

**ACTIVE WAVEFRONT CORRECTION IN LASER
INTERFEROMETRIC GRAVITATIONAL WAVE
ANTENNAE:
THESIS PROGRESS REPORT #1
LIGO-G010218-00-R**

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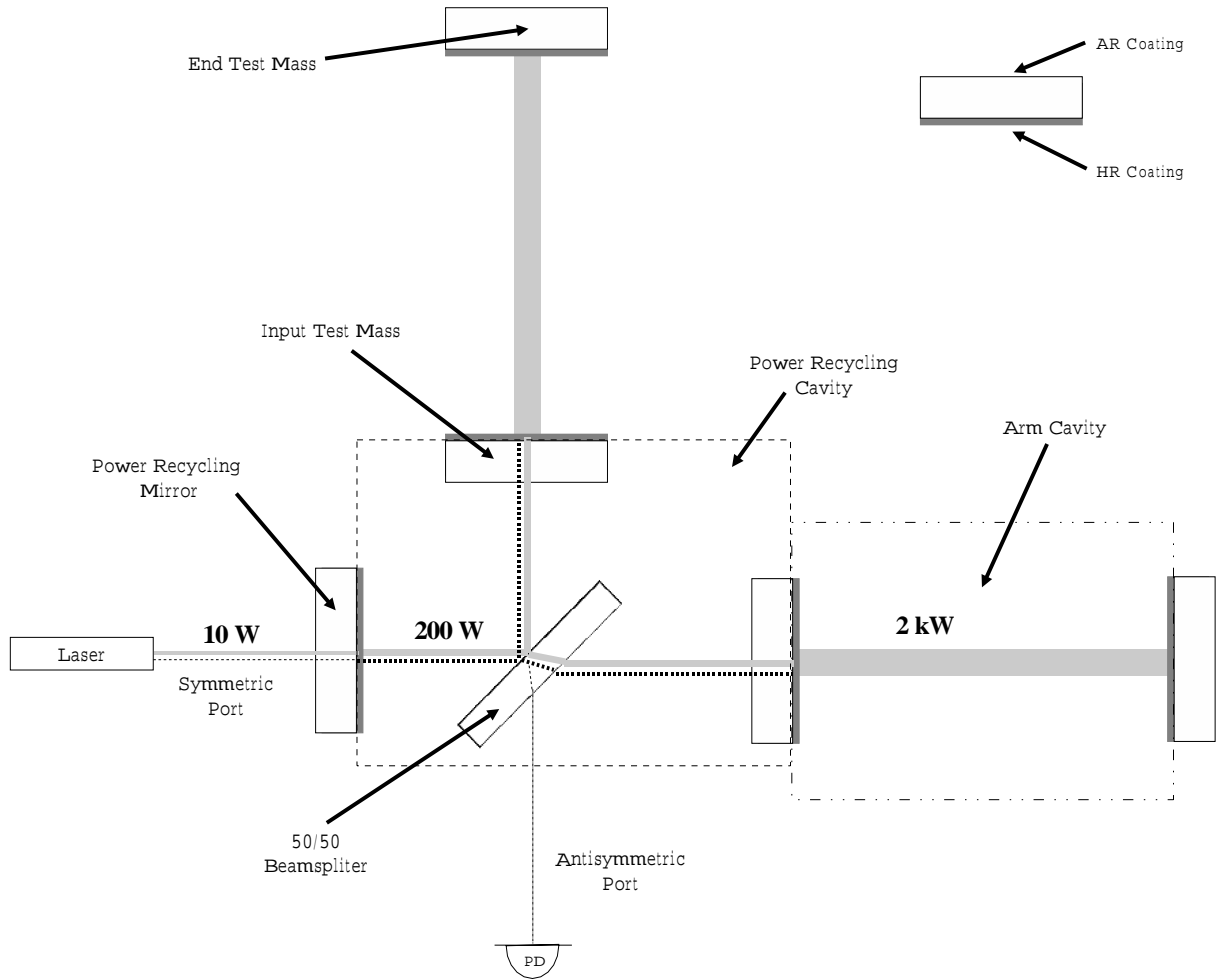
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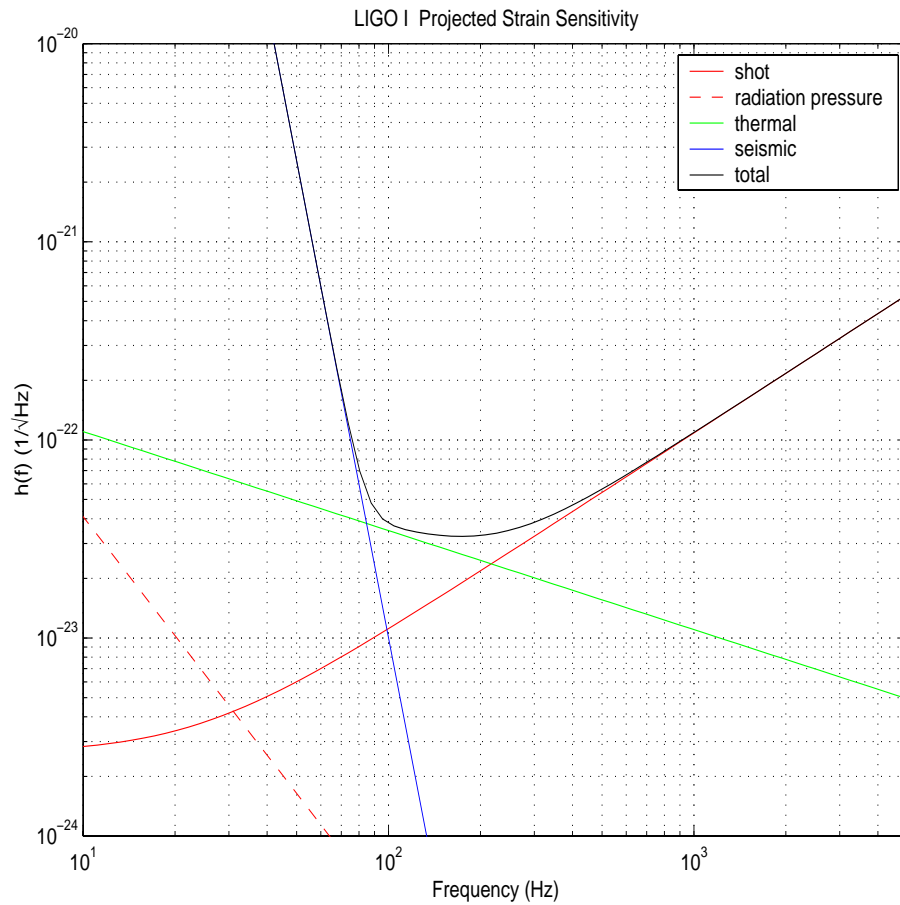
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Introduction



Achieving Higher Strain Sensitivity



- Seismic Noise

Passive mass-spring "stack" in LIGO I.

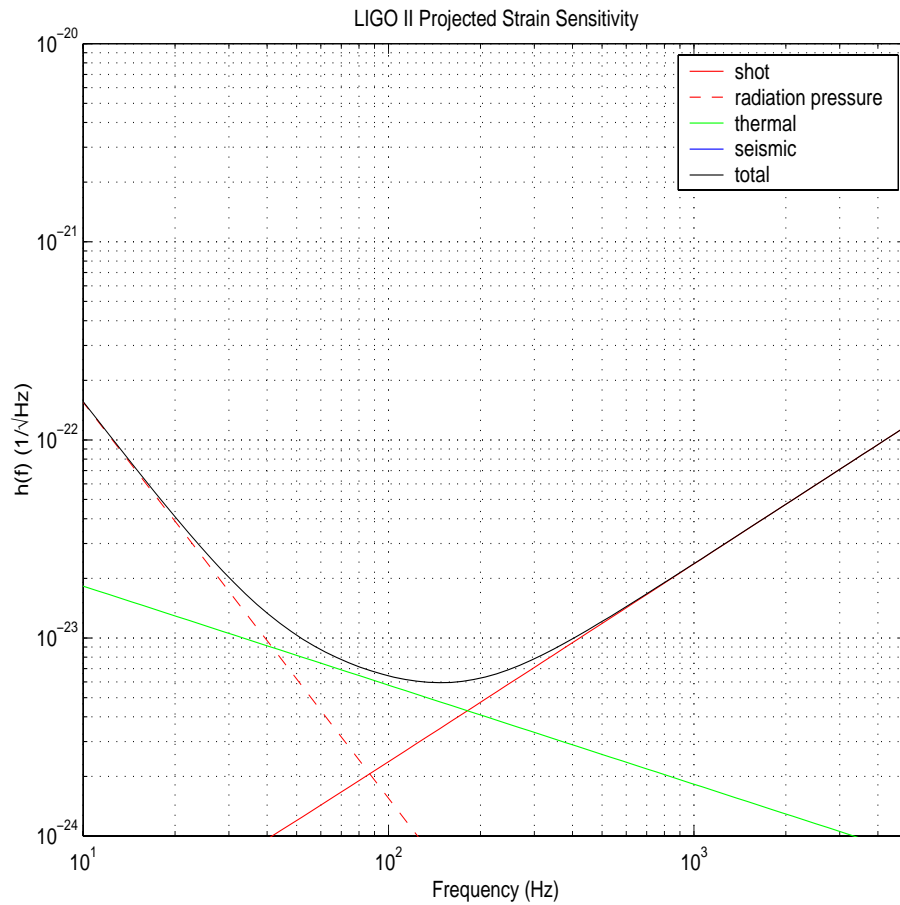
- Thermal Noise (Suspension and Internal)

Thermal vibration of mirror suspension and mirror surfaces.

- Shot Noise

Photon counting statistics (Poissonian).

Achieving Higher Strain Sensitivity



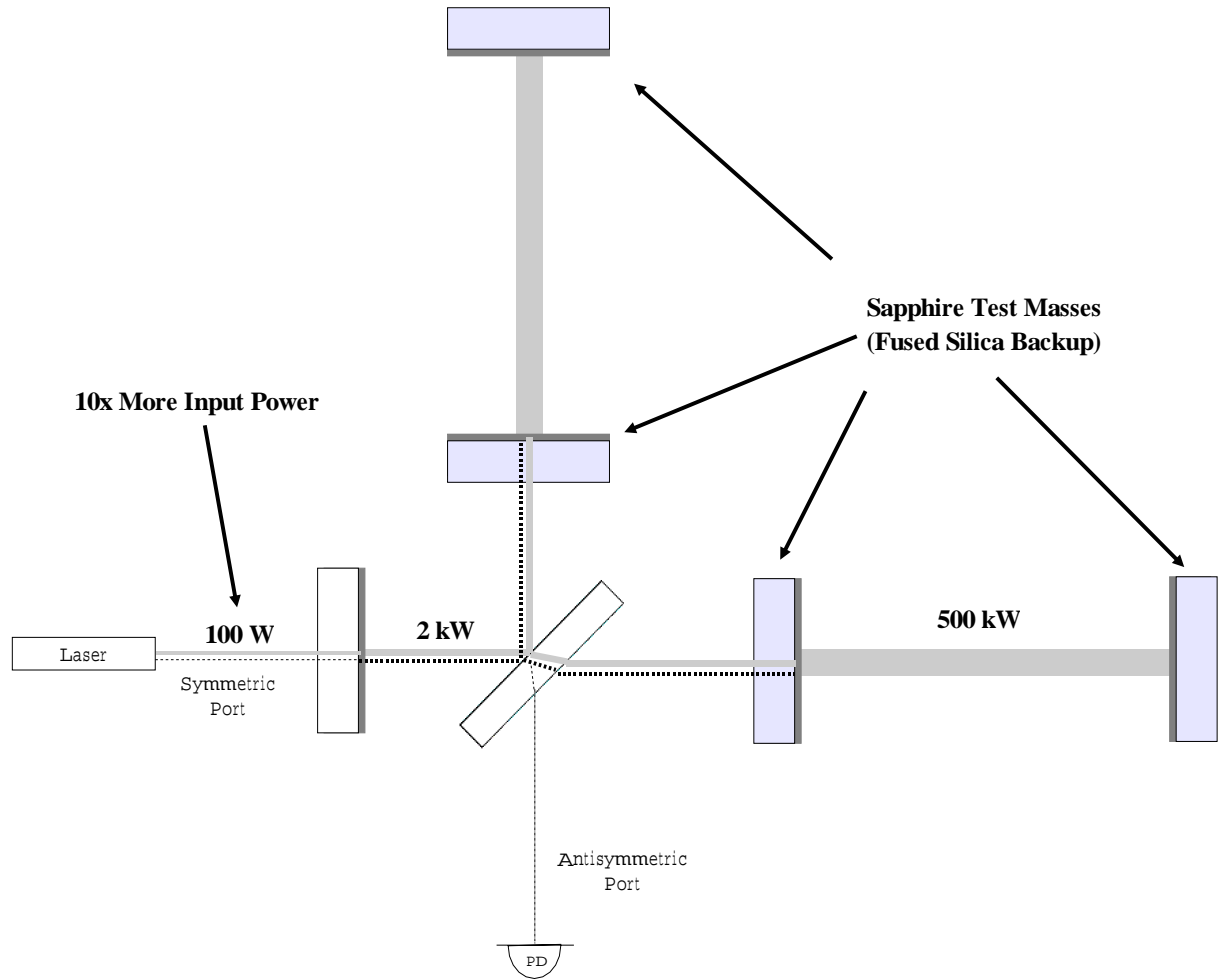
⇒ More stacks, or go to active isolation.

⇒ Make mirrors of a material w/ lower internal loss (Sapphire)

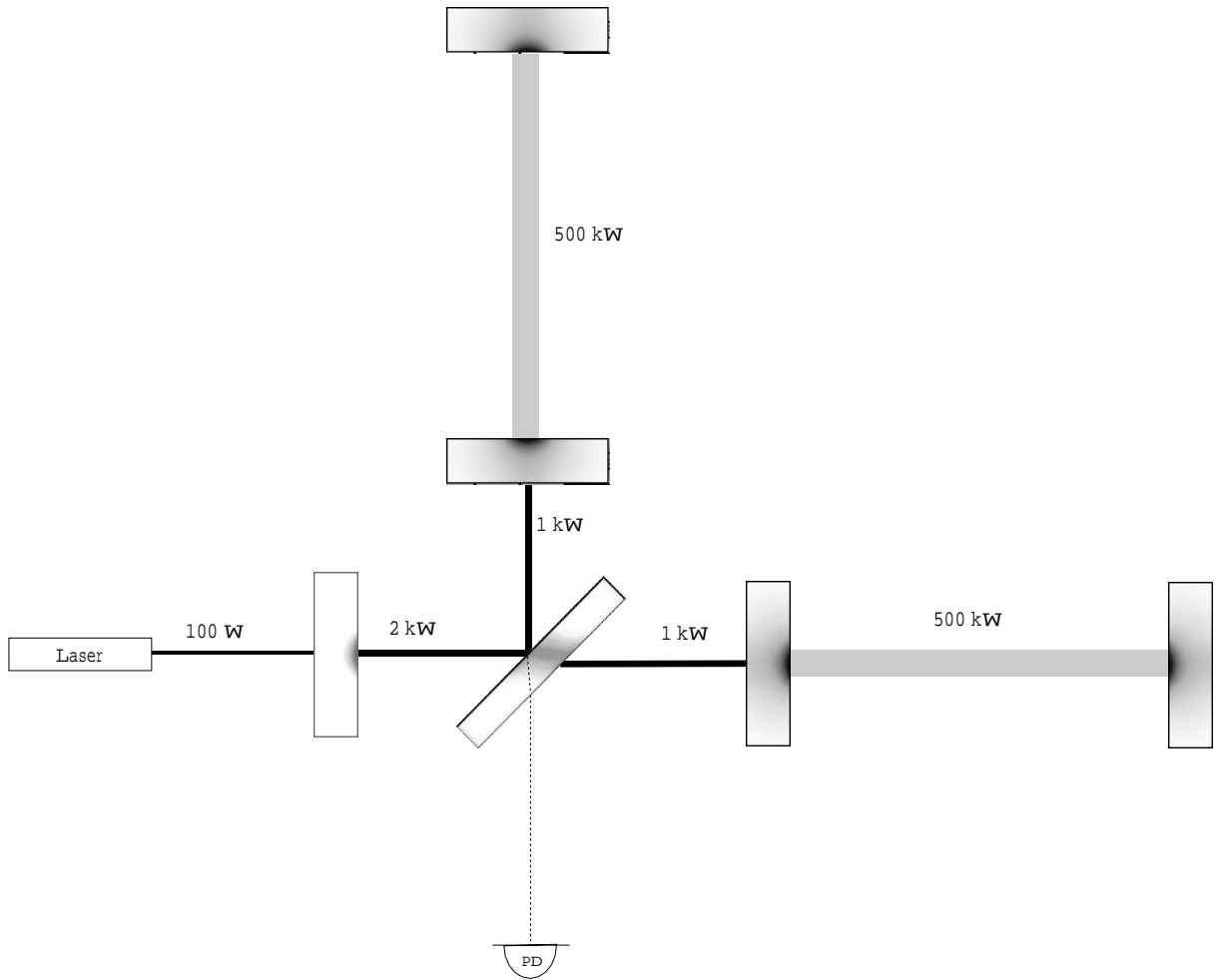
⇒ Construct all-silica suspensions.

⇒ Increase optical power circulating in the interferometer.

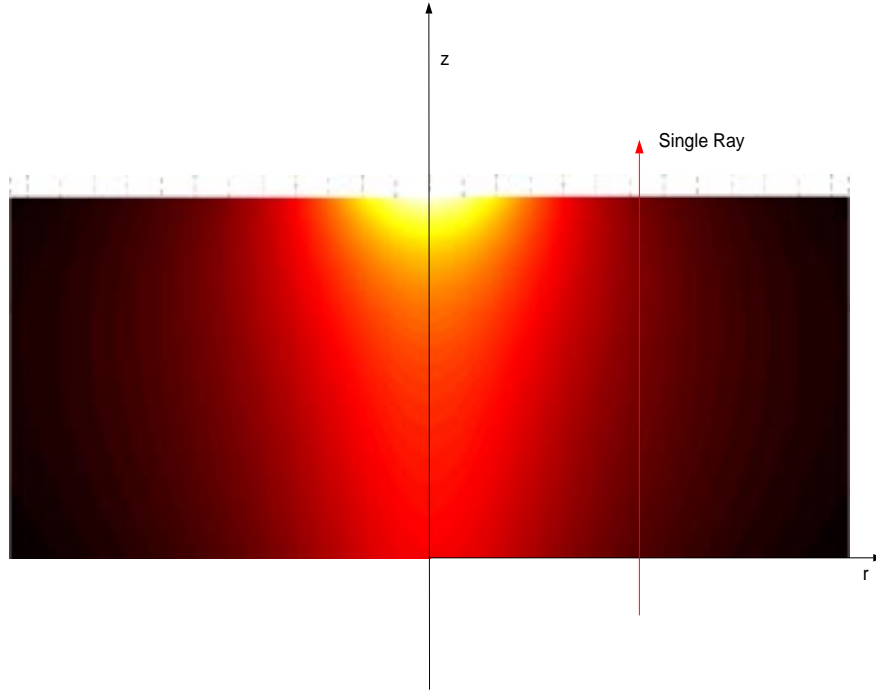
LIGO II Schematic



Thermal Effects?



Thermal Lensing



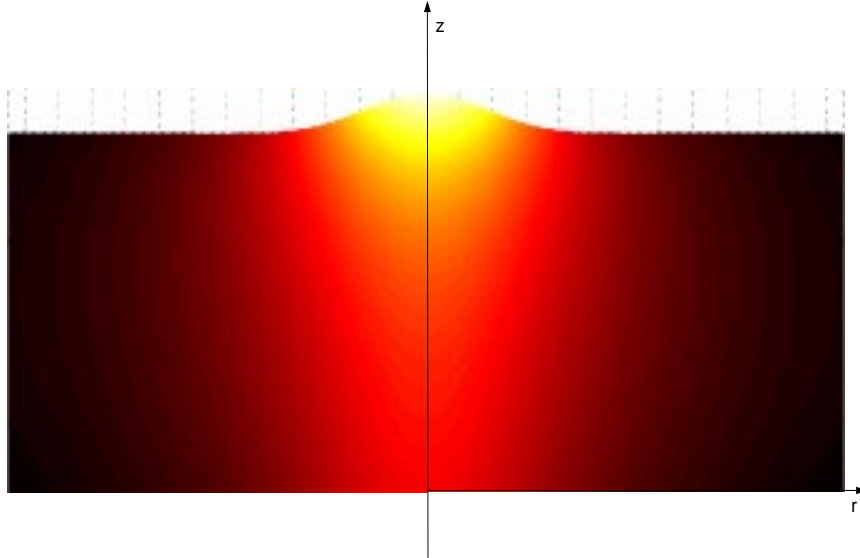
The total optical path $\Phi(r)$ for a single ray at radius r is:

$$\begin{aligned}\Phi(r) &= \int_0^H n(T(z, r)) dz \\ &\simeq n(T_0)H + \frac{dn}{dT} \int_0^H (T(r, z) - T_0) dz \\ &= n(T_0)H + \phi(r)\end{aligned}$$

where T_0 is the external temperature, $\frac{T(r, z) - T_0}{T_0} \ll 1$, and $\phi(r)$ is the optical path distortion at radius r :

$$\phi(r) \equiv \frac{dn}{dT} \int_0^H (T(r, z) - T_0) dz$$

Thermoelastic Deformation



Approximate expansion of the optic's surface:

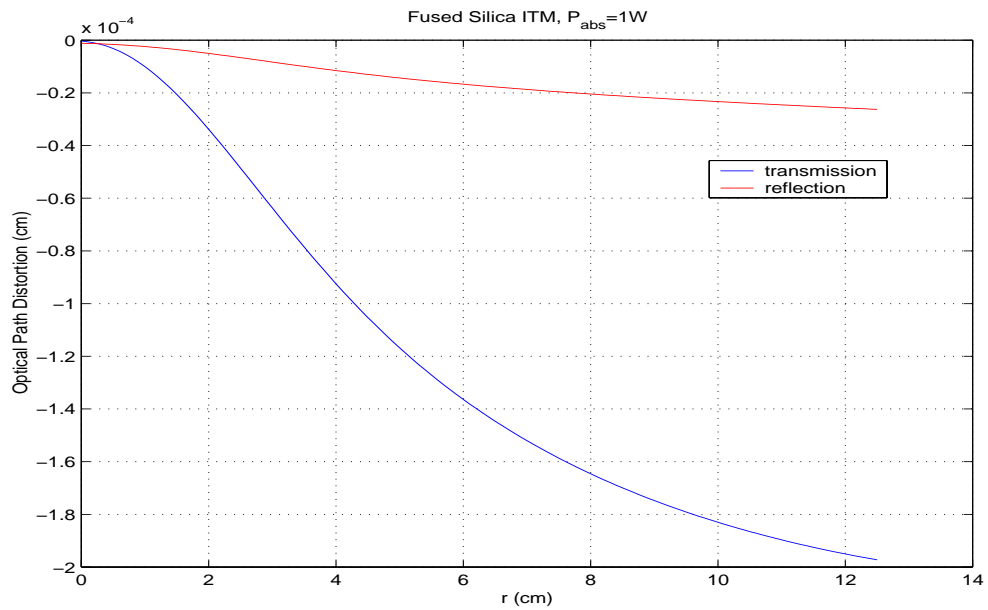
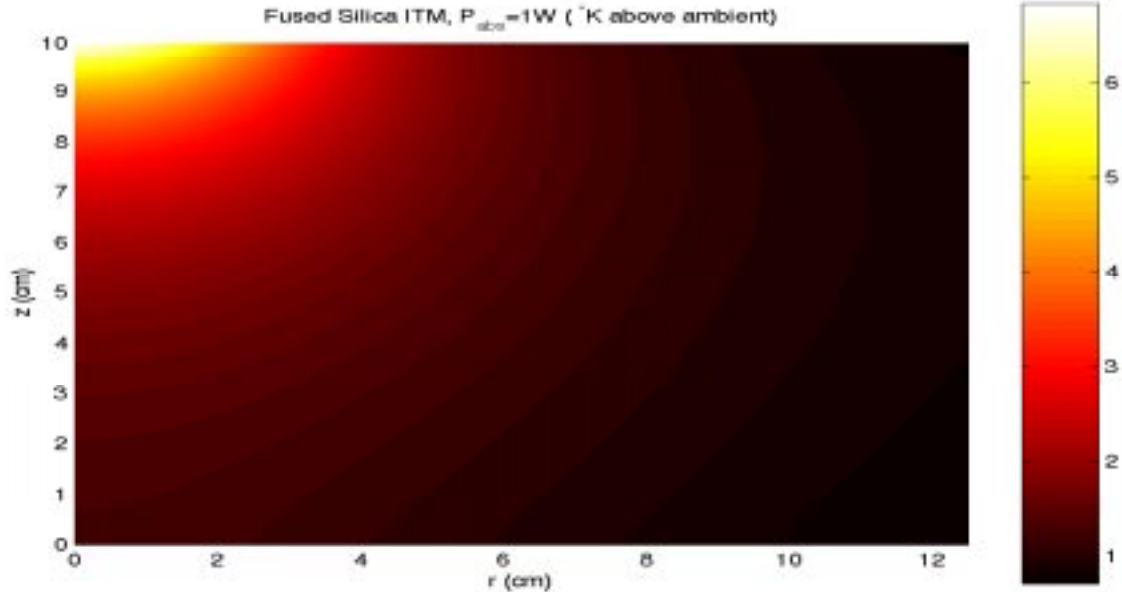
$$\begin{aligned}\delta s &\lesssim \int_0^H \alpha (T(z, r) - T_0) dz \\ &= \beta \phi(r)\end{aligned}$$

where α is the thermal expansion coefficient and $\beta \equiv \alpha / \frac{dn}{dT}$ is the relative strength of the deformation $\delta s(r)$ with respect to the corresponding thermal lens $\phi(r)$.

For Sapphire: $\beta \sim 1$
For Fused Silica: $\beta \sim 0.05$

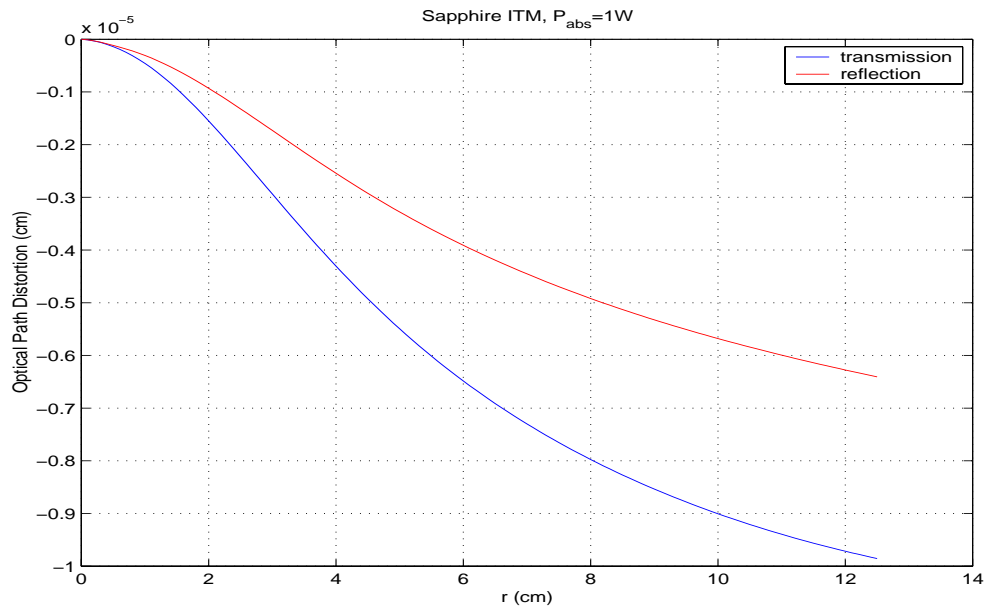
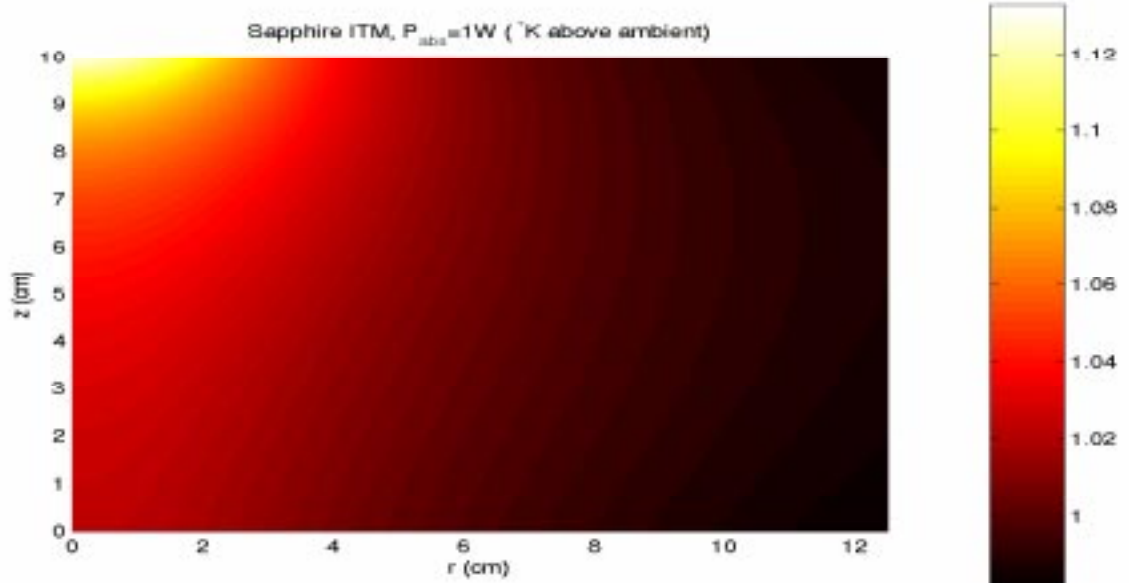
Fused Silica

$\alpha_b = 0.5 ppm/cm$	$\alpha_s = 0.6 ppm$
$\kappa = 1.38 W/m/^{\circ}K$	$\frac{dn}{dT} = 11.8 \times 10^{-6}/^{\circ}K$

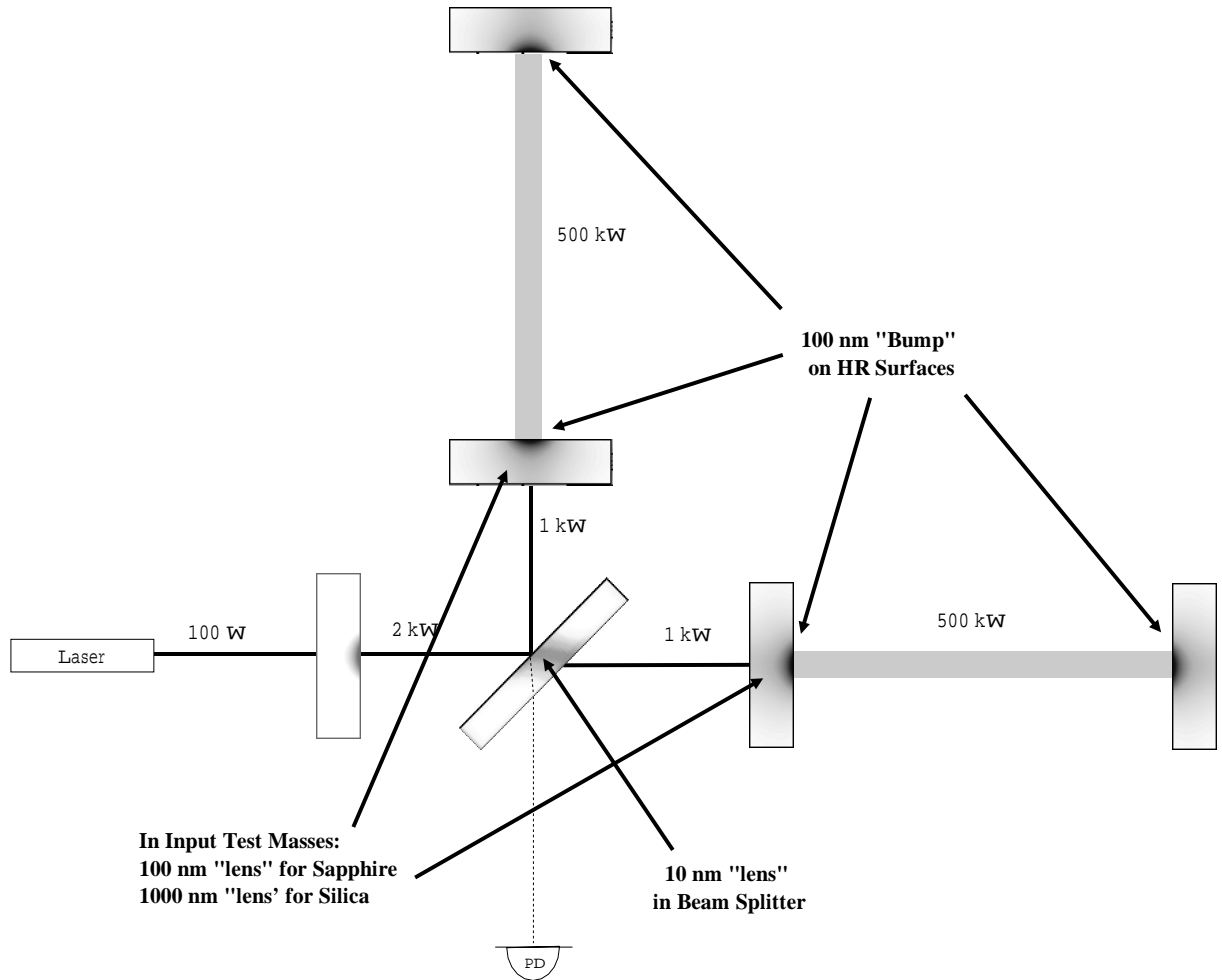


Sapphire

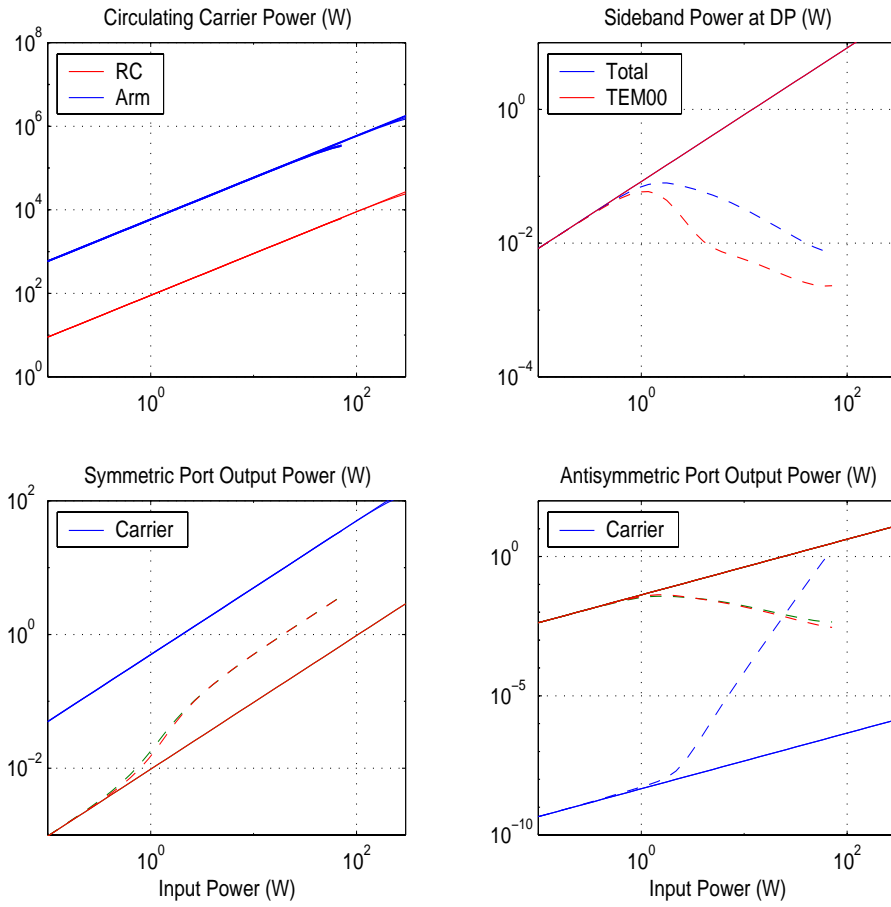
$\alpha_b = 40 \text{ ppm/cm}$	$\alpha_s = 0.6 \text{ ppm}$
$\kappa = 41.4 \text{ W/m/}^\circ\text{K}$	$\frac{dn}{dT} = 12 \times 10^{-6} / ^\circ\text{K}$



Thermal Effects in LIGO II

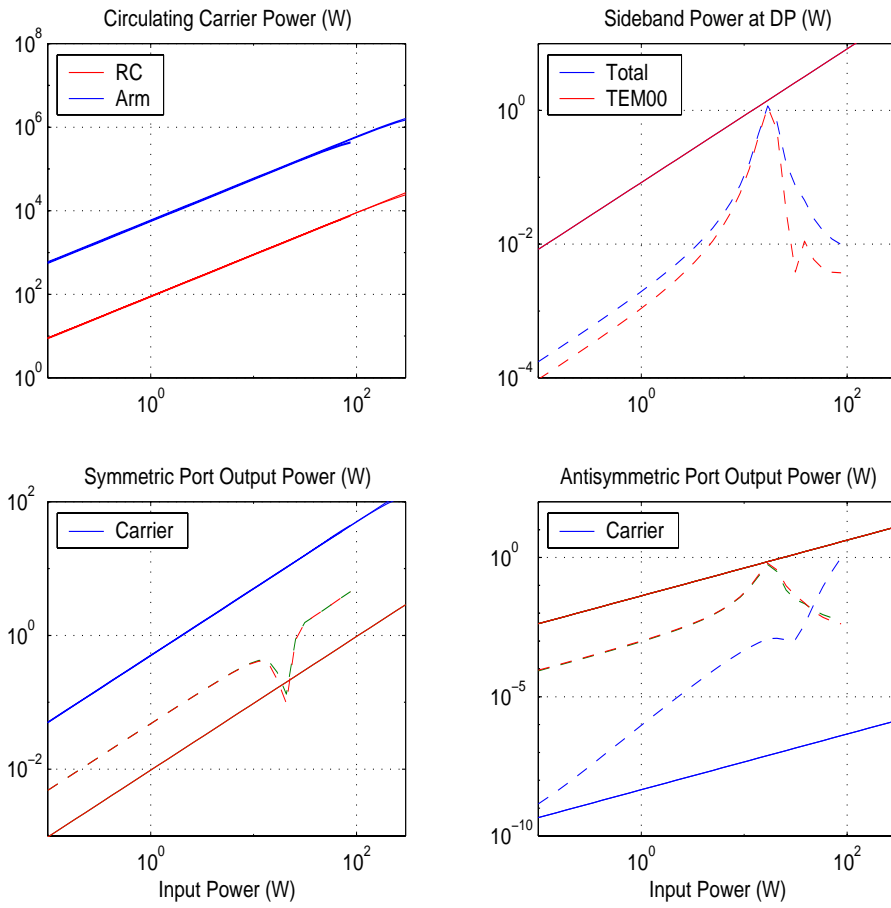


Thermal Effects in LIGO



- LIGO I with low absorption (Heraeus SV) Fused Silica (0.3 ppm/cm).
- Sideband power loss at 1 Watt input power.

Fixing Thermal Effects in LIGO



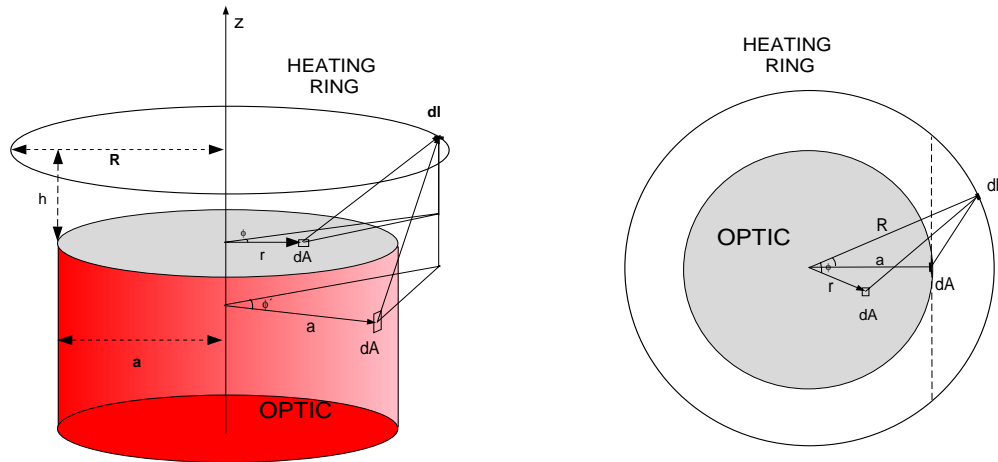
- Change the curvature of recycling mirror to optimize for a set input power.

- Works for input powers < 20 Watts.

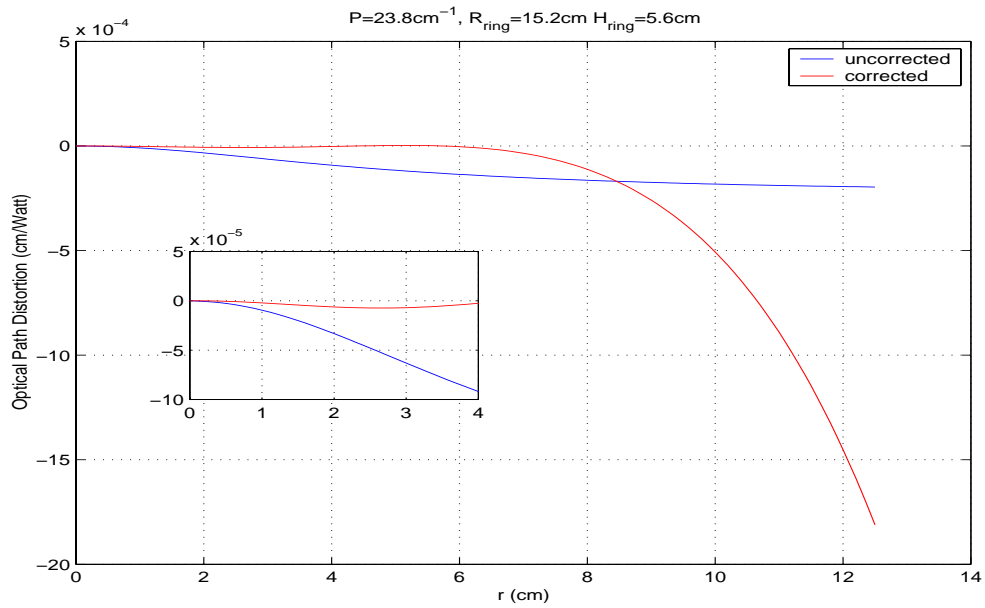
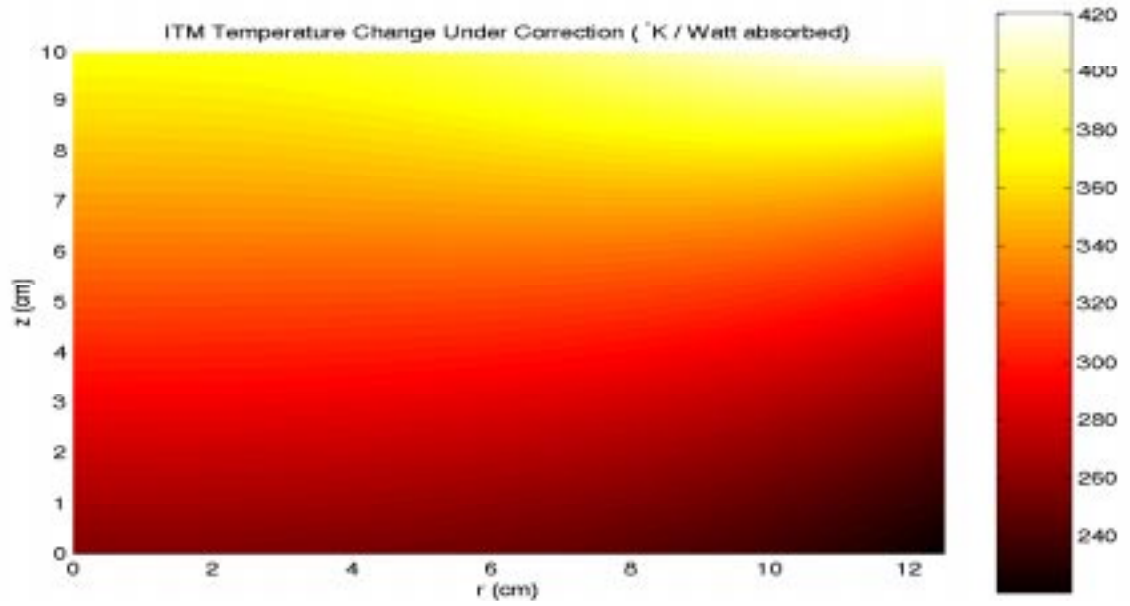
Fixing Thermal Effects in LIGO II *Use “Thermal Compensation”*

- Directly control the optical properties of a test mass by depositing energy (radiatively) in a well defined pattern.
- Can only *add* optical path (you can put heat in, but you can't extract it).
- Two methods: Static (heating pattern tailored to generate a wavefront of fixed shape)
Dynamic (adjustable heating pattern, able to generate an “arbitrary” wavefront)

Static, Axisymmetric Thermal Compensation



Bare Nichrome Ring (Fused Silica ITM)



The Figure of Merit

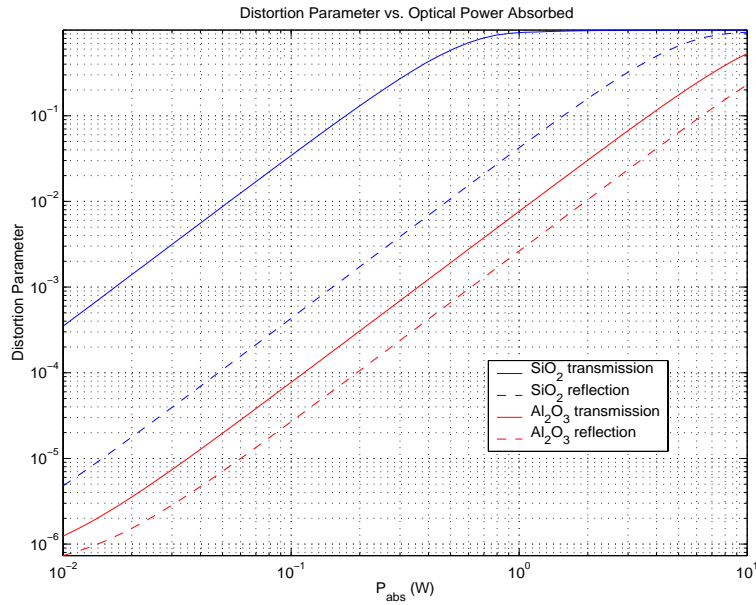
The fractional power lost out of the TEM 00 mode (for a single pass through the optic) is given by the “Distortion Parameter” \mathcal{G} :

$$\mathcal{G} = 1 - \left| \langle 00 | e^{i \frac{2\pi}{\lambda} \phi(r)} | 00 \rangle \right|^2$$

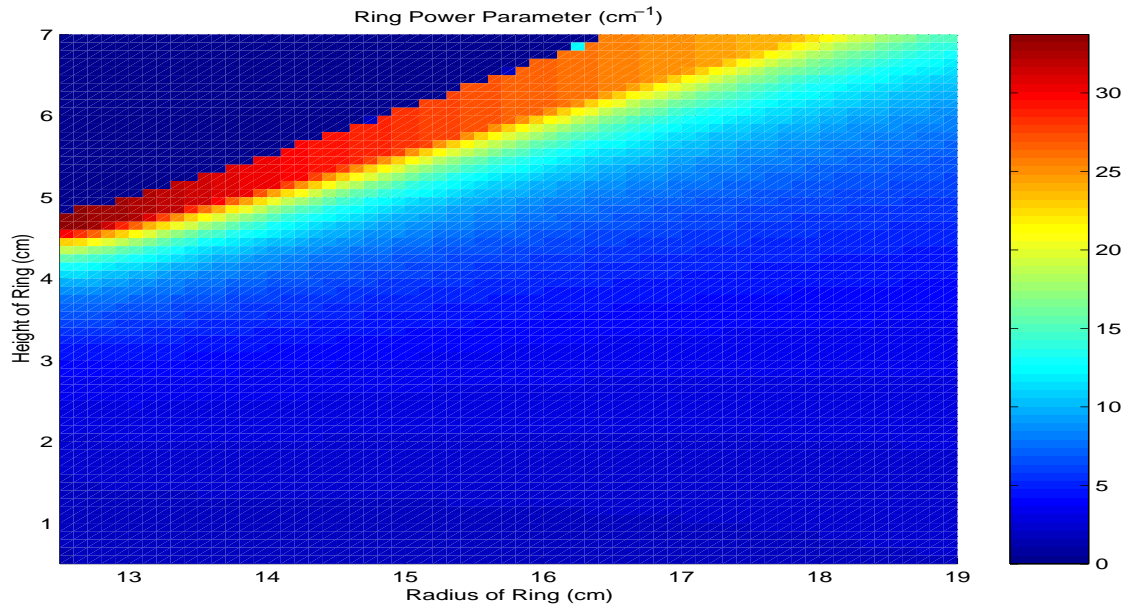
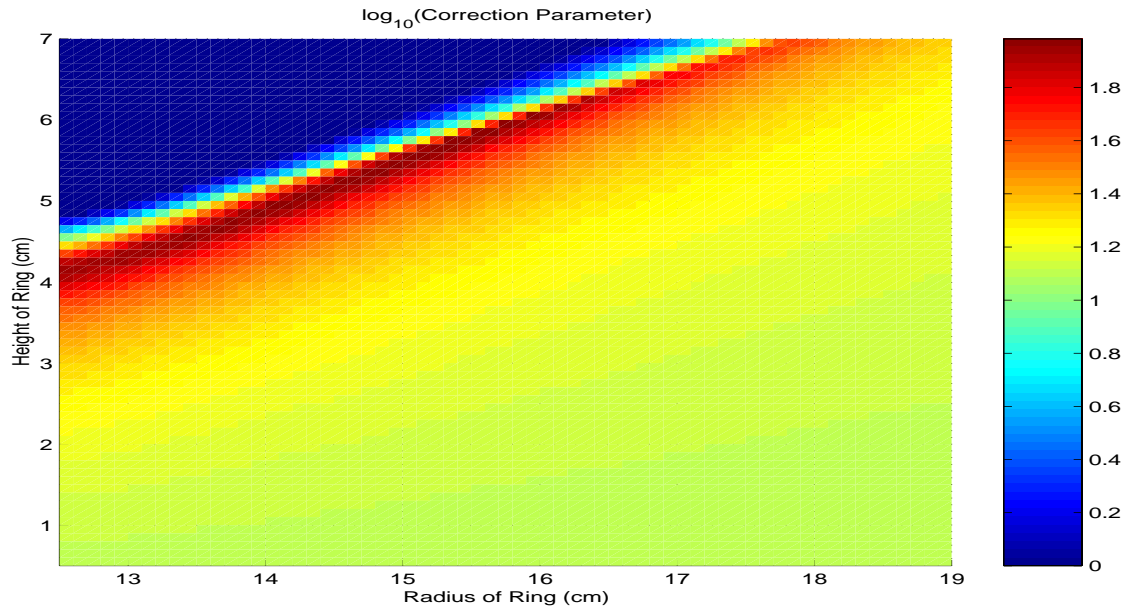
$$= 1 - \frac{16}{w^4} \left| \int_0^\infty e^{i \frac{2\pi}{\lambda} \phi(r)} e^{-2\frac{r^2}{w^2}} 2\pi r dr \right|^2$$

The degree of correction, termed the “correction Parameter” \mathcal{C} is defined as:

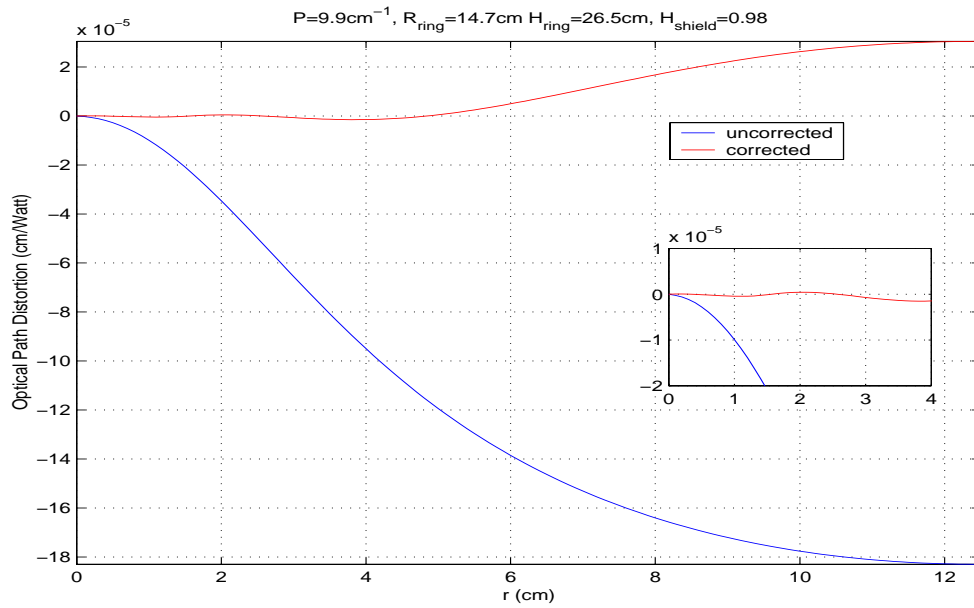
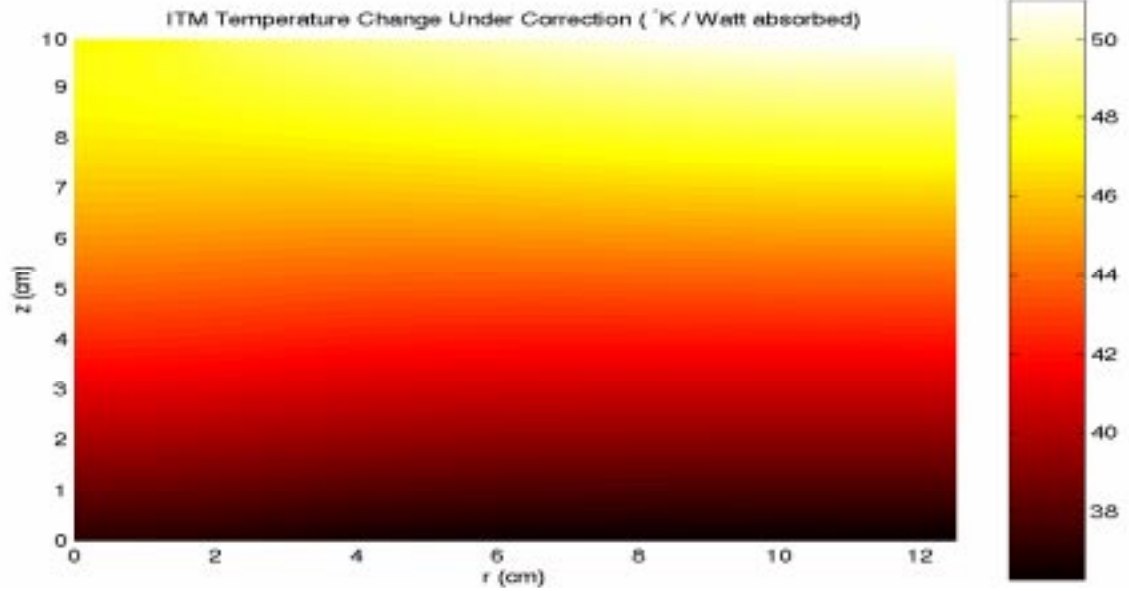
$$\mathcal{C} \equiv \frac{\mathcal{G}_0}{\mathcal{G}}$$



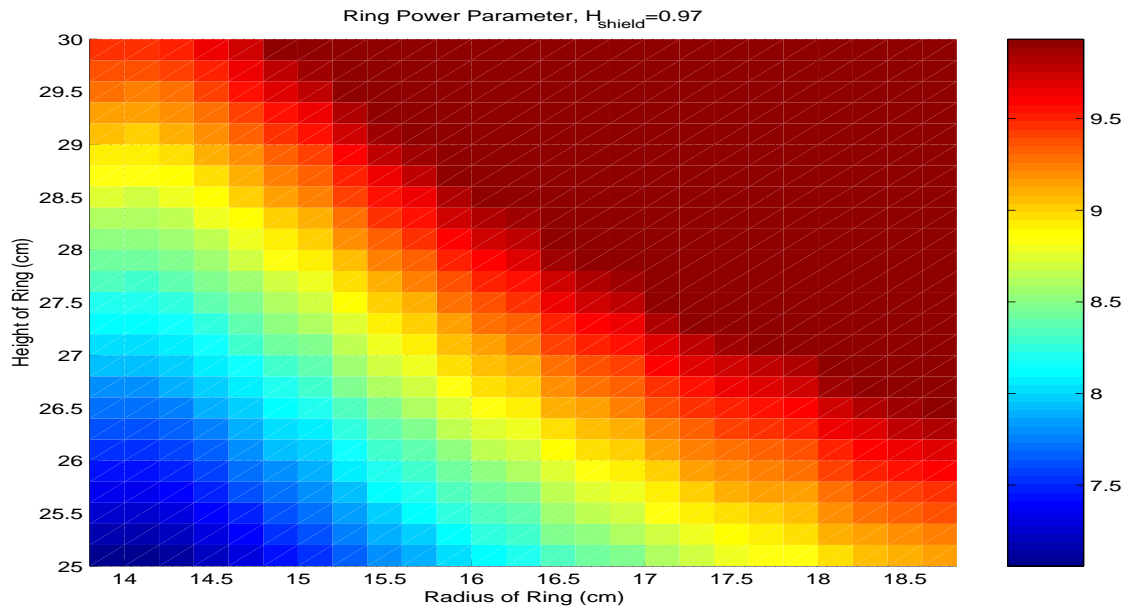
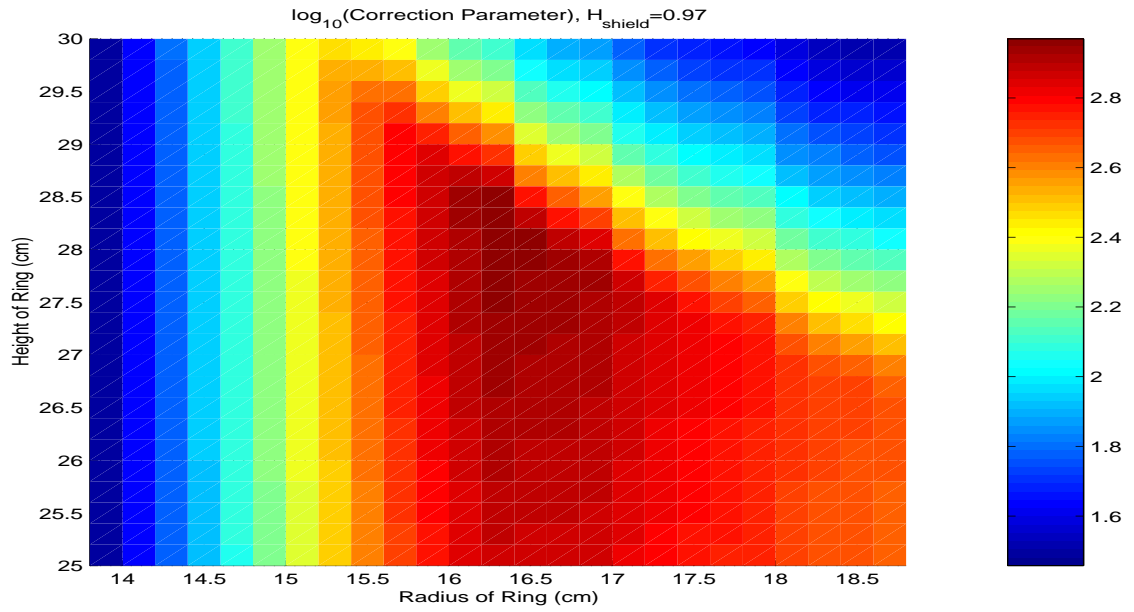
Bare Nichrome Ring (Fused Silica ITM)



Shielded Ring, Insulated Optic (Fused Silica ITM)



Shielded Ring, Insulated Optic (Fused Silica ITM)



Static, Axisymmetric Compensation in Sapphire

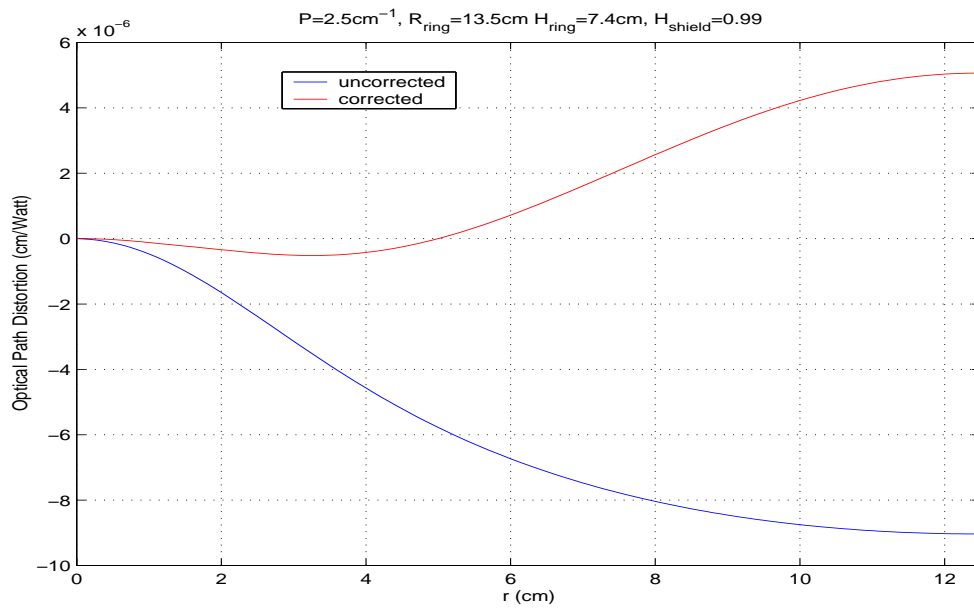
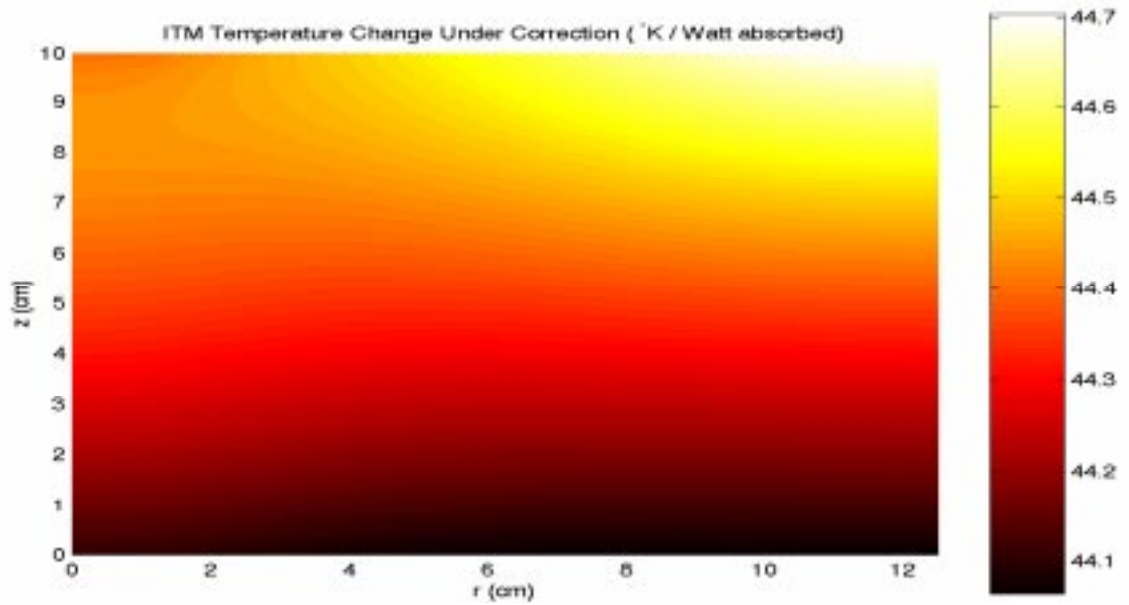
- Thermal Conductivity is $\sim 30\times$ that of Fused Silica.
 \Rightarrow A correspondingly larger heater power is required to maintain a similar wavefront correction.

- At the same time, Sapphire is highly transmissive for $\lambda \leq 5\mu m$
 \Rightarrow Must keep the heater temperature low ($\sim 500^\circ K$).

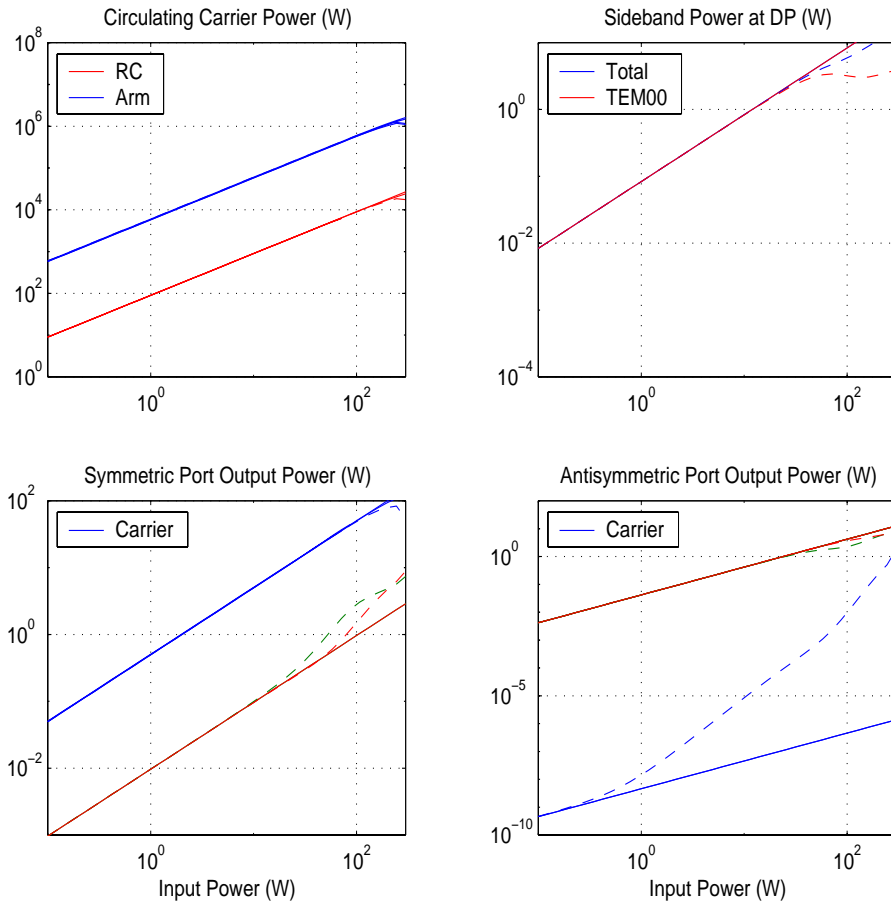
For the bare ring, with Sapphire's current absorptivity, one needs $\sim 1kW$ (!) to remove the thermal lens. It is thus impractical.

For the shielded ring, we are saved by the broad parameter space. We can simply move the ring closer to the optic, at the price of a poorer (but still acceptable) correction.

Shielded Ring, Insulated Optic (Sapphire ITM)

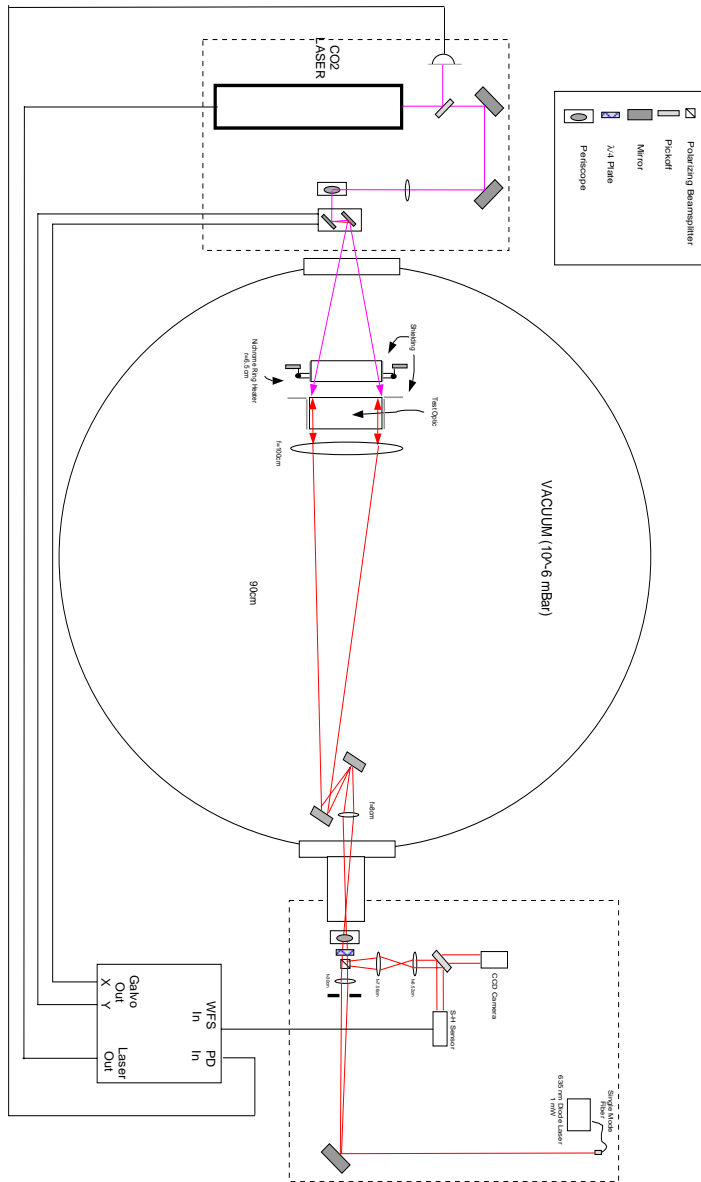


So What?

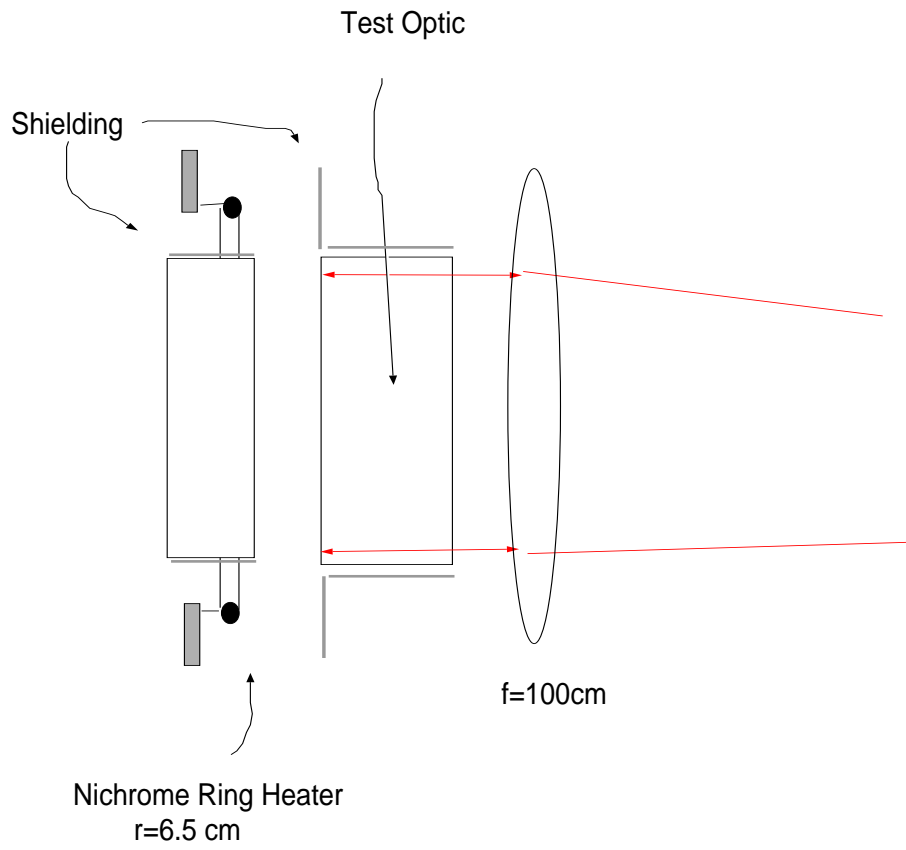


- Same situation as shown originally (slide 12), now with “realistically” compensated fused silica ITM’s.
- Optic curvatures are “cold optimized”.
- Sideband *TEM00* power loss at 60 Watts input power.

The Experimental Effort



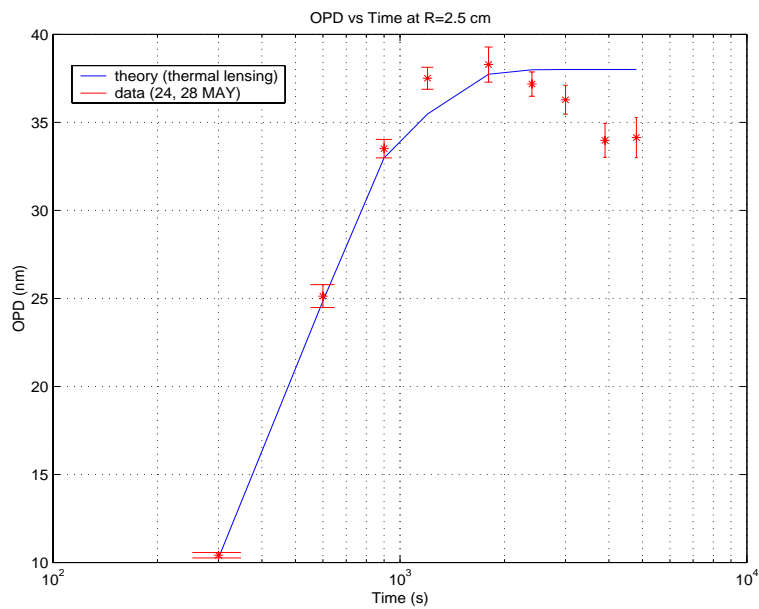
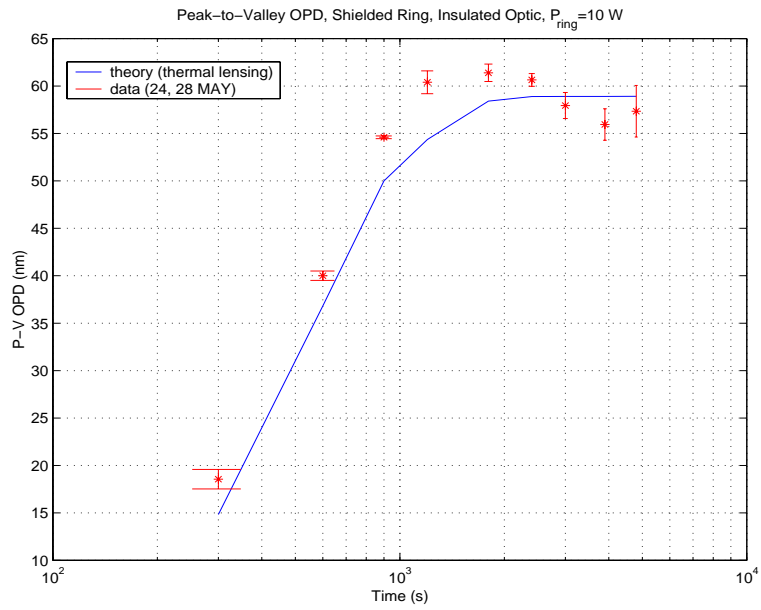
Experiment #1 Static Thermal Compensation



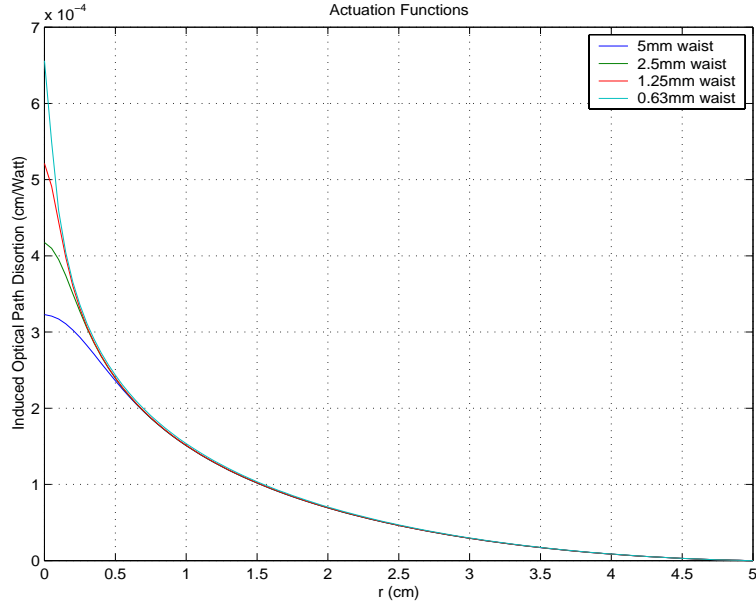


The Data (So Far)

Shielded Ring, Insulated Optic



Dynamic Thermal Compensation Theory



- Work in a basis of 2D functions that are orthogonal over the measured aperture (e.g. Zernike polynomials, $Z_{nm}(r, \theta)$).

- Work in the basis of “actuation functions” ($\mathcal{A}_k(r, \theta)$, the net distortion generated by the laser actuating with unit power on the k th scan point).

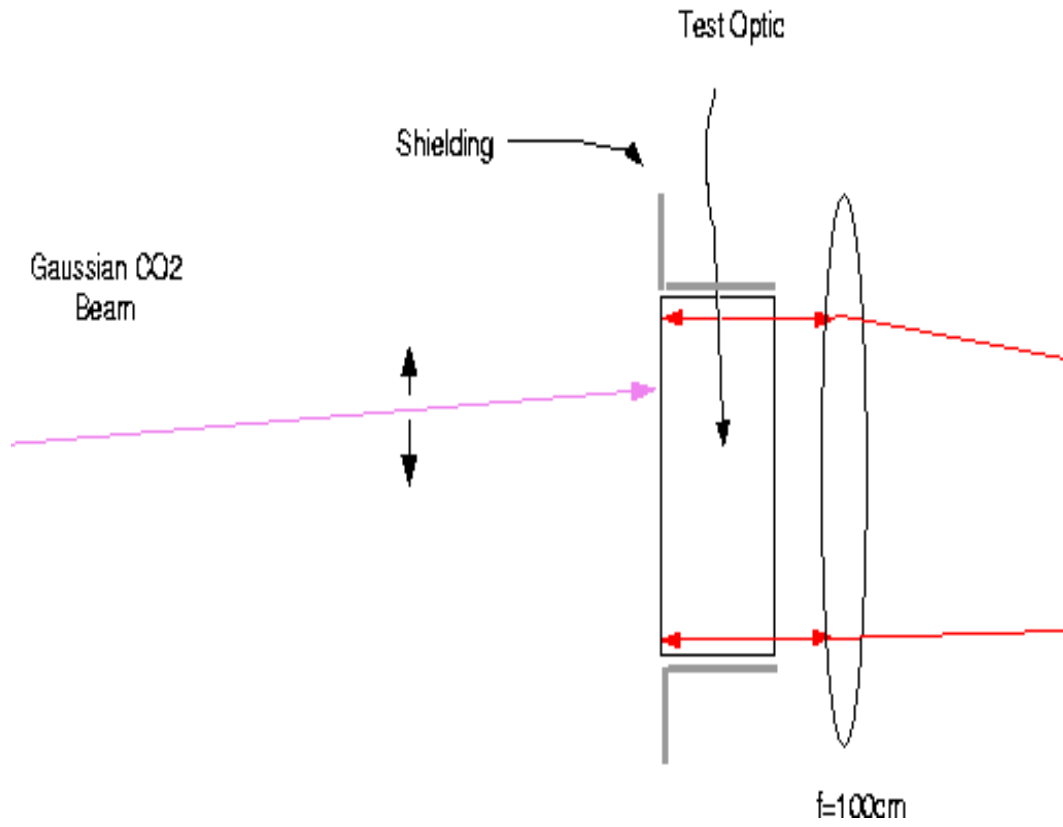
In either case, you calculate (or measure) the *response matrix* \underline{A} :

$$\vec{d} = \underline{A} \cdot \vec{P}$$

Then invert to get the *actuation matrix* \underline{A}^{-1} , so that:

$$\vec{P} = \underline{A}^{-1} \cdot \vec{d}$$

Experiment #2
Generation of Arbitrary Curvature
with Scanning CO2 Beam

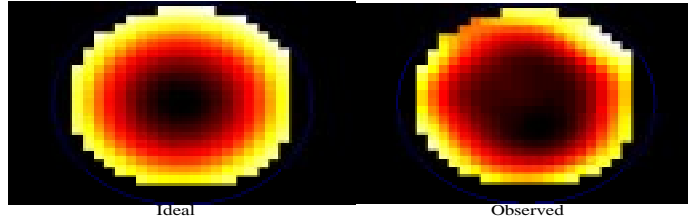


Dynamic Thermal Compensation Data (Phil Marfuta, '01)

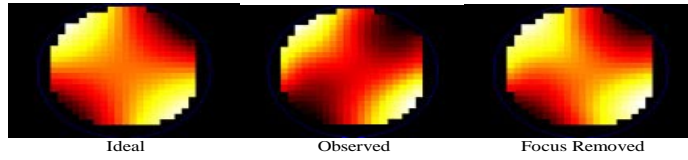
- Actuator beam waist of 5mm, Optical aperture radius of 2.5cm, maximum power of 2.5 Watts.
- Demonstrated Zernikes up to Z_{33} ($N=10$). Higher order terms could not be generated.
- Persistnet focus term, approximately constant for each data run.
⇒ Explained by thermoelastic “bowing” of the test optic.

Dynamic Thermal Compensation Data (Phil Marfuta, '01)

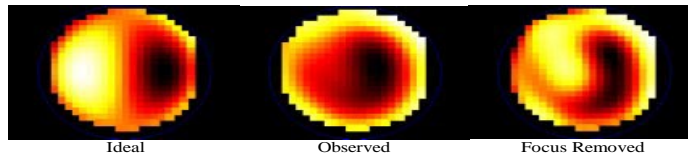
N=5 (focus)



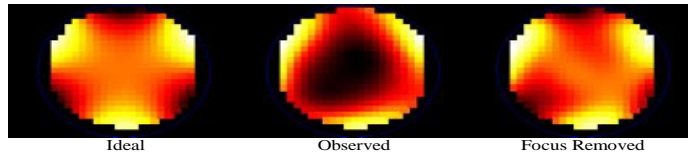
N=4



N=8



N=10



The Bottom Line:

- For Fused Silica:

	\mathcal{C}	T_{max}/P_{abs}	Notes
Bare Ring	100	400 °K/Watt	Steep Parameter Space
Shielded Ring, Insulated Optic	1600	44 °K/Watt	Broad, Flat Parameter Space
Scanning Beam	?	?	More Modeling Required

- For Sapphire:

	\mathcal{C}	T_{max}/P_{abs}	Notes
Bare Ring	-	-	Impractical Due to Ring Power Required
Shielded Ring, Insulated Optic	60	44 °K/Watt	Limited By Ring Power Required
Scanning Beam	-	-	Impractical Due to Laser Power Required

Measuring Thermo-Optical Parameters

- We have built detailed thermal and thermoelastic models, as well as an apparatus to measure these effects.

- Abruptly turn on the heating beam, and examine the Peak to Valley optical path distortion as a function of time.

- The peak-to-valley optical path distortion for a unit power pump beam can be written:

$$\phi_{PV}(t) = A_{tl}f(\beta t) + A_z g_z(\beta t) + A_r g_r(\beta t)$$

$$\begin{aligned} A_{tl} &\equiv \frac{dn}{dT}/k & A_z &\equiv \frac{\alpha_z}{k} \\ A_r &\equiv \frac{\alpha_r}{k} & \beta &\equiv \frac{k}{c\rho} \end{aligned}$$

Where: k =thermal conductivity

c = heat capacity

ρ = density

α_x = thermal expansion in the x direction

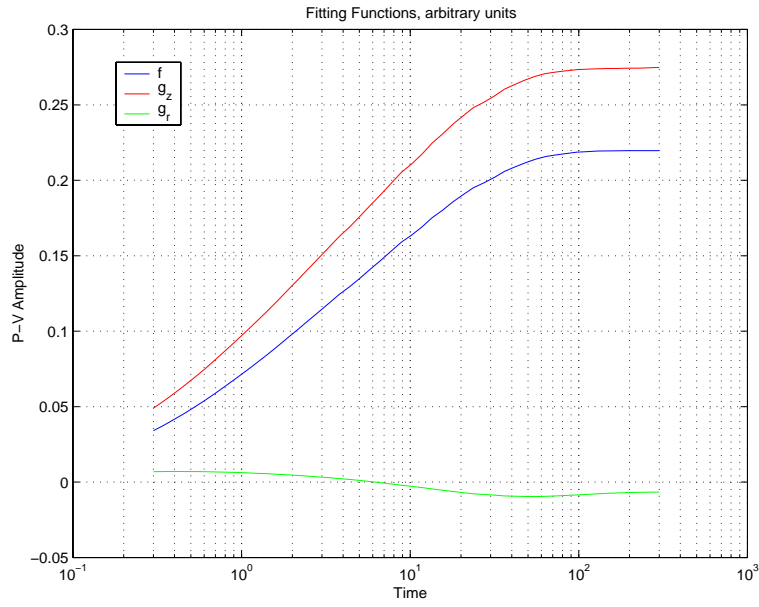
$\frac{dn}{dT}$ = index derivative w.r.t temperature

f = fitting function for thermal lensing

g_r = fitting function for radial thermal expansion coupling into axial expansion

g_z = fitting function for axial thermal expansion

Fitting Functions f , g_r , and g_z (Numerically Calculated)

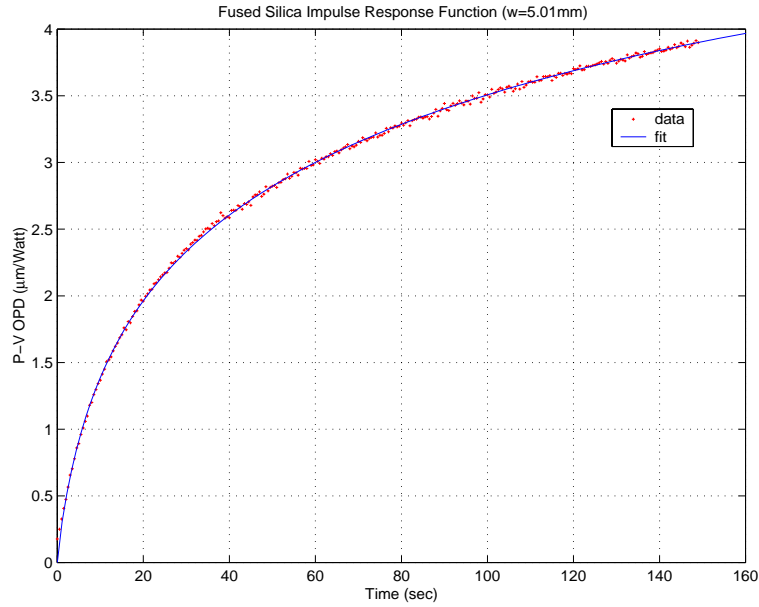


- We can find constants A and B such that $f(t) \simeq Ag_z(Bt)$.

- g_r is small compared to f and g_z .

\Rightarrow We can realistically fit two parameters to the data (β and one of the A 's).

Fused Silica

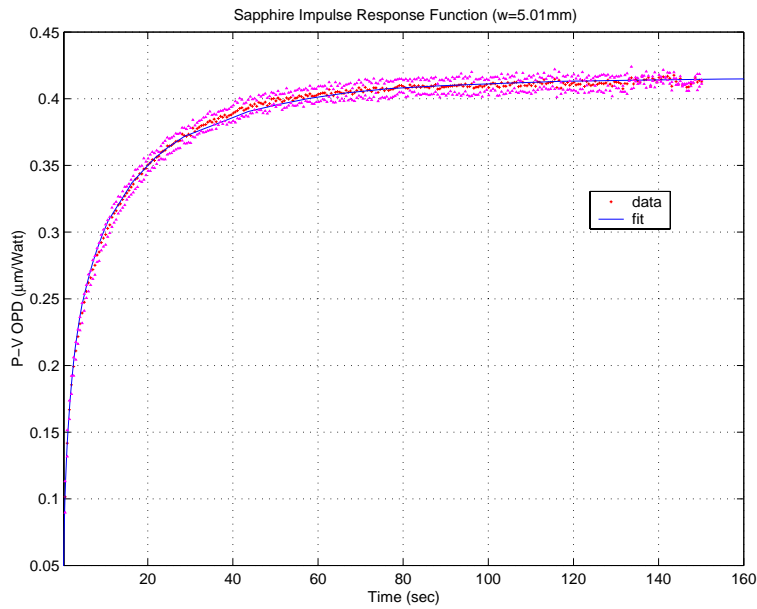


If we assume $c = 0.74 \text{ J/g/}^\circ\text{K}$ and 2.2 g/cm^3 , then:

$$k = (1.25 \pm 0.07) \text{ W/m/}^\circ\text{K}$$

$$\frac{dn}{dT} = (8.9 \pm 0.10) \times 10^{-6} / ^\circ\text{K}$$

Sapphire

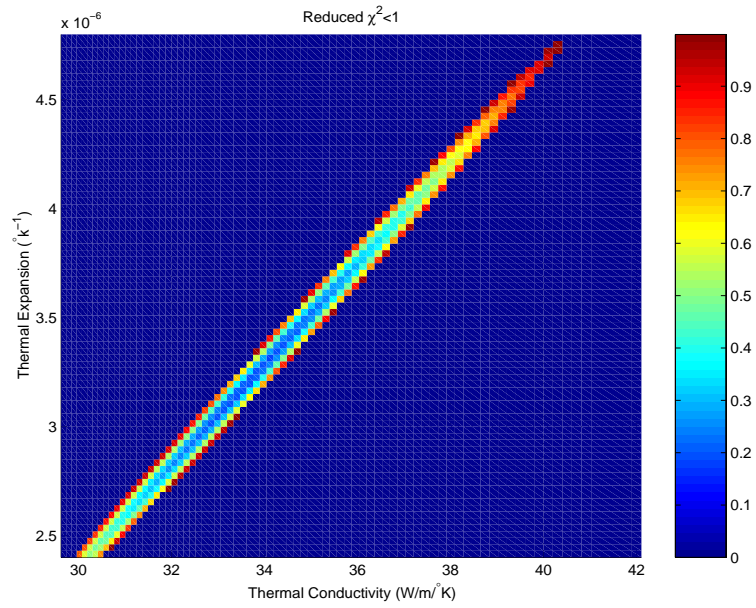


If we assume $c = 0.775 \text{ J/g/}^\circ\text{K}$, 3.98 g/cm^3 , and $\frac{dn}{dT} = 1.26 \times 10^{-5}/^\circ\text{K}$ then:

$$k = (33.8 \pm 4.0) \text{ W/m/}^\circ\text{K}$$

$$\alpha_z = (3.2 \pm 1.0) \times 10^{-6}/^\circ\text{K}$$

Sapphire Error



Future Directions

(0) Finish the thermo-optical parameter measurement (Stabilize the pump laser more effectively).

(1) Modeling of ring solution complete, now integrating it into the full interferometer thermal model.

(2) More modeling of the scanning beam solution (optimize the scan pattern and the actuation matrix).

(3) Experimental test of the optimized scanning beam solution.

(4) Want to test the ring heater in the presence of a heating beam.