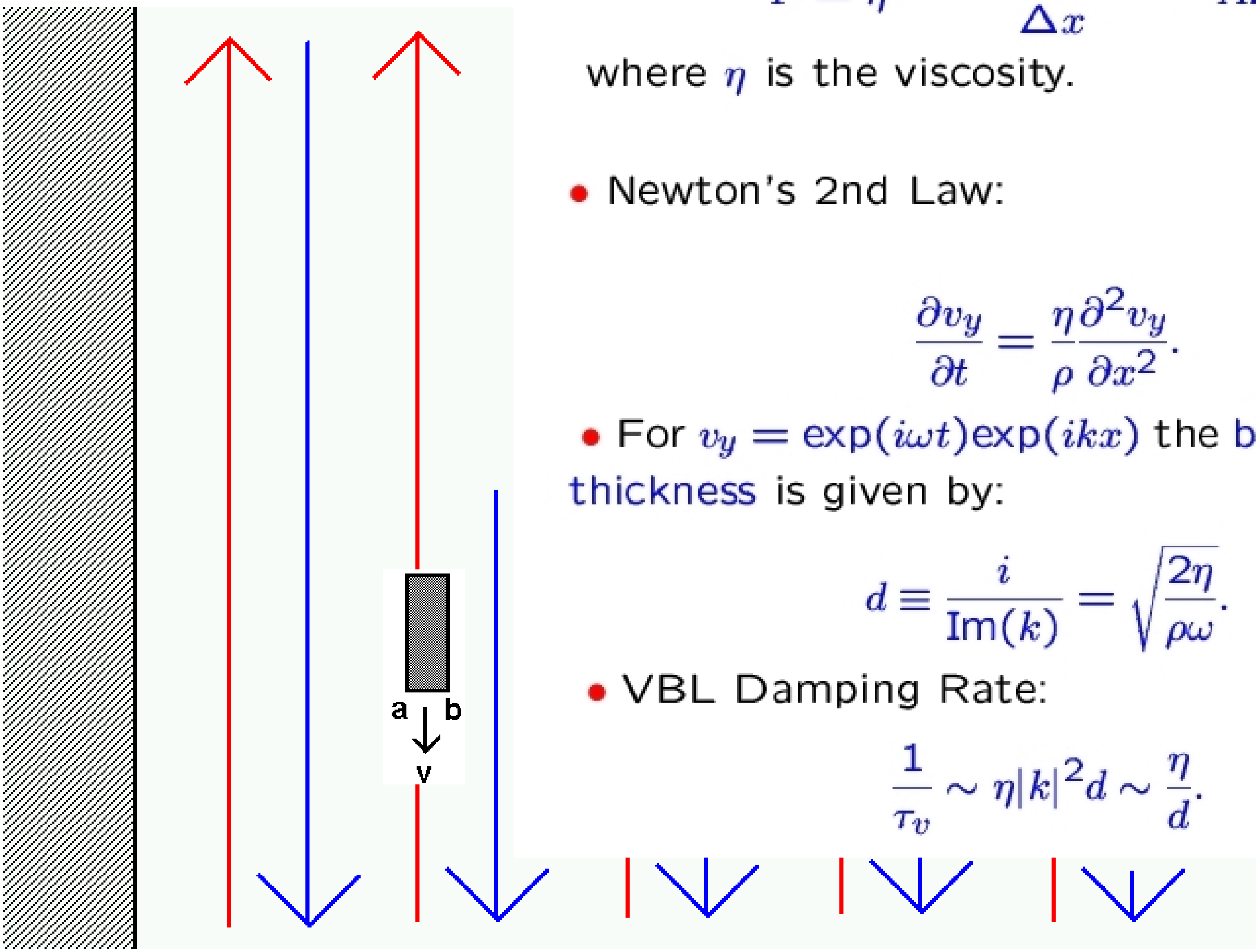
R-mode Instability Calculations

- Gravitation radiation tends to make the r-modes grow on a time scale τ_{GR}
- Internal friction (e.g., viscosity) in the star tends to damp the r-modes on a time scale τ_F
- The shorter time scale wins:
  - $\tau_{\mathbf{GR}} < \tau_{\mathbf{F}}$: Unstable!
  - $\tau_{\mathbf{GR}} > \tau_{\mathbf{F}}$: Stable!

Key Parameters to Understanding the R-mode Instability

- Critical angular velocity for the onset of the instability
- Saturation amplitude

LIGO



- The Shear Force:

$$F = \eta \frac{\left(\frac{\Delta v_y}{\Delta x}\right)_b - \left(\frac{\Delta v_y}{\Delta x}\right)_a}{\Delta x} A \Delta x,$$

where η is the viscosity.

- Newton's 2nd Law:

$$\frac{\partial v_y}{\partial t} = \frac{\eta}{\rho} \frac{\partial^2 v_y}{\partial x^2}.$$

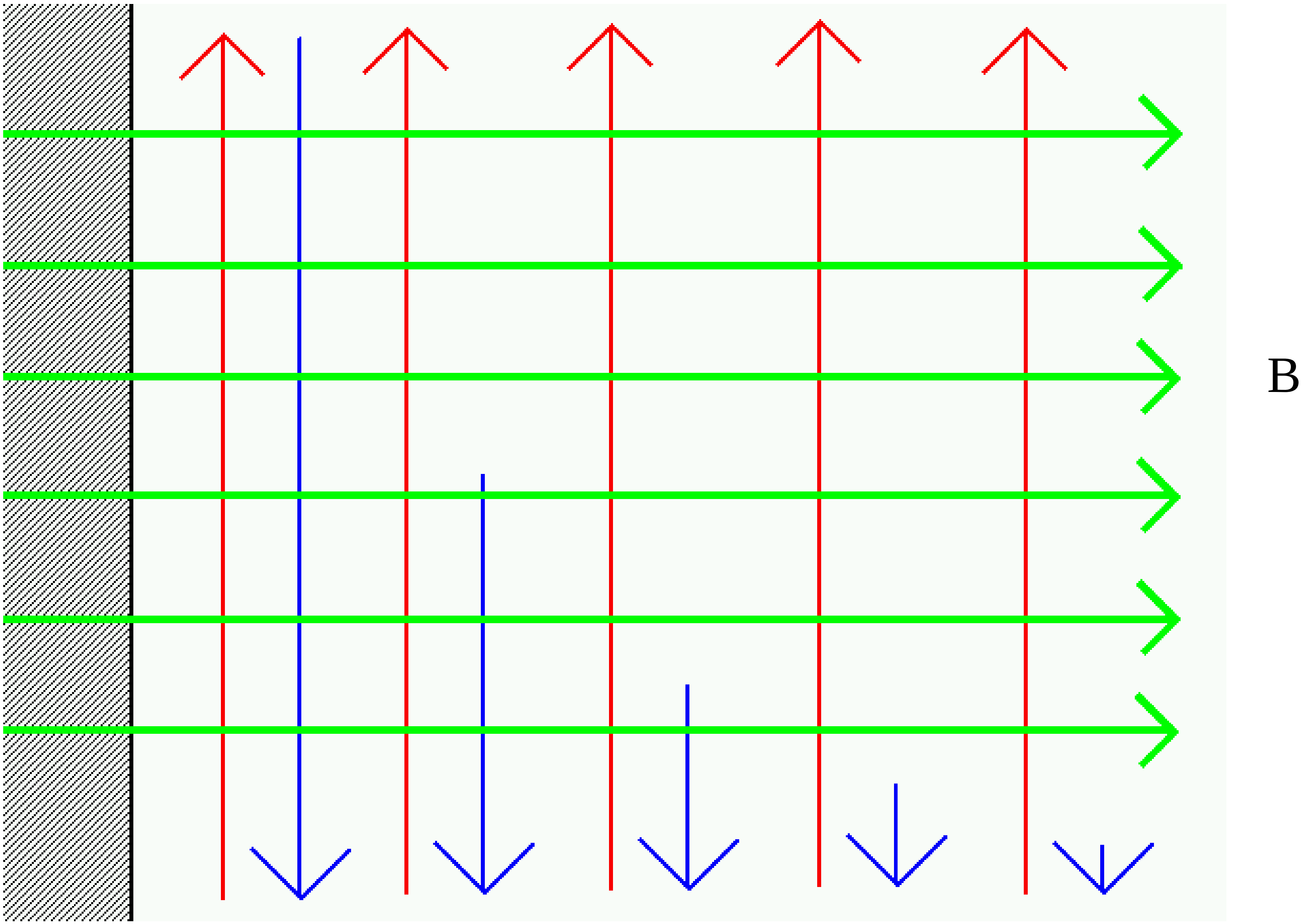
- For $v_y = \exp(i\omega t)\exp(ikx)$ the boundary layer thickness is given by:

$$d \equiv \frac{i}{\mathrm{Im}(k)} = \sqrt{\frac{2\eta}{\rho\omega}}.$$

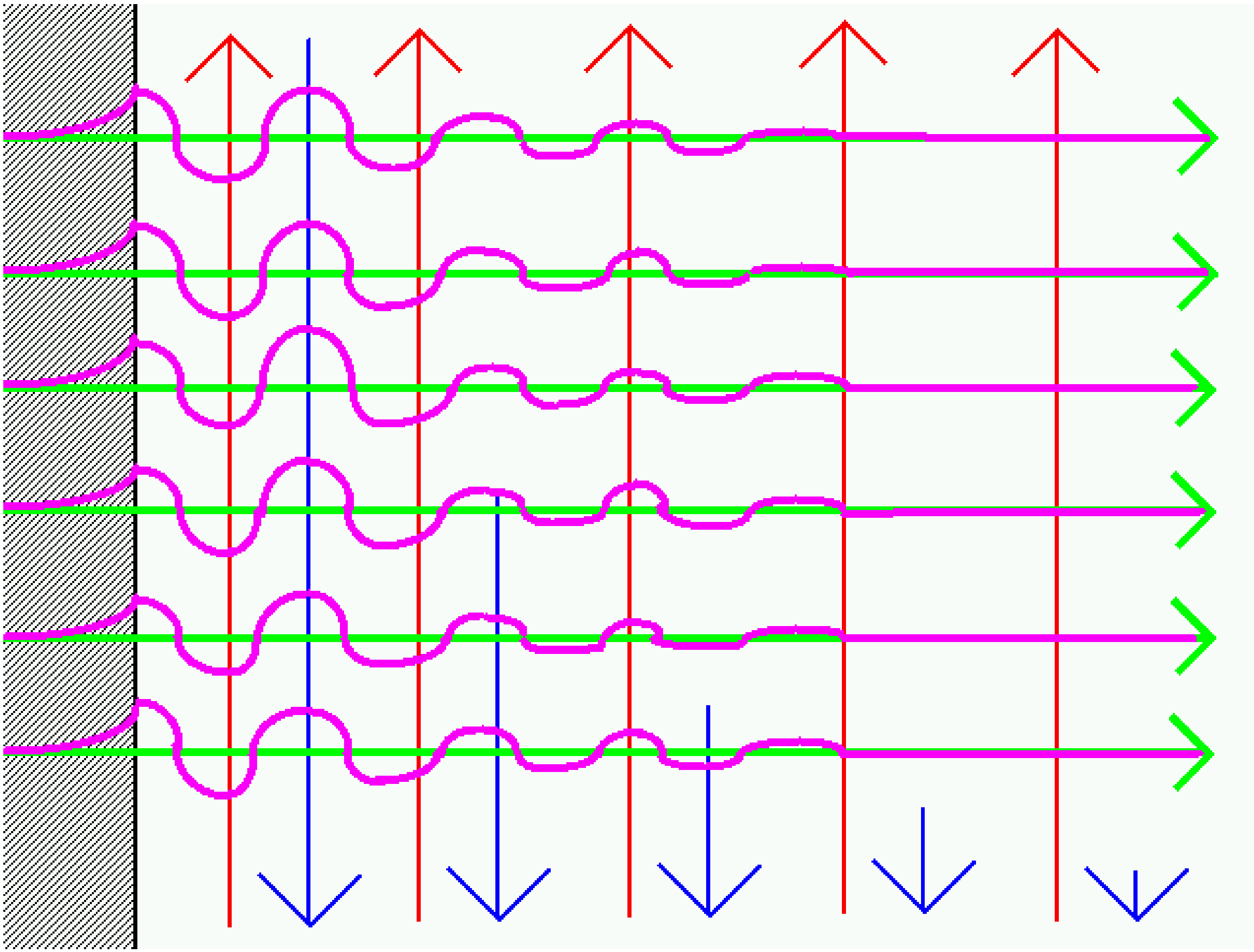
- VBL Damping Rate:

$$\frac{1}{\tau_v} \sim \eta |k|^2 d \sim \frac{\eta}{d}.$$

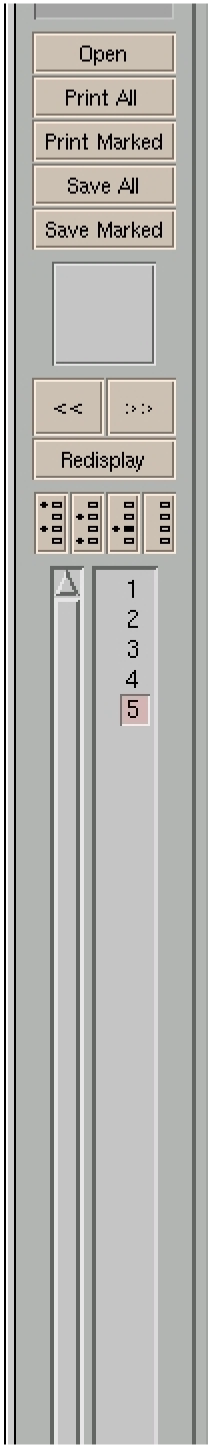
Add Magnetic Field…



Magneto-viscous Boundary Layer With Alfven Waves



LIGO



- Equation for the critical angular velocity:

$$\tau_{\mathrm{GR}} = \tau_v.$$

- Gravitational-radiation growth rate for the $m = 2$ r-modes:

$$\frac{1}{\tau_{\mathrm{GR}}} = 0.24\,\mathrm{s}^{-1}\left(\frac{\Omega}{\Omega_o}\right)^6,$$

where

$$\Omega_o = \sqrt{\pi G \bar{\rho}},$$

$$\Omega_{\mathrm{max}} \cong \frac{2}{3}\Omega_o,$$

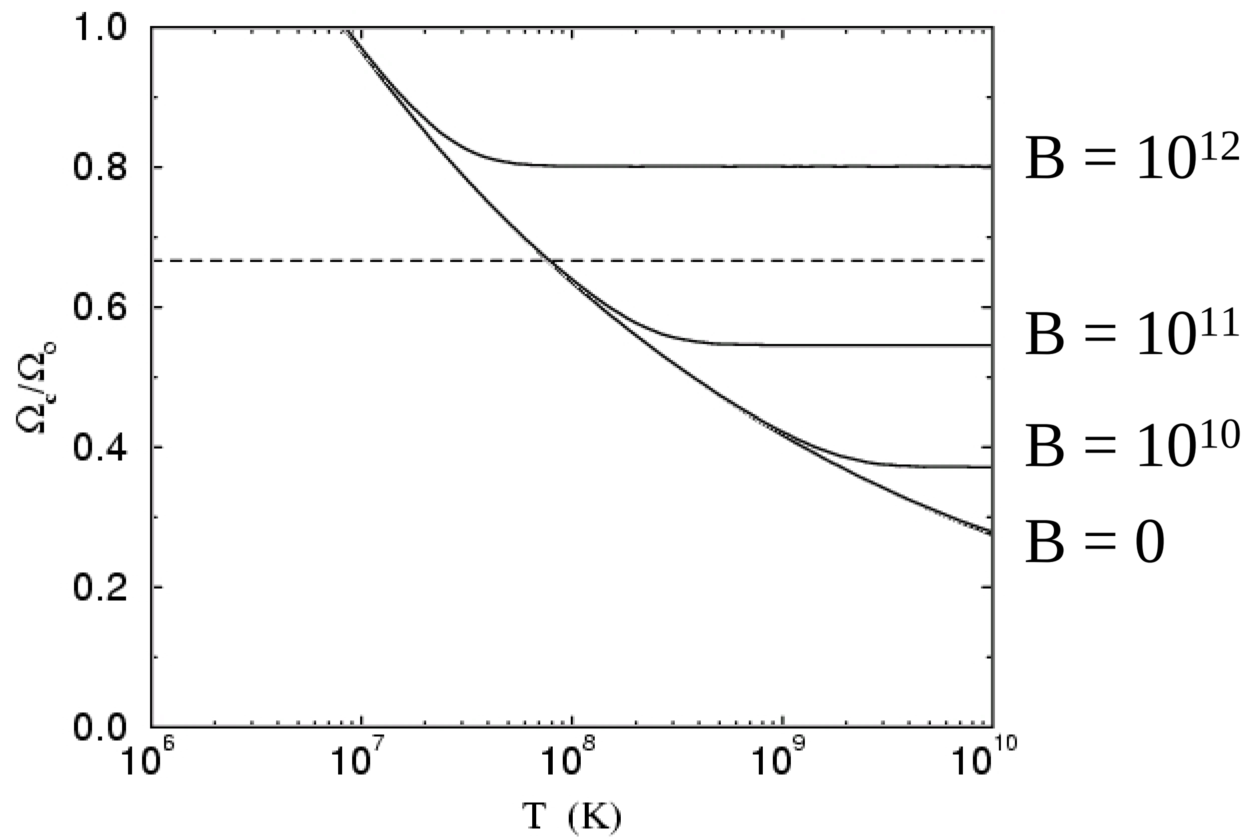
- Magneto-viscous boundary layer damping rate for the $m = 2$ r-modes:

$$\frac{1}{\tau_v} \sim \eta |k|^2 d \sim \eta \frac{d}{\lambda^2} \sim 0.062\,\mathrm{s}^{-1}\,B_{12}.$$

- Critical angular velocity:

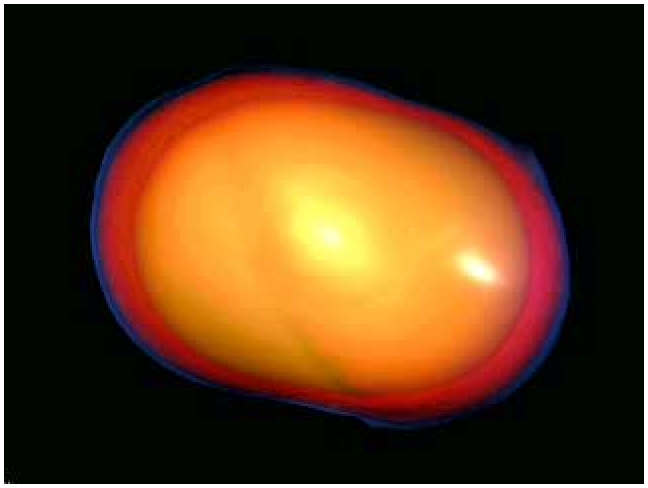
$$\frac{\Omega_c}{\Omega_o} = 0.8 B_{12}^{1/6}.$$

MVBL Critical Angular Velocity



Mendell 2001, Phys Rev D64 044009; gr-qc/0102042

R-mode Movie



See: http://www.cacr.caltech.edu/projects/hydrligo/rmode.html

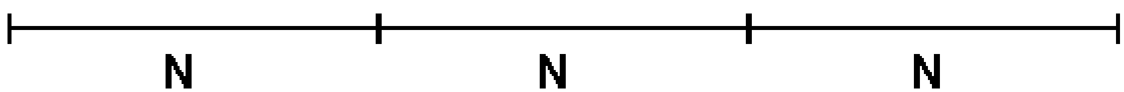
Lee Lindblom, Joel E. Tohline and Michele Vallisneri (2001), Phys. Rev. Letters 86, 1152-1155 (2001). R-modes in newborn NS are saturated by breaking waves.

Computed using Fortran 90 code linked wtih the MPI library on CACR's HP Exemplar V2500.

Owen and Lindblom gr-qc/0111024: r-modes produce 100 s burst with f ~ 940-980 Hz and optimal SNR ~ 1.2-12 for r = 20 Mpc and enhanced LIGO.

Basic Detection Strategy

- Coherently add the signal
- Signal to noise ratio ~ sqrt(T)
- Can always win as long as
  - Sum stays coherent
  - Understand the noise
  - Do not exceed computational limits



$$X_b = \sum_{a=0}^{NM-1} x_a A_a e^{-2\pi i \Phi_{ab}}$$

Amplitude Modulation

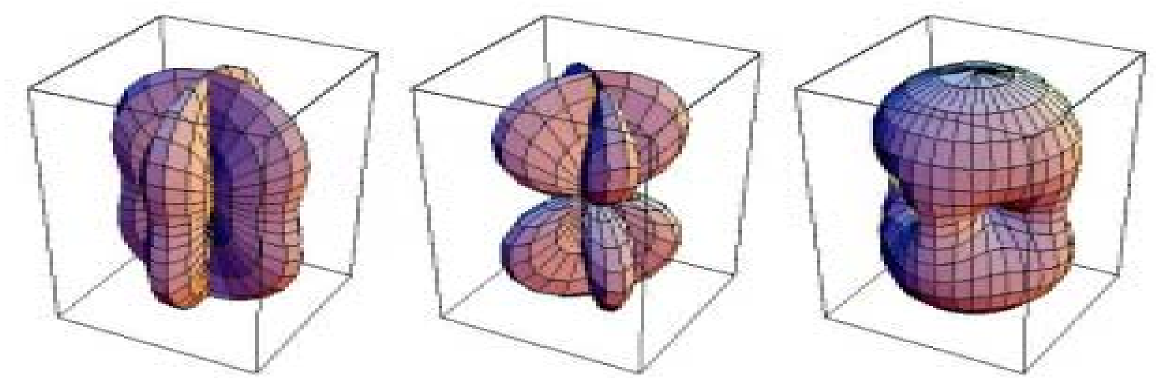


Figure 9. Antenna response function for an interferometric gravitational wave detector. The interferometer is placed at the center of the surrounding box with Michelson arms oriented along the horizontal axes. The distance from a point of the plot surface to the center of the box is a measure for the gravitational wave sensitivity in this direction. The plot to the left is for + polarization, the middle one for × polarization and the right one for unpolarized waves.

$$h(t) = \hat{x} \cdot (M h^{TT} M^{t}) \cdot \hat{x} - \hat{y} \cdot (M h^{TT} M^{t}) \cdot \hat{y}$$

$$\begin{aligned} h(t) = {} & h_{+}[0.5(1+\cos^{2}\theta)\cos 2\varphi \cos 2\psi - \cos\theta \sin 2\varphi \sin 2\psi] \\ & + h_{\times}[0.5(1+\cos^{2}\theta)\cos 2\varphi \sin 2\psi - \cos\theta \sin 2\varphi \cos 2\psi] \end{aligned}$$

Figure: D. Sigg LIGO-P980007-00-D

Phase Modulation

$$\Phi = \int_0^t f_0 (1 + \sum_n f_n t^n)(1 + \frac{\vec{v}}{c} \cdot \hat{n}) dt$$

- The phase is modulated by the intrinsic frequency evolution of the source and by the Doppler effect due to the Earth's motion
- The Doppler effect can be ignored for

$$T \leq 5.5 \times 10^3 \sqrt{\frac{300Hz}{f_0}} \,\text{sec}.$$

DeFT Algorithm

$$X_b = \sum_{\alpha=0}^{M-1} \sum_{j=0}^{N-1} x_{\alpha j} A_\alpha e^{-2\pi i \Phi_{\alpha j b}}$$

$$x_{\alpha j} = \sum_{k=0}^{N-1} X_{\alpha j}^{SFT} e^{-2\pi i \Phi_{ab}}$$

$$X_b = \sum_{\alpha=0}^{M-1} A_\alpha Q_{\alpha b} \sum_{k=0}^{N-1} X_{\alpha k}^{SFT} P_{\alpha k b}$$

AEI: Schutz & Papa gr-qc/9905018; Williams and Schutz gr-qc/9912029; Berukoff and Papa LAL Documentation

Taylor expand the phase

$$\Phi_{\alpha jb} = \Phi_{\alpha,1/2,b} + f_{\alpha,1/2,b}(t_{\alpha j} - t_{\alpha,1/2})$$

$$P_{\alpha kb} = \frac{\sin u_{\alpha kb}}{u_{\alpha kb}} - i\frac{1-\cos u_{\alpha kb}}{u_{\alpha kb}}$$

$$Q_{\alpha b} = e^{iv_{\alpha b}}$$

$$u_{\alpha kb} = 2\pi\left(\frac{T}{M} f_{\alpha,1/2,b} - k\right)$$

$$v_{\alpha b} = -2\pi\left(\Phi_{\alpha,1/2,b} - \frac{T}{2M} f_{\alpha,1/2,b}\right)$$

Advantages of DeFT Code

- $P_{\alpha kb}$ is peaked. Can sum over only 16 k's
- Complexity reduced from O(MN $\times$ number of phase models) to O(MN$\log_2$N + M $\times$ number of phase models).
- Unfortunately, number phase models increased by factor of M/$\log_2$MN over FFT of modulated data. FFT is O(MN$\log_2$MN $\times$ number of phase models/MN.)
- But memory requirements much less than FFT, and easy to divide DeFT code into frequency bands and run on parallel computing cluster.
- Need 10^{10} – 10^{20} phase models, depending on frequency band & number of spin down parameters, for no more than 30% power loss due to mismatch.

Basic Confidence Limit

- Probability stationary white noise will result in power greater than or equal to P_f:

$$1-\alpha = e^{-P_f / P_n}$$

- Threshold needed so that probability of false detection = $1 - \alpha$.

$$P_f / P_n > \ln[N_p / (1-\alpha)]$$

Brady, Creighton, Cutler, Schutz gr-qc/9702050.

Maximum Likehood Estimator

$$h = F_+ h_+ + F_\times h_\times$$

$$S = \frac{4}{T} \frac{A|F|^2 + B|G|^2 - 2C\,\mathrm{Re}(FG^*)}{D}$$

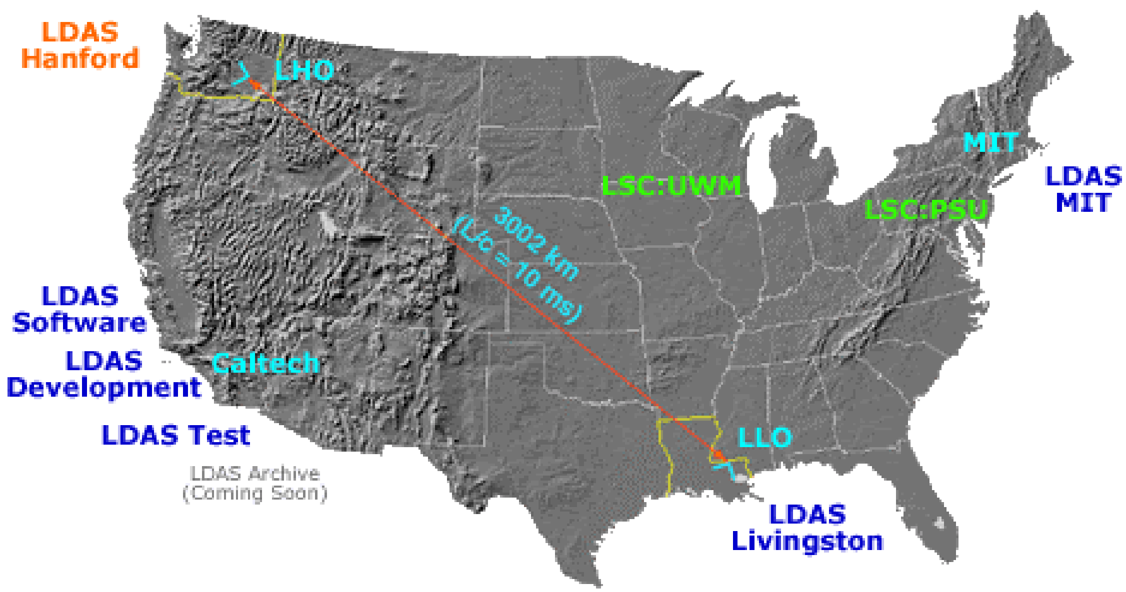
$$p = \frac{1}{\pi^2 D} e^{-S}$$

$$F = \sum_{a=0}^{NM-1} x_a f_a e^{-2\pi i \Phi_{ab}}, \qquad G = \sum_{a=0}^{NM-1} x_a g_a e^{-2\pi i \Phi_{ab}}$$

$$f = F_+ \cos 2\Psi - F_\times \sin 2\Psi, \qquad g = F_+ \sin 2\Psi + F_\times \cos 2\Psi$$

Jaranowski, Krolak, Schutz gr-qc/9804014.

LDAS = LIGO Data Analysis Systems

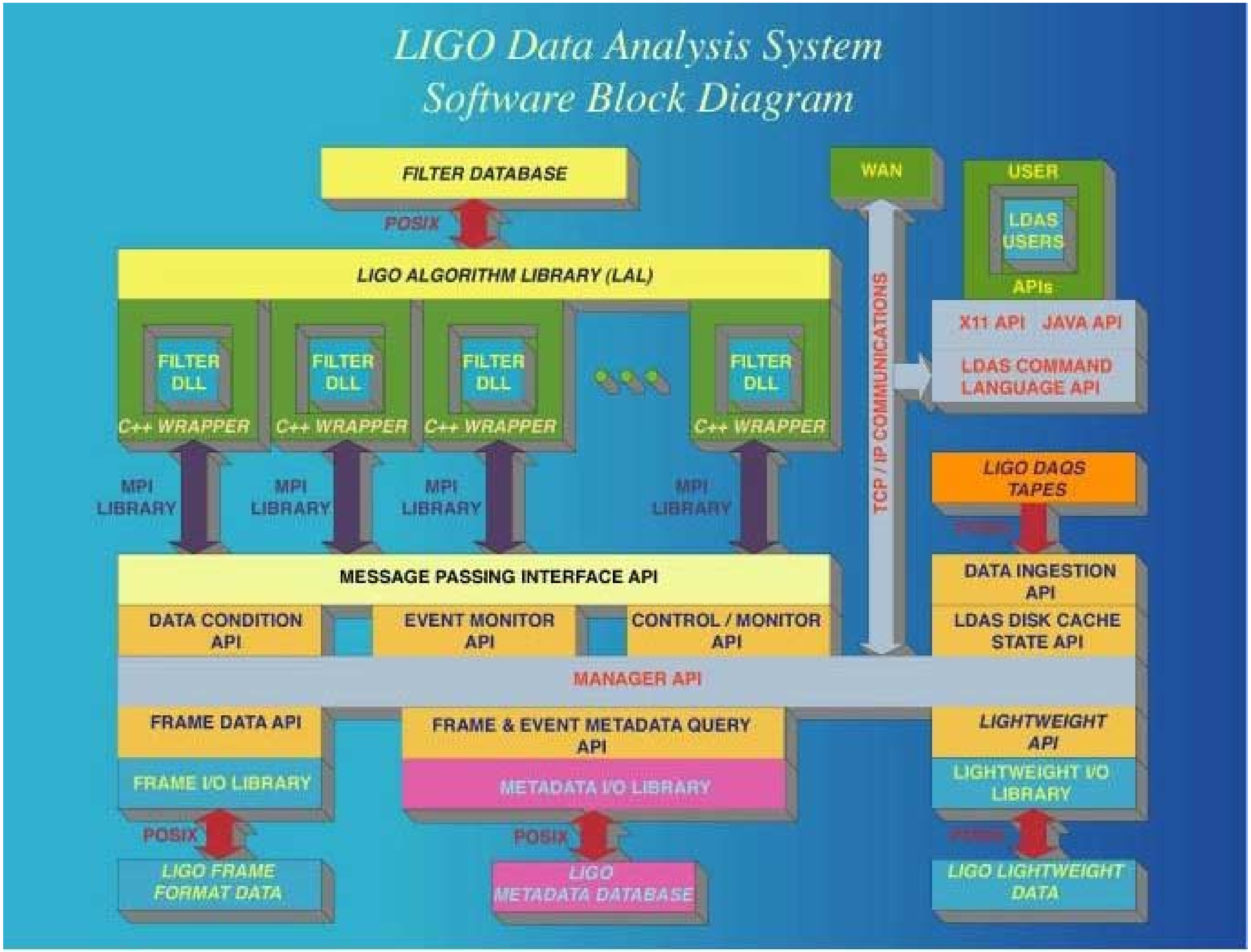


LDAS Hardware

14.5 TB Disk Cache

Beowulf Cluster

LDAS Software



Interface to the Scientist