Setting an Upper Limit on Stochastic Background Signals

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LSC Meeting, LIGO Hanford Observatory (LHO)
August 15, 2001
LIGO-G010304-00-Z

Cross-correlation statistic

Define:

$$Y = \int_{-T/2}^{T/2} dt \int_{-T/2}^{T/2} dt' \, h_1(t) \, Q(|t-t'|) \, h_2(t') = \langle h_1, h_2 \rangle$$

where

$h_1(t) = s_1(t) + n_1(t)$: output of GW detector 1

$h_2(t) = s_2(t) + n_2(t)$: output of GW detector 2

$Q(|t-t'|)$: optimal filter, which maximizes the SNR of Y

$$\tilde{Q}(f) = \lambda \frac{\gamma(f)\Omega_{\mathrm{gw}}(f)}{f^3 P_1(f) P_2(f)}$$

Assumptions/Properties

1. Stochastic background:
   (a) Gaussian, stationary
   (b) unpolarized and isotropic
   (c) $\Omega_{\text{gw}}(f) = \Omega_0 = \text{const}$
2. Detector noise:
   (a) Gaussian, stationary
   (b) noise power $\gg$ stochastic background signal strength
   (c) uncorrelated between the detectors
3. Cross-correlation statistic:
   (a) Gaussian random variable
   (b) mean: $\mu \propto \Omega_0^2$
   (c) variance: σ^2 dominated by autocorrelated detector noise

Measurements

$Y_1, Y_2, \cdots, Y_N$: measured values of the CC statistic for each $T \sim 1$ min stretch of data ($N > 10^4$ for E6)

Histogram:

$\overline{Y} = \frac{1}{N}\sum_{i=1}^{N} Y_i$: sample mean of Y_i

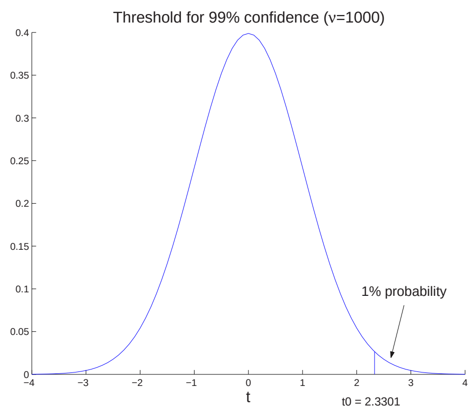
$s^2 = \frac{1}{N-1}\sum_{i=1}^{N}(Y_i - \overline{Y})^2$: sample variance of Y_i

Method I

Use Student's t-test to rule on the presence or absence of a SB signal

1. Pose the null hypothesis H_0: $\mu = 0$

2. Set a threshold t_0 using the t-distribution so that, when H_0 is true, $t > t_0$ in less than a fraction α (e.g., 1%) of all observations.



3. The test:

   (a) Define:

$$t = \frac{\overline{Y}}{s/\sqrt{N}}$$

   (b) If $t > t_0$, reject the null hypothesis and conclude we have detected a SB with significance $1 - \alpha$ (e.g., 99%).

   (c) If $t \leq t_0$, accept the null hypothesis and conclude the observed data is consistent with the absence of a SB.

NB: If $\overline{Y}$ has a large cross-correlated noise component, we may falsely claim the presence of a SB.

Method II

Use Feldman-Cousins approach to set an upper limit or confidence interval on μ given the measurement $\overline{Y}$.

1. Analytically: Assuming Y is Gaussian distributed with variance $\sigma^2 = s^2$ for all μ.

2. Numerically: Injecting simulated SB signals of known strengths into the data streams.

NB: Conservative upper limit since we are assuming no cross-correlated environmental or instrumental noise.

Refinements/Alternatives

1. Estimate the cross-correlated noise component by analyzing data stretches shifted in time by amounts $>$ light travel-time between the two detectors.

   NB: Only persistent, long-term cross-correlated noise components are accounted for.

2. Throw away outliers in the measured data $Y_1, Y_2, \cdots, Y_N$ (e.g., by looking at a $\log(n)$ vs. $\log(Y)$ plot) before calculating $\overline{Y}$, s^2, ...

   NB: Must always be careful when discarding data.

3. Use Bayesian methods to set an upper limit on the SB signal strength.

   NB: Choice of prior.

4. Others??