LIGO: The Laser Interferometer Gravitational-wave Observatory

General Relativity…

Einstein…

A new window…

Astrophysics…

The Search For Gravitation Radiation From Periodic Sources

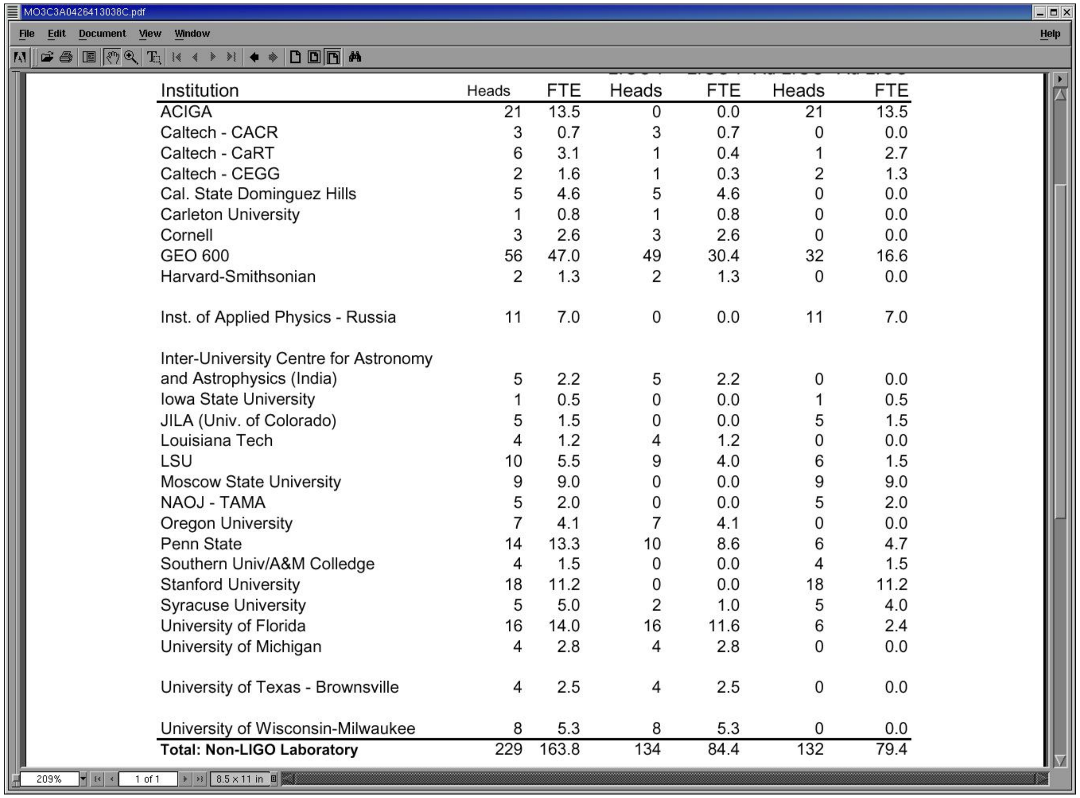
Gregory Mendell

LIGO Hanford Observatory

LIGO-G020008-00-W

Who’s Involved?

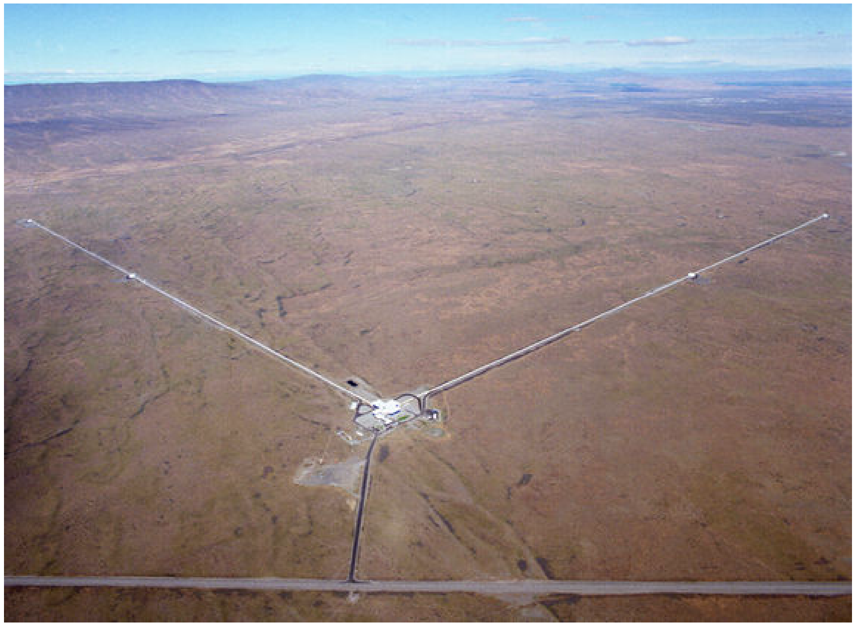
Caltech, MIT, and the LIGO Science Collaboration



Institution	Heads	FTE	Heads	FTE	Heads	FTE
ACIGA	21	13.5	0	0.0	21	13.5
Caltech - CACR	3	0.7	3	0.7	0	0.0
Caltech - CaRT	6	3.1	1	0.4	1	2.7
Caltech - CEGG	2	1.6	1	0.3	2	1.3
Cal. State Dominguez Hills	5	4.6	5	4.6	0	0.0
Carleton University	1	0.8	1	0.8	0	0.0
Cornell	3	2.6	3	2.6	0	0.0
GEO 600	56	47.0	49	30.4	32	16.6
Harvard-Smithsonian	2	1.3	2	1.3	0	0.0
Inst. of Applied Physics - Russia	11	7.0	0	0.0	11	7.0
Inter-University Centre for Astronomy and Astrophysics (India)	5	2.2	5	2.2	0	0.0
Iowa State University	1	0.5	0	0.0	1	0.5
JILA (Univ. of Colorado)	5	1.5	0	0.0	5	1.5
Louisiana Tech	4	1.2	4	1.2	0	0.0
LSU	10	5.5	9	4.0	6	1.5
Moscow State University	9	9.0	0	0.0	9	9.0
NAOJ - TAMA	5	2.0	0	0.0	5	2.0
Oregon University	7	4.1	7	4.1	0	0.0
Penn State	14	13.3	10	8.6	6	4.7
Southern Univ/A&M Colledge	4	1.5	0	0.0	4	1.5
Stanford University	18	11.2	0	0.0	18	11.2
Syracuse University	5	5.0	2	1.0	5	4.0
University of Florida	16	14.0	16	11.6	6	2.4
University of Michigan	4	2.8	4	2.8	0	0.0
University of Texas - Brownsville	4	2.5	4	2.5	0	0.0
University of Wisconsin-Milwaukee	8	5.3	8	5.3	0	0.0
Total: Non-LIGO Laboratory	229	163.8	134	84.4	132	79.4

Sponsored by the National Science Foundation

The Observatories

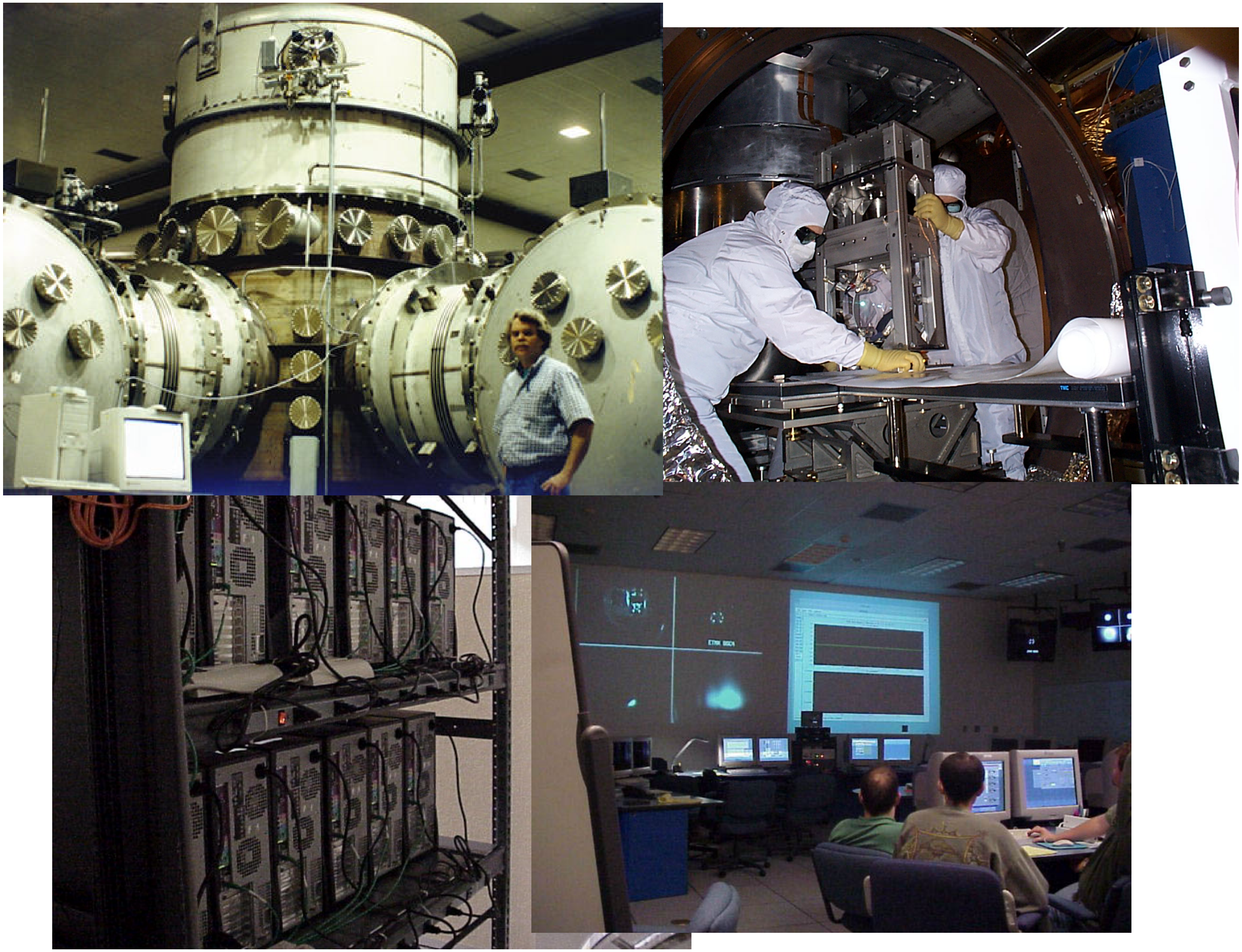


LIGO Hanford

LIGO Livingston

Photos: http:// www.ligo.caltech.edu; http://www.ligo-la.caltech.edu

Inside



Gravitational Waves

- Gravitation = spacetime curvature described by the metric tensor: $ds^2 = g_{\mu\nu} dx^\mu dx^\nu$
- Weak Field Limit: $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$

$$\left(\nabla^2 - \frac{1}{c^2} \frac{\partial^2}{\partial t^2} \right) \bar{h}^{\mu\nu} = 0$$

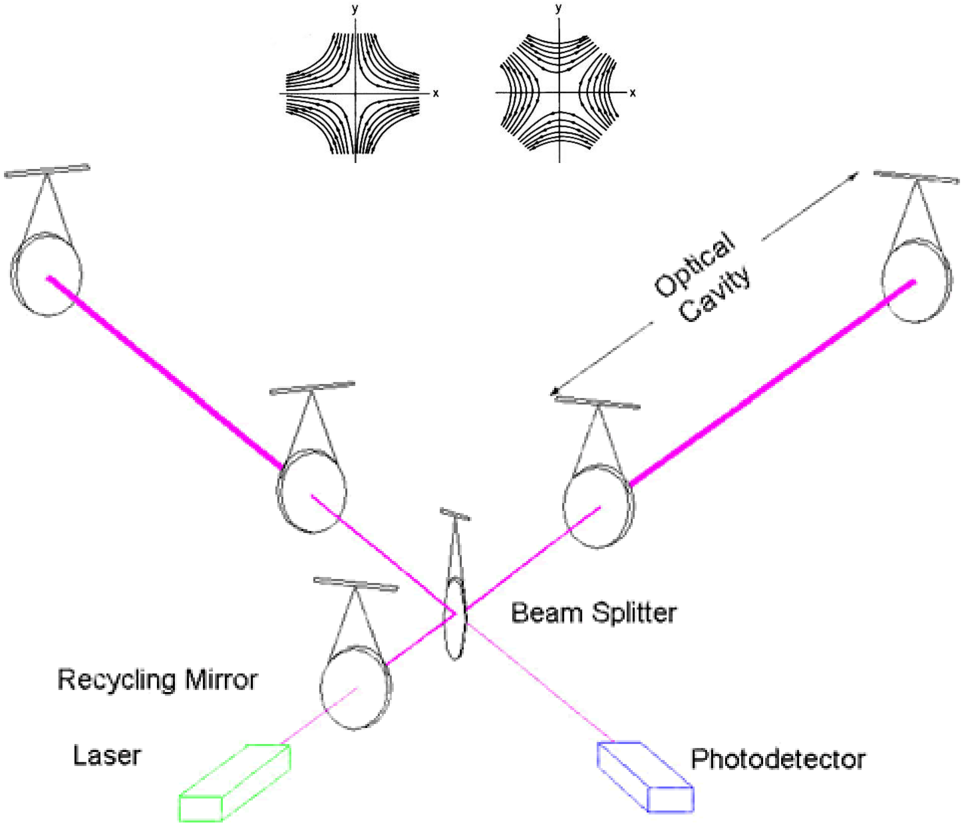
- TT Gauge:

$$h_{\mu\nu}^{TT} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & h_+ & h_\times & 0 \\ 0 & h_\times & -h_+ & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} e^{2\pi i f t - ikz}$$

How Does LIGO Work?

Gravitational-wave Strain: $h = \Delta L / L$

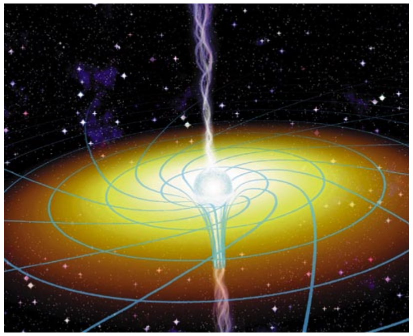
LIGO is a lab looking for GW’s.



LIGO is an “ear” on the universe, listening for cosmic spacetime vibrations.

Figures: K. S. Thorne gr-qc/9704042; D. Sigg LIGO-P980007-00-D

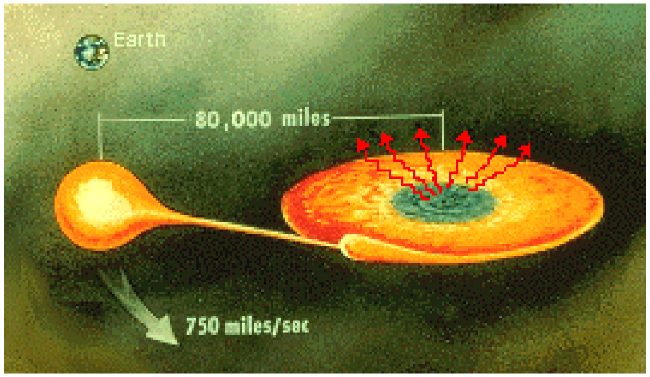
Astrophysical Sources

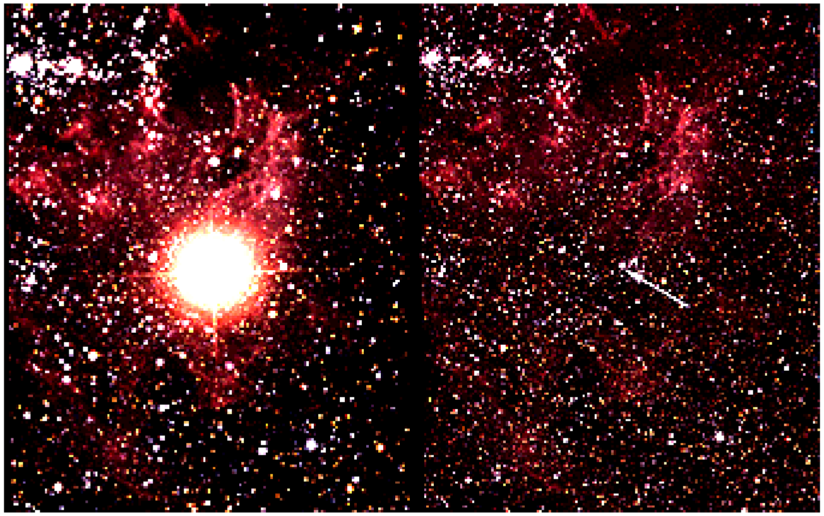


Black Holes

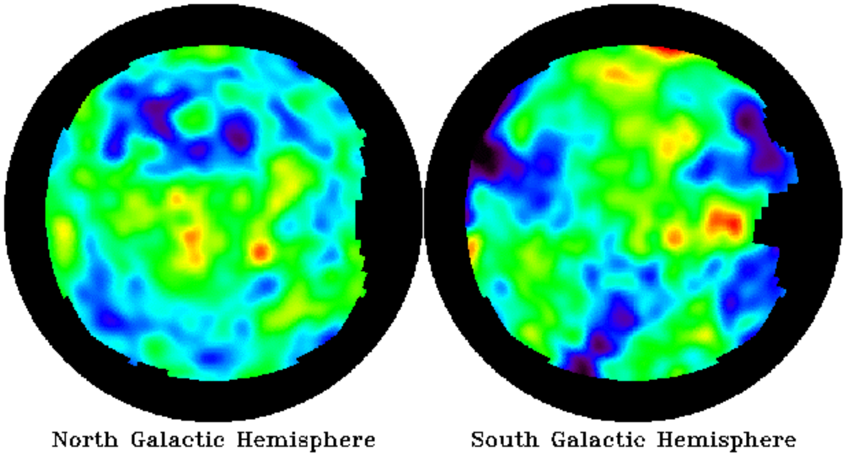
Pulsars

LMXBs





Supernovae



Stochastic Background

Photos: http://antwrp.gsfc.nasa.gov; http://imagine.gsfc.nasa.gov

$$h = \Delta L / L \approx (G / c^4)(\ddot{Q} / r)$$

"Newtonian"
quadrupole formula.

Burst (SN at distance of Virgo Cluster: h = 10^{-23} – 10^{-21}; rate = 1/yr)

•Stochastic (limit Ω_{GW}; cosmic strings; BH from massive pop III stars: h = 10^{-23} – 10^{-21})

•Inspiral (h_{max} = 10^{-22} for NS-NS@ 200 Mpc; rate = 3/yr; NS-BH; BH-BH)

•Periodic (h = 10^{-25} for 10 ms pulsar with maximum ellipticity at 1 Kpc; h = 10^{-27} for 2 ms LMXB in equilibrium at 1 Kpc)

Reviews: K. S. Thorne 100 Yrs of Gravitation; P. R. Saulson, Fund. of Interferometric GW Detectors

Noise Curves

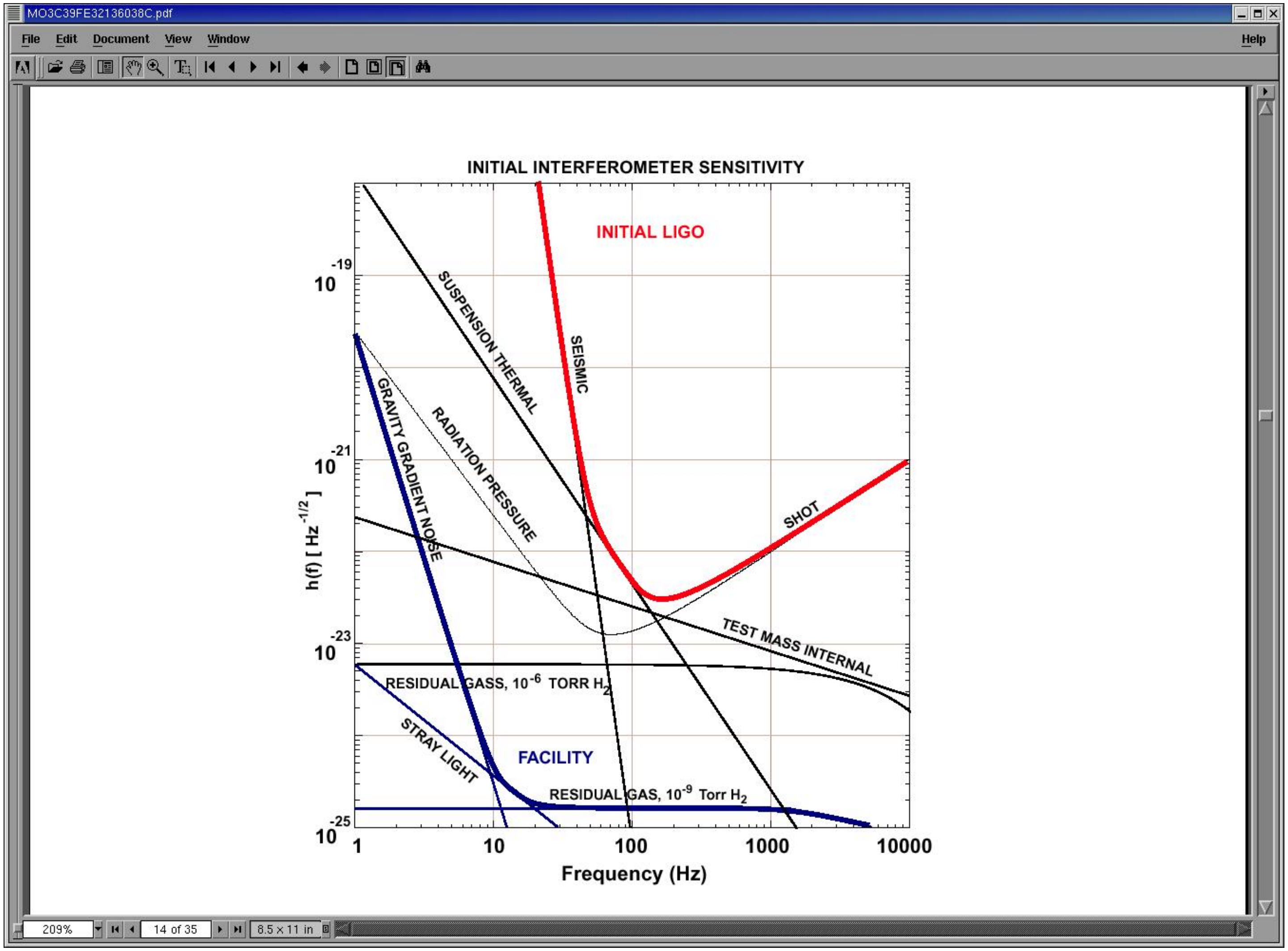


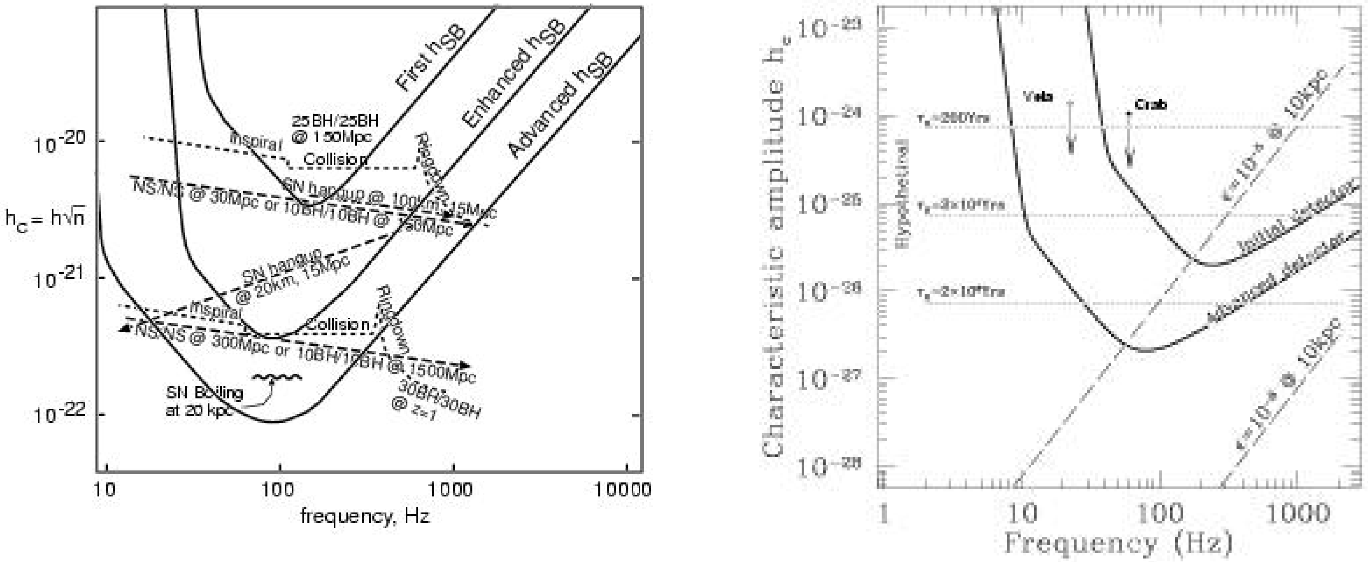
Figure: D. Sigg LIGO-P980007-00-D

Signal to Noise Ratio

$$\frac{S}{N} \cong \frac{h\sqrt{T}}{\sqrt{< n^2(f_c) >}}$$

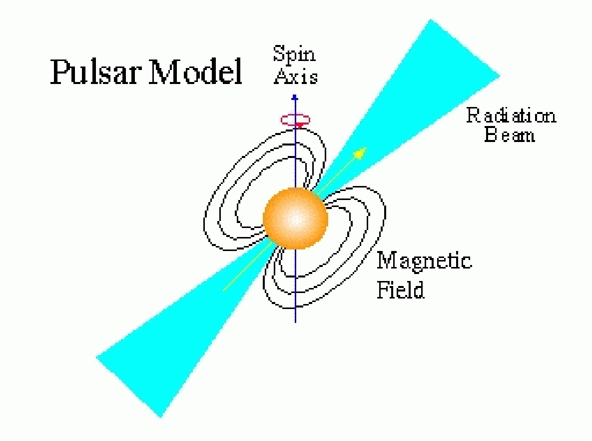
- **h = signal amplitude**
- **T = observation time or duration of signal or period of the characteristic frequency of the signal.**
- **n^2 = power spectrum of the noise**

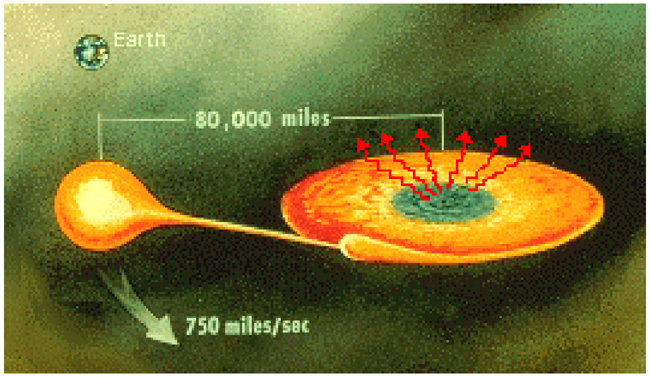
Sensitivity Curves



Figures: K. S. Thorne gr-qc/9704042; Brady, Creighton, Cutler, Schutz gr-qc/9702050.

Known Possible Periodic Sources





LMXBs

- Are neutron stars: the sun compress to size of city. Compact ($2GM/Rc^2 \sim .2$) and ultra dense (10^{14} g/cm^3).
- Are composed of (superfluid) neutrons, (superconducting) protons, electrons, + exotic particles (e.g., hyperons) or strange stars composed of an even more exotic up, down, and strange quark soup.
- Spin Rapidly (~ .1 Hz to 642 Hz i.e., within the LIGO band.)

Periodic sources emit GWs due to…

- Rotation about nonsymmetry axis
- Strain induced asymmetry: $\varepsilon = \dfrac{I_1 - I_2}{I}$
- Accretion induced emission
- Unstable oscillation modes

Sensitivity Curves

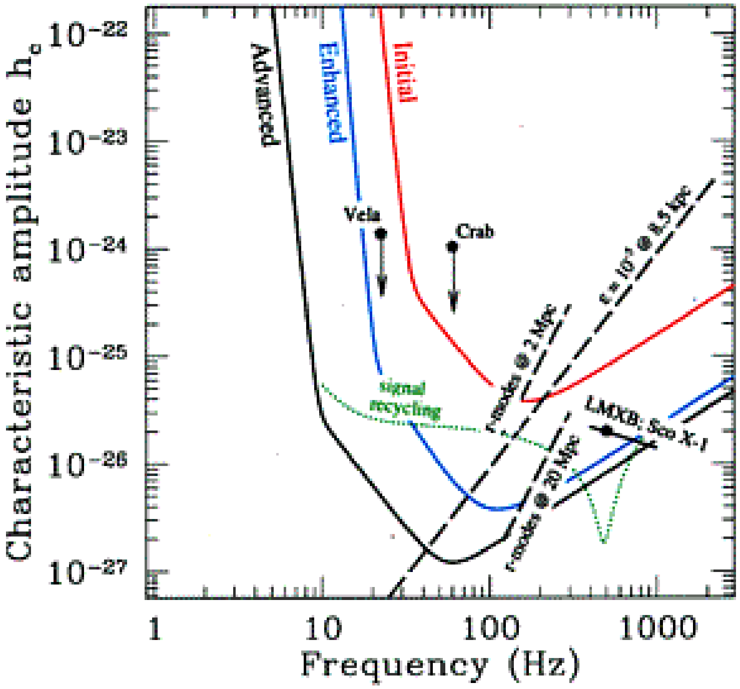


Figure: Brady ITP seminar summer 2000

Amplitude Modulation

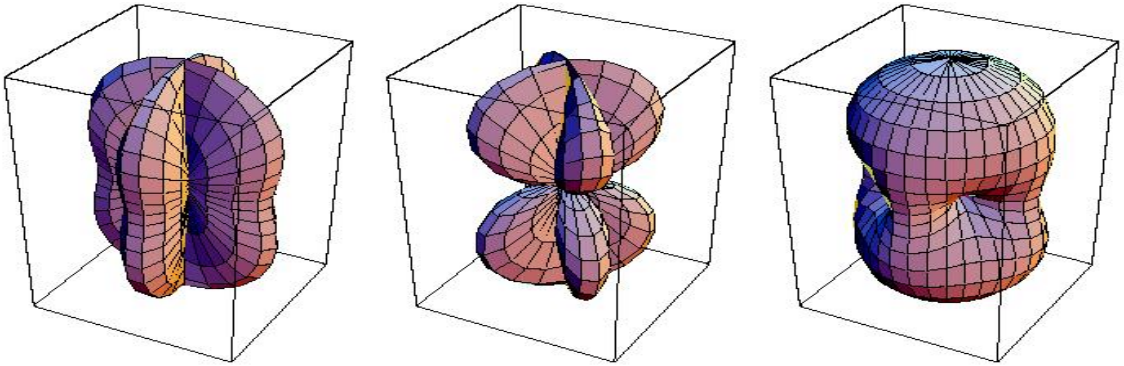


Figure 9. Antenna response function for an interferometric gravitational wave detector. The interferometer is placed at the center of the surrounding box with Michelson arms oriented along the horizontal axes. The distance from a point of the plot surface to the center of the box is a measure for the gravitational wave sensitivity in this direction. The plot to the left is for + polarization, the middle one for × polarization and the right one for unpolarized waves.

$$h(t) = \hat{x}\cdot(Mh^{TT}M^{t})\cdot\hat{x} - \hat{y}\cdot(Mh^{TT}M^{t})\cdot\hat{y}$$

$$\begin{aligned} h(t) = {} & h_{+}[0.5(1+\cos^{2}\theta)\cos 2\varphi\cos 2\psi - \cos\theta\sin 2\varphi\sin 2\psi] \\ & + h_{\times}[0.5(1+\cos^{2}\theta)\cos 2\varphi\sin 2\psi - \cos\theta\sin 2\varphi\cos 2\psi] \end{aligned}$$

Figure: D. Sigg LIGO-P980007-00-D

Phase Modulation

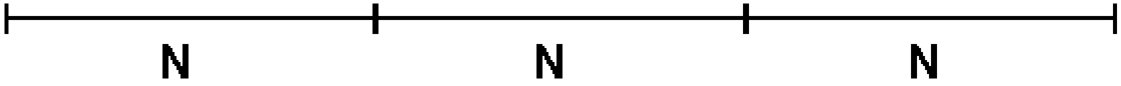
$$\Phi = \int_0^t f_0 (1 + \sum_n f_n t^n)(1 + \frac{\vec{v}}{c} \cdot \hat{n}) dt$$

- The phase is modulated by the intrinsic frequency evolution of the source and by the Doppler effect due to the Earth's motion
- The Doppler effect can be ignored for

$$T \le 5.5 \times 10^3 \sqrt{\frac{300Hz}{f_0}} \sec.$$

Basic Detection Strategy

- Coherently add the signal
- Signal to noise ratio ~ sqrt(T)
- Can always win as long as
  - Sum stays coherent
  - Understand the noise
  - Do not exceed computational limits



$$X_b = \sum_{a=0}^{NM-1} x_a A_a e^{-2\pi i \Phi_{ab}}$$

DeFT Algorithm

$$X_b = \sum_{\alpha=0}^{M-1} \sum_{j=0}^{N-1} x_{\alpha j} A_\alpha e^{-2\pi i \Phi_{\alpha j b}}$$

$$x_{\alpha j} = \sum_{k=0}^{N-1} X_{\alpha j}^{SFT} e^{-2\pi i \Phi_{ab}}$$

$$X_b = \sum_{\alpha=0}^{M-1} A_\alpha Q_{\alpha b} \sum_{k=0}^{N-1} X_{\alpha k}^{SFT} P_{\alpha k b}$$

AEI: Schutz & Papa gr-qc/9905018; Williams and Schutz gr-qc/9912029;
Berukoff and Papa LAL Documentation

Taylor expand the phase

$$\Phi_{\alpha jb} = \Phi_{\alpha,1/2,b} + f_{\alpha,1/2,b}(t_{\alpha j} - t_{\alpha,1/2})$$

$$P_{\alpha kb} = \frac{\sin u_{\alpha kb}}{u_{\alpha kb}} - i\frac{1-\cos u_{\alpha kb}}{u_{\alpha kb}}$$

$$Q_{\alpha b} = e^{iv_{\alpha b}}$$

$$u_{\alpha kb} = 2\pi\left(\frac{T}{M} f_{\alpha,1/2,b} - k\right)$$

$$v_{\alpha b} = -2\pi\left(\Phi_{\alpha,1/2,b} - \frac{T}{2M} f_{\alpha,1/2,b}\right)$$

Advantages of DeFT Code

- $P_{\alpha kb}$ is peaked. Can sum over only 16 k's
- Complexity reduced from O(MN × number of phase models) to O(MN$\log_2$N + M × number of phase models).
- Unfortunately, number phase models increased by factor of M/$\log_2$MN over FFT of modulated data. FFT is O(MN$\log_2$MN × number of phase models/MN.)
- But memory requirements much less than FFT, and easy to divide DeFT code into frequency bands and run on parallel computing cluster.
- Need $10^{10} – 10^{20}$ phase models, depending on frequency band & number of spin down parameters, for no more than 30% power loss due to mismatch.

Basic Confidence Limit

- Probability stationary white noise will result in power greater than or equal to P_f:

$$1-\alpha = e^{-P_f / P_n}$$

- Threshold needed so that probability of false detection = 1 – α.

$$P_f / P_n > \ln[N_p /(1-\alpha)]$$

Brady, Creighton, Cutler, Schutz gr-qc/9702050.

Maximum Likehood Estimator

$$h = F_+ h_+ + F_\times h_\times$$

$$S = \frac{4}{T} \frac{A|F|^2 + B|G|^2 - 2C\,\mathrm{Re}(FG^*)}{D}$$

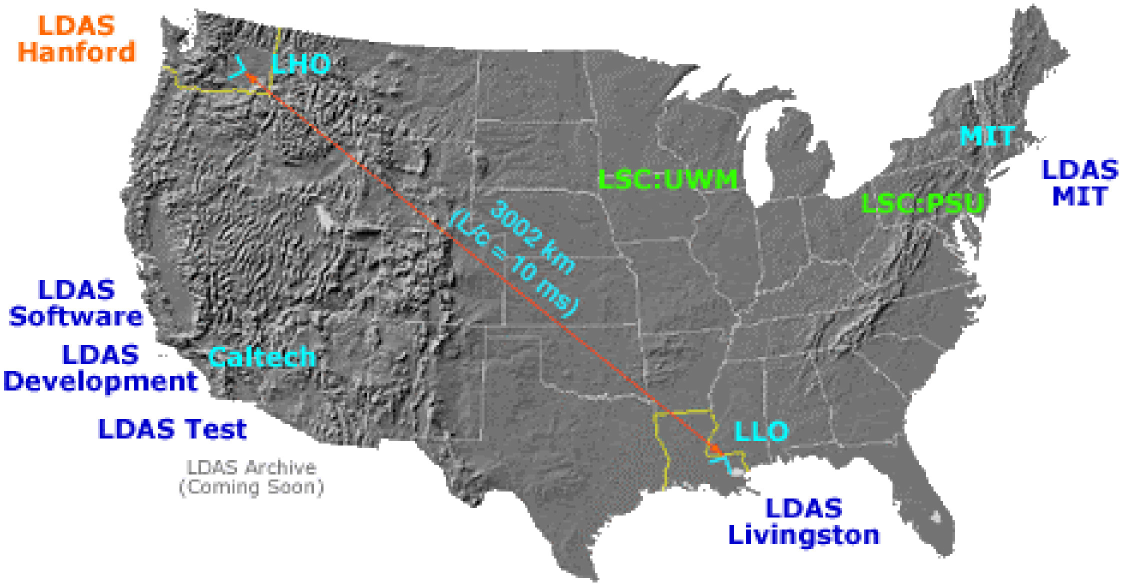
$$p = \frac{1}{\pi^2 D} e^{-S}$$

$$F = \sum_{a=0}^{NM-1} x_a f_a e^{-2\pi i \Phi_{ab}}, \qquad G = \sum_{a=0}^{NM-1} x_a g_a e^{-2\pi i \Phi_{ab}}$$

$$f = F_+ \cos 2\Psi - F_\times \sin 2\Psi, \qquad g = F_+ \sin 2\Psi + F_\times \cos 2\Psi$$

Jaranowski, Krolak, Schutz gr-qc/9804014.

LDAS = LIGO Data Analysis Systems

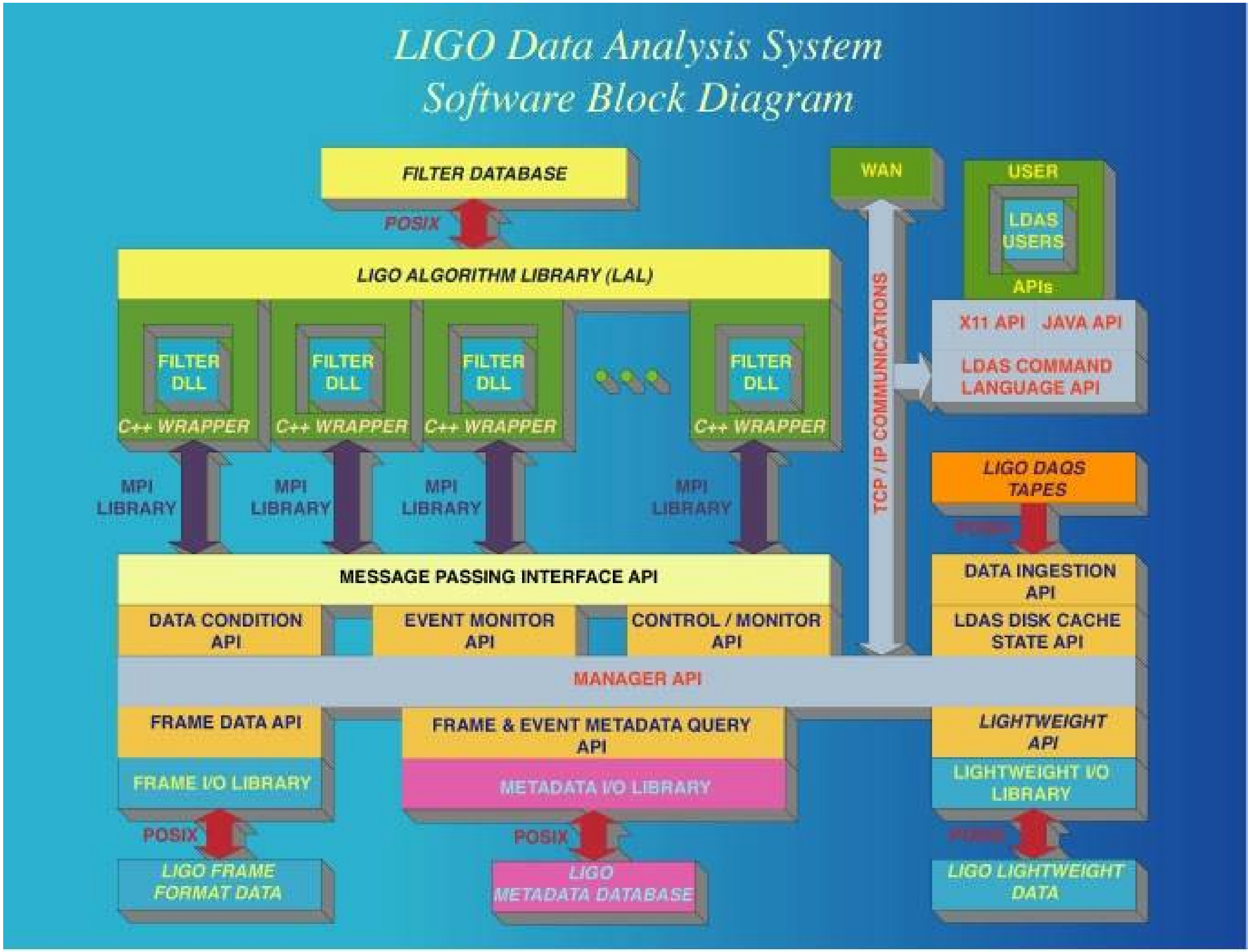


LDAS Hardware

Beowulf Cluster

14.5 TB Disk Cache

LDAS Software



Interface to the Scientist