LLO + Allegro: A Unique Opportunity to Experimentally Modulate the Stochastic Gravitational Wave Background

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LIGO

Experimental Modulation of the Stochastic Background Correlation

- Work in progress performed in collaboration with L. S. Finn (Penn State University)
- References:

  » P.F. Michelson, *Mon. Not. Roy. Astron. Soc.* **227**, 933 (1987).

  » N. Christensen, *Phys. Rev.* **D46**, 5250 (1992)

  » E. Flanagan, *Phys. Rev.* **D48**, 2389 (1993), astro-ph9305029

  » B. Allen and J. Romano, *Phys. Rev.* **D59**, 102001 (1999), gr-qc9710117

  » M. Maggiore, Trieste, June 2000: Gravitational Waves: A Challenge to Theoretical Astrophysics, gr-qc-0008027

LLO + Allegro: A Unique Opportunity

- Idea: perform an improved measurement of the stochastic gravitational wave background (SGWB) between a cryogenic resonant bar and one of the LIGO interferometers by introducing a modulation scheme into the measurement - *rotate the bar w.r.t. the interferometer…*
  - + Observation in a regime with little experimental information
  - + Uses a pair of (very nearly) collocated detectors
    - + good geometric overlap
  - + Ability to identify and remove a class of terrestrial backgrounds
  - - Relatively high frequency (920 Hz), narrowband measurement
    - - Less than optimal sensitivity

Stochastic GW Background Detection

- Cross-correlate the output of two (*independent*) detectors with a suitable filter kernel:

$$C(T) = \int_{-T/2}^{T/2} dt \int_{-\tau/2}^{\tau/2} d\tau' \; s_1(t) s_2(t-\tau') Q(\tau')$$

- Requires:
  - (i) Two detectors must have overlapping frequency response functions i.e., $s_1(f)s_2(f) \neq 0, \; \{f\} \notin \emptyset$
  - (ii) Detectors sensitive to same polarization state (+, x) of radiation field, h_{GW}.
  - (iii) Baseline separation must be suitably “short”:

$$L < \lambda_{GW}(f) \Rightarrow \frac{fL}{c} < 1$$

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Modulation of the Stochastic Background Correlation

- Ideally, the stochastic background correlation increases with integration time as:

$$SNR \propto \frac{3H_0^2}{10\pi^2}\sqrt{T_{\text{int}}}\left[\frac{\gamma^2(f_0)\Omega_{GW}^2 \Delta f}{f^6 S_{1,n}|f| S_{2,n}|f|}\right]^{\frac{1}{2}}$$

  - » Assumes no additional sources of correlated noise
    - – ***cannot discriminate with a single measurement***
  - » Mutual orientation dependence of GW background signal may be exploited to discriminate among possible correlated sources

Optimal filtering in the presence of background correlation

$$C(T) = \int_{-T/2}^{T/2} dt \int_{-T/2}^{T/2} dt' \; s_1(t) s_2(t') Q(t-t') \;;\; s_i(t) = h_i(t) + n_i(t)$$

h_i=GW signal in detector i

n_i= noise in detector i

$$C(T) = \int_0^{\infty} df \int_0^{\infty} df' \; \delta_T(f-f') \tilde{s}^*_1(f) \tilde{s}_2(f') Q(f') \;;\; \delta_T(f) \equiv T \left\{ \frac{\sin(\pi f T)}{\pi f T} \right\}$$

$$\left\langle \tilde{h}_1^*(f) \tilde{h}_2(f') \right\rangle = \delta(f-f') \frac{3H_0^2}{20\pi^2 |f|^3} \Omega_{GW}(|f|) \gamma(|f|, \vec{\Omega}_1, \vec{\Omega}_2)$$

$$\left\langle \tilde{n}_i^*(f) \tilde{n}_2(f') \right\rangle = \frac{1}{2} \delta(f-f') S_{ij}(|f|)$$

$$\begin{aligned} \left\langle \tilde{n}_i^*(f) \tilde{n}_2(f') \tilde{n}_i^*(f'') \tilde{n}_2(f''') \right\rangle = \frac{1}{4} \Big(& S_{ii}(|f|) S_{jj}(|f'|) \delta(f+f'') \delta(f'+f''') + \\ & S_{ij}(|f|) S_{ij}(|f''|) \delta(f+f'') \delta(f'+f''') + \\ & S_{ii}(|f|) S_{jj}(|f'|) \delta(f+f'') \delta(f'+f''') \Big) \end{aligned}$$

$\Omega_{GW}(f) = 1/\rho_0 \, d\rho_{GW}/d(\ln[f])$

$\gamma(f, \Omega_1, \Omega_2)$ = geometric overlap reduction factor depends on antenna orientations

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Optimal filtering in the presence of background correlation

$$\left\langle C(T,\vec{\Omega}_1,\vec{\Omega}_2)\right\rangle = T\int_0^\infty df\ \left(\pm\frac{3H_0^2}{20\pi^2|f|^3}\Omega_{GW}(|f|)\gamma\left(|f|,\vec{\Omega}_1,\vec{\Omega}_2\right)+S_{12}(|f|)\right)\tilde{Q}(f)\ ;$$

Choose two orientations of one detector { Ω_1, Ω_1' }, for which $\gamma(f, \Omega_1, \Omega_2) = -\gamma(f, \Omega_1', \Omega_2)$, denote C_+, C_- values of integrated correlation in these two orientations:

$$\langle C(T)\rangle = \langle C_+(T/2) - C_-(T/2)\rangle$$

$$\langle C(T)\rangle = T\int_0^\infty df\ \left(\frac{3H_0^2}{20\pi^2|f|^3}\Omega_{GW}(|f|)\gamma\left(|f|,\vec{\Omega}_1,\vec{\Omega}_2\right)\right)\tilde{Q}(f)$$

$$\sigma_C^2 = \left\langle C^2\right\rangle - \langle C\rangle^2 = 2\sigma_{C+,-}^2$$

$$\sigma_C^2 = \frac{T}{2}\int_0^\infty df\ \left(S_1(|f|)S_2(|f|)+S_{12}^2(|f|)\right)\left[\tilde{Q}(f)\right]^2$$

$$SNR = \frac{\langle C\rangle}{\sigma_C}\ \overset{max}{\Rightarrow}\ \frac{\delta[SNR]}{\delta\left[\tilde{Q}\right]} = 0\ \Rightarrow\ \tilde{Q}(f) = \left(\frac{\gamma\left(|f|,\vec{\Omega}_1,\vec{\Omega}_2\right)\Omega_{GW,\mathrm{model}}(|f|)}{|f|^3\left(S_1(|f|)S_2(|f|)+S_{12}^2(|f|)\right)}\right)$$

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Modulation of the Stochastic Background Correlation

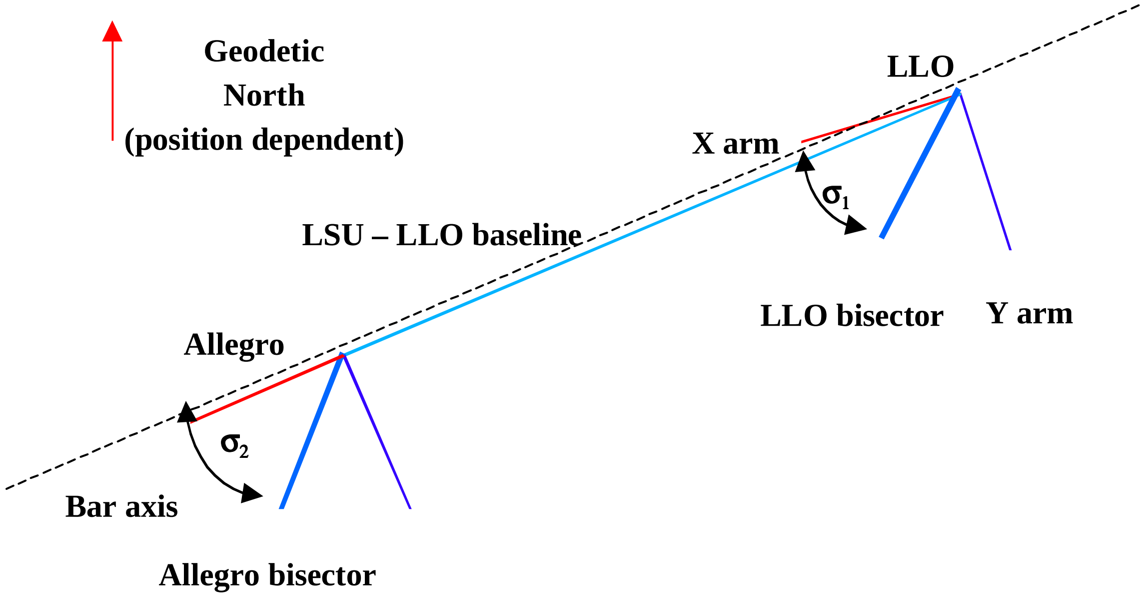

Figure 1: Schematic showing Allegro and LLO orientations with respect to geodetic north and the LLO-LSU baseline.

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Modulation of the Stochastic Background Correlation

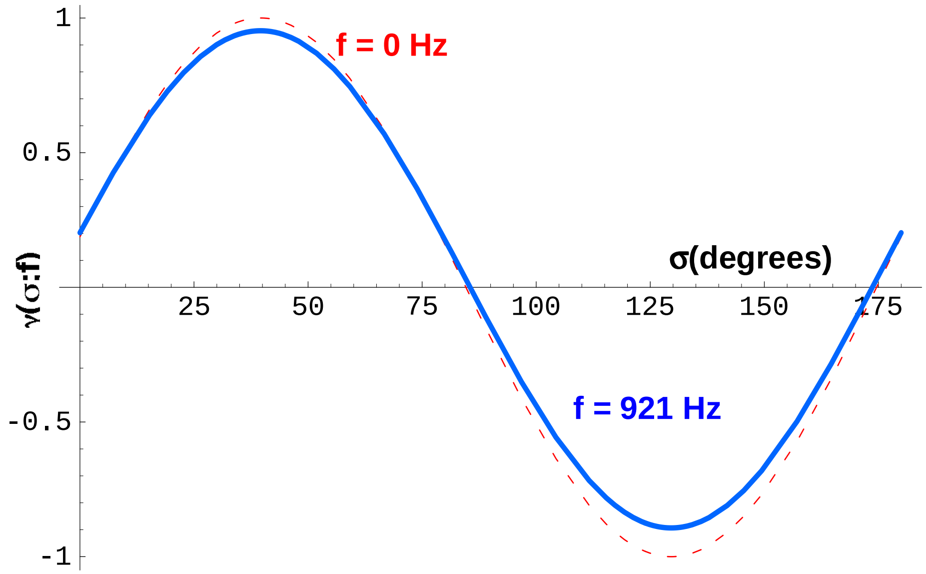

Figure 2: Dependence of the Allegro-LLO correlation function on the angle between the Allegro bar bisector and the LLO-to-LSU baseline (refer to Figure 1). Dashed line is for DC and the solid line is for the Allegro resonant frequency.

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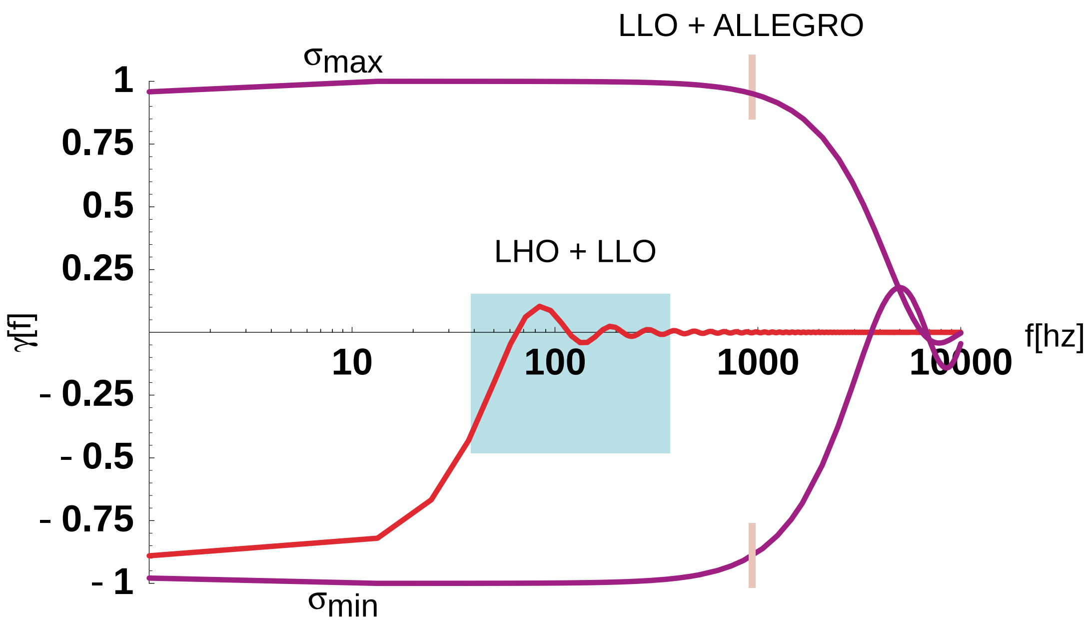
Modulation of the Stochastic Background Correlation



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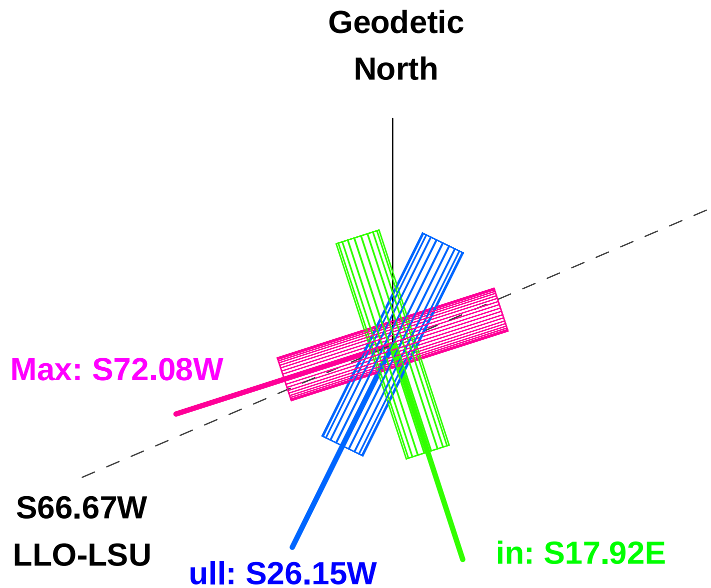
Modulation of the Stochastic Background Correlation



igure 3: Schematic showing Allegro orientations with espect to geodetic north and the LLO-LSU baseline

$$\boldsymbol{C_+}(\sigma_2 = 39.6°) \approx T_{\text{int}}\Delta f\left(\frac{3H_0^2}{20\pi^2}\frac{\gamma^2\Omega_{GW}(f_0)\Omega_{GW,\text{model}}(f_0)}{f_0^6\left(S_A(f_0)S_L(f_0)+S_{LA}(f_0)^2\right)}+S_{LA}(f_0)\frac{\gamma\ \ \Omega_{GW,\text{model}}(f_0)}{f_0^3\left(S_A(f_0)S_L(f_0)+S_{LA}(f_0)^2\right)}\right)$$

$$\boldsymbol{C_-}(\sigma_2 = 129.6°) \approx T_{\text{int}}\Delta f\left(-\frac{3H_0^2}{20\pi^2}\frac{\gamma^2\Omega_{GW}(f_0)\Omega_{GW,\text{model}}(f_0)}{f_0^6\left(S_A(f_0)S_L(f_0)+S_{LA}(f_0)^2\right)}+S_{LA}(f_0)\frac{\gamma\ \ \Omega_{GW,\text{model}}(f_0)}{f_0^3\left(S_A(f_0)S_L(f_0)+S_{LA}(f_0)^2\right)}\right)$$

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Modulation of the Stochastic Background Correlation

- After total a observation time T_{int}

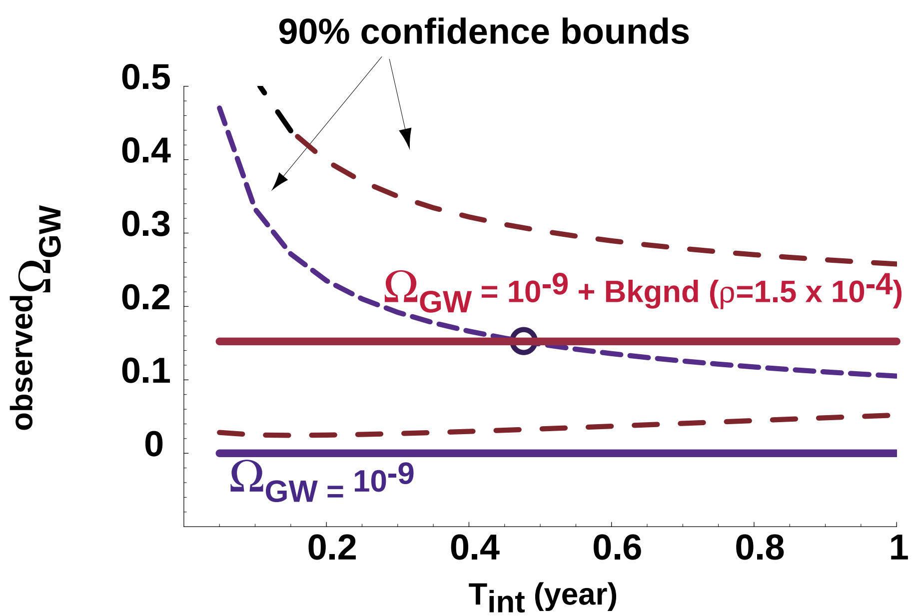
$$\mathbf{C} = \mathbf{C}_{+}(\sigma_2 = 39.6°) - \mathbf{C}_{-}(\sigma_2 = 129.6°) = T_{\text{int}}\Delta f\left(\frac{3H_0^2}{20\pi^2}\frac{\gamma^2\Omega_{GW}(f_0)\Omega_{GW,\text{model}}(f_0)}{f_0^6\left(S_A(f_0)S_L(f_0)+S_{LA}^2(f_0)\right)}\right)$$

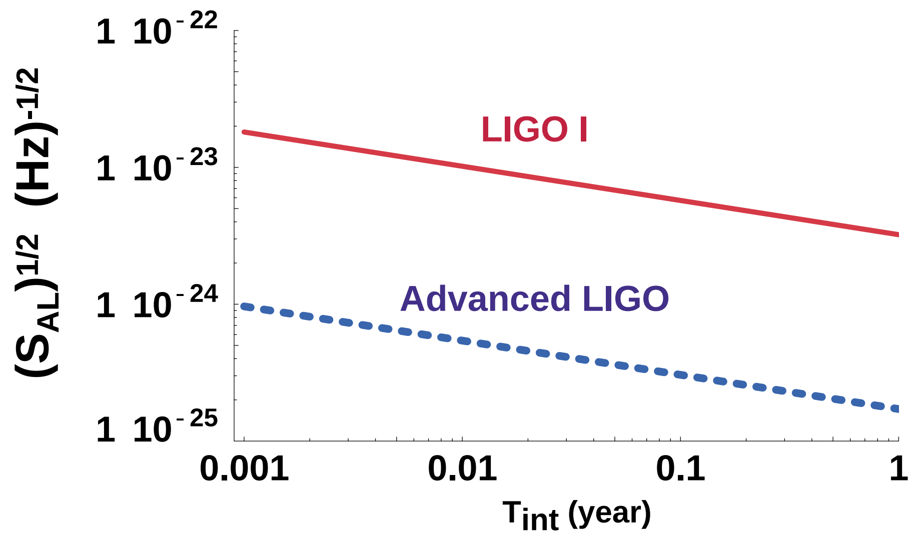
$$\sigma_C^2 = T_{\text{int}}\Delta f\left(\frac{\gamma^2\Omega_{GW,\text{model}}^2(f_0)}{f_0^6\left(S_A(f_0)S_L(f_0)+S_{LA}^2(f_0)\right)}\right)$$

$$\mathbf{C} = \left(\frac{3H_0^2}{20\pi^2}\frac{\Omega_{GW}(f_0)}{\Omega_{GW,\text{model}}(f_0)}\right)\sigma_C^2$$

$$SNR = \frac{\mathbf{C}}{\sigma_C} = \sqrt{T_{\text{int}}\Delta f}\left(\frac{3H_0^2}{10\pi^2}\frac{\gamma\ \Omega_{GW}(f_0)}{f_0^3\sqrt{\left(S_A(f_0)S_L(f_0)+S_{LA}^2(f_0)\right)}}\right)$$

Effect of correlated background on observable upper limits for Ω_{GW}





$$\rho \equiv \frac{S_{12}}{\sqrt{S_1 S_2}}$$

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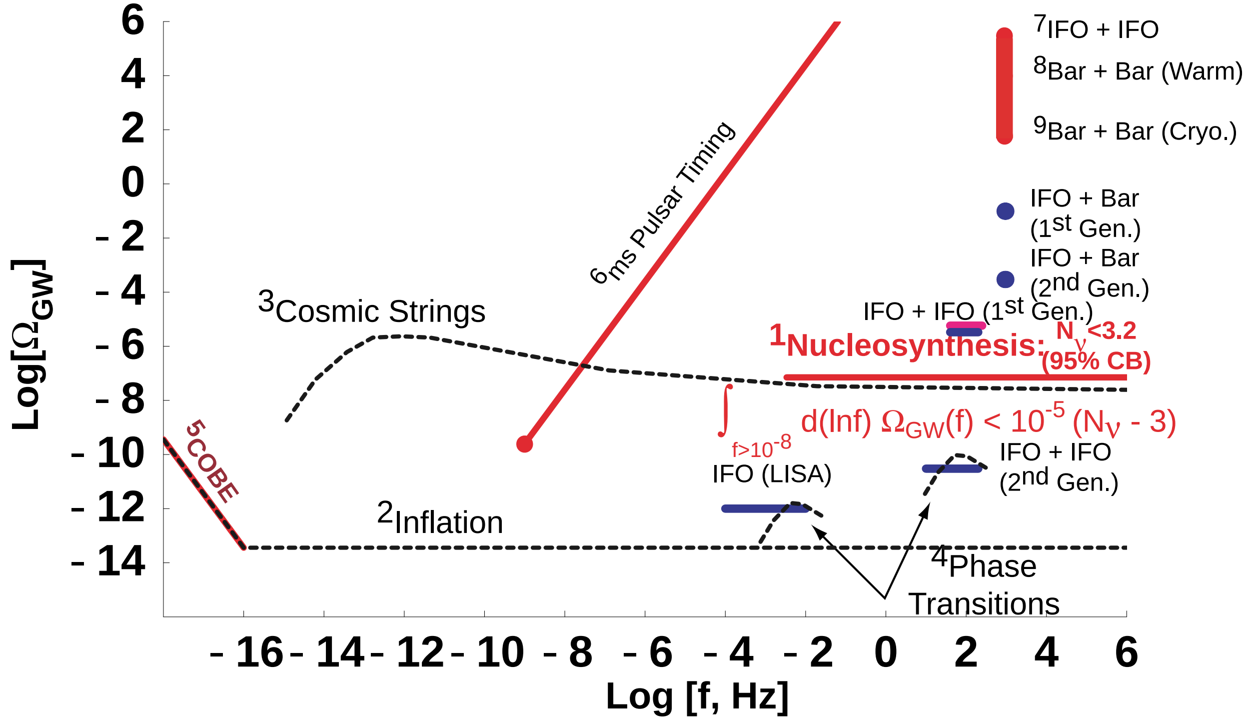
Measurements of the Stochastic Background



[1] Kolb & Turner (The Early Universe, 1990)
Burles, Nollet,Trunan, Turner (PRL 82, 1999)

[2] Grishchuk (SPJETP 40, 1975)

[3] Allen & Brustein (gr-qc9609013)
Allen (gr-qc9604033)

[4] Kamionkowski, Kosowoski &Turner (PRD 49, 1994)

[5] Allen & Koranda (PRD 50, 1994)

[6] Thorsett & Dewey (PRD 53, 1996)
Kaspi, Taylor, Ryba (ApJ 428, 1994)

[7] Compton, Nicholson, Schutz, Proc. MG7 (1994)

[8] Hough, Pugh, Bland, Drever, Nature 254 (1975)

[9] Astone, et. al., Astr. Astroph. 351 (1999)

Conclusions

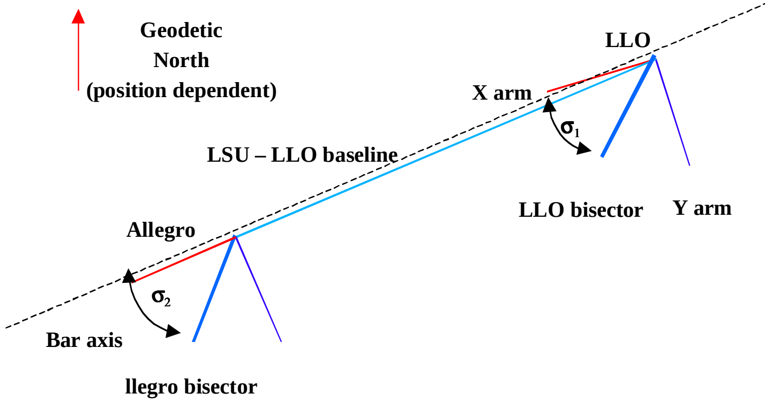
Modulation of the Stochastic Background Correlation

- It is possible to account for correlated detector noise background in deriving the Optimal Wiener filter
- In the presence of correlated detector noise background, the upper limit will be a biased estimate:
  - Level of $\rho \sim 10^{-4}$ begins to limit measurement after less than 1 year
  - Current best published upper limit of $\Omega_{GW} < 60$ can be improved by LIGO+ALLEGRO
- Move of ALLEGRO to new quarters was used to modify bar to allow this measurement. *Considerations:*
  - » Modulation period >> detector settling time => *dead-time*
  - » Modulation period < total integration time => *multiple orientations*
  - » Choose period of ~ 3 - 5 months (not commensurate with seasonal/annual cycles)

Experimentally modulating the stochastic background

Finn & Lazzarini

gr-qc 0104040; submitted to PRD



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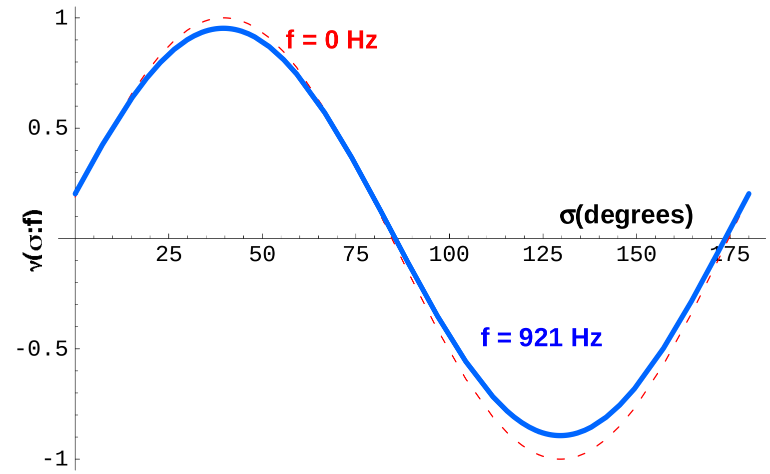

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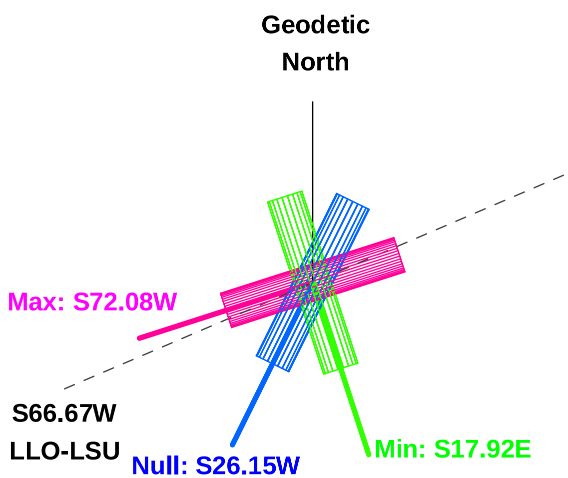

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