LIGO-G010313-00-Z

Coherence of LLO and LHO Power Monitors

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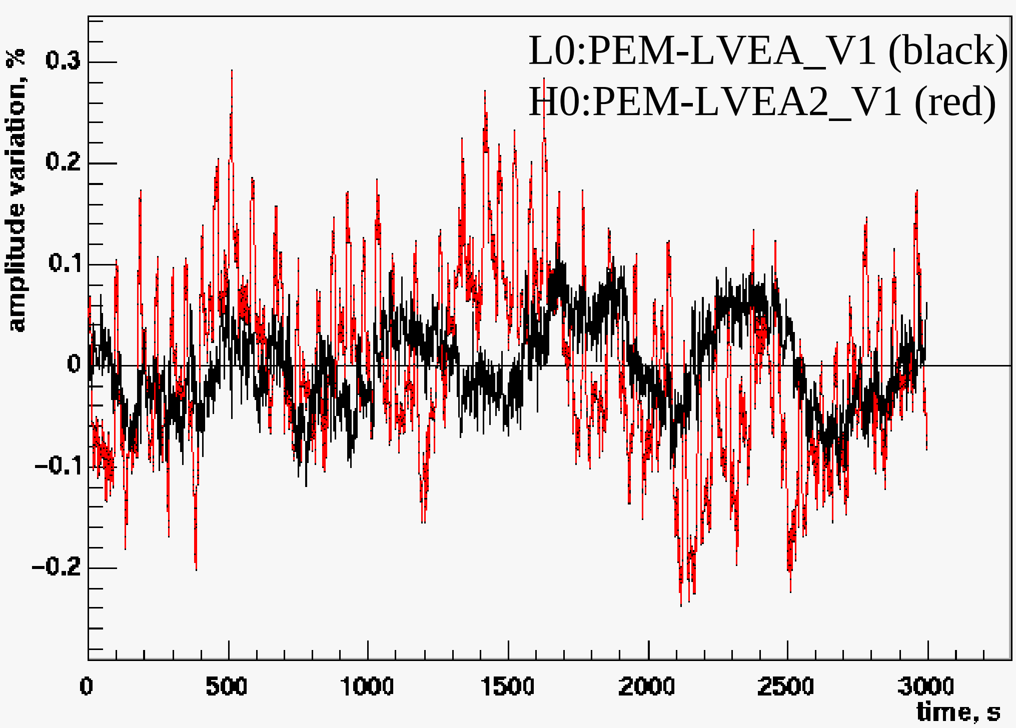
- **Outline**
  - Parameters of PM signals
  - Coherence
  - Coherence time scale
  - Results
  - Conclusion

Parameters of 60Hz signal

- Approximately 10 hour of data, starting at GPS time 668212508 was used in this analysis.
- The parameters of 60Hz PM signals were measured using data segments of one second long.
- For each data segment , the average amplitude and phase Ψ were measured with LineMonitor
- The power frequency was estimated as a derivative of the phase Ψ(t).

Amplitude variation ~10^{-3}

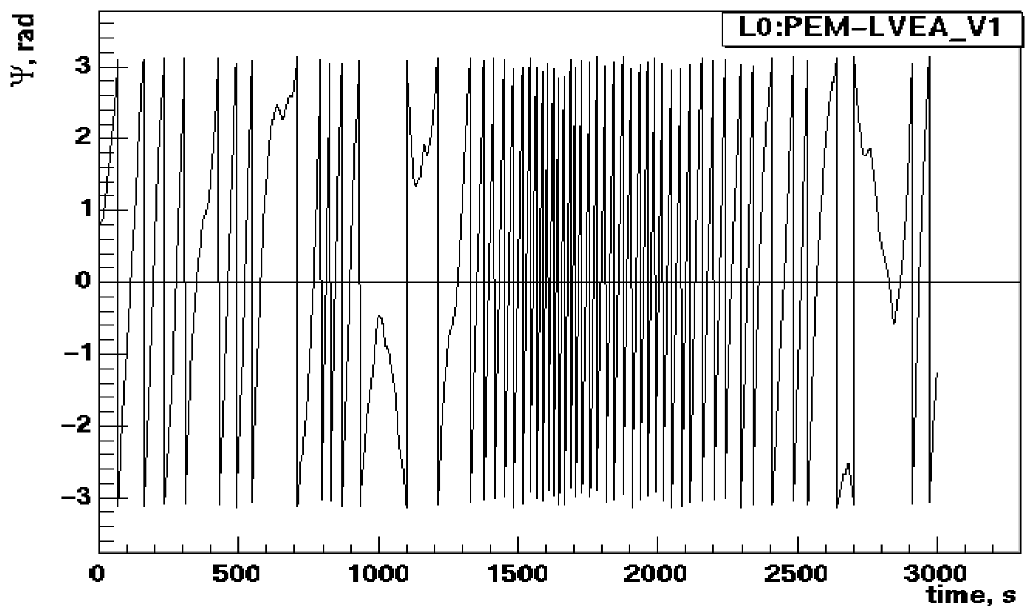


Phase

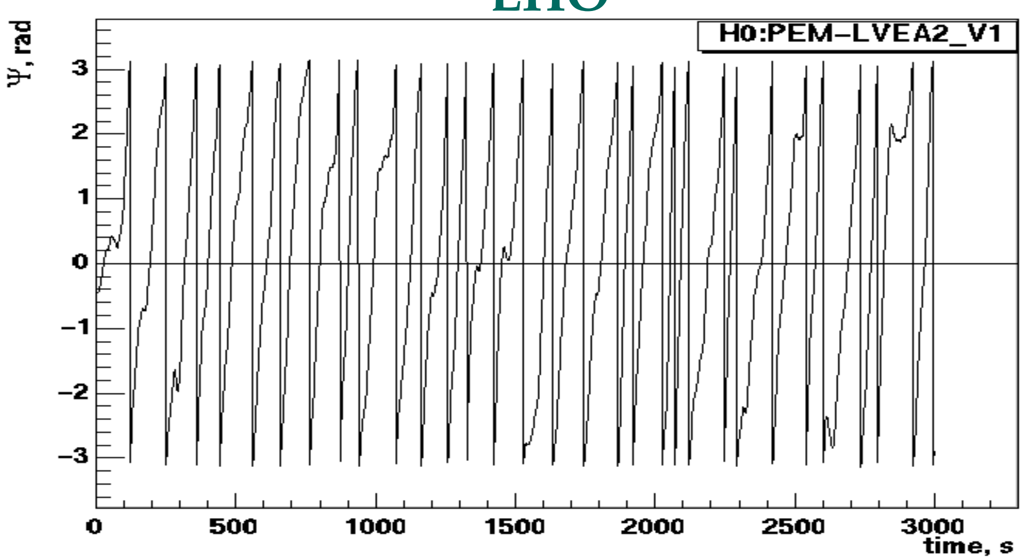
$$S(t) = a(t)\cos(\Psi(t))$$

- for monochromatic signals $\Psi(t)$ is a linear function of time

LLO

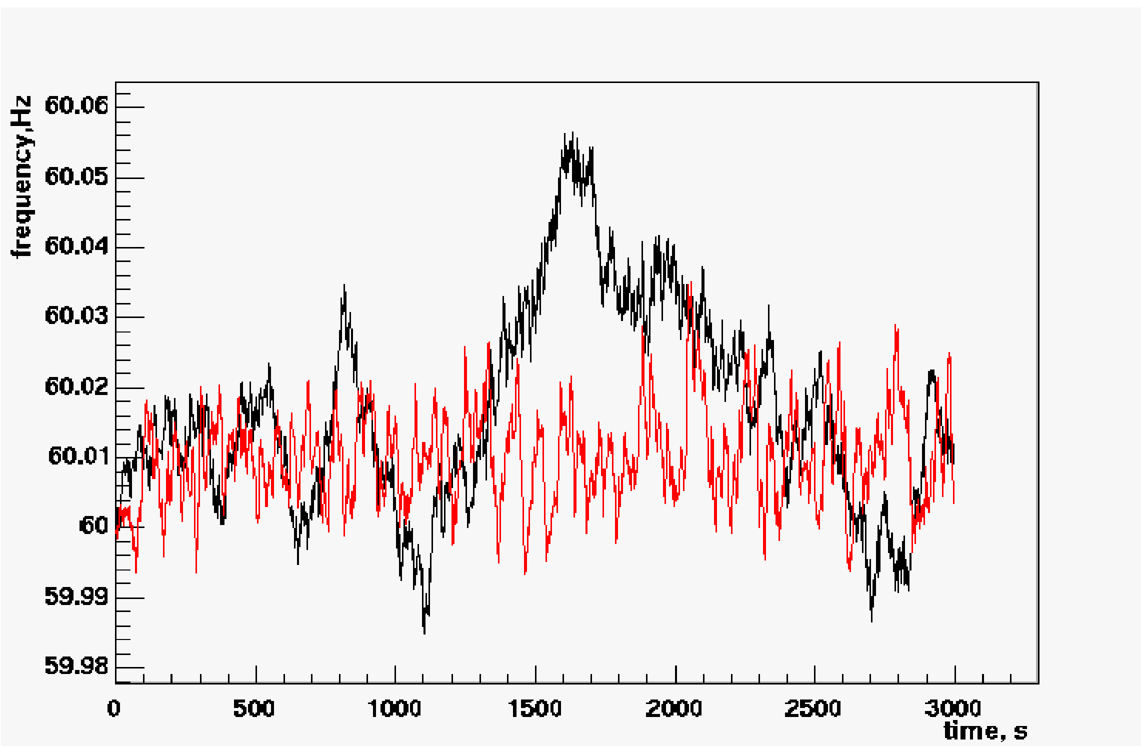


LHO



Frequency

- During one second time interval the power frequency doesn't change much.
- The Line Monitor measures average frequency, which is used as an estimate of instantaneous power frequency.
- The measured frequency may vary with time



Power frequency for the L0:PEM-LVEA_V1 (black) and H0:PEM-LVEA2_V1 (red) channels

Coherence

- A sum of two harmonic oscillations $s_L(t)$ and $s_H(t)$ with the same frequency is also a harmonic oscillation

$$s(t) = s_L(t) + s_H(t) = A \cdot \sin(wt + q)$$

- the amplitude A is given by

$$A^2 = a_L^2 + a_H^2 + 2a_L a_H \cos(f_L - f_H),$$

- the average (over the time interval T) square
- amplitude is

$$\overline{A}^2 = a_L^2 + a_H^2 + 2a_L a_H \frac{1}{T}\int \cos(f_L - f_H)dt.$$

$$\overline{A}^2 = a_L^2 + a_H^2 + 2a_L a_H \overline{\cos(\Delta f)}.$$

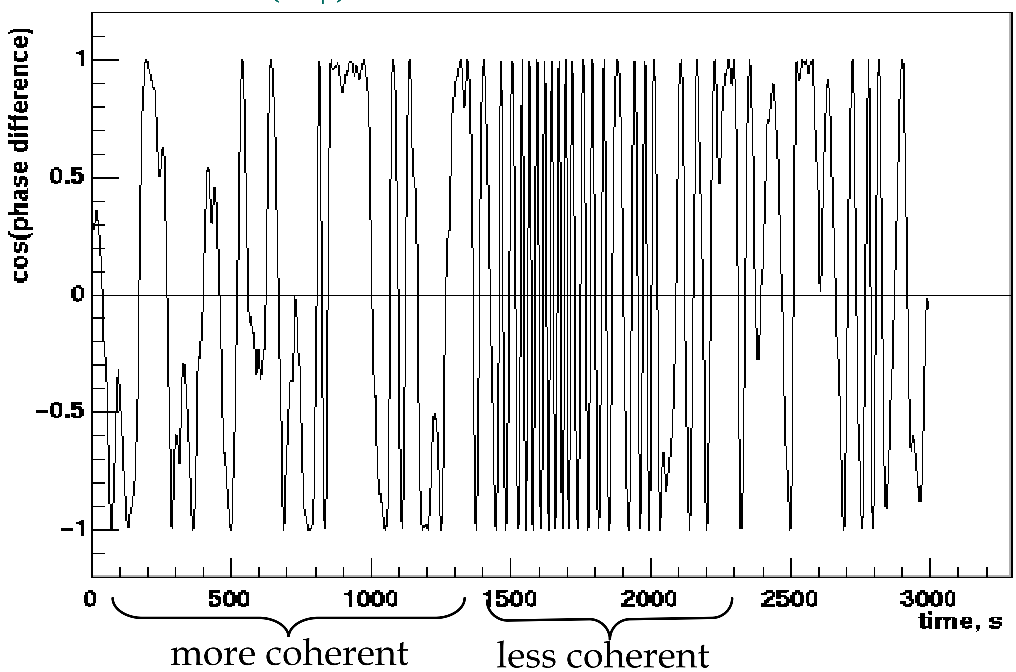
- If the phase difference remains constant during time T, the signals $s_L(t)$ and $s_H(t)$ are coherent.
- If the phase difference changes randomly in time the $s_L(t)$ and $s_H(t)$ are not coherent and the interference term is zero.
- Coherence coefficient

$$g = \tfrac{1}{N}\left|\sum_{k=1}^{N} \exp\left(i\Delta f_k\right)\right| \qquad N = T_{tot}/T$$

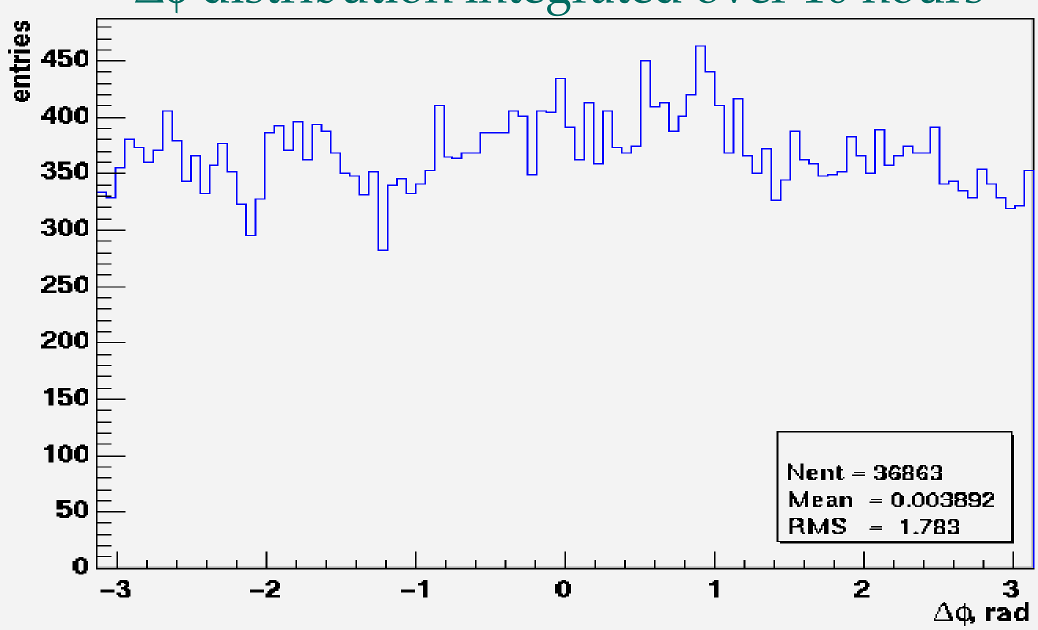
- Coherence time scale

Phase Difference

cos(Δϕ) as a function of time



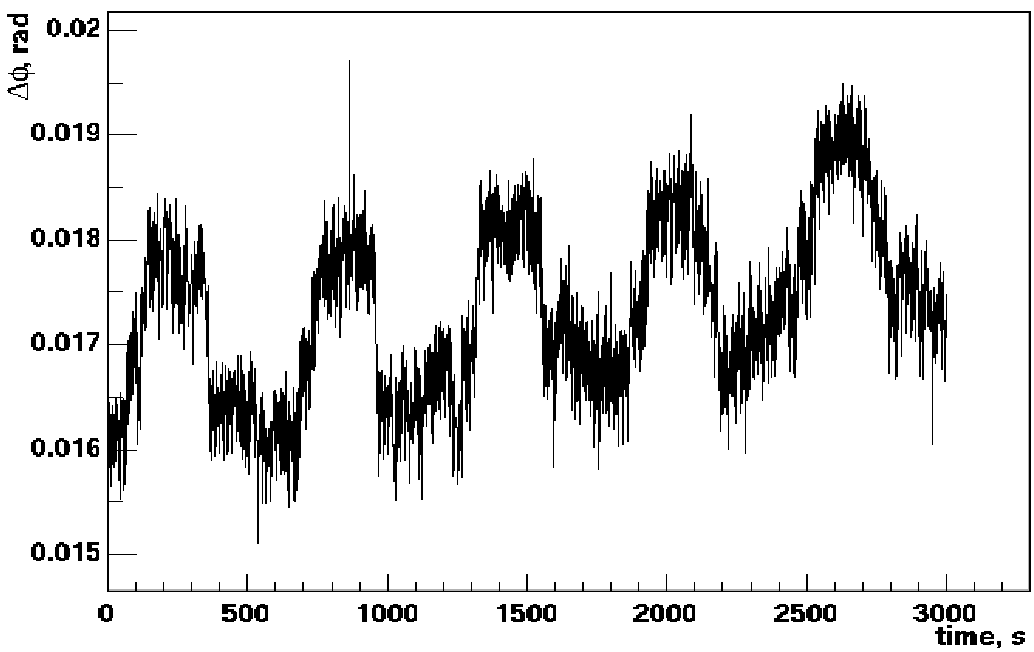
Δϕ distribution integrated over 10 hours

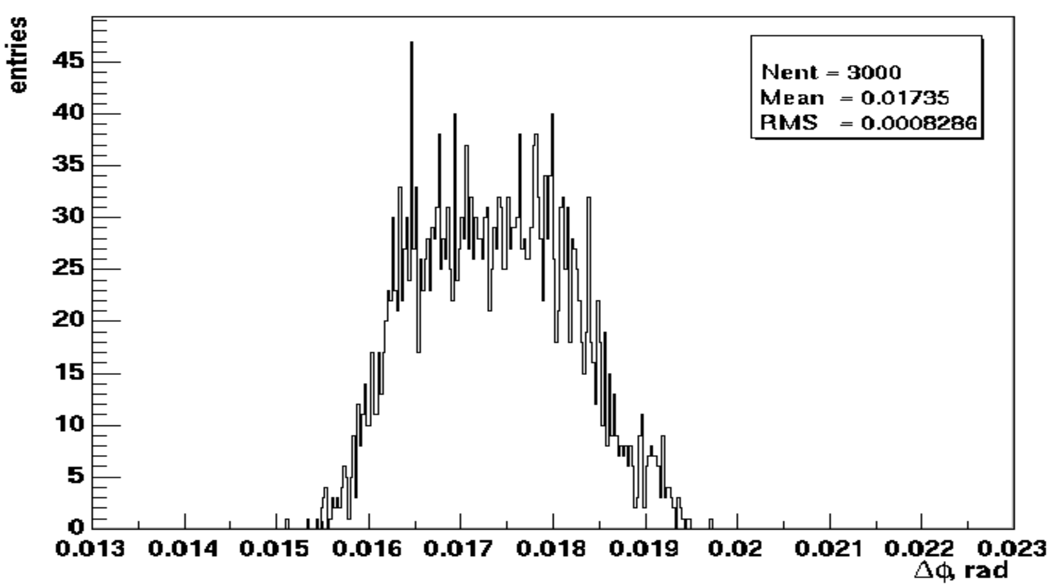


Power coherence for the same site

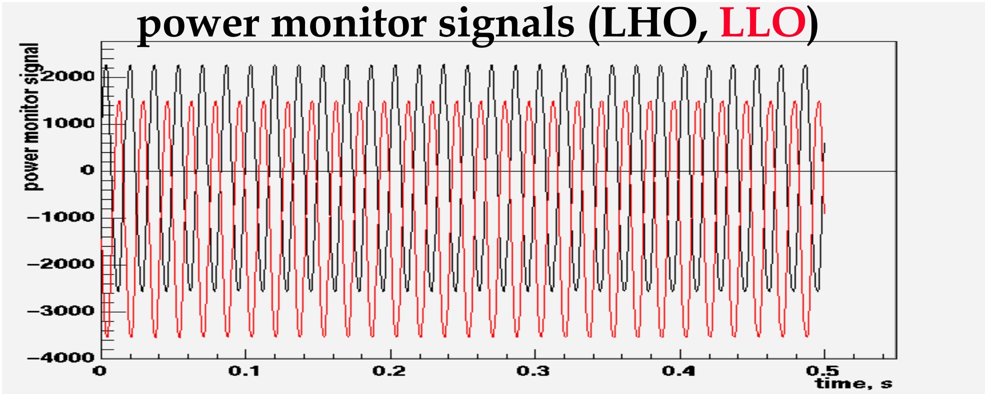
- coherence of L0:PEM-LVEA_V1 and L0:PEM-EX_V1

$\Delta\phi$ rms ~ 1mrad



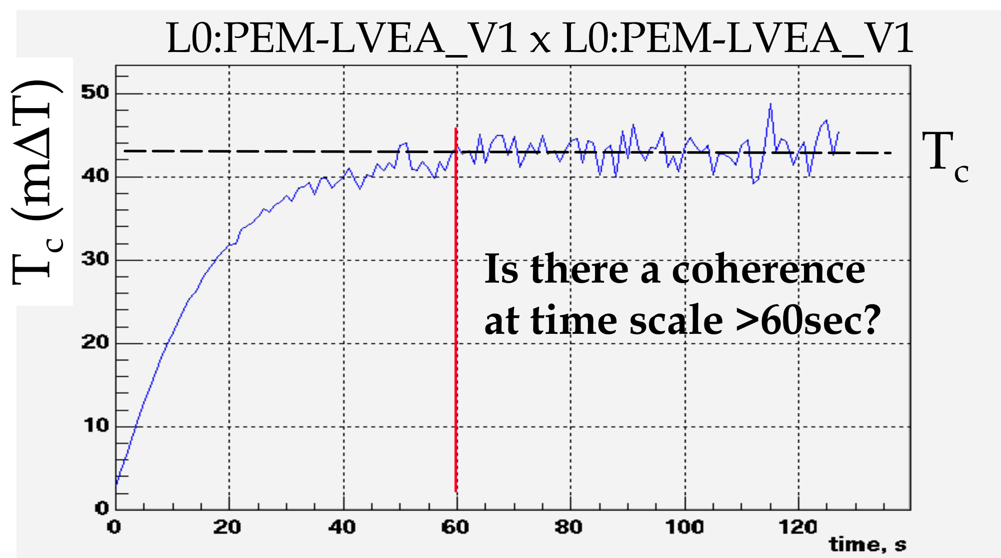


Coherence Time Scale



$$T_c(T_s) = T_s \cdot \overline{g(m)^2}$$

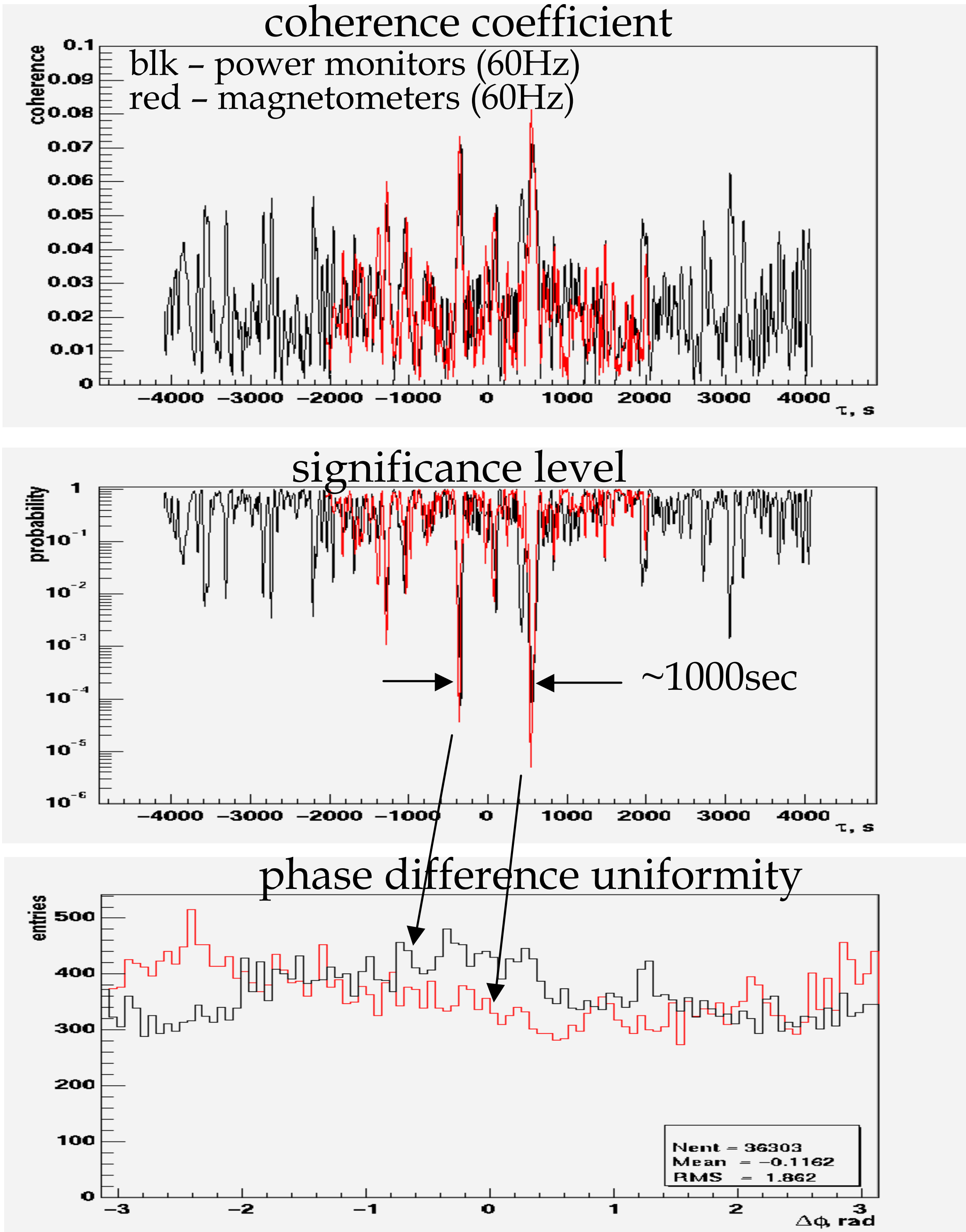
- T_s – time scale
- m $=T_s/\Delta T$ – number of samples
- ΔT – sampling interval
- $T_c = \Delta T =$ const for white noise



- T_{tot}/T_c – effective number of samples

Coherence & Significance Level

- the coherence of signals $s_L(t)$ and $s_H(t+\tau)$, where τ is a time delay between two signals.



Interpretation

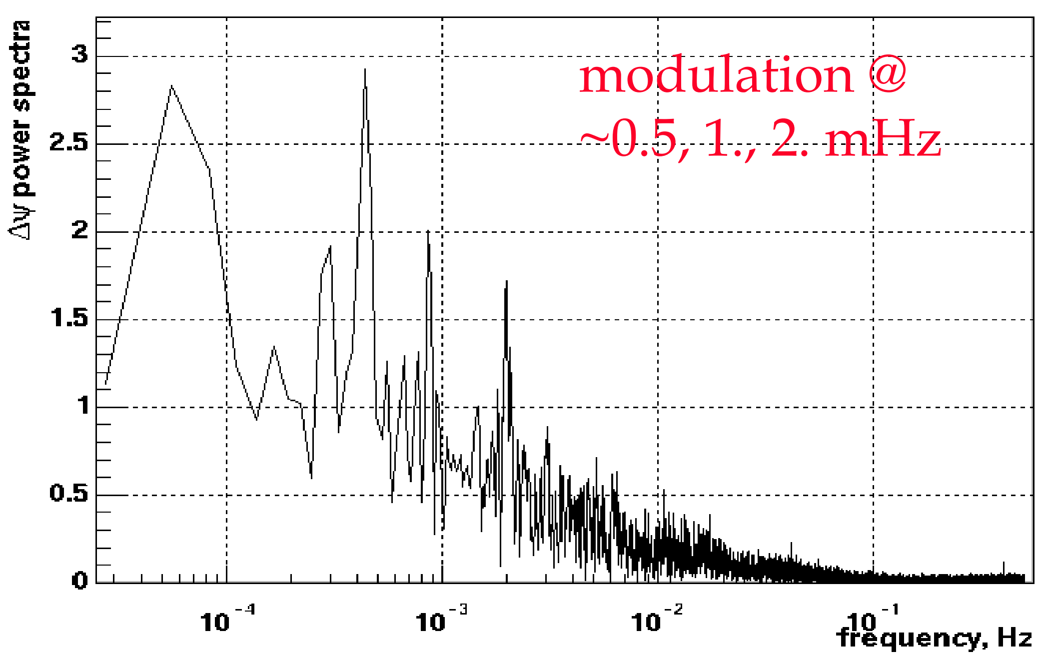
- One possible explanation of the observed coherence
  - In ideal case the phase of each monitor is

$$f(t)=w_0 t + const$$

  - In real life: $f(t)=w_0 t + j(t) + const$, where $j(t)$ is (hopefully) a random phase
  - If $j(t)=r\cos(nt+q) + h(t)$ and ν is the same for LLO & LHO,

$$\Delta f = r_L \cos(nt + q_L) - r_H \cos(nt + q_H) + h_L(t) - h_H(t)$$

  - If θ_L and θ_H are constant, frequency ν can be seen at Δφ Fourier spectra.



Conclusion

- 60Hz power line is coherent at time scale <1min. The coherence time is ~42sec
- Long term (>1min) correlation between LHO and LLO 60 Hz power lines is observed
- Although, there is some indication of power correlation between the LHO and LLO sites for this particular interval of time, they may not be coherent in a longer run.
- To conclude if there are periods of time when the LLO-LHO coherence time is much longer then 1 minute, 24 hours of data for different days of week should be analyzed.