



*Data Analysis and the
Detection of Gravitational Waves with LIGO*

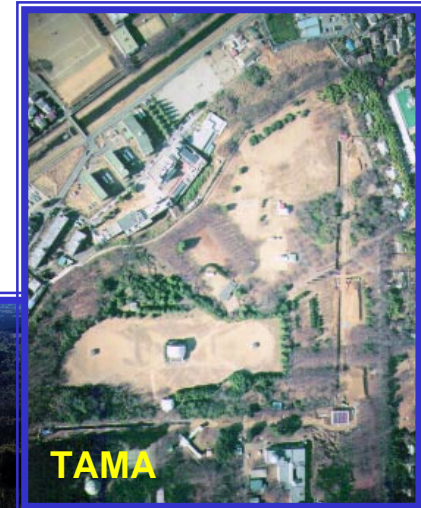
LIGO Summer SURF Seminar Series

*Albert Lazzarini
LIGO Laboratory
California Institute of Technology
Pasadena, California 91125*



The Opening of a New Observational Window on the Universe

- LIGO, VIRGO, GEO, TAMA ...
 - 4000m, 3000m, 2000m, 600m, 300m interferometers built to detect gravitational waves from compact objects



LIGO-G010259-00-E

LIGO LABORATORY CALTECH



Ultimate Goals for the Observation of Gravitational Waves

- Tests of Relativity
 - ◊ *Black holes & strong-field gravity (ringdown of excited BH)*
 - ◊ *Spin character of the radiation field (polarization of radiation from CW sources)*
 - ◊ *Wave propagation speed (delays in arrival time of bursts)*
- Gravitational Wave Astronomy
 - ◊ *Compact binary inspirals*
 - ◊ *Gravitational waves and gamma ray burst associations*
 - ◊ *Black hole formation*
 - ◊ *Supernovae in our galaxy*
 - ◊ *Newly formed neutron stars - spin down in the first year*
 - ◊ *Pulsars and rapidly rotating neutron stars*
 - ◊ *LMXBs*
 - ◊ *Stochastic background*



Observing the Galaxy with Different Electromagnetic Wavelengths

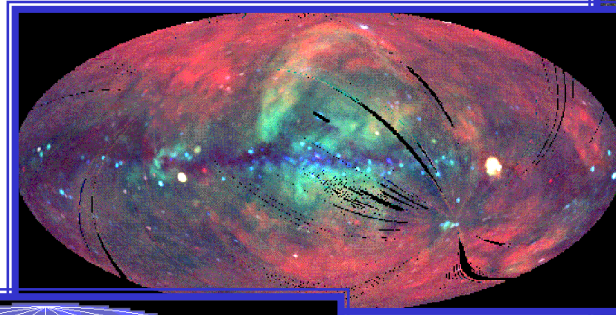
http://antwrp.gsfc.nasa.gov/apod/image/SagSumMW_dp_big.gif

$\lambda = 5 \times 10^{-7} \text{ m}$



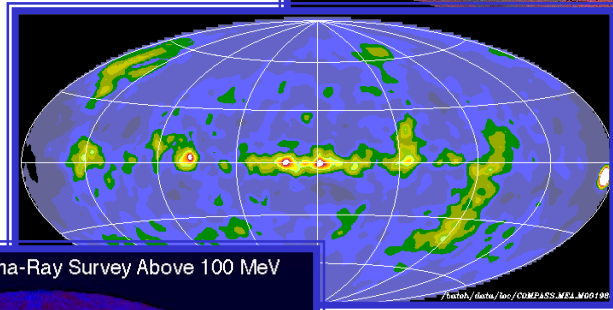
http://antwrp.gsfc.nasa.gov/apod/image/xallsky_rosat_big.gif

$\lambda = 5 \times 10^{-10} \text{ m}$



http://antwrp.gsfc.nasa.gov/apod/image/comptel_allsky_1to3_big.gif

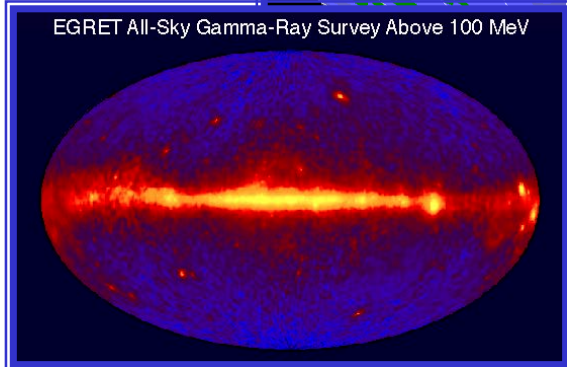
$\lambda = 6 \times 10^{-13} \text{ m}$



DIRBE 1.25, 2.2, 3.5 μm Composite

<http://www.gsfc.nasa.gov/astro/cobe>

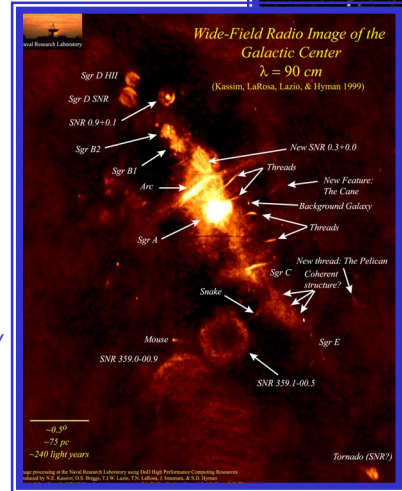
$\lambda = 2 \times 10^{-6} \text{ m}$



EGRET All-Sky Gamma-Ray Survey Above 100 MeV

<http://cossc.gsfc.nasa.gov/cossc/egret/>

$\lambda = 1 \times 10^{-14} \text{ m}$



Wide-Field Radio Image of the Galactic Center
 $\lambda = 90 \text{ cm}$
(Kassim, LaRosa, Lazio, & Hyman 1999)

<http://rsd-www.nrl.navy.mil/7213/lazio/GC/>

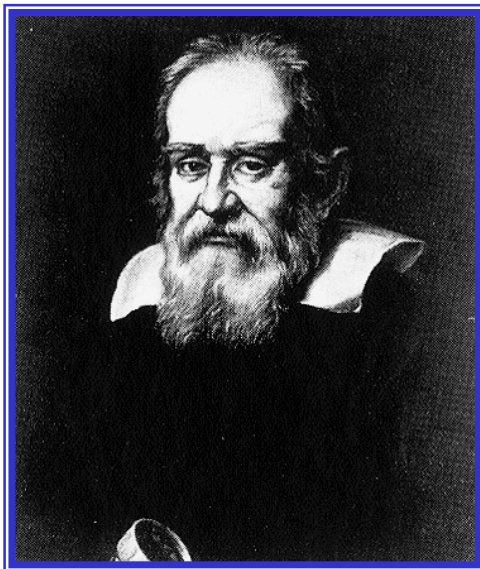
$\lambda = 9 \times 10^{-1} \text{ m}$

LIGO-G010259-00-E

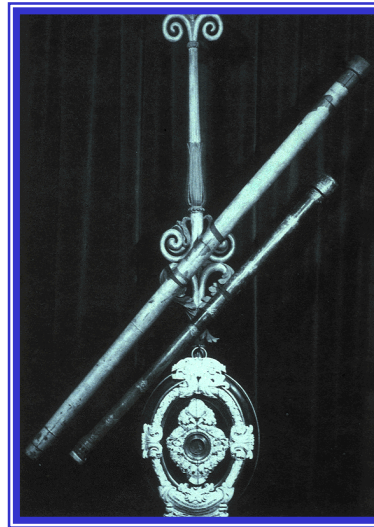
LIGO LABORATORY CAL TECH

The Opening of a New Observational Window on the Universe

- Galileo Galilei, 1610
 - Improves on an invention by Hans Lipperhey to build a 9X telescope
 - ! *Discovers the “Galilean” moons of Jupiter*



<http://es.rice.edu:80/ES/humsoc/Galileo/>



<http://photojournal.jpl.nasa.gov>

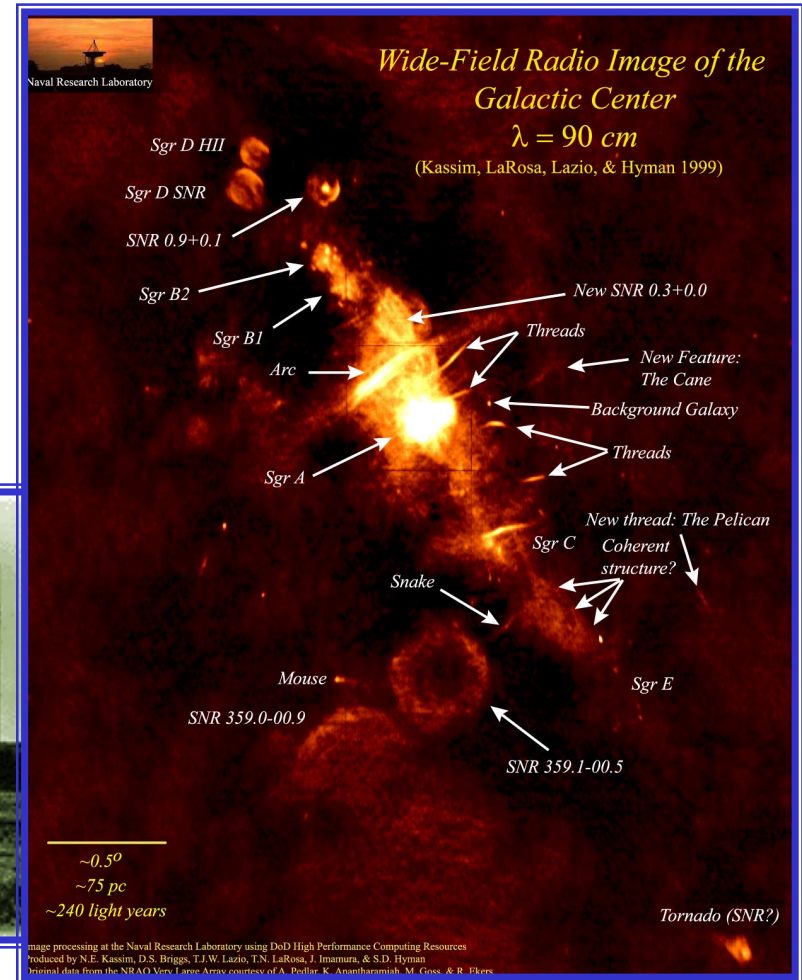
The Opening of a New Observational Window on the Universe

- Karl Jansky, 1933
 - Builds a radio antenna array to study interference in transatlantic telecommunications

! Discovers radio emissions from the galactic center



<http://www.lucent.com/museum/1933rt.html>



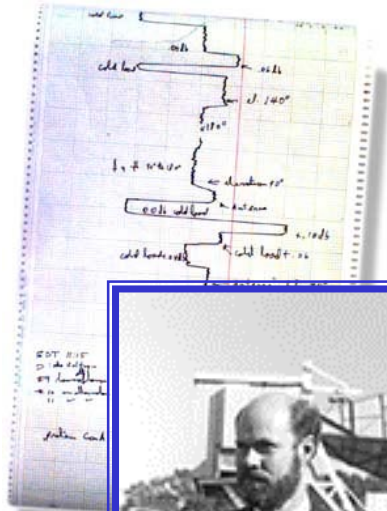
<http://rsd-www.nrl.navy.mil/7213/lazio/GC/>



The Opening of a New Observational Window on the Universe

- Penzias & Wilson, 1963
 - Track down excess antenna noise

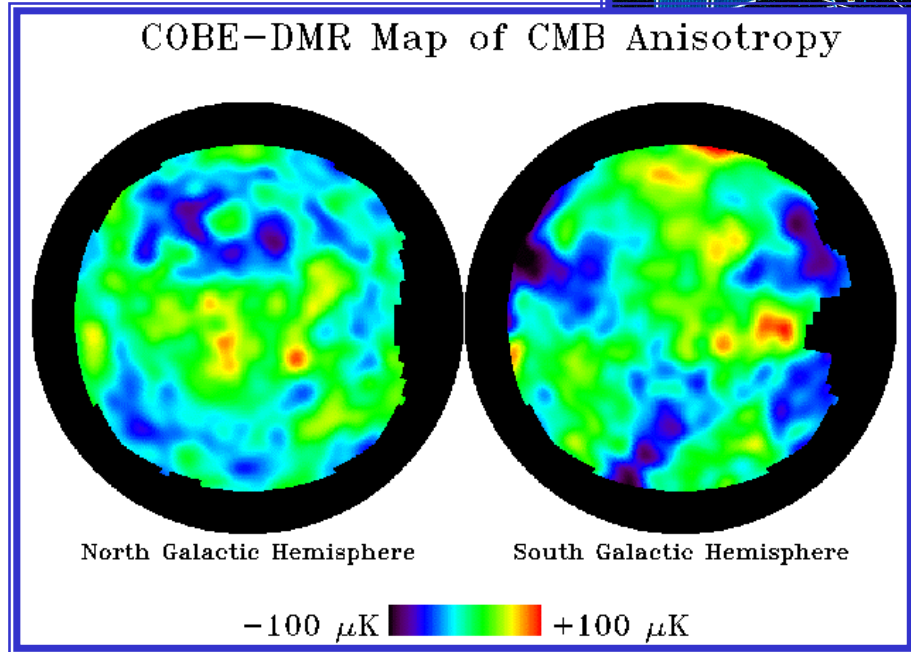
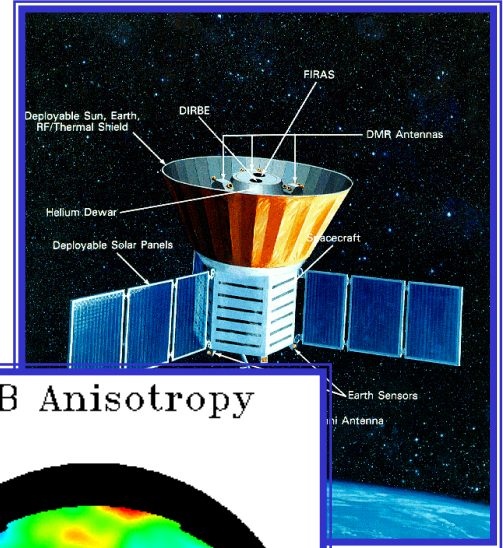
! Observe the cosmic microwave background radiation (CMBR)



<http://www.lucent.com/museum/1964bang.html>

LIGO-G010259-00-E

http://www.gsfc.nasa.gov/astro/cobe/cobe_home.html

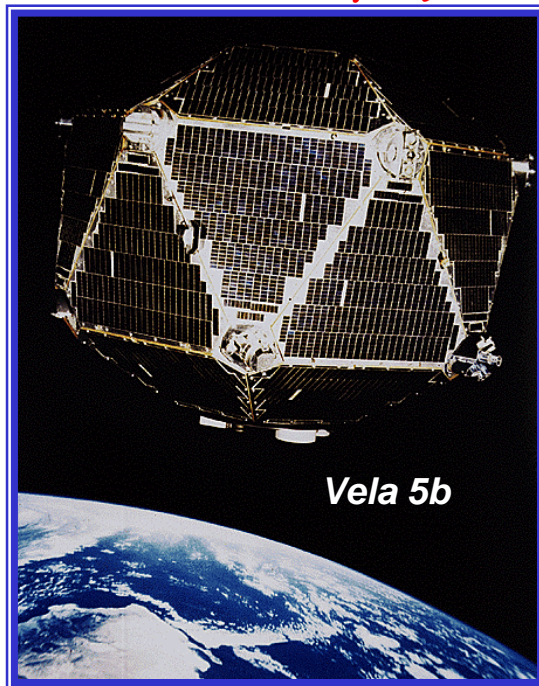


LIGO LABORATORY CALTECH



The Opening of a New Observational Window on the Universe

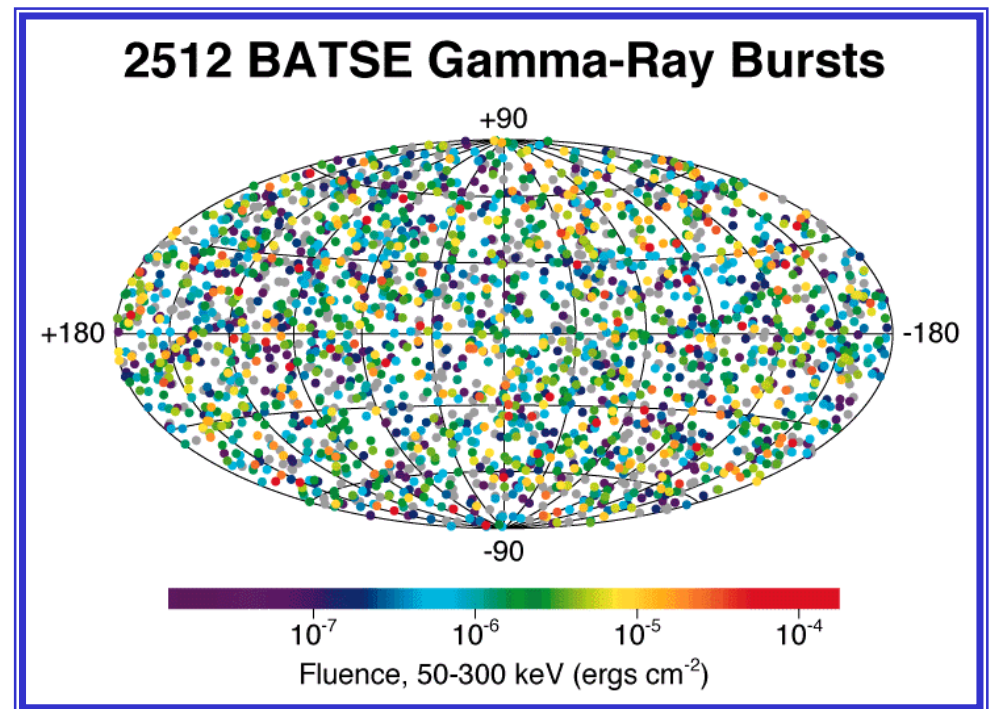
- Klebesadel, Strong & Olsen (LANL), 1969
 - Review of Vela 5 satellite data from 1967.07.02 shows a γ event of non-terrestrial origin
- ! Discover γ -ray bursts (GRBs), X-ray sources**



Vela 5b

http://science.msfc.nasa.gov/newhome/headlines/ast19sep97_2.htm

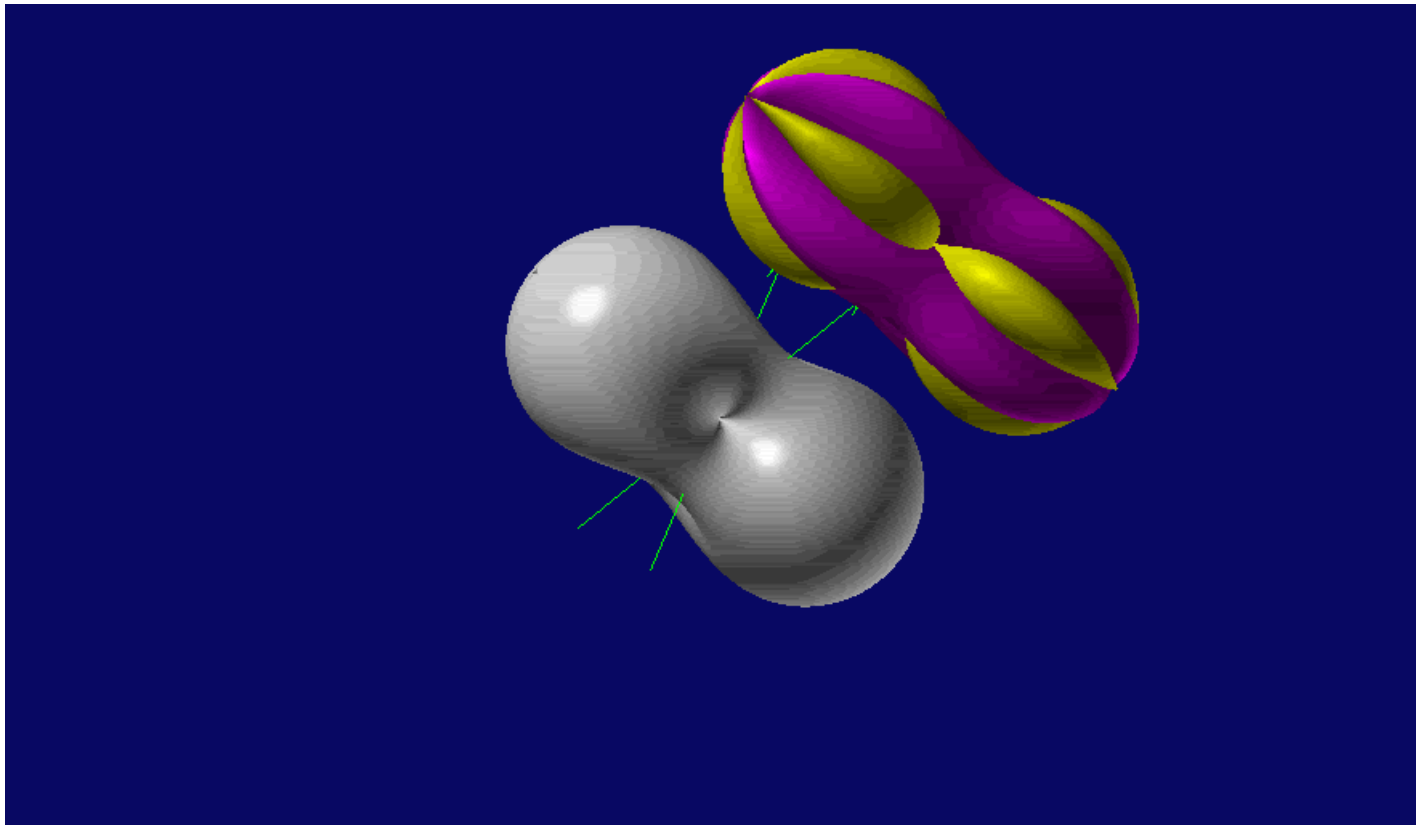
LIGO-G010259-00-E



LIGO LABORATORY CALTECH

<http://www.batse.com/>

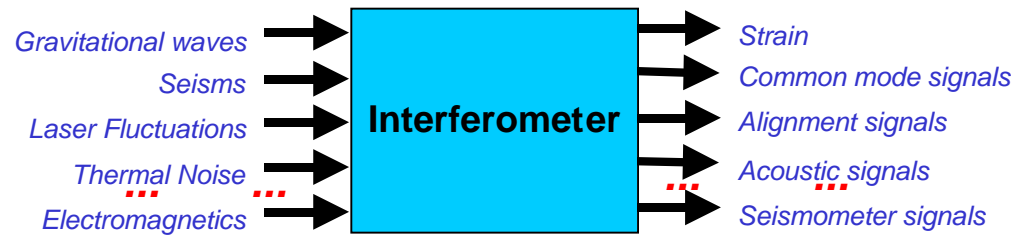
Interferometer GW Antenna Pattern



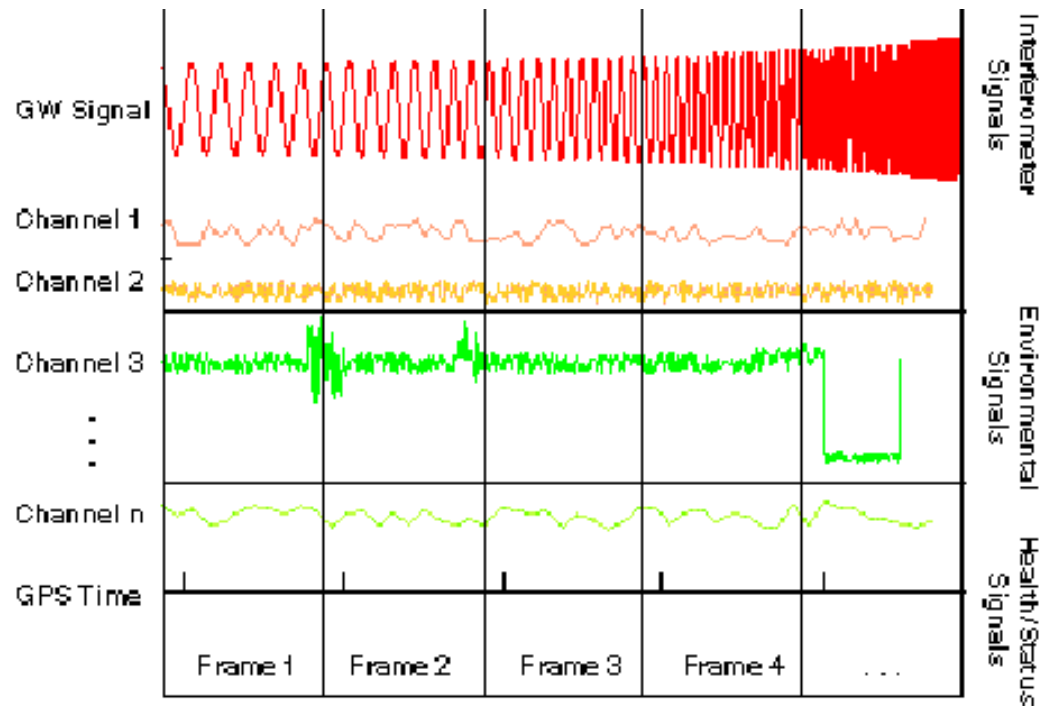
- ***Omni-directional*** - multiple detectors, lots of signal processing to determine direction
- ***Extremely weak signal*** - lots of environmental, instrumental monitoring to determine validity



Interferometer Data Channels



- All interferometric detector projects have agreed on a standard data format
- Anticipates joint data analysis
- LIGO Frames for 1 interferometer are ~3MB/s
 - 32 kB/s strain
 - ~2 MB/s other interferometer signals
 - ~1MB/s environmental sensors
- Strain is ~1% of all data





Mathematical Interlude ...
Random Processes & Signal Noise

- $n(t)$ is a randomly varying signal

– **Gaussian process:**

$$P(n) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(n-\mu)^2}{2\sigma^2}}$$

$$\langle n \rangle = \mu = \int_{-\infty}^{\infty} nP(n)dn$$

$$\langle n^2 \rangle = \sigma^2 + \mu^2 = \int_{-\infty}^{\infty} n^2 P(n)dn$$

– **Can assume $\mu=0$ without loss of generality**

- **Ensemble averages, time averages**

– **Time average:**

$$\langle n(t) \rangle_t = \lim_{T \rightarrow \infty} \frac{1}{T} \int_{-T/2}^{T/2} n(t)dt$$

– **Ensemble average:**

$$\langle n \rangle_p = \int_{-\infty}^{\infty} nP(n)dn$$

– **Stationarity:** μ, σ , etc. do not vary with time

– **Ergodicity:** Probability distribution of $n(t)$ over a long period T is the same as $P(n)$ at one instant, t'

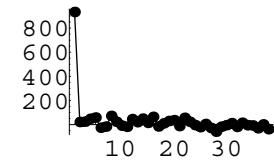
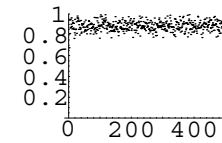
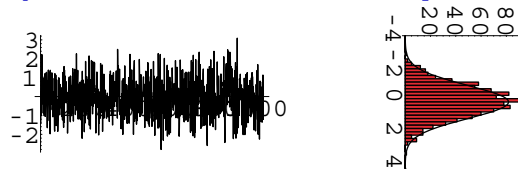


Characteristics of Random Processes

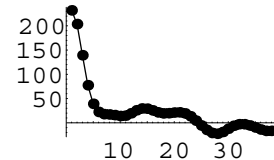
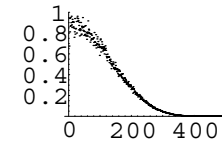
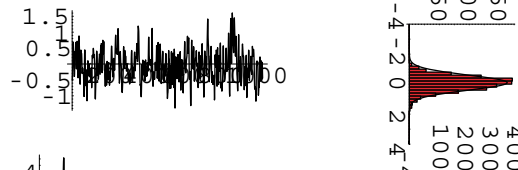
- $n(t)$ is a (white) random Gaussian process; $b(t)$ is a transient or burst process
- $H(f)n(f)$ is a (colored) random Gaussian process; $H(f)b(f)$ is a filtered burst process

FINITE POWER

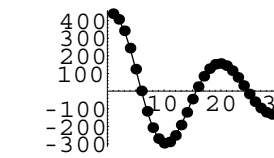
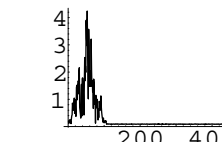
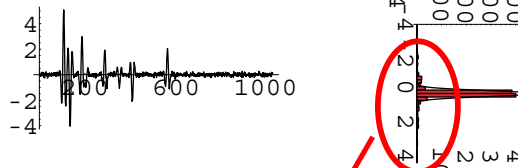
$n(t)$ or $n(f)$



$H(t-t') \otimes n(t')$ or $H(f)n(f)$

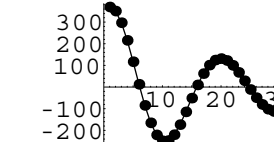
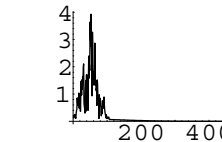
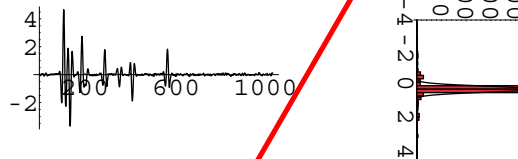


$b(t)$ or $b(f)$



FINITE ENERGY

$b(t-t') \otimes b(t')$ or $b(f)b(f)$



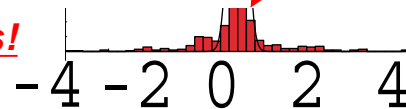
Time series

Histogram of data

Power Spectrum
[avg of 100 samples of $n(t)$]

Autocorrelation Fn.

Outliers!





Mathematical Interlude ...
Fourier Transforms of Time Dependent Signals

- Continuous-time
(infinite time duration)

$$\hat{R}(f) = \int_{-\infty}^{\infty} R(t) e^{-2\pi i f t} dt \equiv \mathbf{F} [R(t)]$$

$$R(t) = \int_{-\infty}^{\infty} \hat{R}(f) e^{2\pi i f t} df \equiv \mathbf{F}^{-1} [\hat{R}(f)]$$

$$\mathbf{F}^{-1} \mathbf{F} [R(t)] = R(t) \Rightarrow \delta(t - t') = \int_{-\infty}^{\infty} e^{-2\pi i f (t-t')} df$$

$$\int_{-\infty}^{\infty} \delta(t - t') dt = 1$$

- Discrete-time
(finite time stretch T)

$$\hat{R}(f_i) = \sum_{n=0}^{N-1} R(t_n) e^{-2\pi i f_i t_n} \Delta t \equiv \mathbf{F} [R(t_n)] ; t_n = n\Delta t = \frac{nT}{N}$$

$$R(t_n) = \sum_{k=0}^{N-1} \hat{R}(f_k) e^{-2\pi i f_k t_n} \Delta f \equiv \mathbf{F}^{-1} [\hat{R}(f_k)] ; f_k = k\Delta f = \frac{k}{T}$$

$$\mathbf{F}^{-1} \mathbf{F} [R(t_j)] = R(t_i) \Rightarrow \delta_{ij}$$

- Computational cost to perform transform (FFT): **$5 N \log_2 [N]$**
 - 100 s of 1024 Sample/s data: 8.5×10^6 floating point operations (FLOP)
 - To keep up with data: 85 kFLOP/s (kFLOPS)



*Mathematical Interlude ...
Useful Theorems and Formulae*

- Parseval's Theorem

- Translation:

$$\mathcal{F}[R(t + t')] = \hat{R}(f)e^{-2\pi i f t'}$$

- Useful identity:

$$\hat{n}(f) = \hat{n}^*(-f) \text{ for real } n(t)$$

- Convolution

$$c(\tau) = \int_{-\infty}^{\infty} a(t)b(\tau - t)dt$$

$$\hat{c}(f) = \int_{-\infty}^{\infty} d\tau \int_{-\infty}^{\infty} dt a(t)b(\tau - t)e^{-2\pi i f \tau}; t' = \tau - t$$

$$\hat{c}(f) = \int_{-\infty}^{\infty} dt' b(t')e^{-2\pi i f t'} \int_{-\infty}^{\infty} dt a(t)e^{-2\pi i f t}$$

$$\hat{c}(f) = \hat{a}(f)\hat{b}(f)$$

LIGO-G010259-00-E

$$\int_{-\infty}^{\infty} R(t)^2 dt = \text{"Integrated Signal Energy"}$$

$$= \int_{-\infty}^{\infty} \left[\int_{-\infty}^{\infty} \hat{R}(f)e^{2\pi i f t} df \right] \left[\int_{-\infty}^{\infty} \hat{R}(f'')e^{2\pi i f'' t} df'' \right] dt$$

$$= \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} df'' df \{ \hat{R}(f)\hat{R}(f'') \} \int_{-\infty}^{\infty} e^{2\pi i (f+f'')t} dt$$

$$= \int_{-\infty}^{\infty} |\hat{R}(f)|^2 df$$

- Autocorrelation

$$R_a(\tau) = \int_{-\infty}^{\infty} a(t)a(\tau + t)dt$$

$$\frac{1}{2}S_a(f) \equiv \int_{-\infty}^{\infty} R_a(\tau)e^{-2\pi i f \tau} d\tau$$

$$= \int_{-\infty}^{\infty} d\tau \int_{-\infty}^{\infty} dt a(t)a(\tau + t)e^{-2\pi i f \tau}$$

$$= \int_{-\infty}^{\infty} dt' a(t')e^{-2\pi i f t'} \int_{-\infty}^{\infty} dt a(t)e^{-2\pi i f t}$$

$$= |a(f)|^2$$

LIGO LABORATORY CALTECH

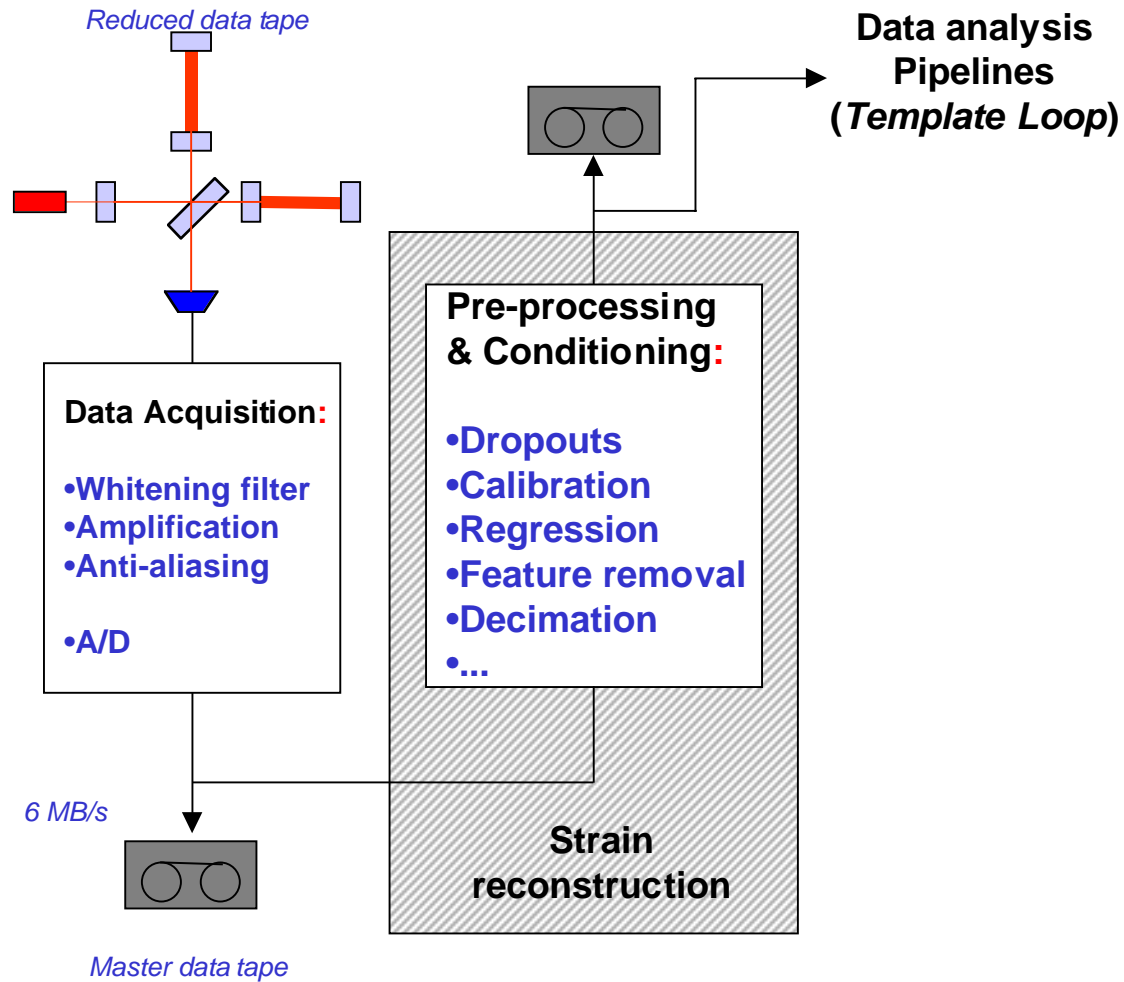


Mathematical Interlude ...
Property of an Ergodic Gaussian Process

- Different Fourier components of $n(t)$ are independent of each other:

$$\begin{aligned}\langle n(f)n^*(f') \rangle_P &= \left\langle \int n(t)e^{-2\pi ift} dt \int n(t+\tau)e^{2\pi if'(t+\tau)} d\tau \right\rangle_t \\ &= \int dt' \int \langle n(t+\tau)n(t) \rangle_t e^{-2\pi ift} e^{2\pi if'(t+\tau)} d\tau \\ &= \int dt' e^{-2\pi i(f-f')t} \int R_n(\tau) e^{2\pi if'\tau} d\tau \\ &= S_n(f)\delta(f-f')\end{aligned}$$

Data Flow: Pre-processing





Data Pre-processing: removing instrumental effects

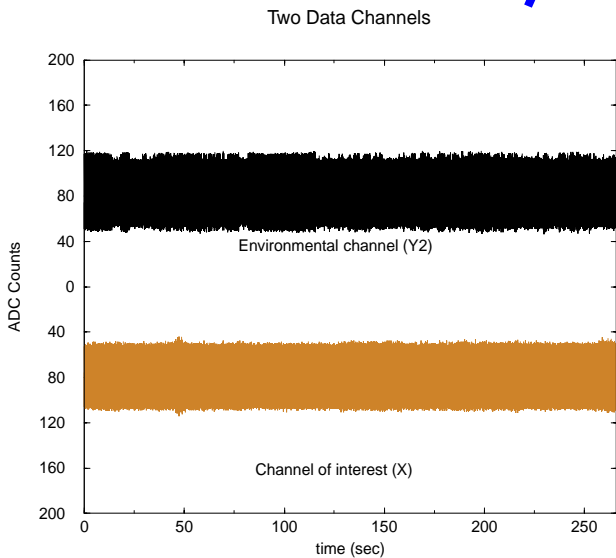
- Cross channel regression will be used to improve signal to noise ratios when possible (need adequate SNR)

Raw channel data (40m prototype)

$$s_a(t) \Rightarrow \hat{s}_a(f)$$

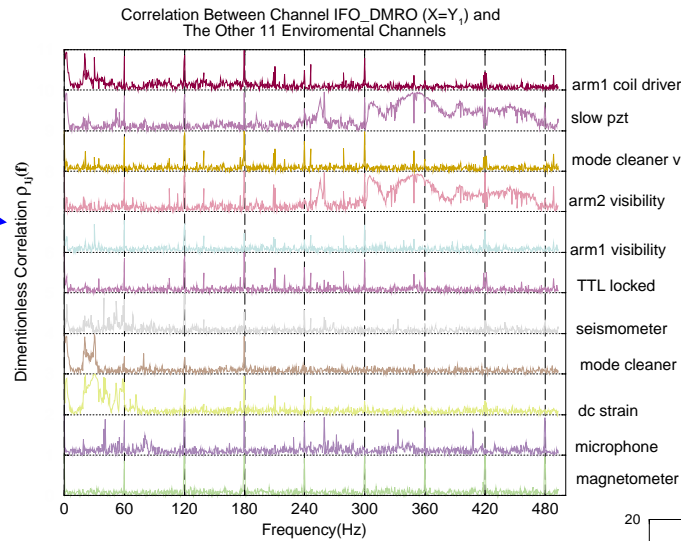
$$s_b(t) \Rightarrow \hat{s}_b(f)$$

$$s_a(t)s_b(t + \tau) \Rightarrow R_{ab}(\tau) \Leftrightarrow \hat{S}_{ab}(f)$$



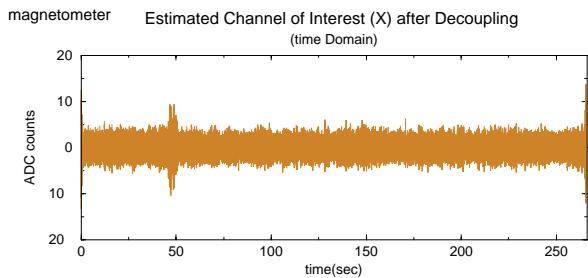
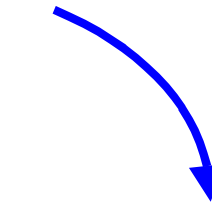
LIGO-G010259-00-E

ref: Allen, Hua, Ottewill (gr-qc/9909083)



Cross channel spectral correlation

$$\Gamma_{ab}(f) \equiv \frac{|\hat{s}_{ab}(f)|^2}{\hat{s}_a(f) \hat{s}_b(f)}$$



$$\begin{pmatrix} \hat{s}'_a(t) \\ \hat{s}'_b(t) \end{pmatrix} \Leftrightarrow \begin{pmatrix} \hat{s}'_a(f) \\ \hat{s}'_b(f) \end{pmatrix} = M(f) \cdot \begin{pmatrix} \hat{s}_a(f) \\ \hat{s}_b(f) \end{pmatrix}$$

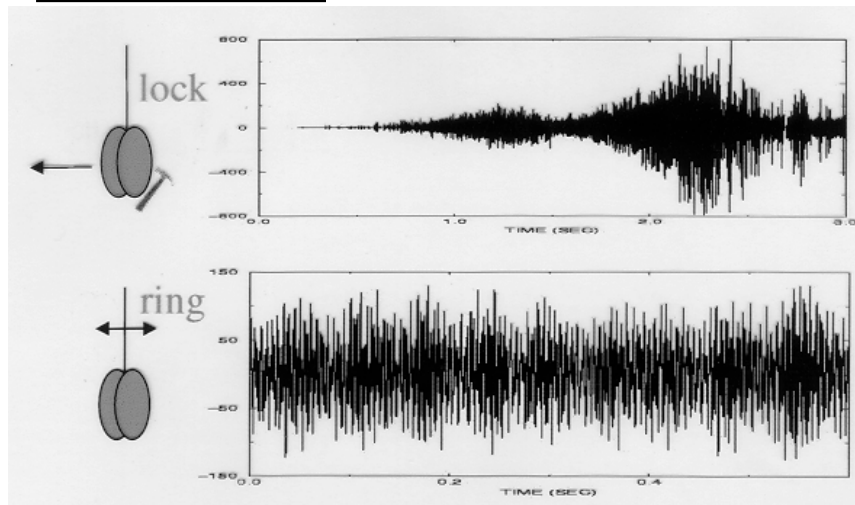
Reduced data channel

LIGO LABORATORY CALTECH

Interferometer Transients -- Examples from 40m Data

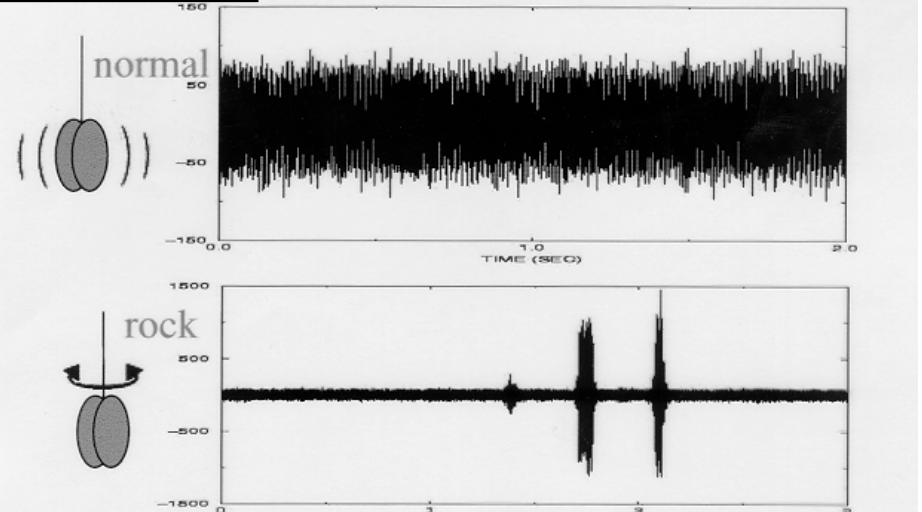
Real interferometer data is UGLY!!!
(Glitches - known and unknown)

LOCKING



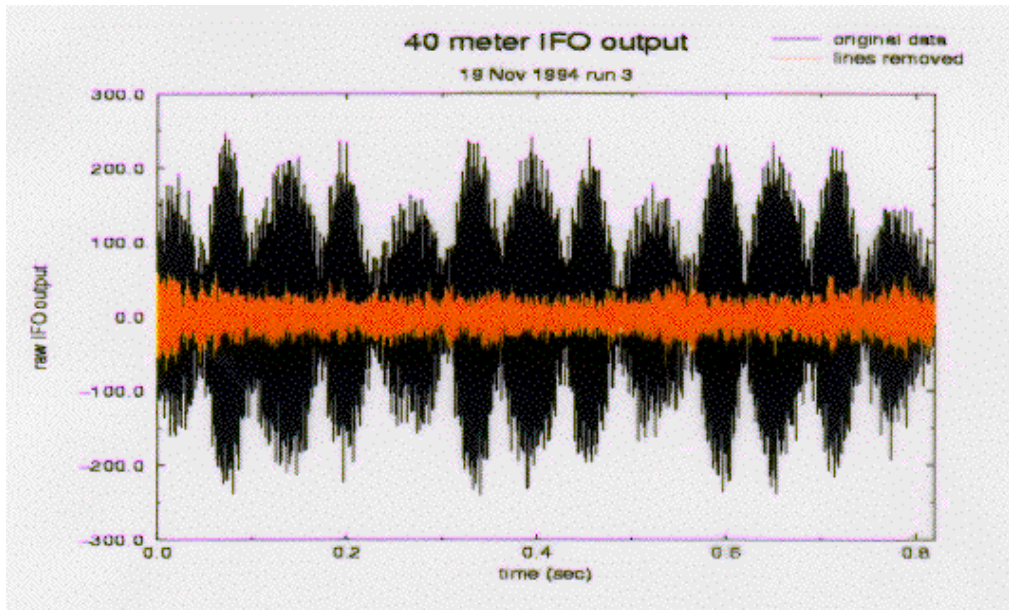
RINGING

NORMAL

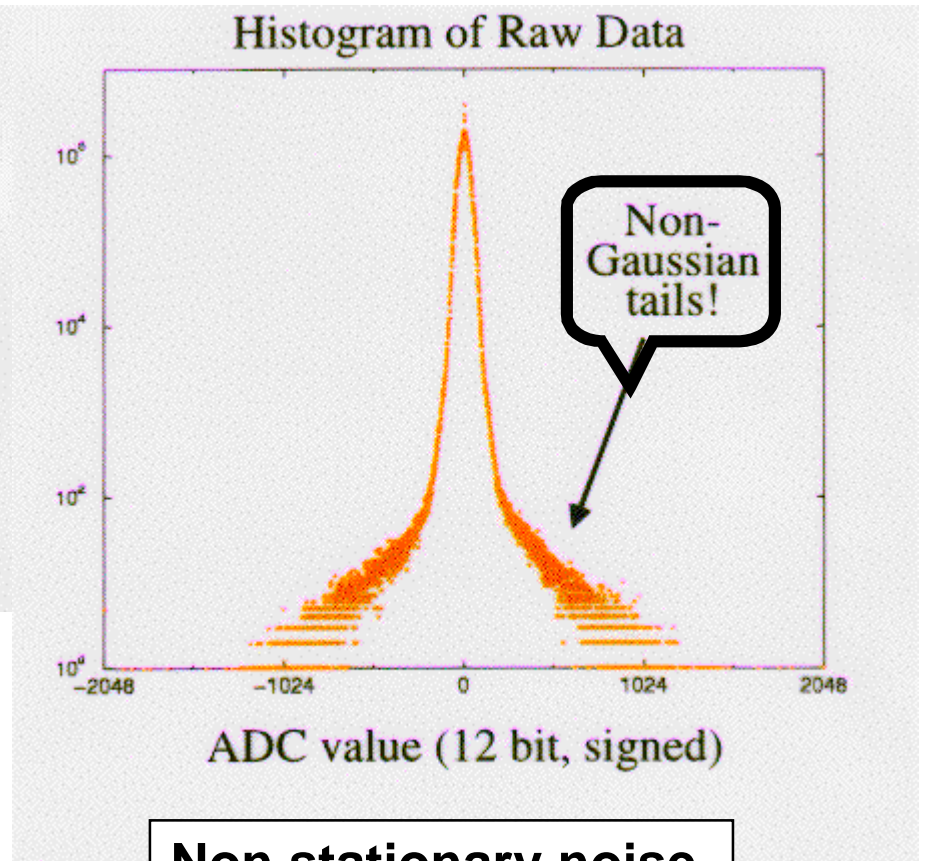


ROCKING

“Clean up” data stream



Effect of removing sinusoidal artifacts using multi-taper methods

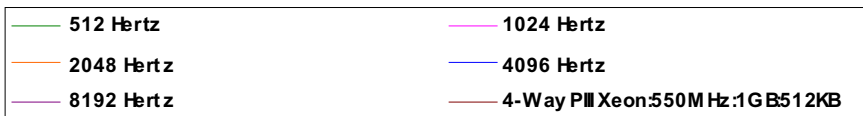
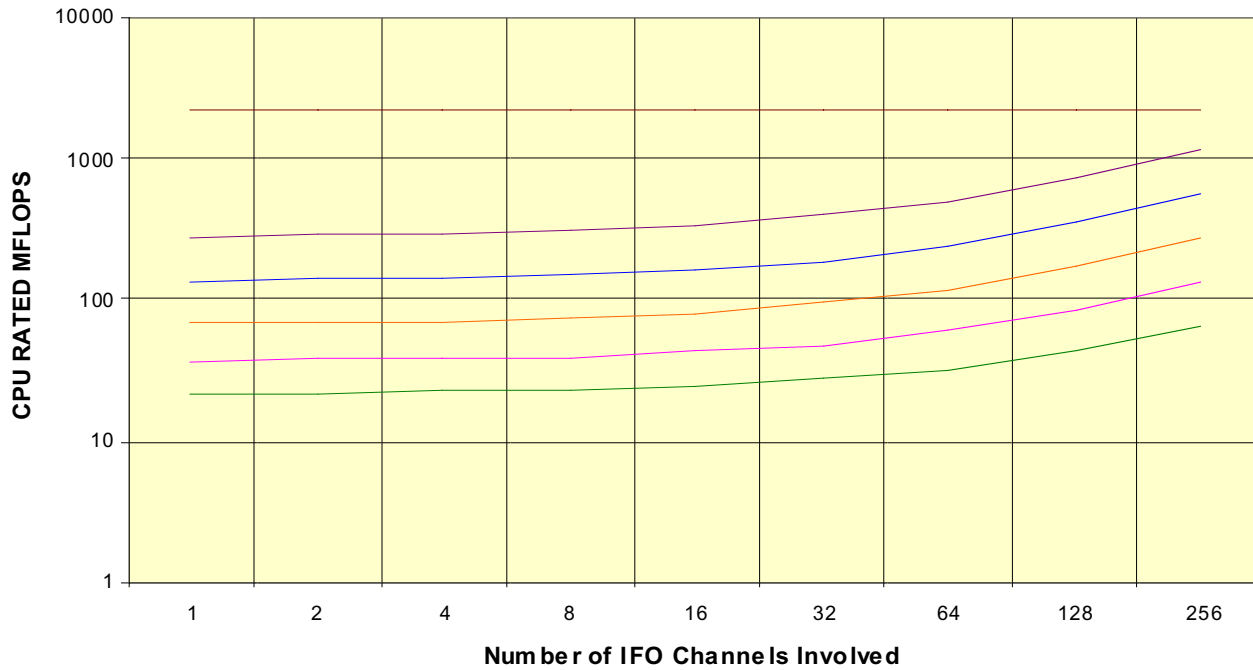


**Non stationary noise
Non gaussian tails**



LIGO Data(Pre)-Conditioning Computational Costs

Data Pre-Conditioning Compute Requirements
(processing 4 seconds of data for a single search)



LIGO LABORATORY CALTECH

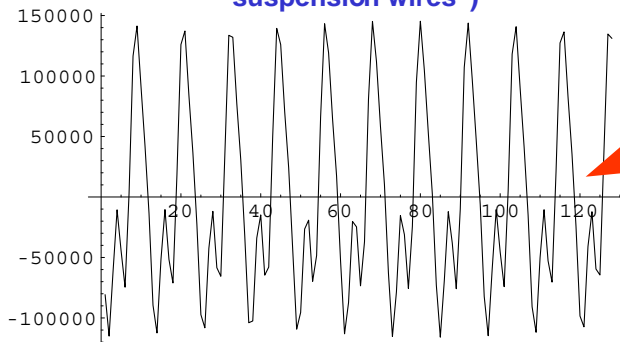
- Pre-Conditioning steps involved:
 - 64K samples of input data per channel
 - Data Drop-Out Correction on 10% of data.
 - Line Removal of 64 lines.
 - Calibration of GW channel.
 - Resampling (see legend).
 - Linear Regression using all input signals.

- Roughly 4-8 unique searches expected to be active in LDAS.

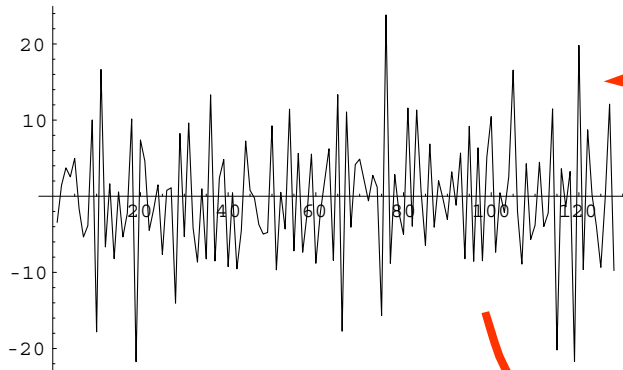
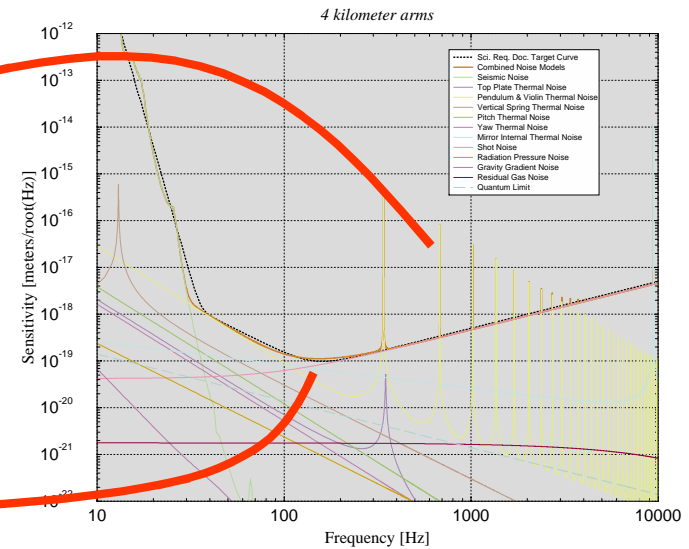


Interferometer Strain Signal (Simulated)

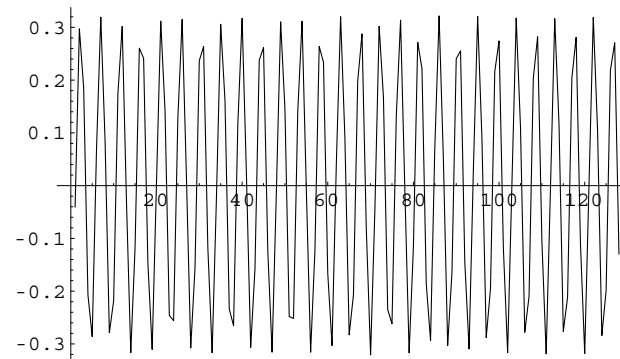
Dominated by narrowband features in spectrum
("violin resonances" of suspension wires)



Design LIGO I limiting strain sensitivity
Initial LIGO Noise Curves



Broadband noise spectrum



Chirped waveform

LIGO-G010259-00-E

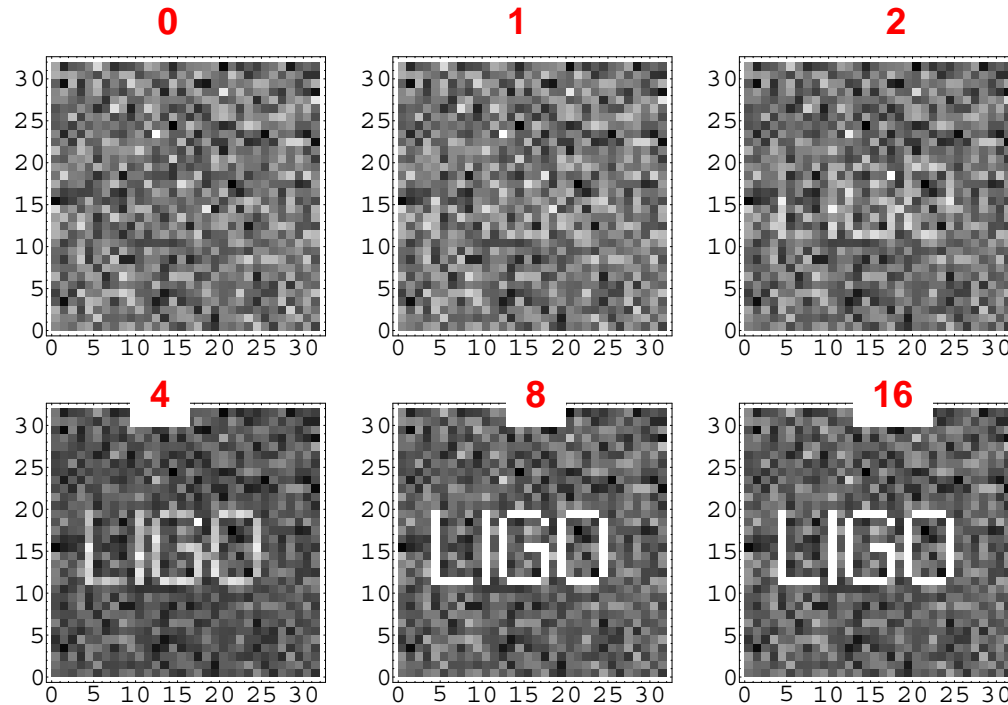
LIGO LABORATORY CALTECH

James Kent Blackburn
Sat Apr 11 20:15:33 1998

Detections vs. Observations

- It is “easier” to detect than to quantitatively observe

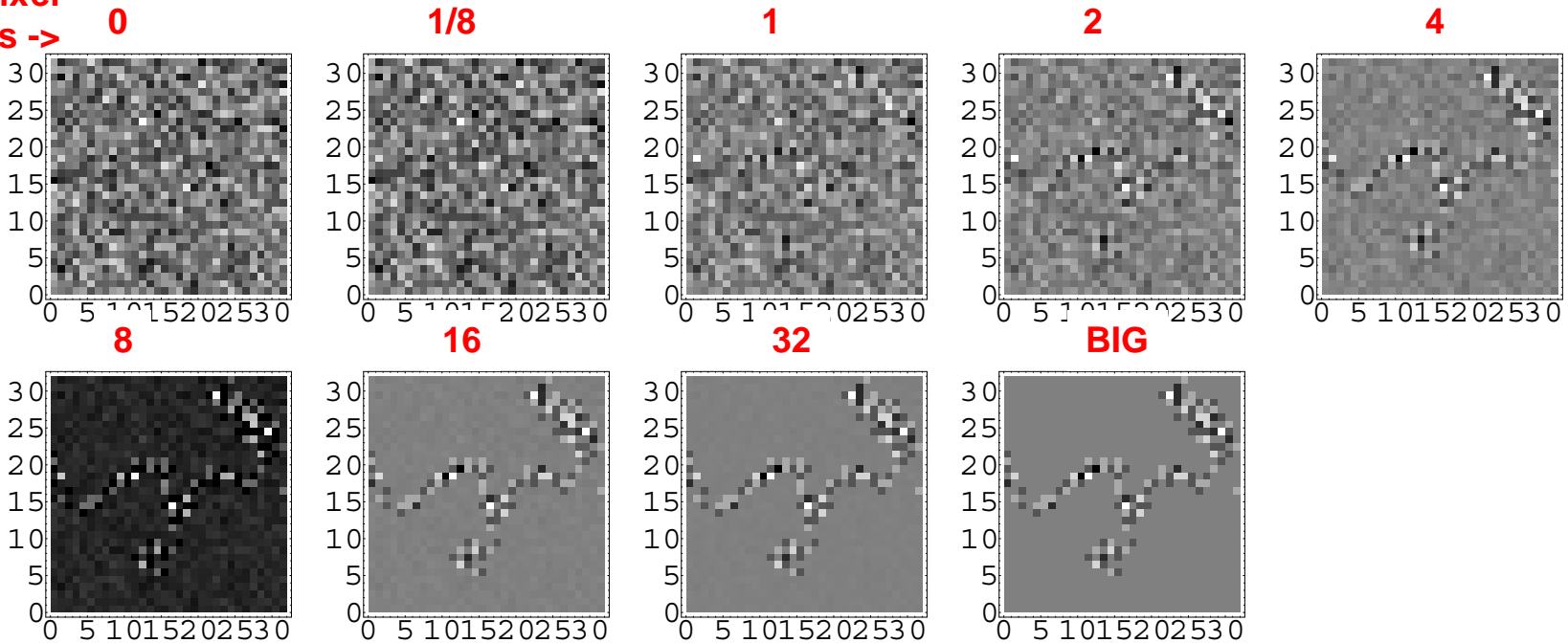
Single-pixel SNRs ->



Detections vs. Parameter Estimation

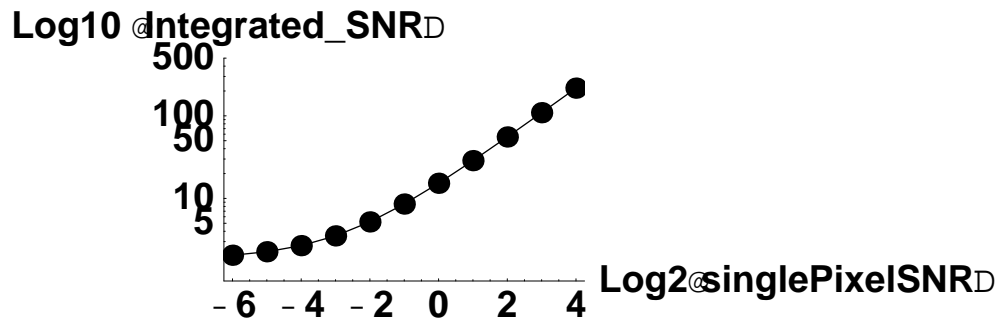
- It is “easier” to detect than to quantitatively observe

Single-pixel
SNRs ->



Detections vs. Parameter Estimation

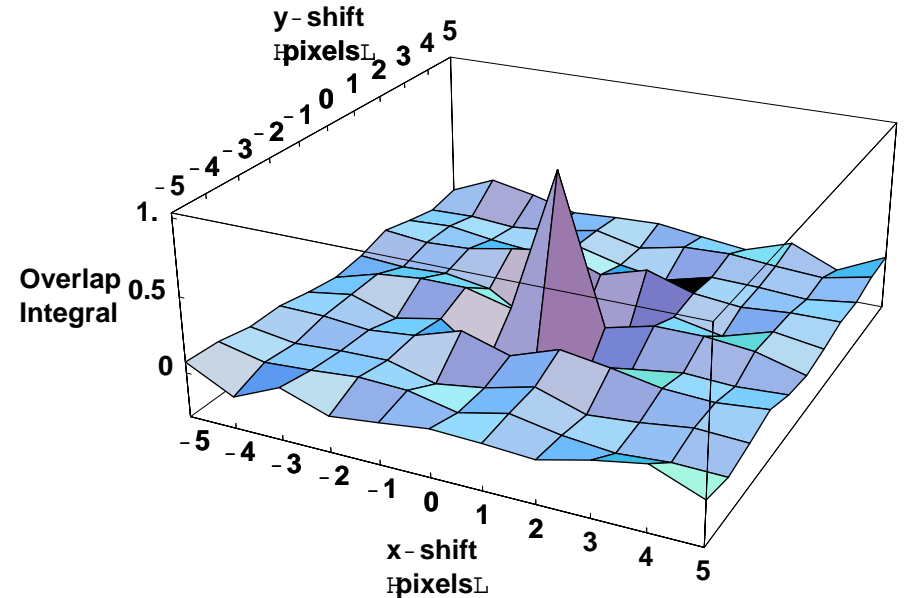
- It is “easier” to detect than to quantitatively observe



$n = \text{noise};$
 $d = \text{noisy data};$
 $T = \text{noiseless template}$

$$SNR = \frac{\sum_{i,j} d(i,j)T(i,j)}{\sqrt{\sum_{i,j} n^2(i,j)T^2(i,j)}}$$

- Integrated SNR == Sum over entire image after it is multiplied by noiseless signal (template)



$$A(\Delta i, \Delta j) = \sum_{i,j} T(i,j)T(i + \Delta i, j + \Delta j)$$

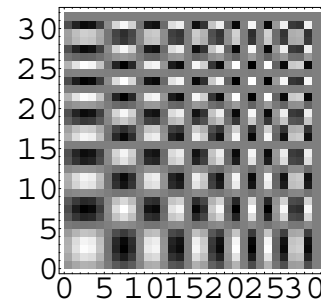
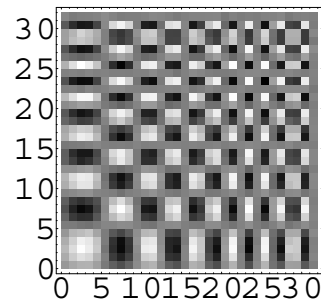
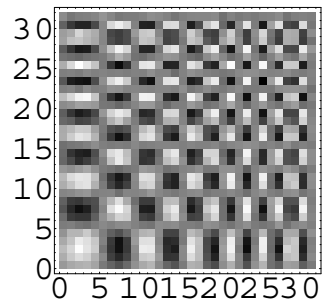
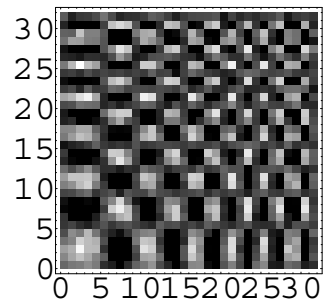
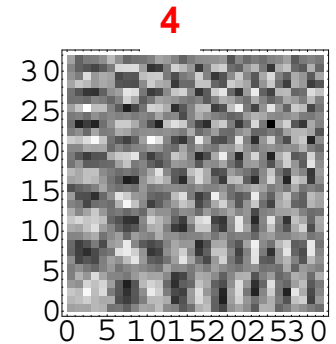
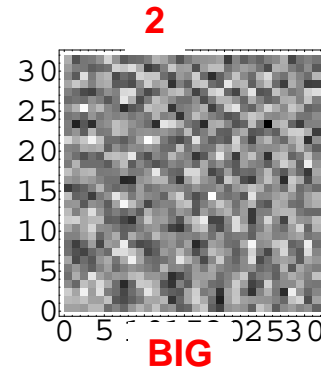
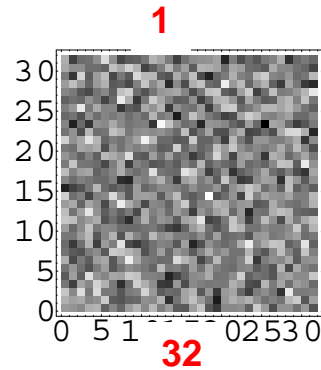
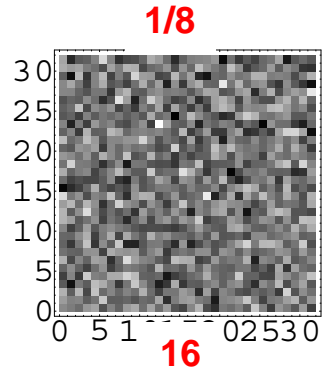
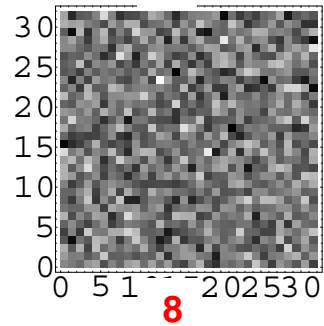
- Effect on Integrated SNR of (unknown) shift between noiseless signal and template
- Shift +/- 1 pixel kills correlations => need 32^2 templates to cover a 32×32 pixel image...

Detections vs. Parameter Estimation

- It is “easier” to detect than to quantitatively observe

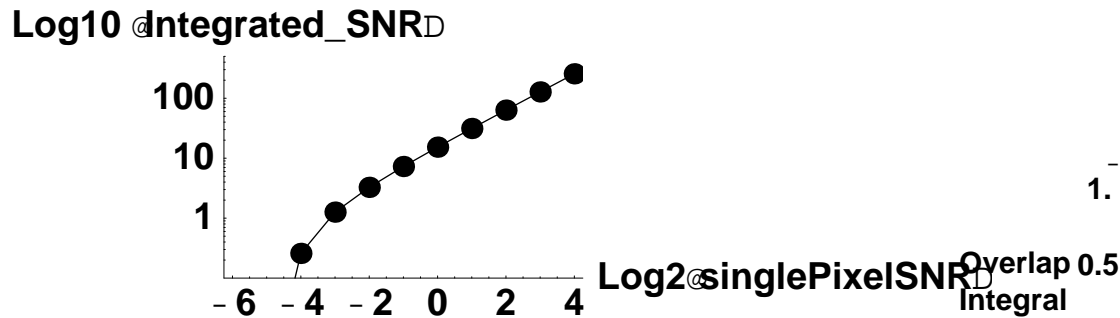
Single-pixel

SNRs \rightarrow 0



Detections vs. Parameter Estimation

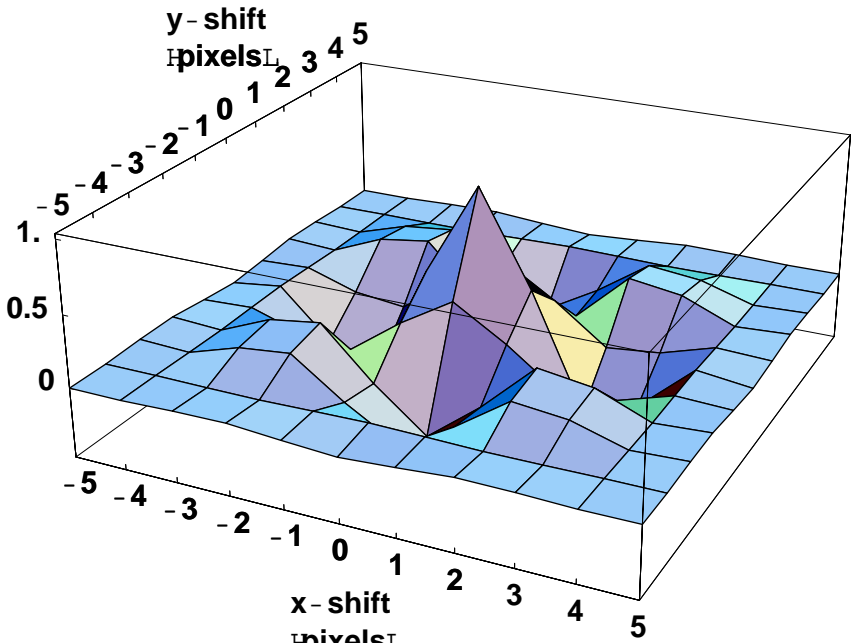
- It is “easier” to detect than to quantitatively observe



$n = \text{noise};$
 $d = \text{noisy data};$
 $T = \text{noiseless template}$

$$SNR = \frac{\sum_{i,j} d(i,j)T(i,j)}{\sqrt{\sum_{i,j} n^2(i,j)T^2(i,j)}}$$

- Integrated SNR == Sum over entire image after it is multiplied by noiseless signal (template)



$$A(\Delta i, \Delta j) = \sum_{i,j} T(i,j)T(i + \Delta i, j + \Delta j)$$

- Effect on Integrated SNR of (unknown) shift between noiseless signal and template
- Shift +/- 1 pixel reduces correlations => need 32^2 templates to cover a 32×32 pixel image...



Optimal Filtering and Signal Processing

- Measurement s , (may contain) signal h , contains noise n :

$$s(t) = h(t) + n(t)$$

$$\langle n(t) \rangle = 0 ; \langle h \rangle = \alpha T_i(t - t_0);$$

- $\alpha T_i(t-t_0)$ is one of a family of fiducial (expected) waveform or **templates** of type i with unknown time origin t_0 and amplitude α .
- Design optimal correlation filter, Q , that maximizes chance of detecting T in $s(t)$:

$$C(t_0) = \int_{-\infty}^{\infty} s(t - t_0) Q(t) dt;$$

$$\langle C \rangle = \int_{-\infty}^{\infty} \alpha T(t - t_0) Q(t) dt$$

$$= \alpha \int_{-\infty}^{\infty} \hat{T}(f) e^{-2\pi i f t_0} \hat{Q}^*(f) df$$

$$N = C - \langle C \rangle; \langle N \rangle \equiv 0$$

$$\langle N^2 \rangle = \langle C^2 \rangle - \langle C \rangle^2$$

$$= \left\langle \int \hat{n}(f) \hat{Q}^*(f) df \int \hat{n}(f') \hat{Q}^*(f') df' \right\rangle$$

$$= \int df \int df' \hat{Q}^*(f) \hat{Q}^*(f') \langle \hat{n}(f) \hat{n}(f') \rangle$$

$$= \frac{1}{2} \int S_n(f) |Q(f)|^2 df$$



Optimal Filtering and Signal Processing

- Maximize P(detection) => Maximize signal-to-noise ratio (SNR):

$$\begin{aligned}
 \text{Max}[SNR = \frac{\langle C \rangle}{\langle N^2 \rangle^{1/2}}] &\Rightarrow \text{Max}[(SNR)^2 = \frac{\langle C \rangle^2}{\langle N^2 \rangle}] \\
 \delta(SNR)^2 &= \frac{\delta\langle C \rangle^2}{\langle N^2 \rangle} - \frac{\langle C \rangle^2 \delta\langle N^2 \rangle}{\langle N^2 \rangle^2} = 0 \\
 &= \frac{2\langle C \rangle}{\langle N^2 \rangle^2} (\langle \delta C \rangle \langle N^2 \rangle - \langle C \rangle \langle N \delta N \rangle) = 0 \\
 \langle \delta C \rangle \langle N^2 \rangle &= \langle C \rangle \langle N \delta N \rangle
 \end{aligned}$$

- Variation and maximization is with respect to optimal filter, δQ :

$$\begin{aligned}
 \langle C \rangle &= \alpha \int_{-\infty}^{\infty} \hat{T}(f) e^{-2\pi i f t_0} \hat{Q}^*(f) df \\
 \langle \delta C \rangle &= \alpha \int_{-\infty}^{\infty} \hat{T}(f) e^{-2\pi i f t_0} \delta \hat{Q}^*(f) df \\
 \langle N^2 \rangle &= \frac{1}{2} \int_{-\infty}^{\infty} S_n(f) |Q(f)|^2 df \\
 \langle N \delta N \rangle &= \frac{1}{2} \int_{-\infty}^{\infty} S_n(f) Q(f) \delta Q^*(f) df
 \end{aligned}$$



Optimal Filtering and Signal Processing

•Equate LHS, RHS:

$$\langle \delta C \rangle \langle N^2 \rangle = \langle C \rangle \langle N \delta N \rangle$$
$$\hat{T}(f) e^{-2\pi i f t_0} \left[\int S_n(f') |Q(f')|^2 df' \right] = S_n(f) Q(f) \left[\int \hat{T}(f') e^{-2\pi i f' t_0} \hat{Q}^*(f') df' \right]$$

•Require coefficients of [...] to be equal:

$$\hat{T}(f) e^{-2\pi i f t_0} = S_n(f) Q(f) \Rightarrow Q(f) = \frac{\hat{T}(f) e^{-2\pi i f t_0}}{S_n(f)}$$

Optimal filter for this problem!

•Check that [...] = [...] with Q(f) from above:

$$\left[\int S_n(f') |Q(f')|^2 df' \right] ? = ? \left[\int \hat{T}(f') e^{-2\pi i f' t_0} \hat{Q}^*(f') df' \right] \quad \text{Yes!!!!}$$

Matched Filtering with Templates

$$C(t_0 : \{p\}) = \int_{-\infty}^{\infty} \frac{\hat{s}(f) \hat{T}_{\{p\}}^*(f)}{S_n(f)} e^{2\pi i f t_0} df$$

- $C(t_0 : \{p\})$ is a family of derived correlation time series
 - $\{p\}$ is a set of parameters used to characterize the templates T
 - ♦ Intrinsic parameters: masses, other GR parameters
 - ♦ Extrinsic parameters: distance, orbital inclination, phase, position in the sky, ...
 - ♦ Dimensions of $\{p\}$ can be **HUGE** (e.g., $10^4 - 10^5$) if one wants a reasonable certainty of detecting a weak signal with unknown parameters
 - If statistics of noise $n(t)$ are sufficiently well understood, it is possible to make a confidence statement of the likelihood that a certain value for $C(t_0)$ is due to a signal characterized by $\{p\}$, and not noise alone.



Science in LIGO I

Data analysis plan

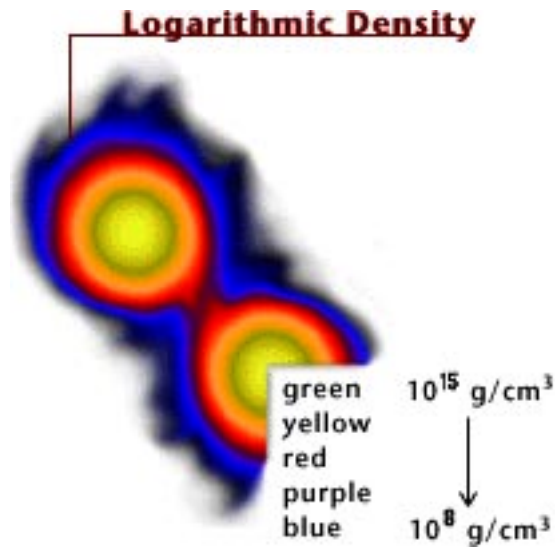
-
- Compact binary inspiral: *“chirps”*
 - *NS-NS waveforms are well described*
 - *BH-BH need better waveforms*
 - *search technique: matched templates*
 - Supernovae / GRBs: *“bursts”*
 - *burst search algorithms – excess power; time-freq patterns*
 - *burst signals - coincidence with signals in E&M radiation*
 - *prompt alarm (~ 1 hr) with ν detectors [SNEWS]*
 - Cosmological Signals *“stochastic background”*
 - *Search technique: optimal Wiener filter for different models*
 - Pulsars in our galaxy: *“periodic”*
 - *search for observed neutron stars (freq., doppler shift) - matched filters*
 - *all sky search (computing challenge)*
 - *r-modes*



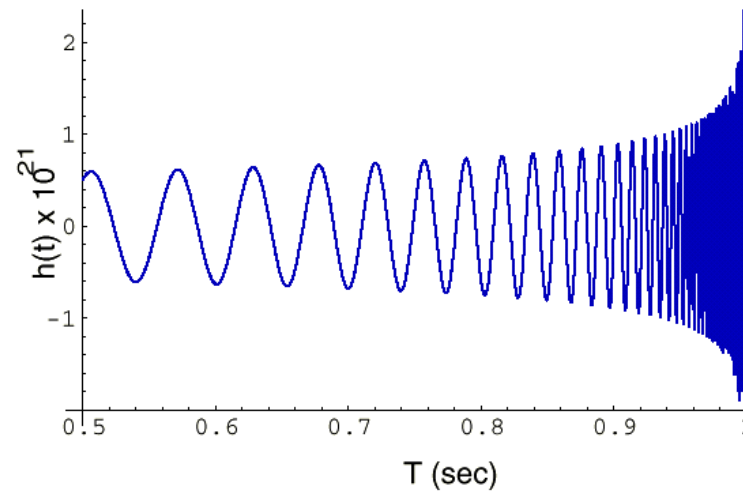
Sources of Gravitational Waves

Closely orbiting compact stars

Inspiral of Neutron Stars

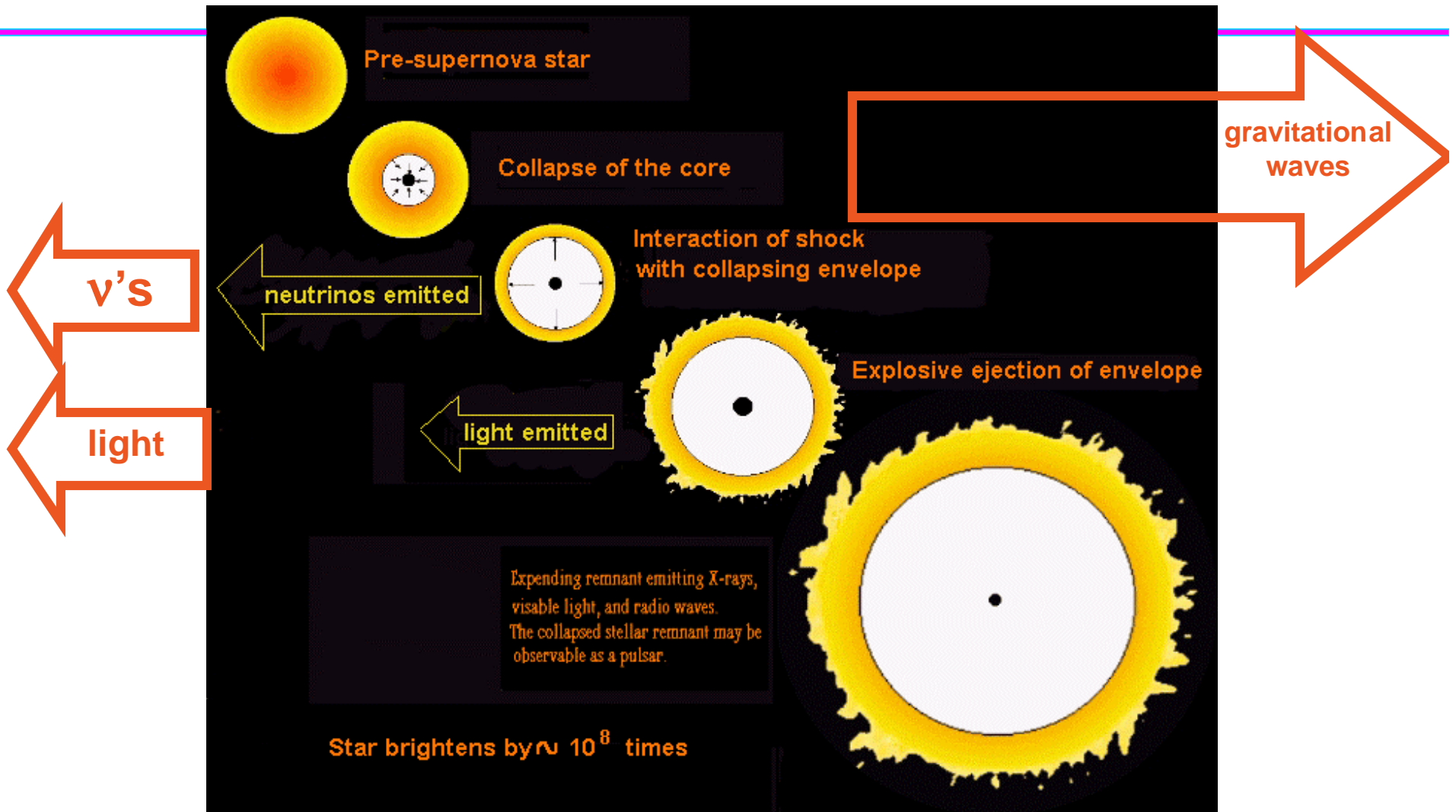


“Chirp Signal”



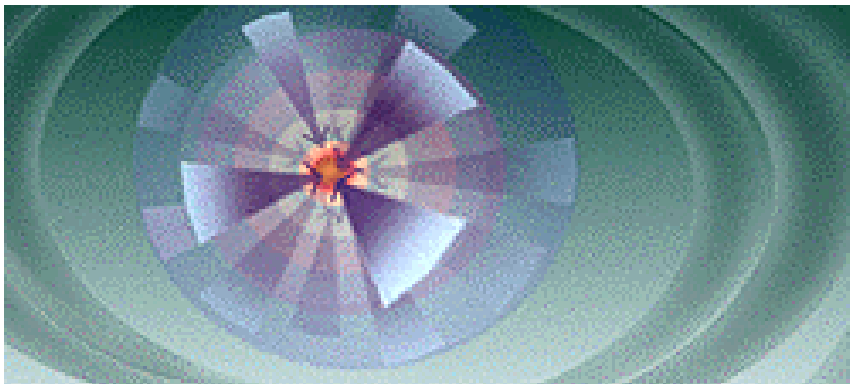


Sources of Gravitational Waves Supernovae



Supernovae Gravitational Waves

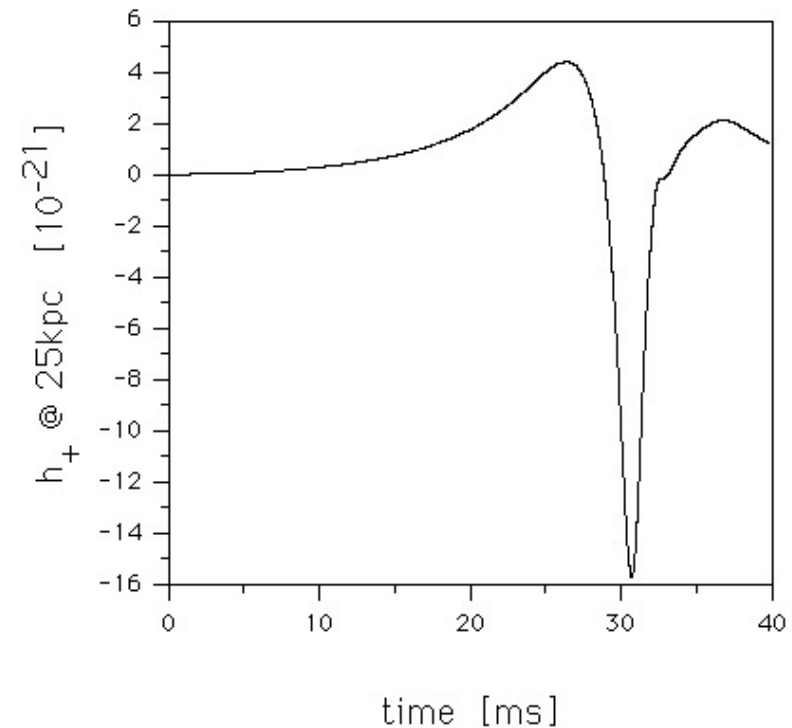
Non axisymmetric collapse



Rate

*1/50 yr - our galaxy
3/yr - Virgo cluster*

'burst' signal

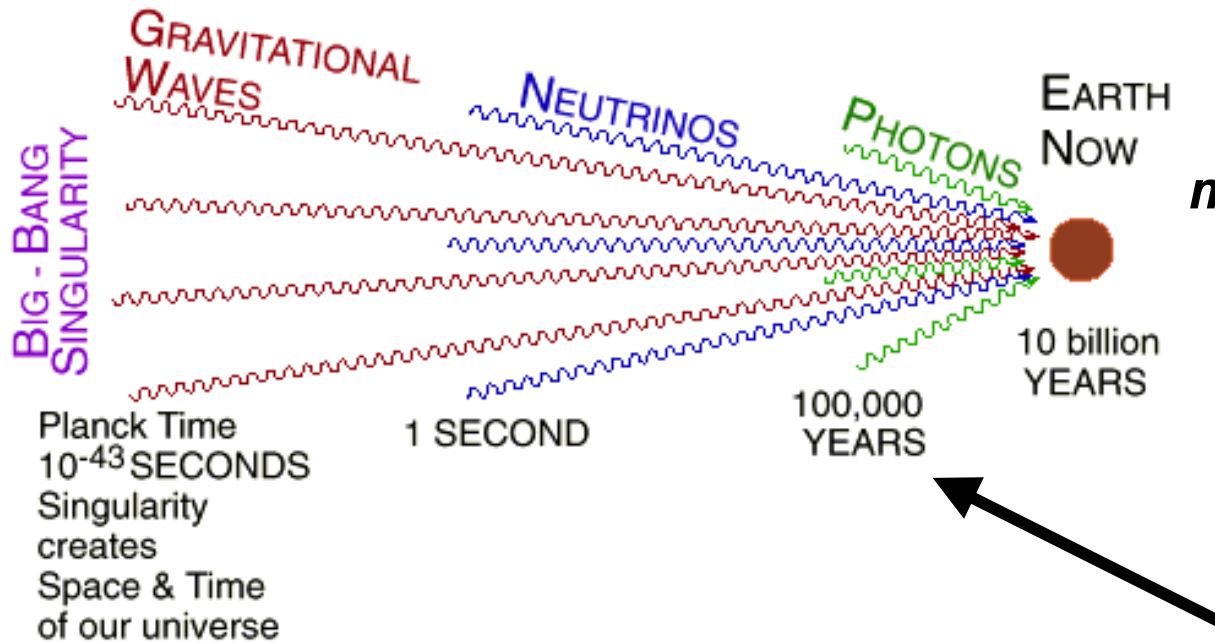


Sources of Gravitational Waves

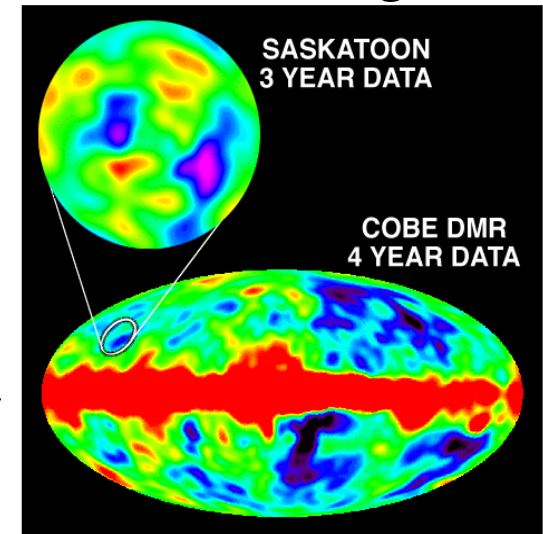
The Big Bang

'Murmurs' from the Big Bang

Signals from the early universe: Gaussian white noise (hiss) that is correlated among multiple detectors in predictable manner (frequency, orientations, etc.)



Cosmic microwave background





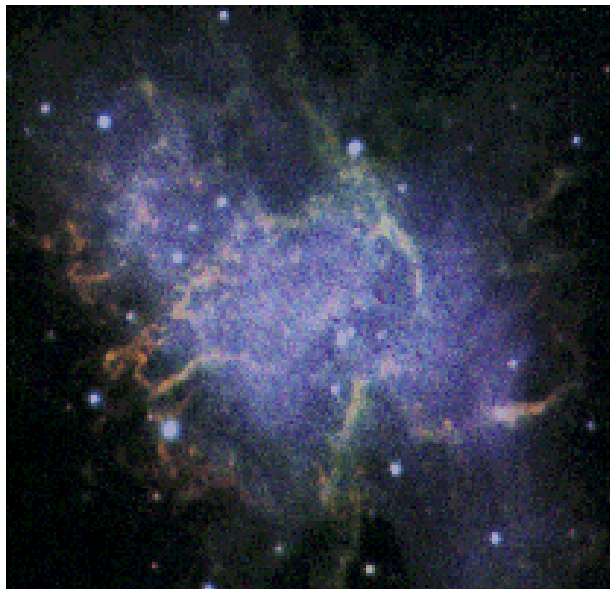
Sources of Gravitational Waves

The Rotating Deformed Neutron Stars

(Pulsars)

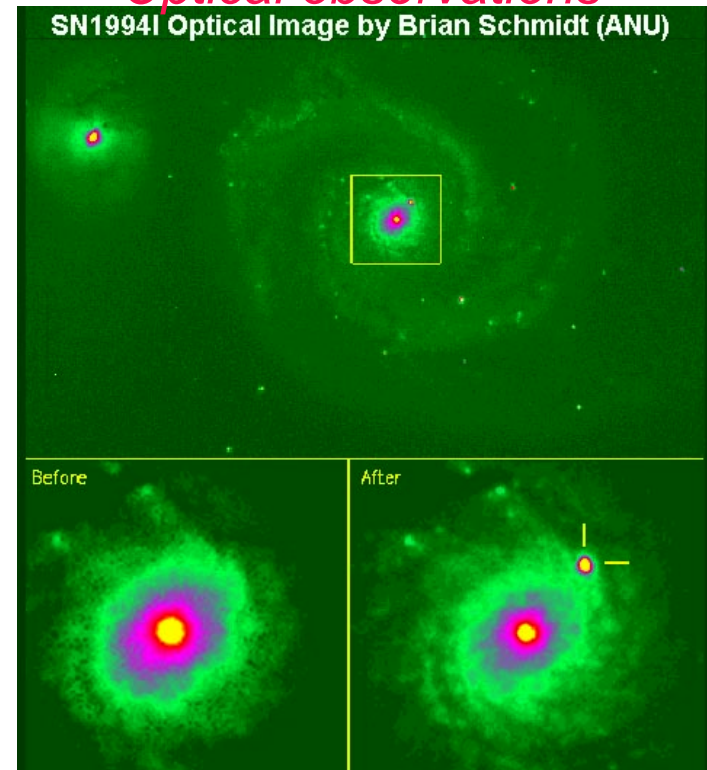
Pulsars (e.g., Crab Nebulaa 1054 AD)

Continuous waves



Supernovae (e.g., SN 1994I)

Optical observations



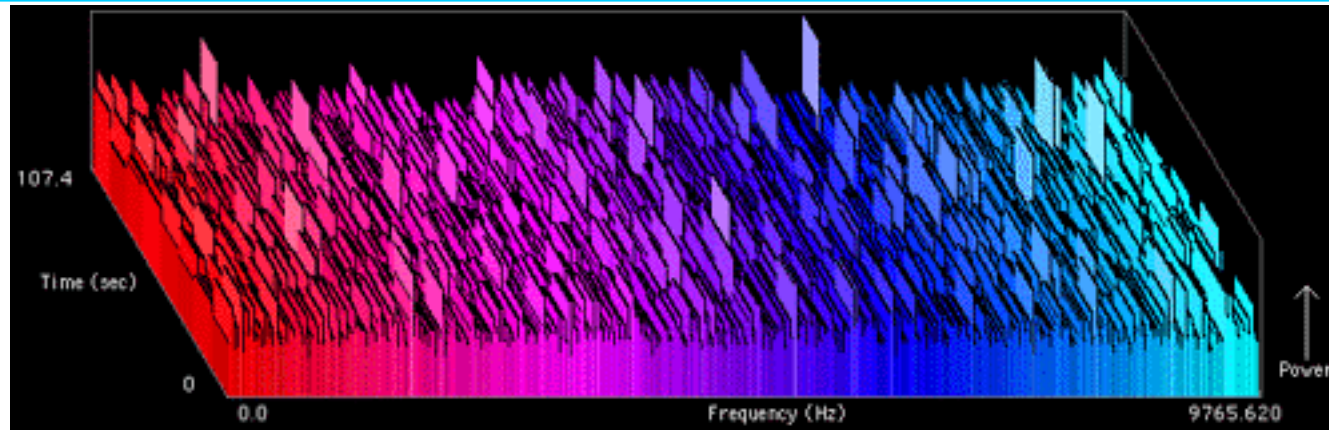
- Periodic sources last “forever” have AM and FM due to motion the earth
 - Look for these signatures
 - Depend on sky position of source
 - Sources will have intrinsic variations (spindown)

LIGO-G010259-00-E

LIGO LABORATORY CALTECH

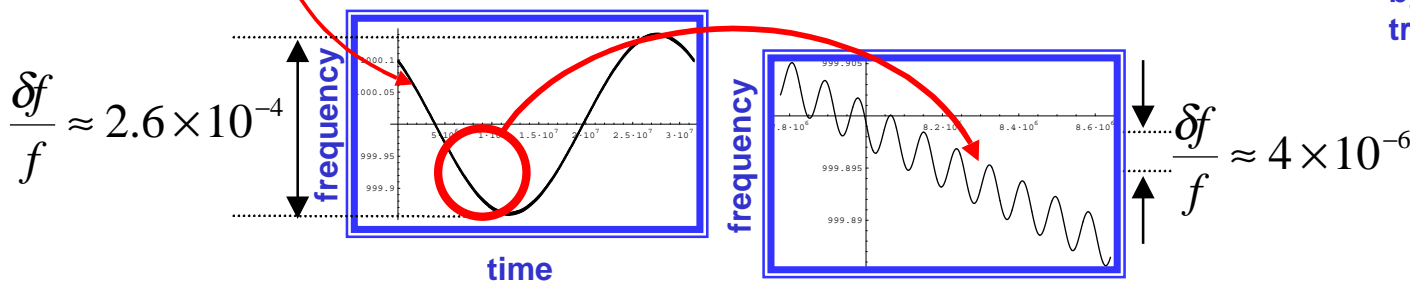
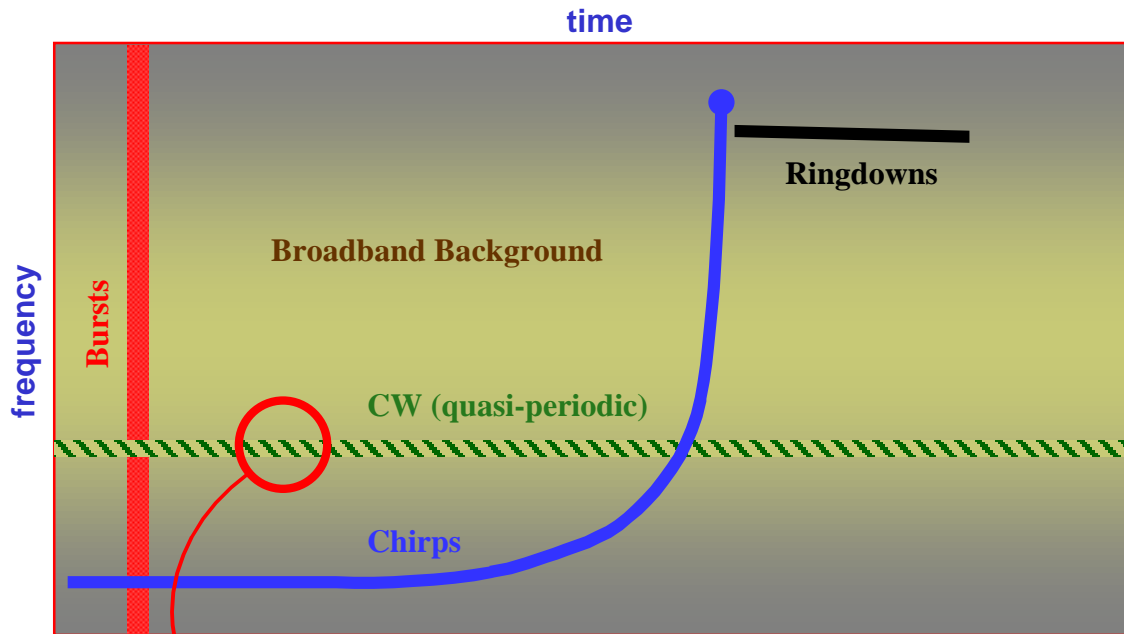


Frequency-Time Maps (“Images”)



SETI@home uses frequency-time analysis methods to detect *unexpected* or *novel* features in otherwise *featureless “hiss”*

Frequency-Time Characteristics of GW Sources



$$\frac{\delta f}{f} \approx 2.6 \times 10^{-4}$$

$$\frac{\delta f}{f} \approx 4 \times 10^{-6}$$

LIGO-G010259-00-E

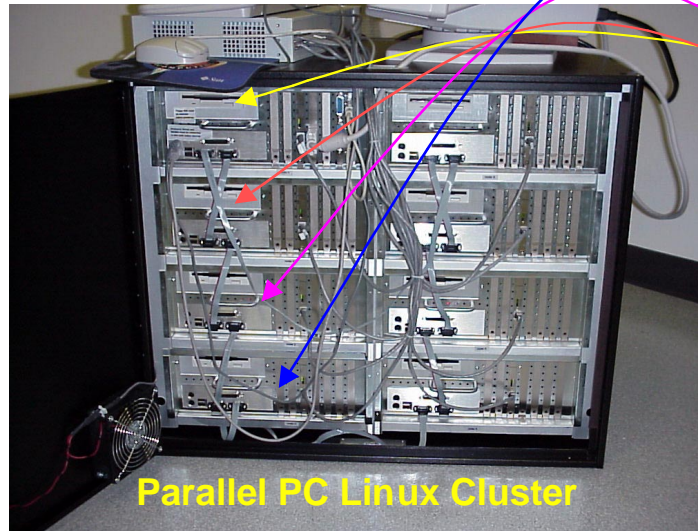
LIGO LABORATORY CALTECH

- Bursts are short duration, broadband events
- Chirps explore the greatest time-frequency area
- BH Ringdowns expected to be associated with chirps
- CW sources have FM characteristics which depend on position on the sky (and source parameters)
- Stochastic background is stationary and broadband
- For each source, the optimal signal to noise ratio is obtained by integrating signal along the trajectory
 - If $SNR \gg 1$, kernel $\propto |\text{signal}|^2$
 - If $SNR \leq 1$, kernel $\propto |\text{template}^* \text{signal}|$ or $|\text{signal}_j^* \text{signal}_k|$
- Optimal filter: kernel $\propto 1/(\text{noise power})$

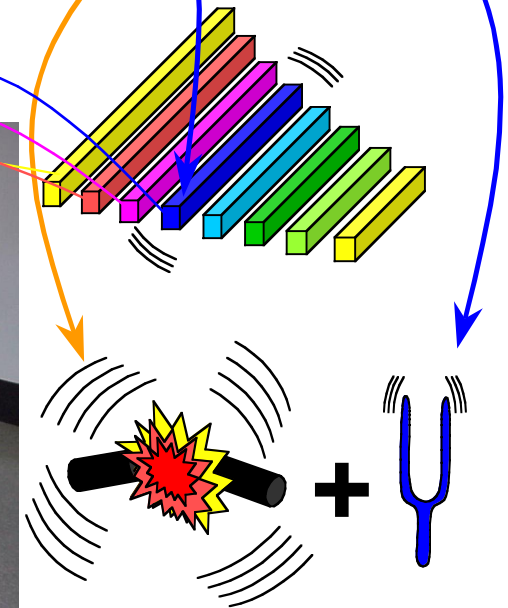
Optimal Wiener Filtering

- Matched filtering (optimal) looks for best overlap between a signal and a set of expected (template) signals in the presence of the instrument noise -- correlation filter
- Replace the data time series with an SNR time series
- Look for excess SNR to flag possible detection

$$\xi_p[t_c] = \int \frac{\hat{T}_p^*(f) \hat{s}(f)}{\hat{S}_n(|f|)} e^{-2\pi i f t_c} df$$

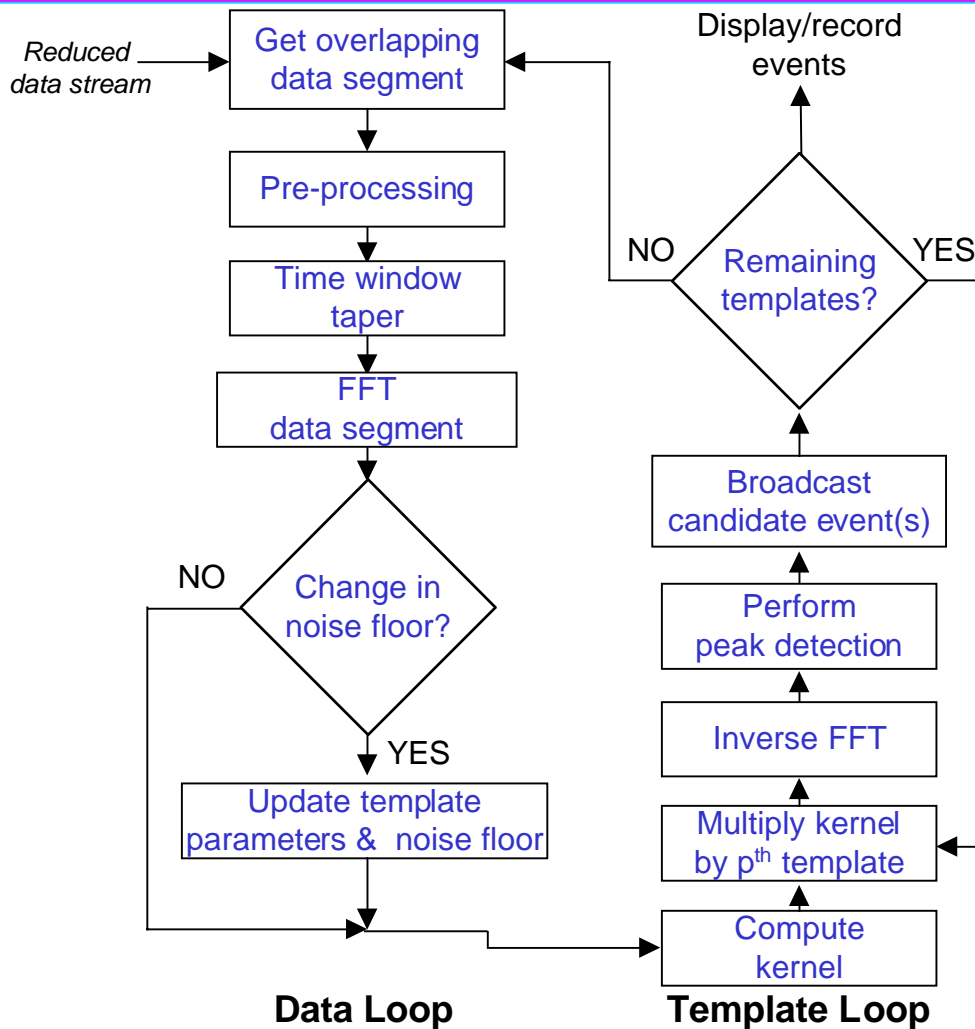


Parallel PC Linux Cluster



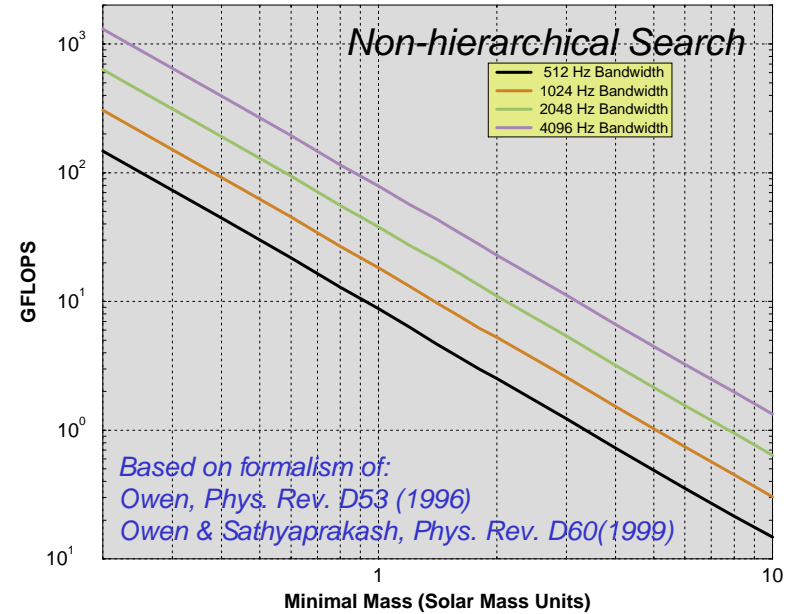


Compact Binary Inspirals Data Analysis Flow



$$\xi_p[t_c] = 2 \int \frac{\hat{T}_p^*(f) \hat{s}(f)}{\hat{S}_n(|f|)} e^{-2\pi i f t_c} df$$

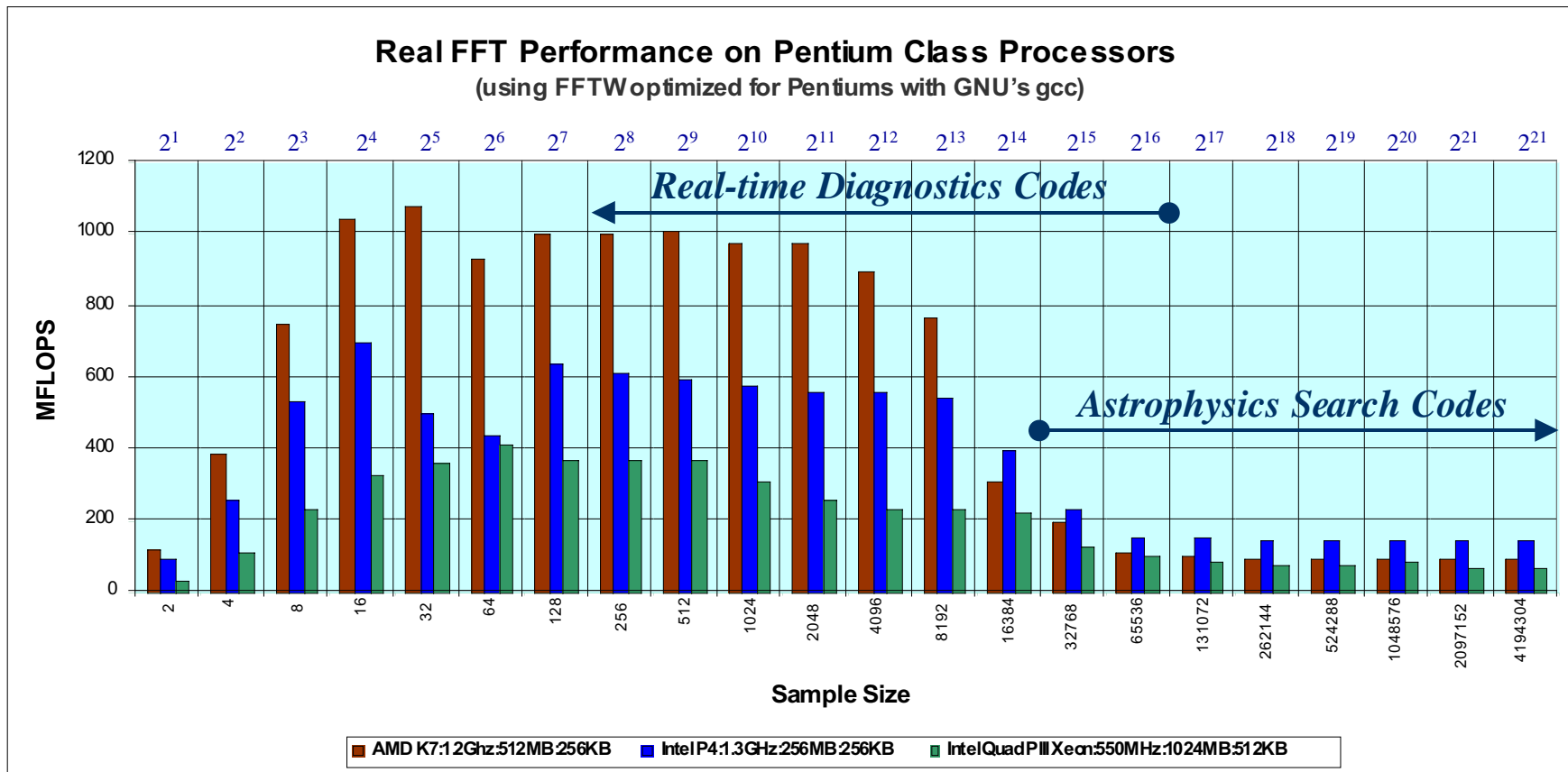
Binary Inspirals Template Compute Requirements
(estimated per interferometer with 8x overlap)



- Process data at real time rate
- Improvements:
 - Hierarchical searches developed
 - Phase coherent analysis of multiple detectors (Finn, in progress)



Recent CPU Node Performance



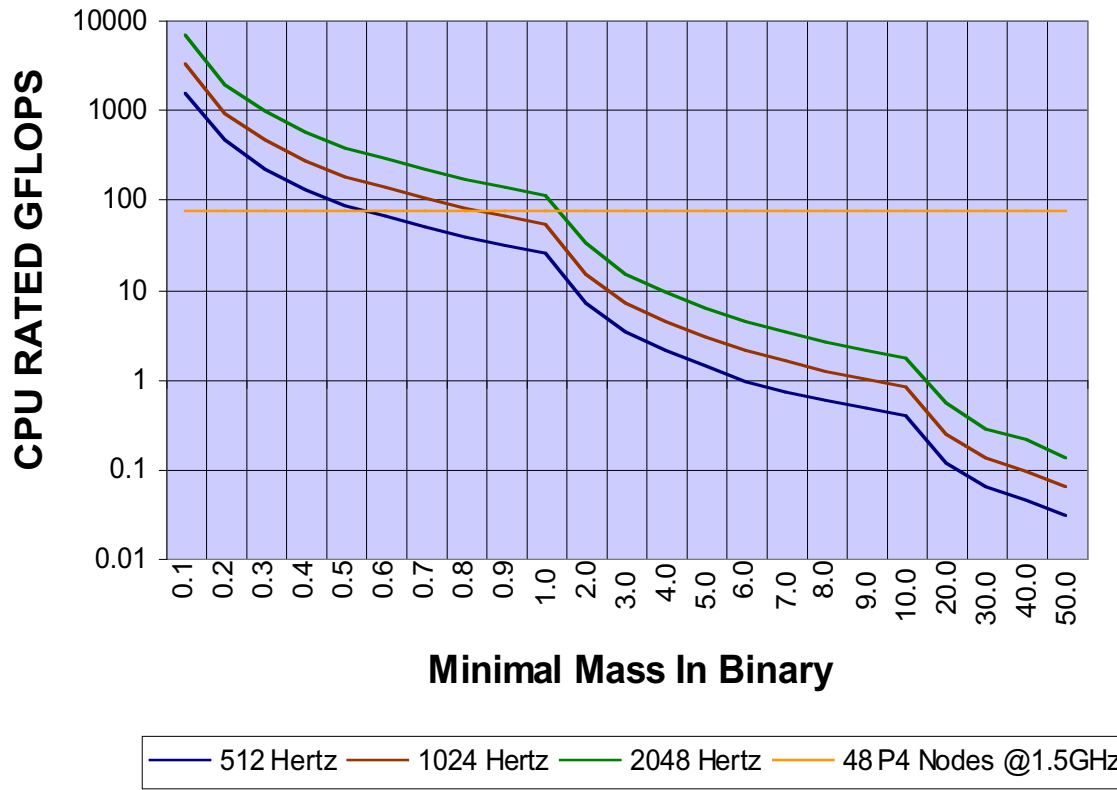
- Pipeline analysis of LIGO data computationally dominated by cost of Fast Fourier Transforms (FFT).
 - Non-Hierarchical Binary Inspiral Search spends an average of ~90% of CPU cycles performing FFT.
- Most practical/efficient data segment size as much as 2²⁰ points for Binary Inspiral Search.
 LIGO-G010259-00-E

LIGO LABORATORY CALTECH



Computation of Templated Binary Inspiral Search

Computation for Binary Inspiral Search Using 1.5GHz P4's
(8x overlap, no hierarchy, 10% detection loss, 10% efficiency for FFTs)



- Non-Hierarchical Search for NS-NS Inspiral ($1.4M_s, 1.4M_s$) ~ {15, 32, 67} GFLOPS.
- 512 Hertz band is adequate for detections (blue curve).
- Hierarchical strategies expected to decrease cost by 5x to 30x.
- Ringdown Search estimated to be roughly 10% as costly.
- Other searches (excluding all-sky pulsar search) are single node compute problems.

LIGO-G070229-00PL

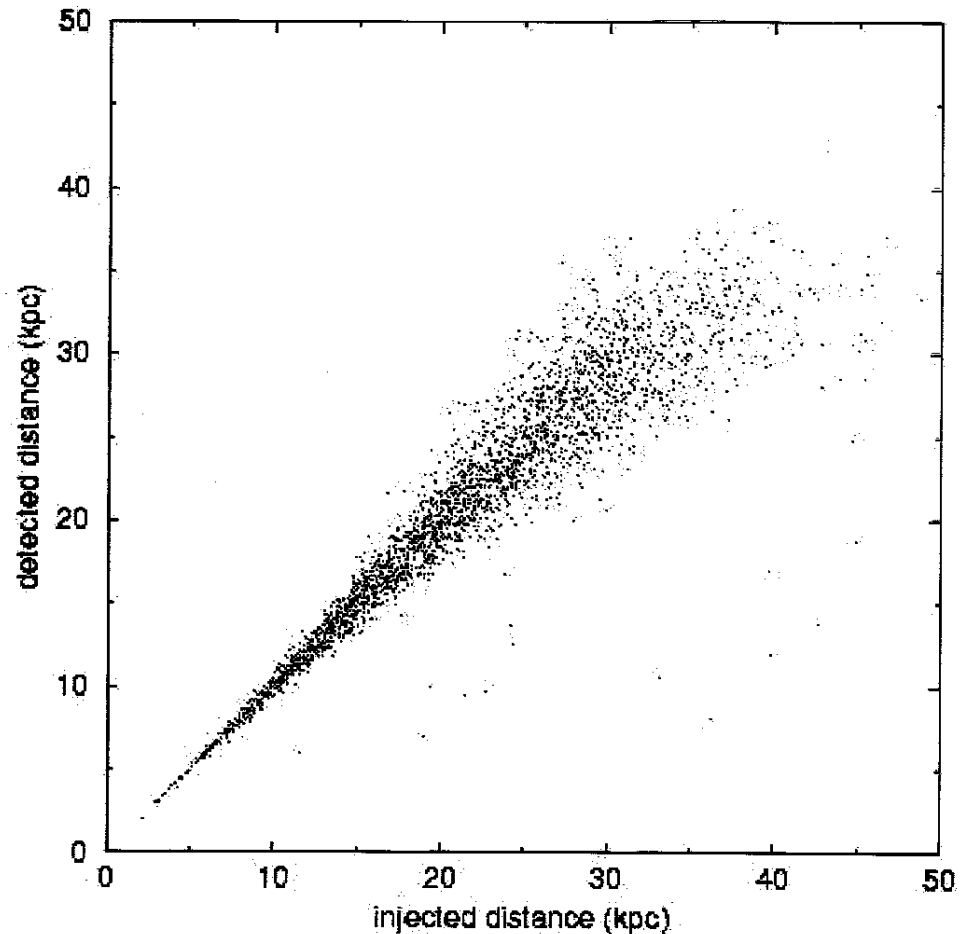
LIGO LABORATORY CALTECH



Detection Efficiency

Monte Carlo (Statistical) techniques are needed to characterize complex detection probabilities

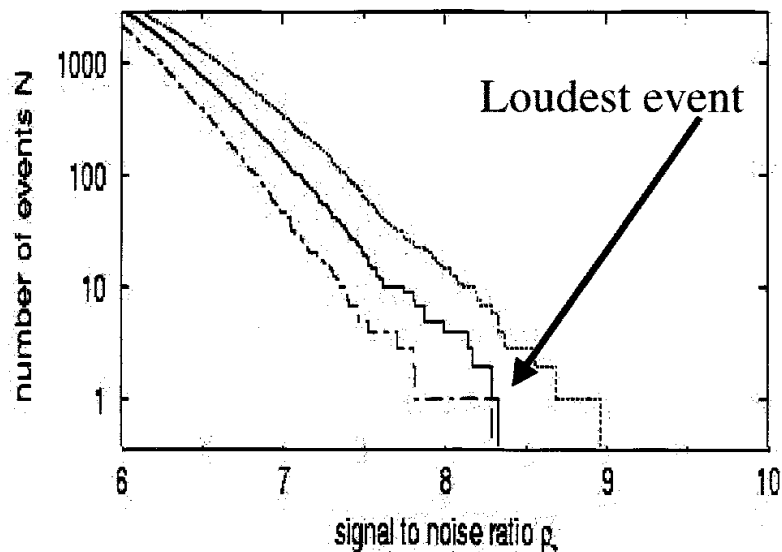
- *Simulated inspiral events provide end to end test of analysis and simulation code for reconstruction efficiency*
- *Errors in distance measurements from presence of noise are consistent with SNR fluctuations*





Setting a limit

Quantitative Science: making a probabilistic statement about the likelihood of an observation (or lack thereof)



..... probability($\chi^2 > 61.2$) = 1%
——— probability($\chi^2 > 49.5$) = 10%
- - - probability($\chi^2 > 41.6$) = 32%

Upper limit on event rate can be determined from SNR of 'loudest' event

Limit on rate:

$R < 0.5/\text{hour}$ with 90% CL

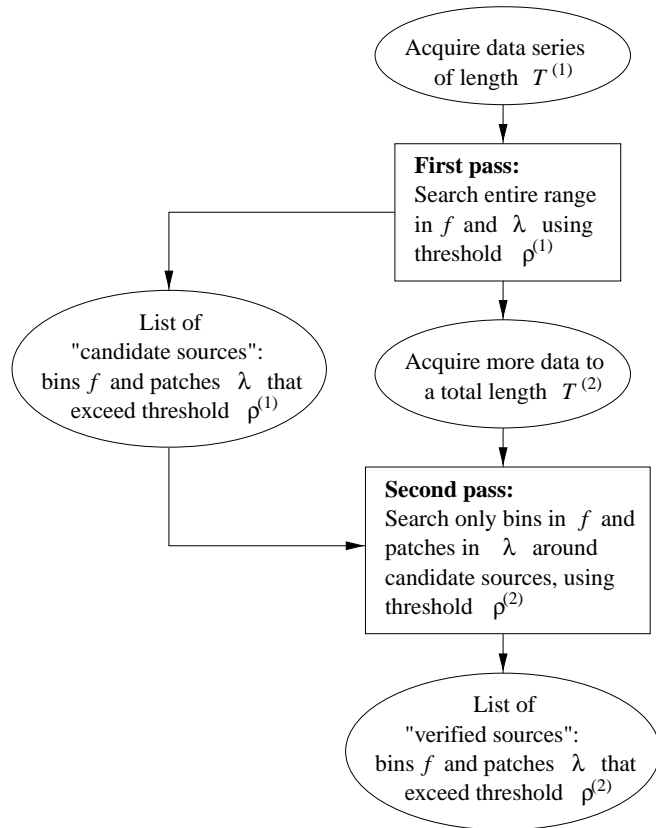
$\epsilon = 0.33 = \text{detection efficiency}$

An ideal detector would set a limit:

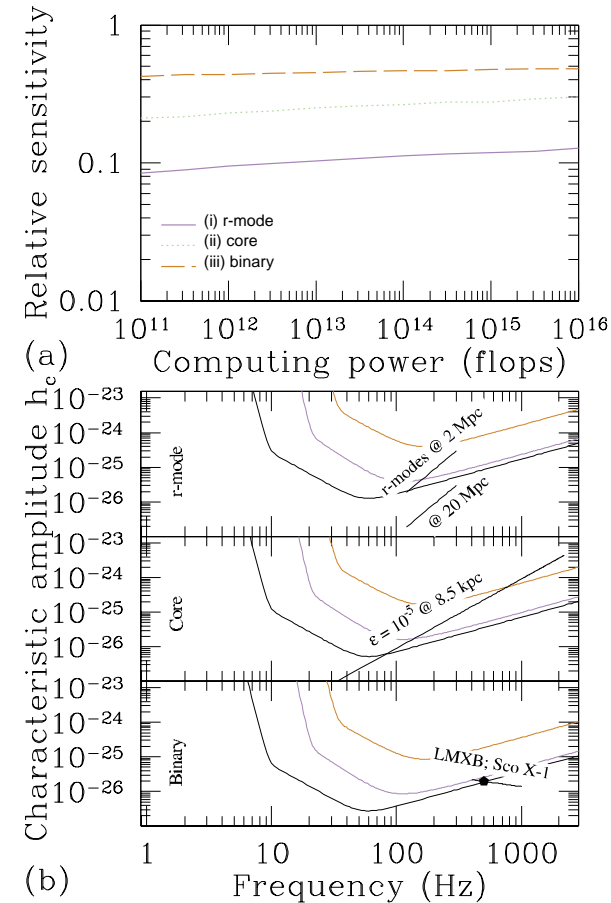
$R < 0.16/\text{hour}$

CW Sources

Hierarchical (Constrained) Search



ref: Brady & Creighton gr-qc/981204





Search Approaches for Other GW Sources

- Burst events (unmodeled)

- ◆ Cross correlate detector outputs over narrow time window
- ◆ Look for excess power
- ◆ Use environmental vetoes
- ◆ Look for few parametric templates (e.g., wavelets)
- ◆ CPU: Workstation(s)

$$S(t) = \iint_0^{T_B} dt' dt'' s_1(t-t') Q(t''-t') s_2(t-t'')$$

$$Q(\tau) = \int df e^{-2\pi i f \tau} \frac{\hat{h}_{B,1}^*(f) \hat{h}_{B,2}(f)}{S_1(|f|) S_2(|f|)}$$

ref: Finn, Mohanty, Romano gr-qc/9903101

- Stochastic background search

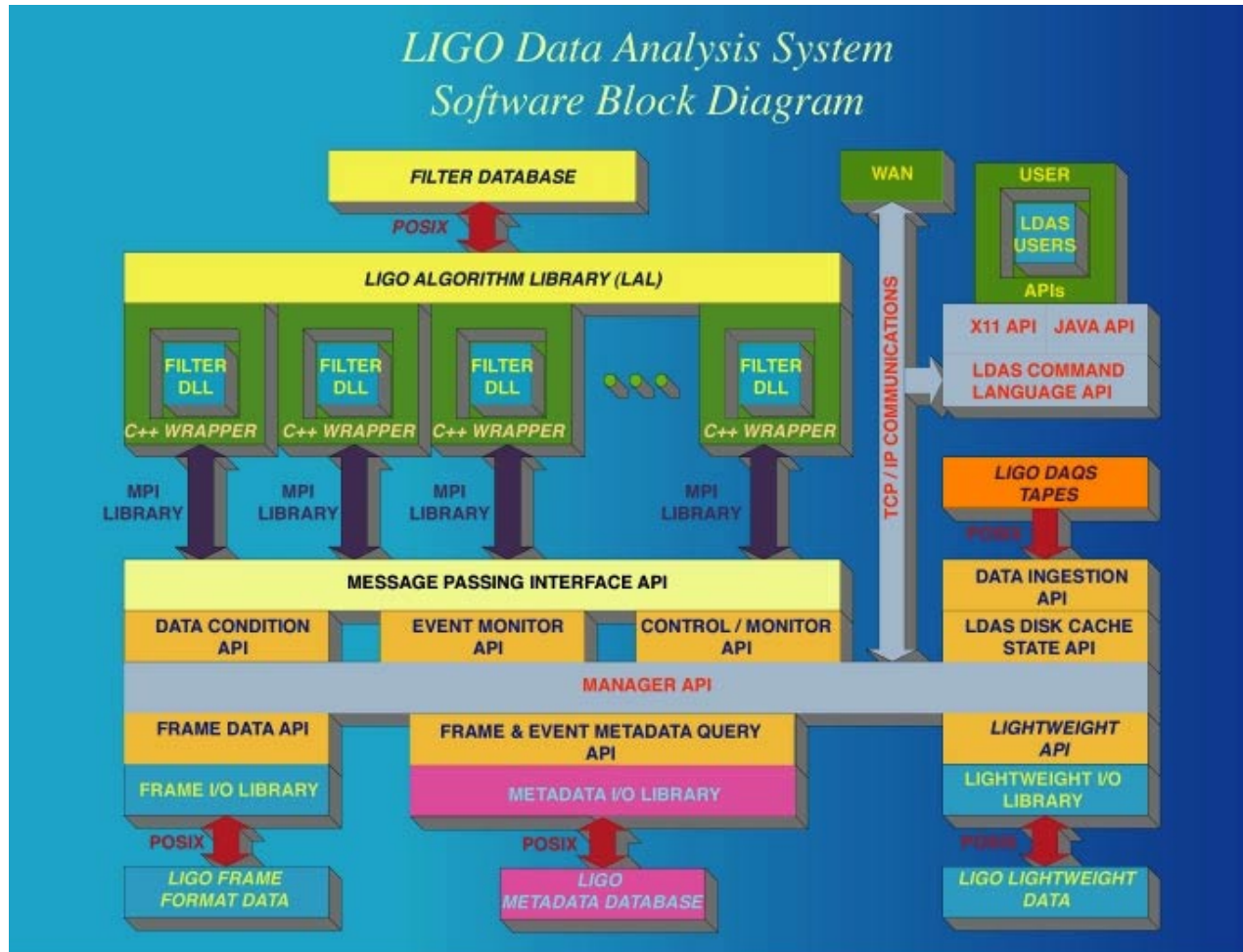
- ◆ Correlate & integrate signals from pairs of interferometers
- ◆ Look for excess power in band consistent with baseline separation
- ◆ CPU: Workstation(s)

$$S_{(\alpha)} = T_{\text{int}} \int_{f_{\text{min}}}^{f_{\text{max}}} \frac{\hat{S}_1^*(f) \hat{S}_2(f) \Omega_{\text{GW}}^{(\alpha)}(|f|) \gamma(|f|)}{f^3 S_1(|f|) S_2(|f|)} df$$

ref:
Michelson, 1987
Christensen, 1992
Flannigan, 1993
Allen & Romano gr-qc/9710117



Software Block Diagram



LIGO-T990101-E
LAL Specification

LIGO-T990097-E
wrapperAPI Requirements

www.lam.org
LAM version 5.6.1

LIGO-T990086-E
mpiAPI Requirements

LIGO-T990002-E
dataConditionAPI Requirements
FFTW version 2.3.1
CLAPACK version

LIGO-T980115-E
managerAPI Requirements

LIGO-T980117-E
frameAPI Requirements

LIGO-T970130-E
LIGO-G010259-
LIGO-VIRGO Frame Specification
FrameCPP Version 4

LIGO-T980119-E
metaDataAPI Requirements

dbEasy Library
ODBC Level 3

LIGO-T990101-E
LDAS Database Tables

LIGO-T990023-E
LIGO-Lightweight Format

GUILD
TCL/TK GUI

UNIX Sockets (TELNET)
User Commands (TCL)

LIGO-T010052-E
dataIngestionAPI Requirements
"In Development"

LIGO-T010051-E
diskCacheAPI Requirements
"In Development"

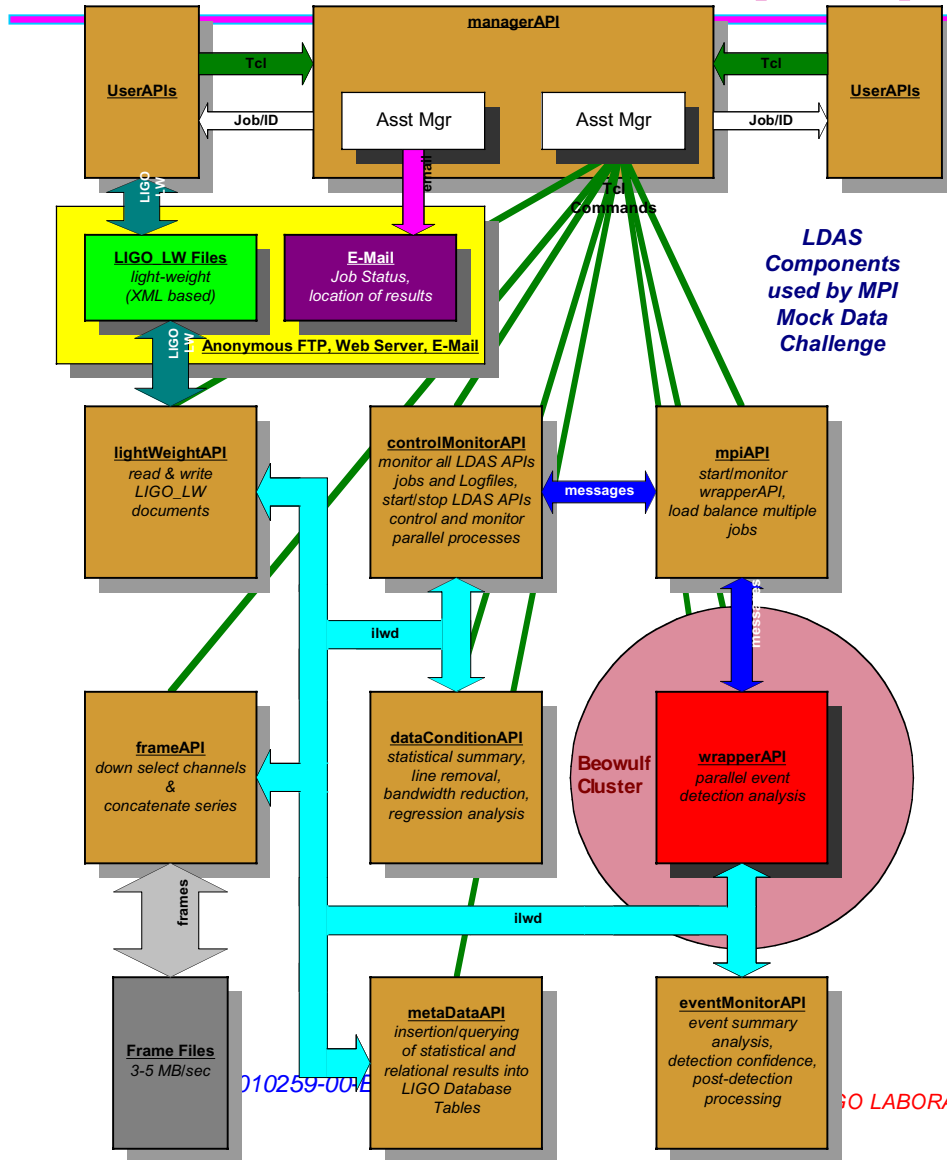
LIGO-T000026-E
controlMonitorAPI Requirements

LIGO-T990037-E
lightWeightAPI Requirements

www.apache.org
C++ XML Parser Library



Simple Inspiral Pipeline Example



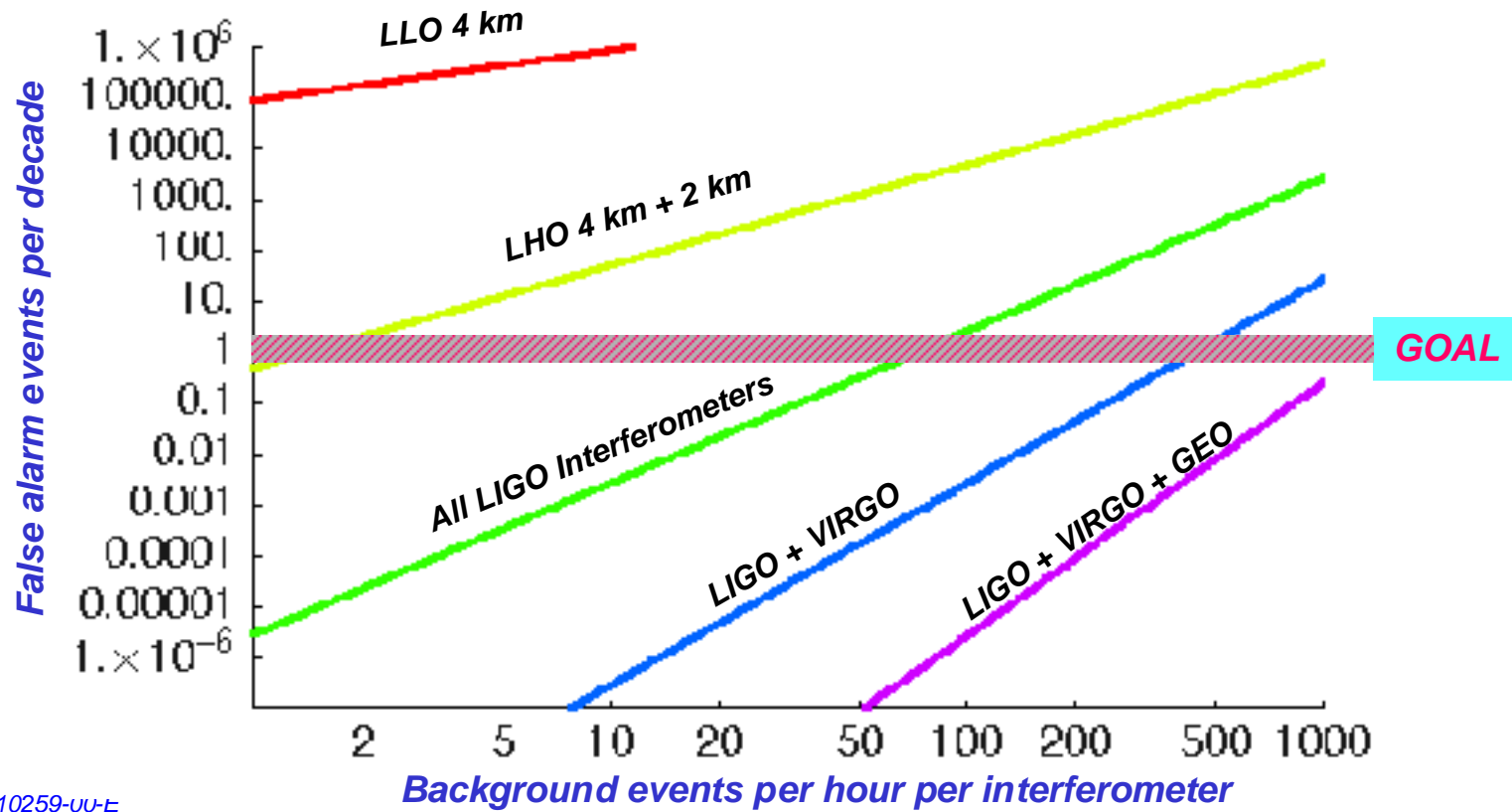
- **UserAPI** connects to **managerAPI** and starts pipeline.
- **managerAPI** assigns *assistant manager* to control job.
- **frameAPI** reads *frames* into the LDAS system, down-selects and sends data to the **dataConditionAPI**.
- **dataConditionAPI** pre-conditions data and forwards this to the **wrapperAPI** under the control of **mpiAPI**.
- **wrapperAPI** performs template based search using a dynamically loaded shared object.
- **eventMonitorAPI** receives search results and directs events to the **metaDataAPI** where they are placed in database.
- **eventMonitorAPI** sends user data products from search to **lightWeightAPI** where they are sent to user via FTP or URI commands.
- **managerAPI** notifies user via email of job completion.

GO LABORATORY CALTECH



Coincidence windows among detectors

- Rejection of statistically uncorrelated random events
 - Coincidence window duration determine by baselines

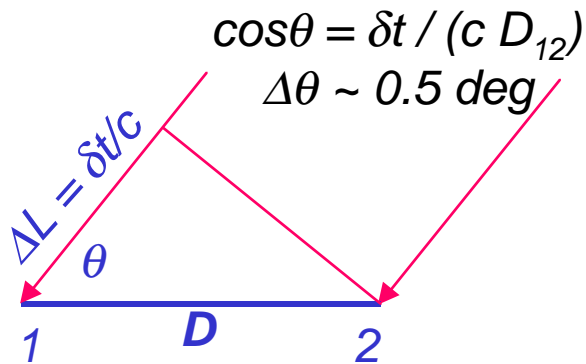


LIGO-G010259-UU-E

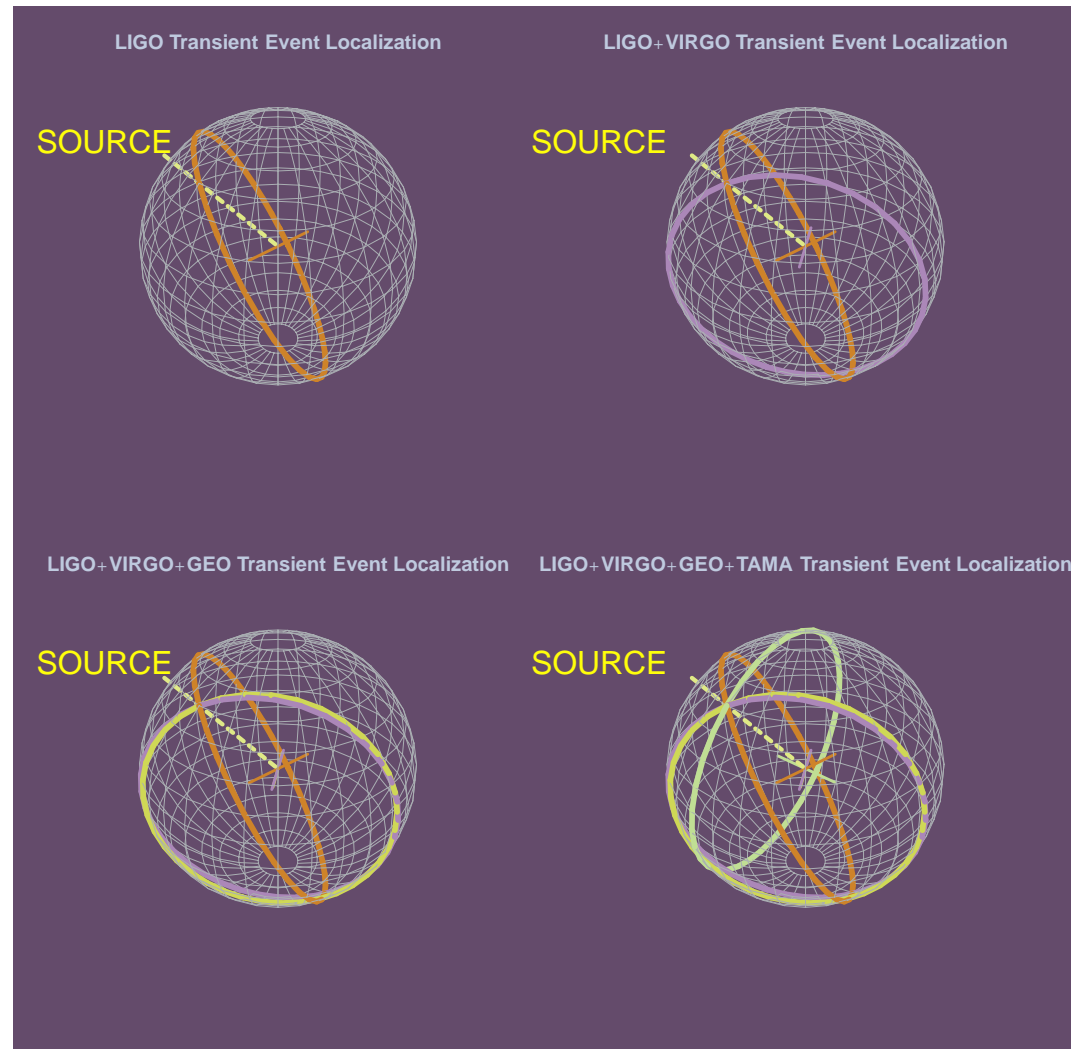
LIGO LABORATORY CALTECH



Event Localization With An Array of GW Interferometers



LIGO-G010259-00-E



LIGO LABORATORY CALTECH



Joint Data Analysis Among GW projects

From detection to validation

- For a *putative* detection:

Environmental, instrumental vetoes?

$(\Delta t_i, \Delta \Omega_i)$: Seen by all detectors within consistent (time, position) windows?

Δh_i : Is the amplitude of the signal consistent among detectors*?

$\Delta \alpha_i$: Are the deduced model parameters consistent?

- Follow up analyses

- ♦ Independent

- ♦ Coherent multi-detector analysis -
maximum likelihood over all detectors: $\{t, \Omega, h, \alpha\}$

- Discrepancies should be explainable, e.g.:

- ♦ Not on line

- ♦ Below noise floor

- ♦ *Different polarization sensitivity, *etc.*

$$h_i \rightarrow \vec{h}$$

$$\ln \Lambda(h_i, \theta_i) \rightarrow \ln \Lambda(\vec{h}, \vec{\theta})$$

$$\sigma_i^2 \rightarrow C_{kl} \equiv \langle \vec{n}_k \otimes \vec{n}_l \rangle$$