Thermal noise from lossy surfaces

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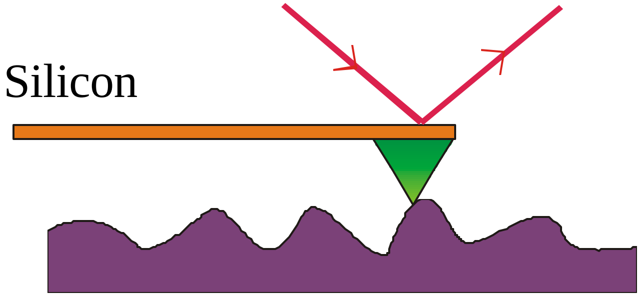
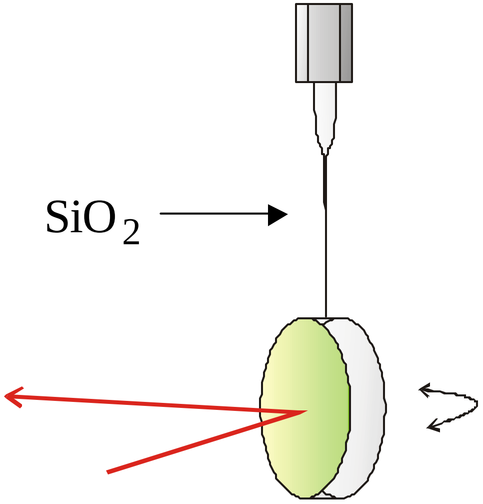
Typical experiments limited by mechanical thermal noise

Atomic force microscope



Thermal force fluctuations in armature limit position accuracy

Torsion pendulum

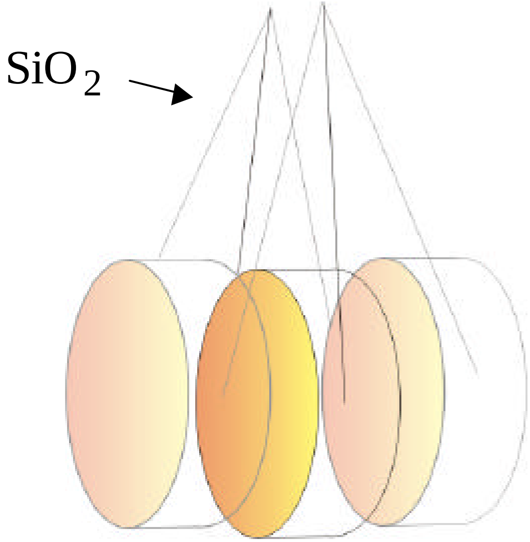


Thermal force fluctuations in fiber limit phase accuracy

Thermal noise in LIGO

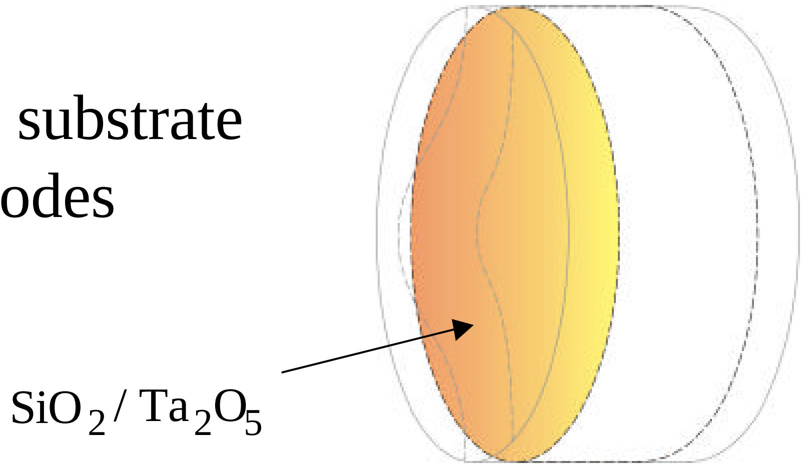
Pendulum mode thermal noise

Fluctuations in the suspending flexures randomly excite the pendulum mode



Internal mode thermal noise

Fluctuations in the coating and substrate randomly excite the internal modes



Fluctuation-Dissipation Theorem

Below resonance

$$\boxed{S_x(f) \approx \frac{2k_B T}{\pi k f}\phi}$$

Loss angle
=1/Q

PSD of thermal fluctuations

Spring constant

Above resonance

$$\boxed{S_x(f) \approx \frac{k_B T f_0^2}{2\pi^5 m f^5}\phi}$$

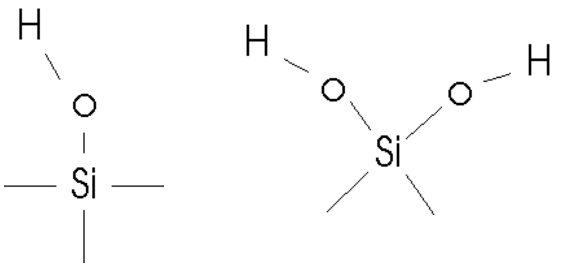
Typical bulk Q's at room temperature

(10-1000 Hz)

Plastics	$10^1 - 10^3$
Metals	$10^3 - 10^4$
Ordinary glass	$10^4 - 10^5$
Fused Silica	$10^6 - 10^8$
Sapphire	$\sim 10^8$
Silicon	$\sim 10^8$

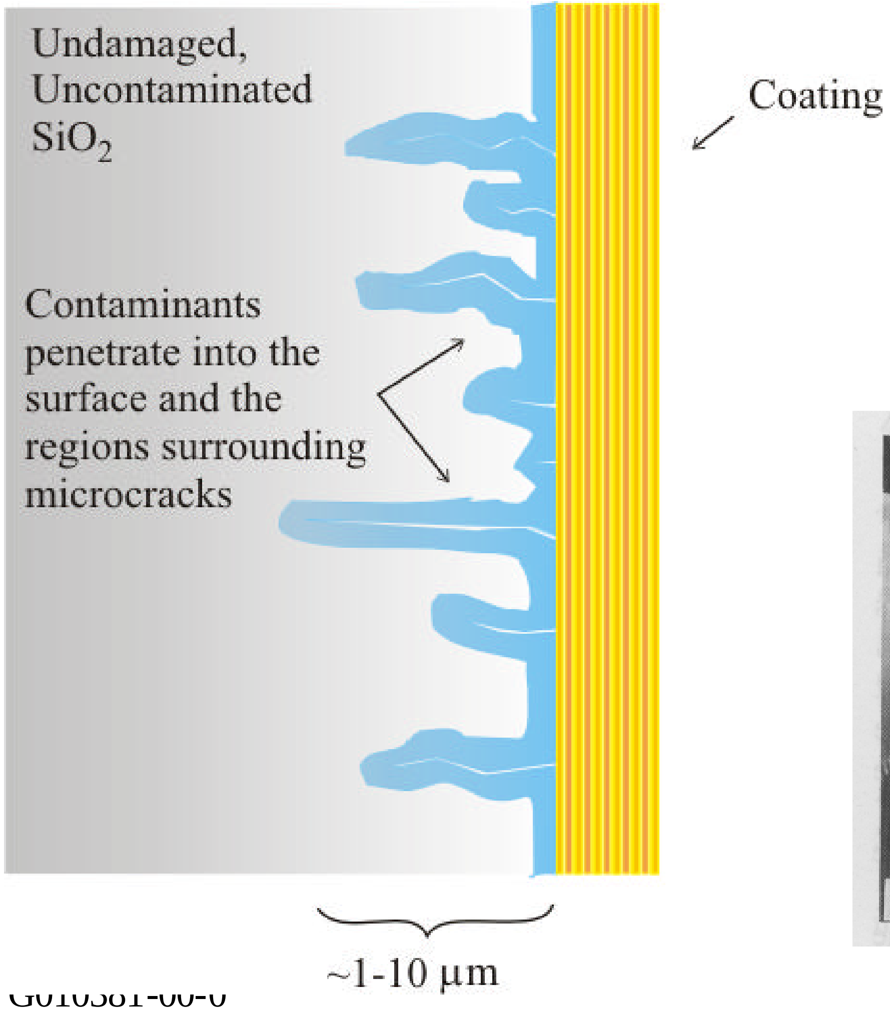
Surface loss

- In low-loss probes, mechanical loss from the *surfaces* often dominates.

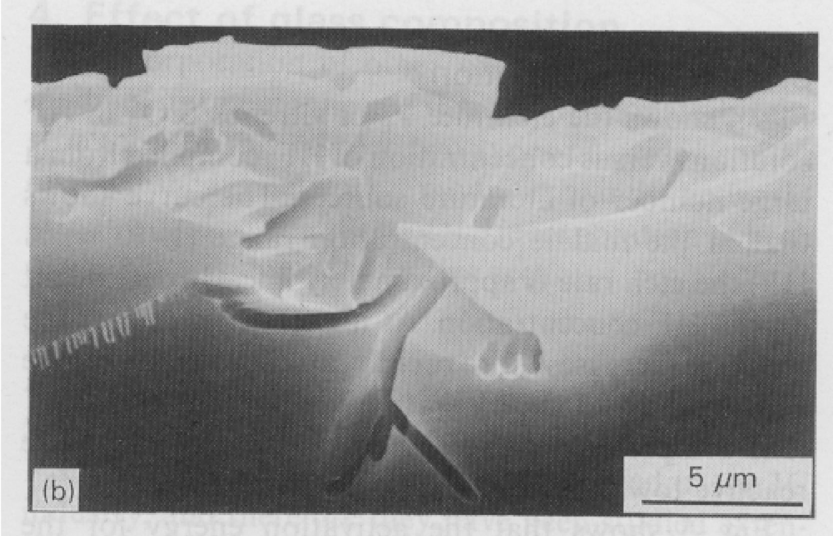


- Sources of surface loss in SiO_2
  - Unsatisfied surface bonds attract H_2O to form SiOH groups.
  - Other chemicals are also present.
  - Polishing leaves a “gel-layer” of damage (Lunin).
  - Polishing creates microcracks (Spierings).
  - Optical coatings may need to be applied to the surface.

Typical polished and coated surface



Microcracks are optically closed but can be revealed with a light HF etch.



G. A. C. M. Spierings, J. Mat. Sci. **28** 6261 (1993).

Quantifying surface loss

- Material loss angle is degraded near the surface

$$\phi_{\text{bulk}} \rightarrow \phi(d), \qquad d = \text{depth.}$$

- For coatings, change is sudden

$$\phi_{\text{bulk}} \rightarrow \phi_{\text{coating}}.$$

- Usually however, we can’t measure the depth-dependence of the loss.
- Need a more practical measure of surface loss.

The surface loss α

- Lump all the "bad stuff" together

$$\boxed{\alpha \equiv \int_0^d \phi(n)\,\mathrm{d}n}$$

d = surface depth << sample thickness

n = distance inwards from surface

- For a homogenous coating of thickness d

$$\alpha = \phi_{\mathrm{coating}}\, d$$

So, a measurement of α is equivalent to a measurement of ϕ_{coating} .

Utility of α

- Quantify the effects of surface-treatments in a sample-independent way.
- Compare the surface loss of different materials.
- $\alpha/\phi_{\mathrm{bulk}}$ is a new measure of the physical condition of a surface. (How does it correlate with other metrics?)
- Need to know the value of α for fiber surfaces and for coatings in order to calculate thermal noise in LIGO.

Measuring the surface loss α

- Total measured loss is something like

$$\phi_{\text{total}} = \frac{U_{\text{bulk}}}{U_{\text{total}}}\phi_{\text{bulk}} + \frac{U_{\text{surface}}}{U_{\text{total}}}\phi_{\text{surface}}$$

Undefined

- correct expression

$$\phi_{\text{total}} \approx \phi_{\text{bulk}} + \frac{\delta U}{U}\alpha$$

Energy density at surface integrated over the surface [J/m]

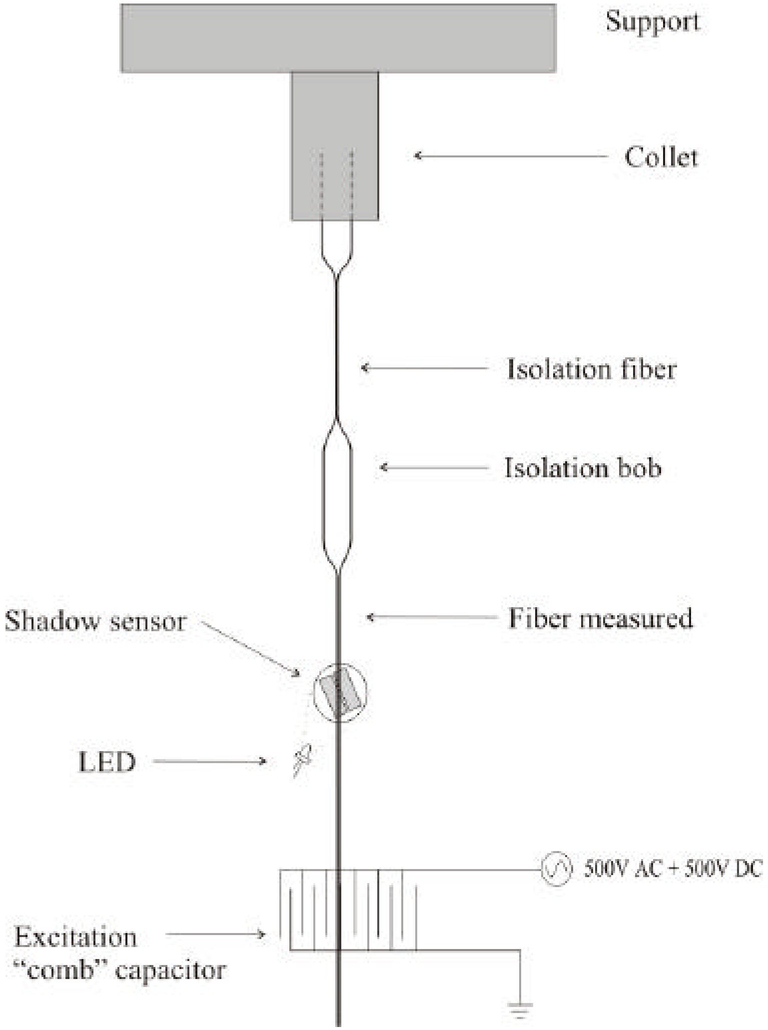
Units [m]

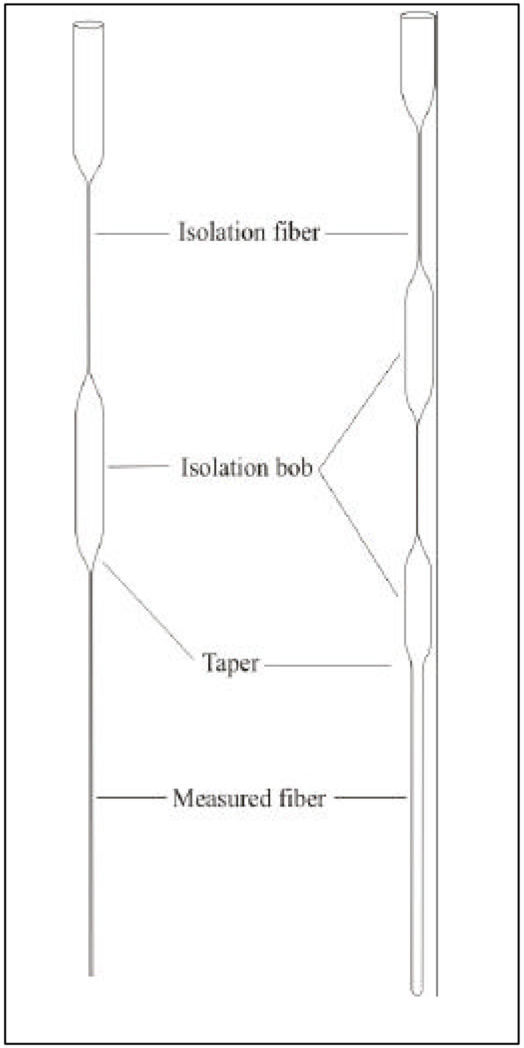
Elastic energy amplitude

Measurement of α for flame-drawn fused silica fibers

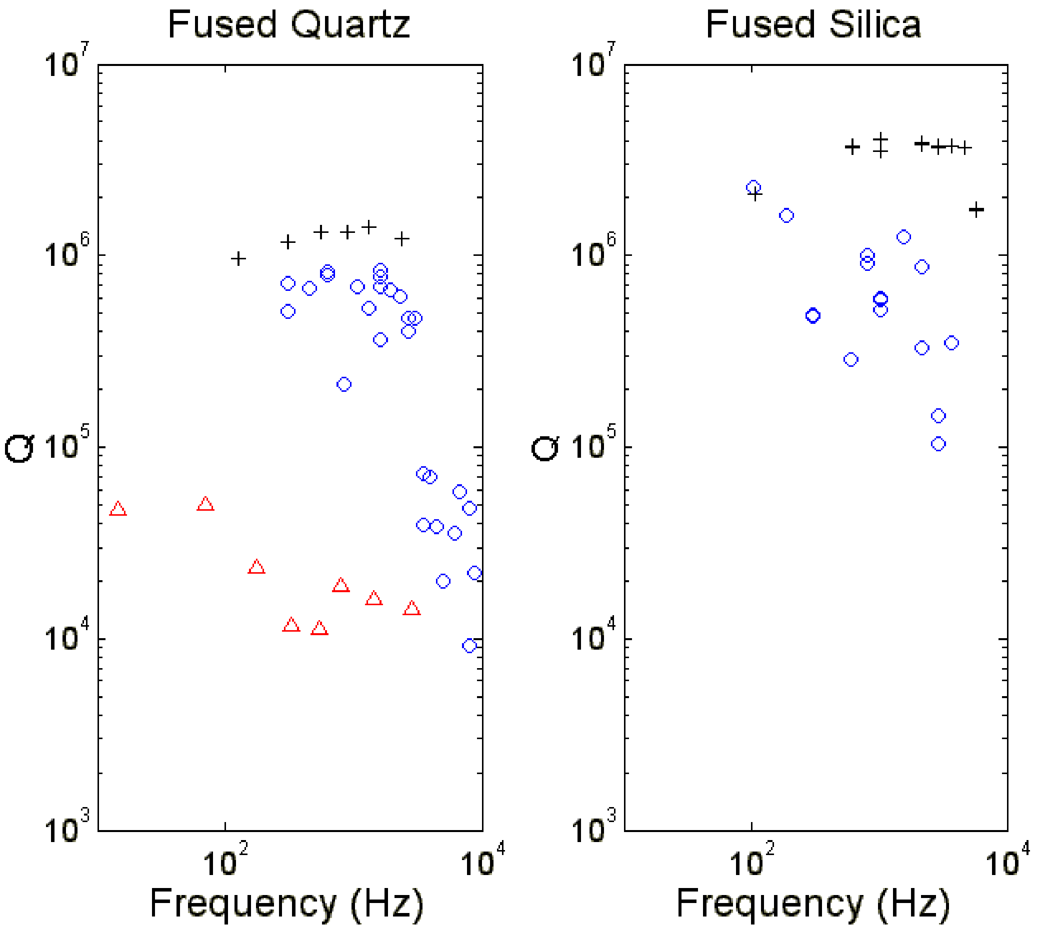
- Measured the Q’s of 16 flame drawn fibers 40 microns - 6 mm in diameter.
- Excited fiber modes and measured their ringdown times.
- Tried not to touch fiber surface but otherwise no special handling.

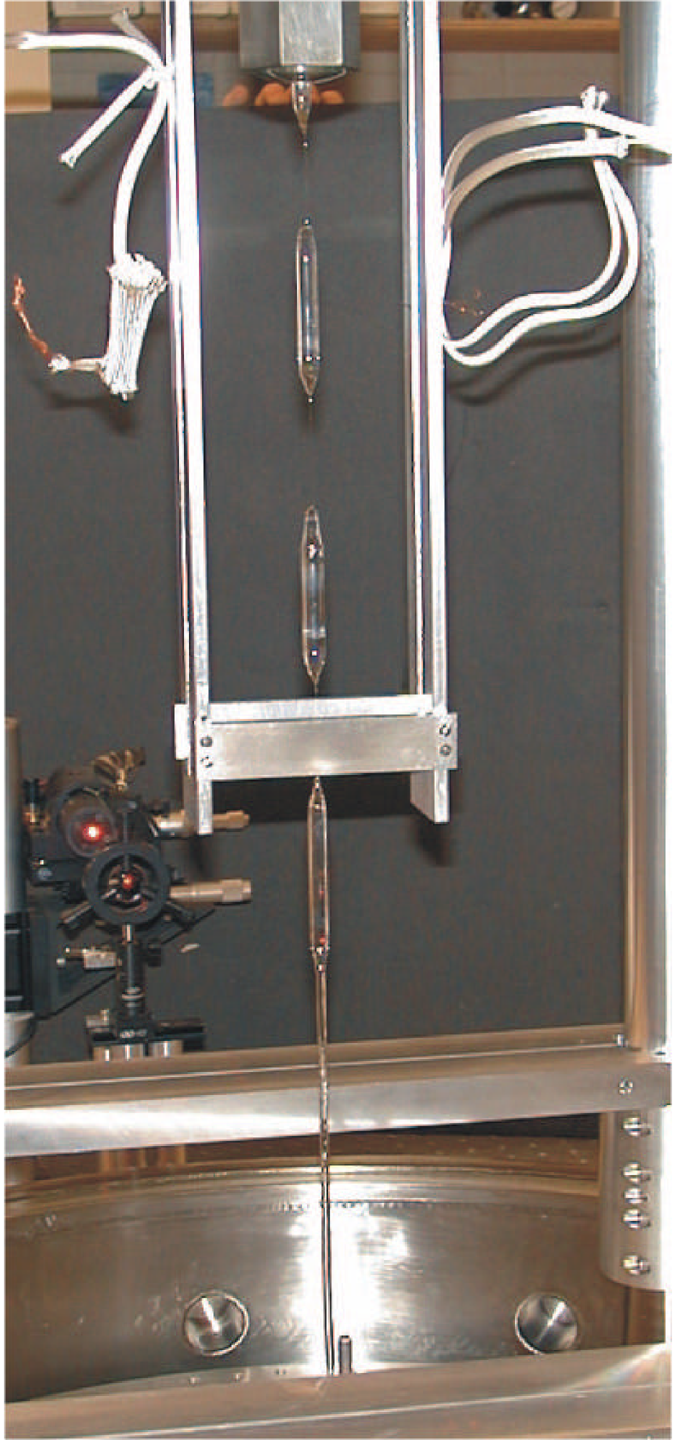
Setup


Support
Collet
Isolation fiber
Isolation bob
Shadow sensor
Fiber measured
LED
500V AC + 500V DC
Excitation
"comb" capacitor


Isolation fiber
Isolation bob
Taper
Measured fiber

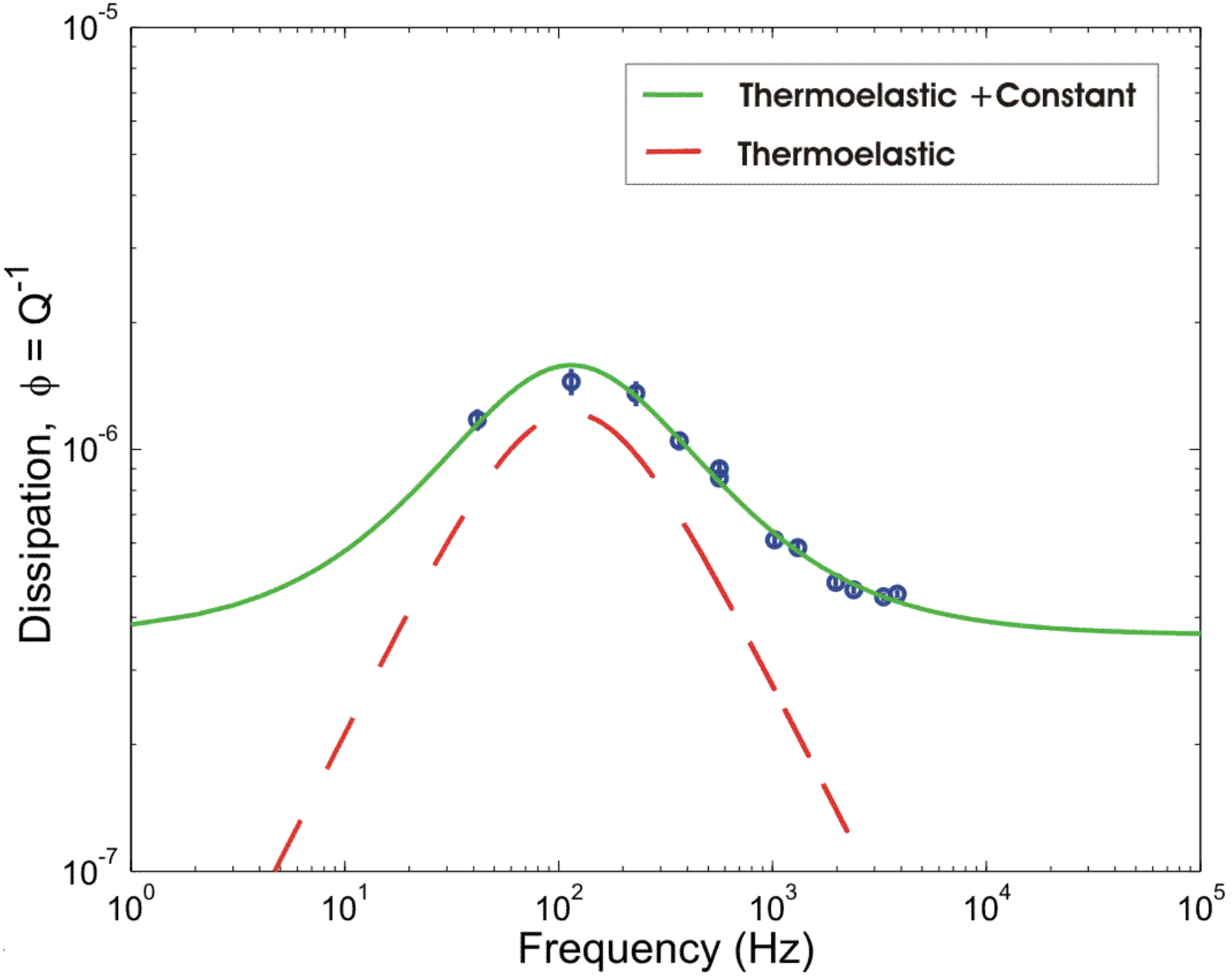
(getting rid of) Excess loss



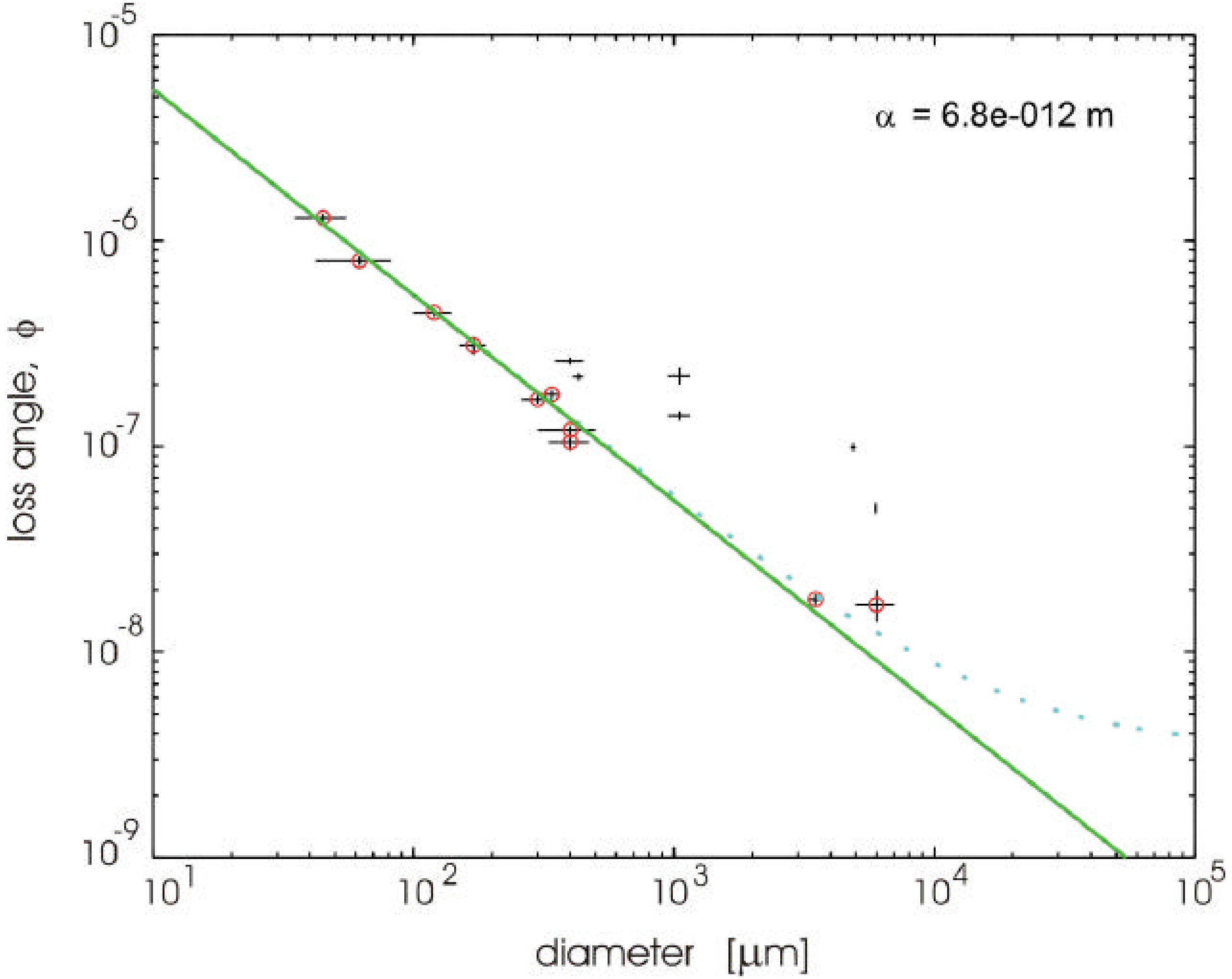


ϕ vs. frequency

(140 µm fiber)

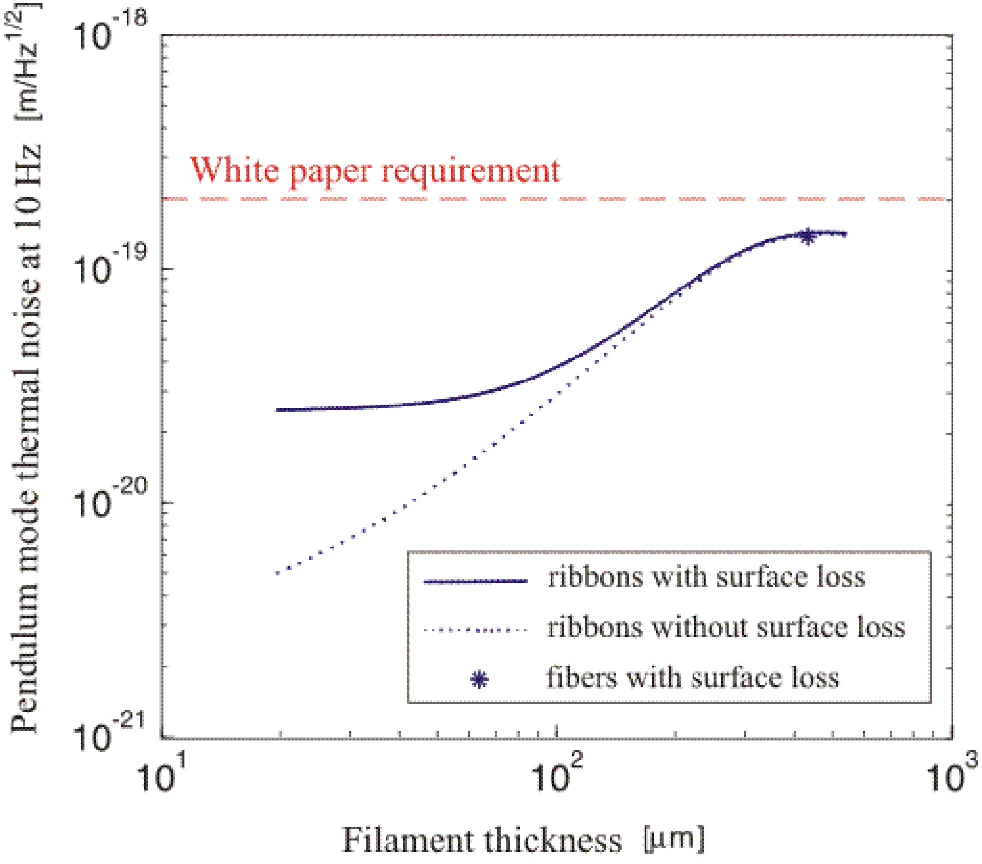


Loss vs. diameter

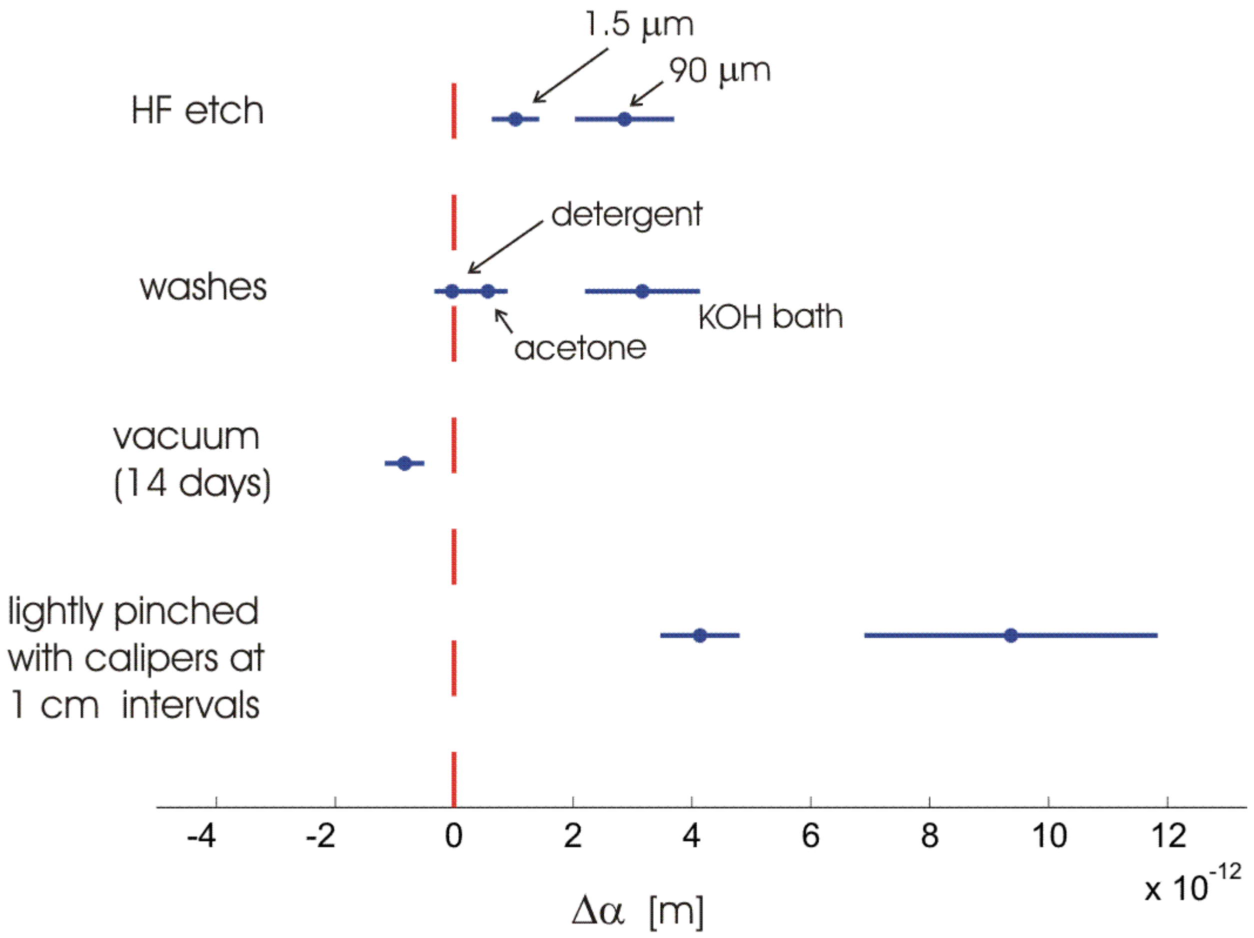


Effect on advanced LIGO thermal noise

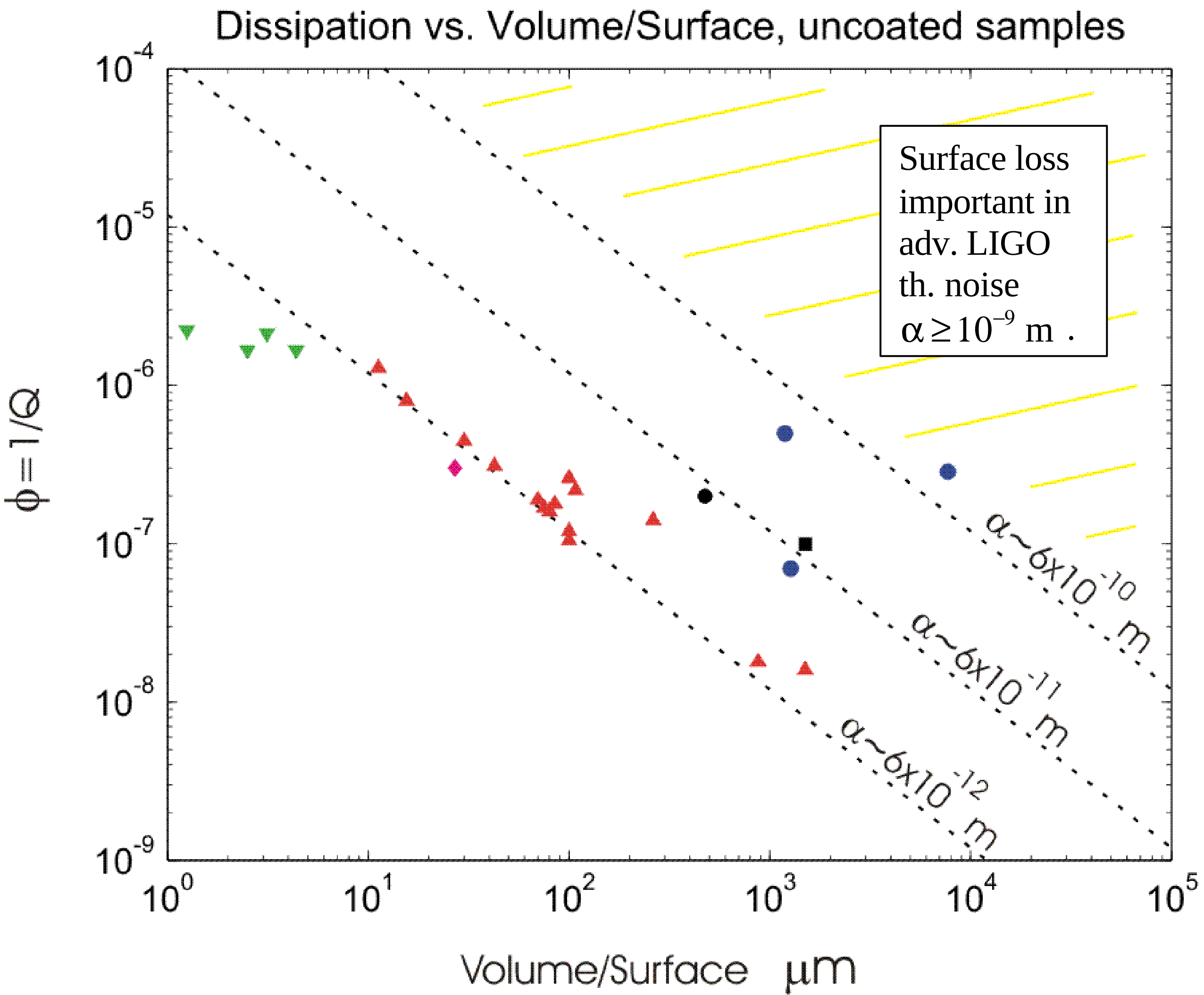
$$S_{\mathrm{pend}}(f) = \frac{k_B T g}{\pi^5 M L^2 f^5} \sqrt{\frac{Y}{12\sigma}} \left(d\phi_{\mathrm{bulk}} + d\phi_{\mathrm{th.el.}} + 6\alpha\right)$$



Surface treatments



Polishing

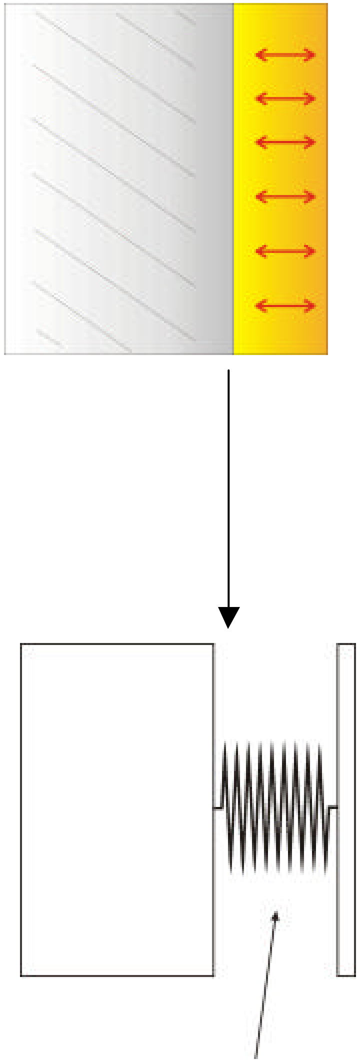


Surface loss important in adv. LIGO th. noise $\alpha \geq 10^{-9}$ m .

Cut and polished surfaces are 10-100x worse than “virgin” surfaces

- ▲ Flame-drawn fibers
- ♦ RF-drawn ribbon
- ▼ Laser-drawn fibers
- ● Highly polished disks
- ● Comm. polished disks
- ■ Comm. polished slides

Coating-induced thermal noise (baby model)



$$k = \frac{Y'\pi w^2}{d}$$

$$S_x(f) \approx \frac{2k_BT}{\pi k f}\phi_{\text{coating}} = \frac{2k_BT}{\pi^2 f}\left(\frac{d\,\phi_{\text{coating}}}{Y' w^2}\right)$$

$$\Rightarrow \sqrt{S_h(100\ \text{Hz})} \approx 4\times 10^{-20}\sqrt{d\,\phi_{\text{coating}}}$$

Need: $\boxed{d\,\phi_{\text{coating}} = \alpha < 2\times 10^{-9}\ \text{m}}$

If $d = 10^{-5}\,\text{m}$, need $\phi_{\text{coating}} < 2\times 10^{-4}$

Need coating material Q > 5000.

Typical bulk Q’s at room temperature

(10-1000 Hz)

Plastics	$10^1 - 10^3$
Metals	$10^3 - 10^4$
Ordinary glass	$10^4 - 10^5$
Fused Silica	$10^6 - 10^8$
Sapphire	$\sim 10^8$
Silicon	$\sim 10^8$

Coating-induced thermal noise (more careful model)

- Apply a cyclic Gaussian pressure distribution to the test mass and calculate the response [Levin, Bondu et al.].

$$p(r,\mathrm{t}) = \frac{2F}{\pi w^2}\exp\left(\frac{-2r^2}{w^2}\right)\sin\left(2\pi f\,\mathrm{t}\right)$$

- Add coatings [Gretarsson, Nakagawa et al.].
- Allow the loss angle of the coating to be anisotropic to reflect the layered structure.

Use the Fluctuation-Dissipation Theorem

Below resonance

$$\boxed{S_x(f) \approx \frac{2k_BT}{\pi k f}\phi}$$

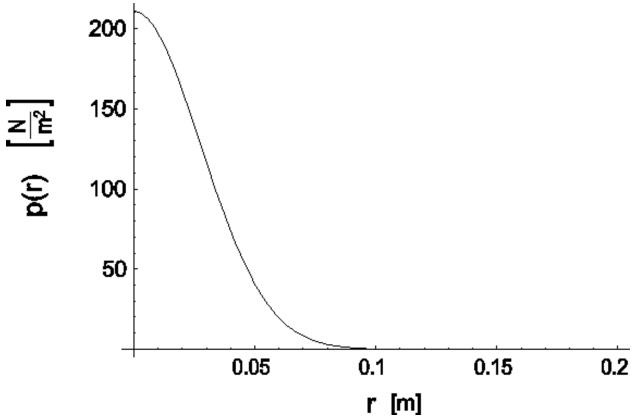
$$k = \frac{F^2}{2U}; \qquad \phi = \phi_{\text{bulk}} + \frac{\delta U_{\parallel}}{U}\phi_{\parallel} d + \frac{\delta U_{\perp}}{U}\phi_{\perp} d$$

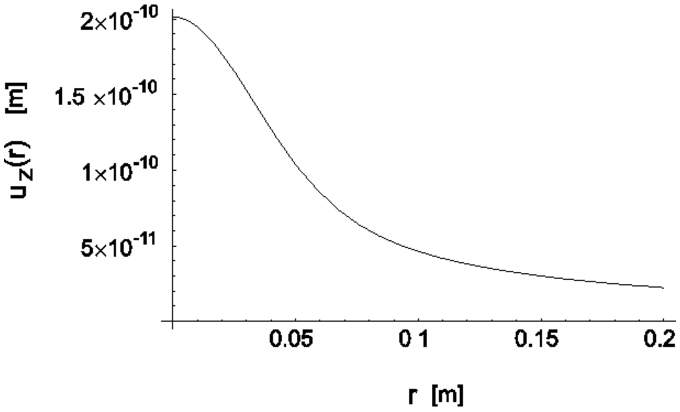
Solve static elastic equations to find U, $\delta U_{\parallel}$, and $\delta U_{\perp}$.

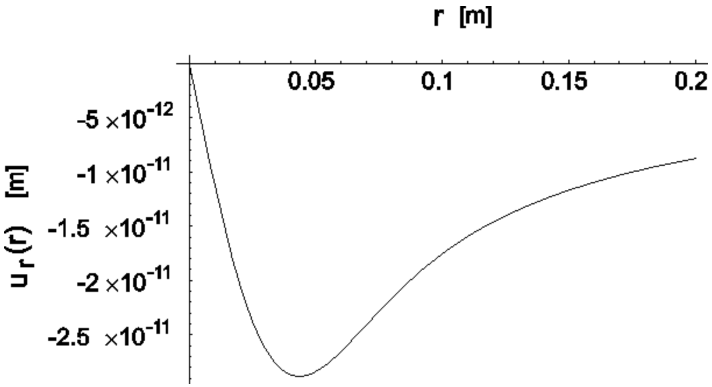
Use half-infinite test mass approximation.

Substitute into FDT to get expression for test mass thermal in noise in terms of $\phi_{\parallel}$ and $\phi_{\perp}$.

Response to Gaussian pressure







Result

$$S_x(f) = \frac{2k_BT}{\pi^{\frac{3}{2}} f}\,\frac{1-\sigma^2}{wY}\left\{\phi_{\text{substrate}} + \frac{1}{\sqrt{\pi}}\,\frac{d}{w}\,\frac{1}{YY'(1-\sigma'^2)(1-\sigma^2)}\right.$$

$$[Y'^2(1+\sigma)^2(1-2\sigma)^2\phi_{\parallel} + YY'(1+\sigma')(1+\sigma)(1-2\sigma)(\phi_{\parallel}-\phi_{\perp})$$

$$\left. + Y^2(1+\sigma')^2(1-2\sigma')\phi_{\perp}]\right\}$$

Limit $\phi_{\parallel} = \phi_{\perp}$ *agrees w/Nakagawa (private comm.)*

$$\boxed{\xrightarrow{\sigma,\sigma'\to 0}\quad \frac{2k_BT}{\pi^{\frac{3}{2}} f}\,\frac{1}{wY}\left\{\phi_{\text{substrate}} + \frac{d}{w\sqrt{\pi}}\left(\frac{Y'}{Y}\phi_{\parallel} + \frac{Y}{Y'}\phi_{\perp}\right)\right\}}$$

Need to measure: $\phi_{\parallel}$, $\phi_{\perp}$, and Y'.

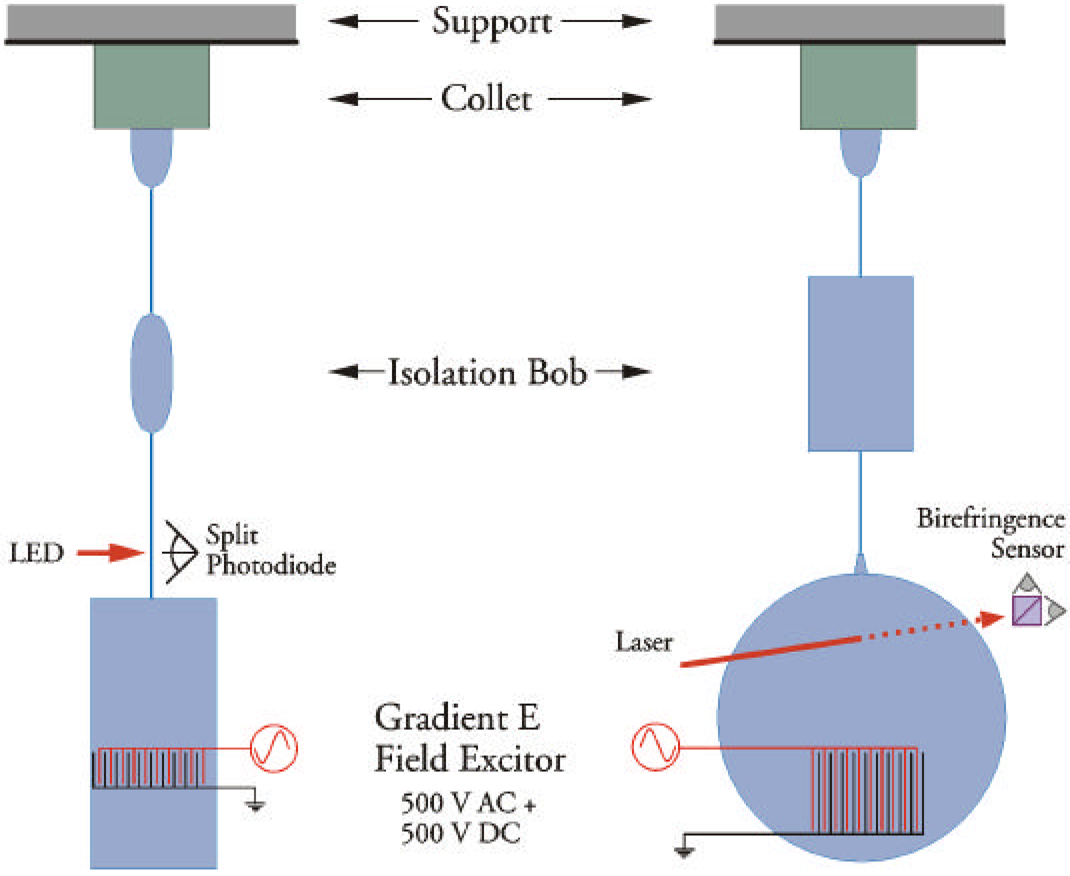
Measuring the loss in Ta_2O_5/SiO_2 coatings

- Compare the ringdown times of coated and uncoated samples.
- Extract $\phi_{\parallel}$ for the coating loss via

$$\phi_{\text{coated}} \approx \phi_{\text{uncoated}} + \frac{\delta U}{U}\phi_{\parallel} d$$

- But can’t measure $\phi_{\perp}$ in ringdown experiments.

Slide and disk Q measurement setup



Slide

Disk

(Figure taken from G. Harry et al., 2001)

Measured Q's

Slides

Slide	Coating	Mode	Frequency	Q
A	HR	2	1022 Hz	$1.1 \pm 0.5 \times 10^5$
	HR	3	1944 Hz	$1.6 \pm 0.1 \times 10^5$
	HR	4	2815 Hz	$1.6 \pm 0.1 \times 10^5$
B	HR	2	962 Hz	$1.3 \pm 0.1 \times 10^5$
C	none	2	1188 Hz	$4.0 \pm 0.2 \times 10^6$
	none	3	2271 Hz	$4.9 \pm 0.3 \times 10^6$

Disk

Hanging Number	Coating	Frequency	Q
1	none	4107 Hz	$3.46 \pm 0.02 \times 10^6$
2	none	4107 Hz	$3.10 \pm 0.007 \times 10^6$
3	HR (45°)	4108 Hz	$1.28 \pm 0.02 \times 10^6$
4[†]	HR (45°)	4121 Hz	$1.24 \pm 0.001 \times 10^6$

[†] Ear was sheared off twice before this hanging.

Final Result

$$\boxed{\phi_{\text{coated}} \approx \phi_{\text{uncoated}} + \frac{\delta U}{U}\phi_{\|} d}$$

$\mathrm{O}(10^{-5}\,\mathrm{m})$

$1/Q_{\text{coated}}$

$1/Q_{\text{unccoated}}$

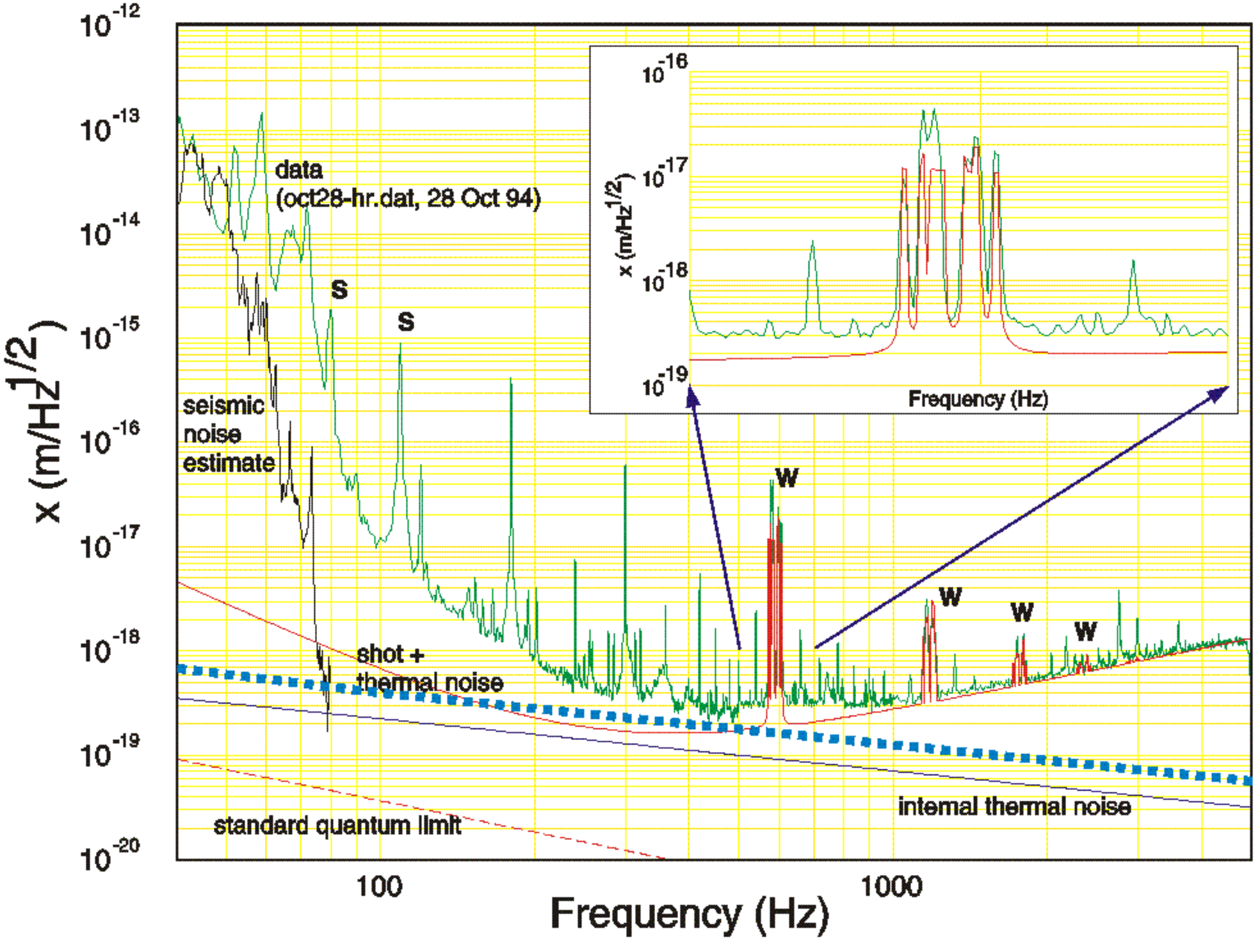
Calculate from mode shape and sample geometry $\mathrm{O}(10^{3}\,\mathrm{m}^{-1})$

Commercial polish slides: $\phi_{\|} = (4.2 \pm 0.3)\times 10^{-4}$

"Superpolished" disk: $\phi_{\|} = (1.0 \pm 0.3)\times 10^{-4}$

Caltech 40 meter (Oct. ’94)

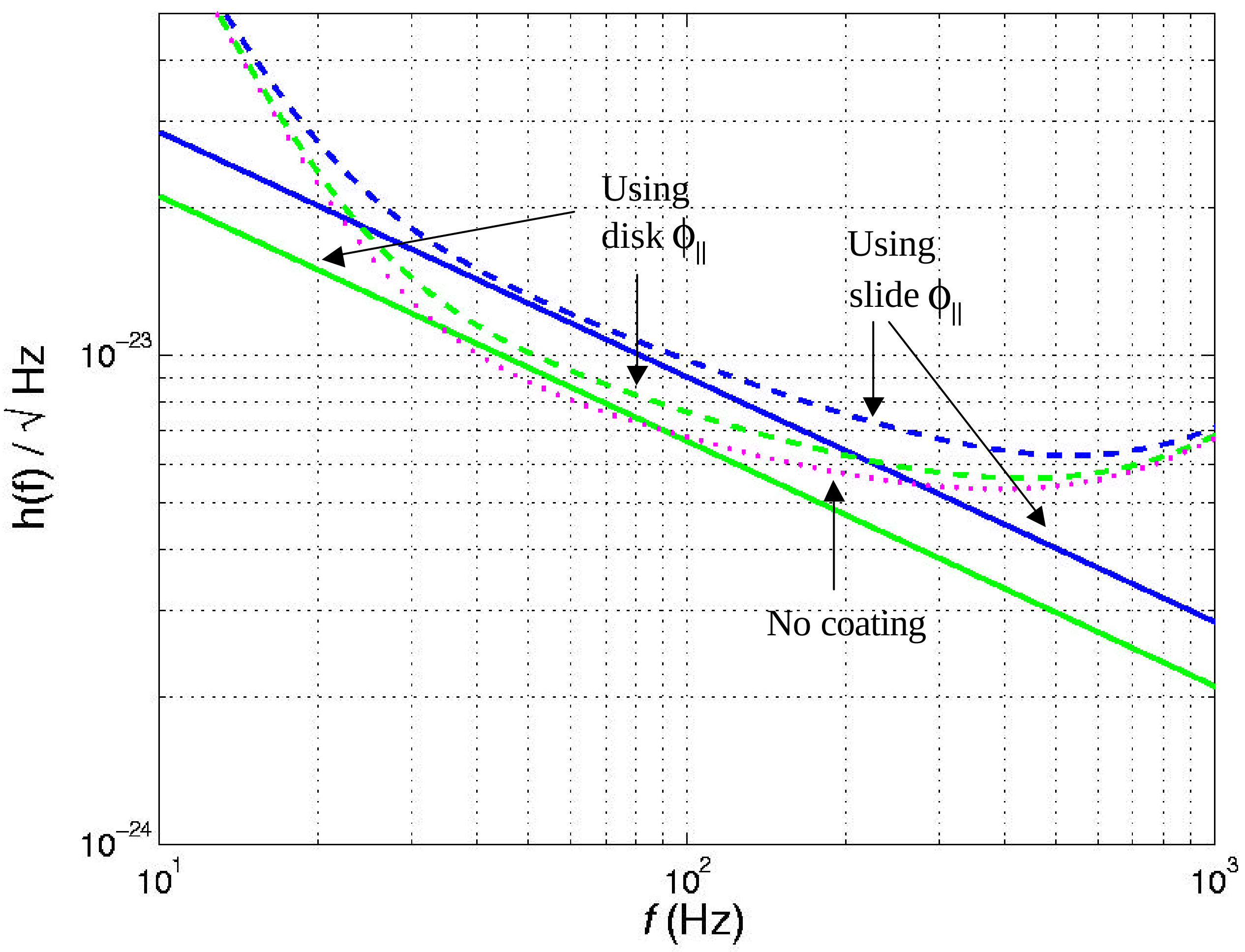
$$\phi_{\parallel} = \phi_{\perp} = 1\times 10^{-4}$$



Implications for advanced LIGO

Test mass material	Coating loss	Q_{eff} $(=1/\phi_{\mathrm{readout}})$	Structural thermal noise at 100 Hz, $\sqrt{S_h}$
Sapphire	none	200×10^6	1×10^{-24}
	$\phi_{\parallel} = 1 \times 10^{-4}$	15×10^6	3×10^{-24}
	$\phi_{\parallel} = 4 \times 10^{-4}$	4×10^6	5×10^{-24}
Fused silica	none	30×10^6	6×10^{-24}
	$\phi_{\parallel} = 1 \times 10^{-4}$	19×10^6	7×10^{-24}
	$\phi_{\parallel} = 4 \times 10^{-4}$	9×10^6	9×10^{-24}

Coated fused silica test masses

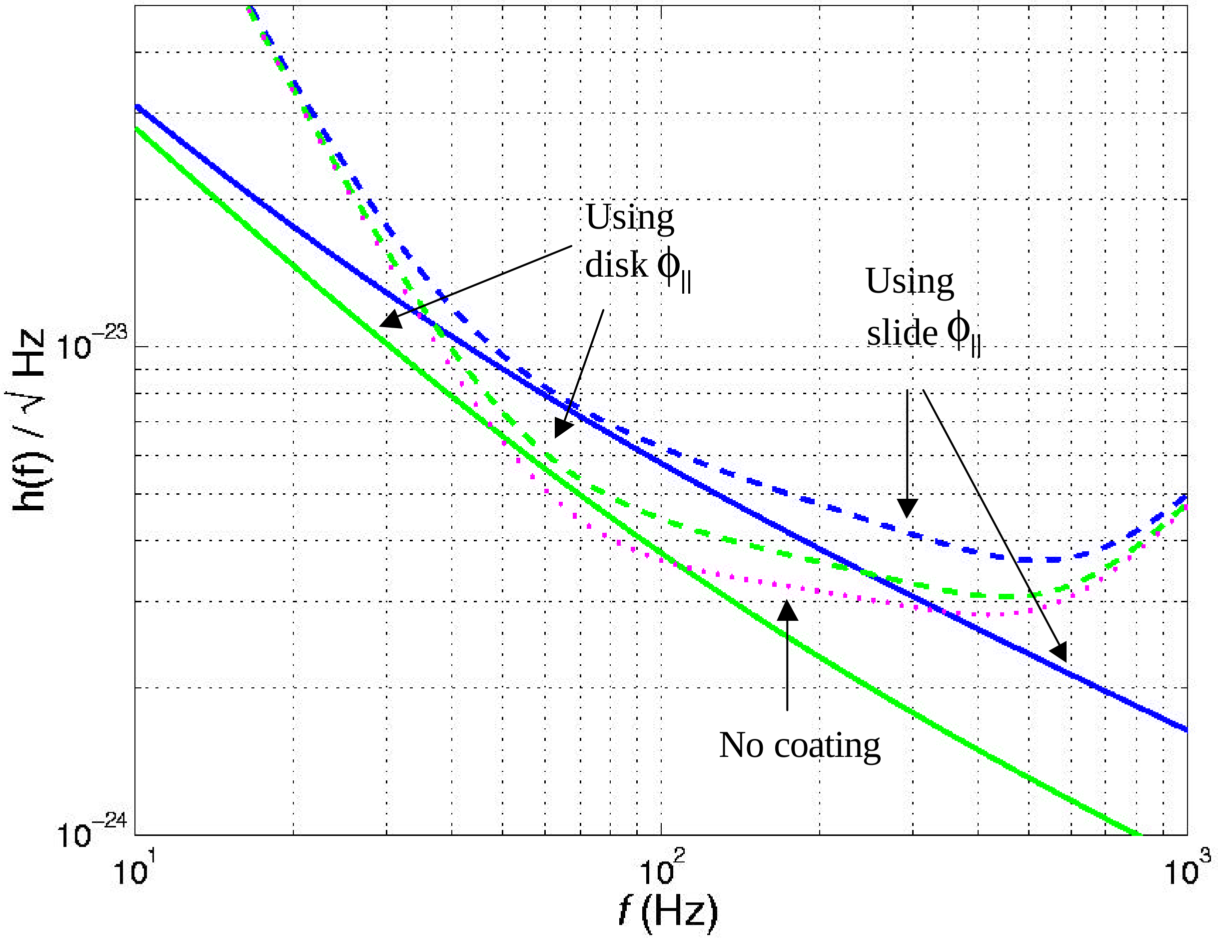


Using disk $\phi_{||}$

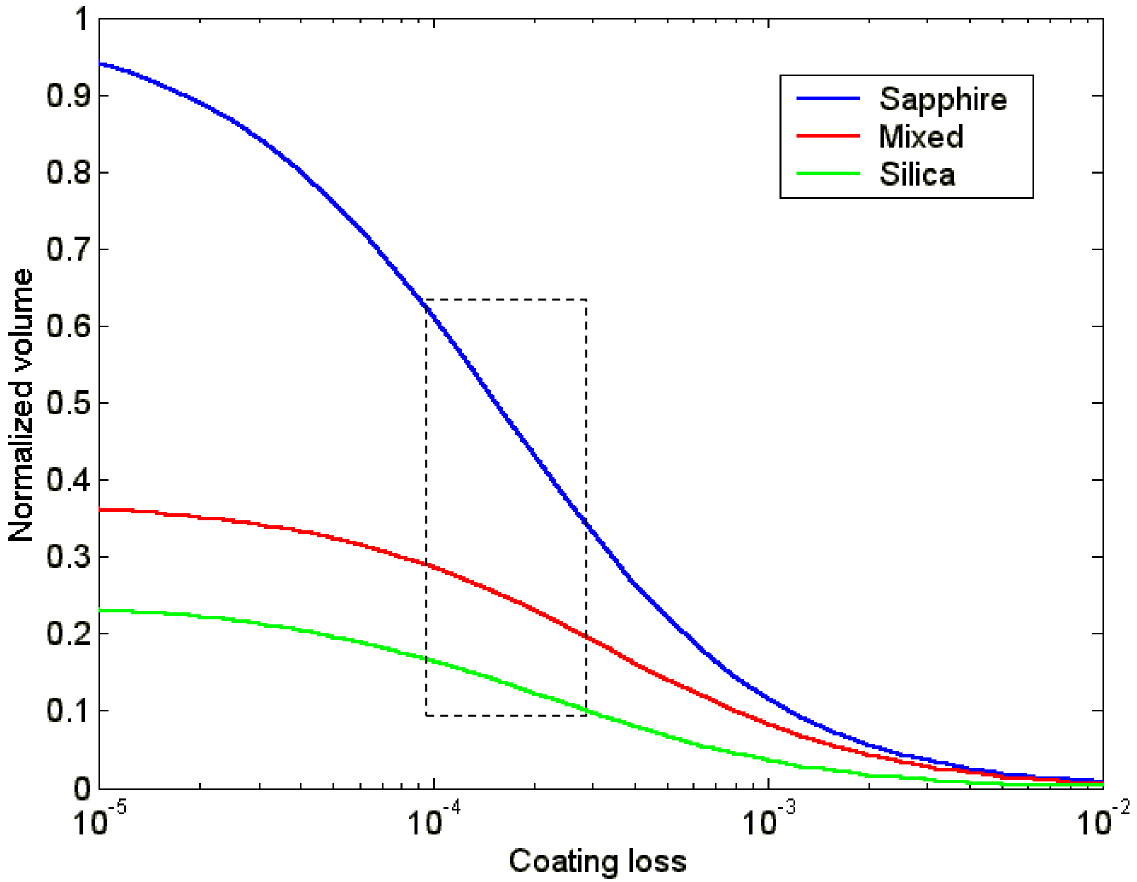
Using slide $\phi_{||}$

No coating

Coated sapphire test masses

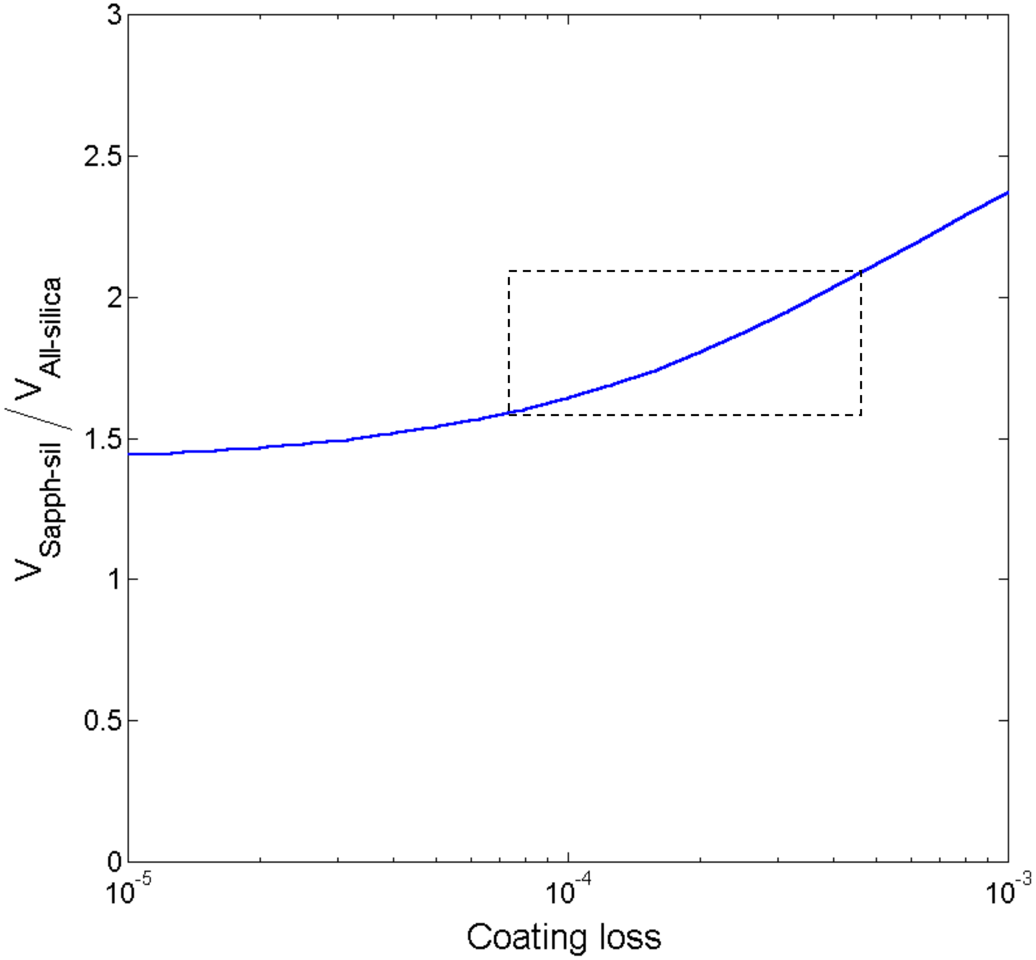


Sensitivity as a function of coating loss



Caveat: This calculation done for half-infinite test masses

Mixed (sapph-sil) versus all-silica



If Sapphire cannot be made with sufficiently low absorbtion/ biref., need to go to silica ITM’s and BS’s.

Th. n. from coatings is dominated by the thick ETM coatings.

Gain by using sapphire ETM’s because of high Young’s modulus.

Comparison of sources of surface loss

Source of loss	α
Contamination by water (Flame drawn fiber surface)	6×10^{-12} m
Accidental knocking of surface (Caliper damage)	$\sim10^{-11}$ m
Abrasion by polishing (polished samples)	10^{-11} - 10^{-9} m
Application of 10μm HR-coating (coated samples: $\alpha=\phi_{\|\|}d$)	$(1\text{-}4)\times10^{-9}$ m

Summary

- Surface loss tends to be dominant source of loss in high Q materials.
- Ribbons thinner than ~80 μm do not lead to lower suspension thermal noise.
- The surface of uncoated, polished silica is sufficiently good for adv. LIGO (but close).
- For adv. LIGO, the addition of coatings on sapphire may decrease the volume sensitivity to ~ 0.4 - 0.8 of the uncoated value.