Violin Modes in Fused Silica Suspensions

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Why Keep Studying Violin Modes?

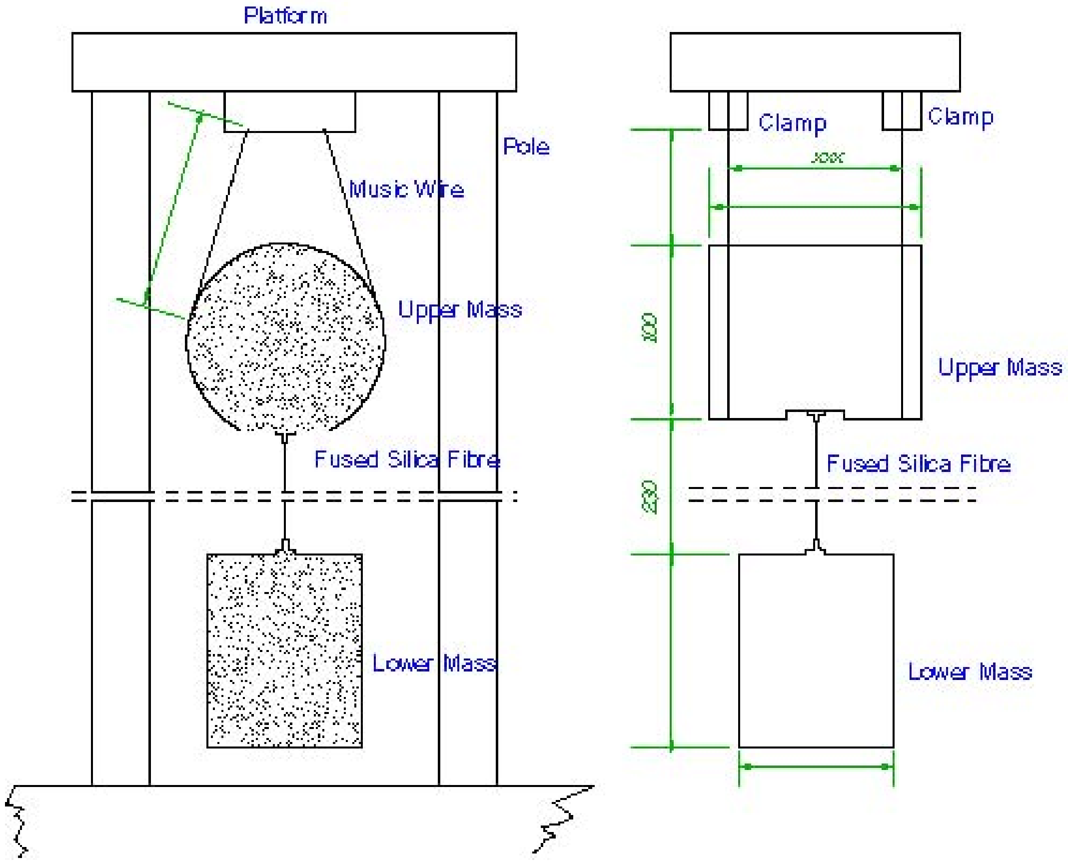
- Choice between ribbons & fibers will require better knowledge of noise performance.
- Suspension thermal noise is not expected to dominate optical noise in Advanced LIGO, but predicted low loss not yet demonstrated.
- Low frequency searches can reduce optical noise by reducing laser power and/or removing signal extraction mirror, thus exposing suspension thermal noise.
- Violin resonances can interfere with locking and control.

Suspensions Apparatus

Automated fiber pulling lathe

Q-measurement rig

Suspensions Apparatus



Violin Mode Frequencies

$$f_n = \frac{n}{2L}\sqrt{\frac{P}{\rho_L}}\left[1 + \frac{2}{k_e L} + \frac{1}{2}\left(\frac{n\pi}{k_e L}\right)^2\right]$$

Measured Mode Frequencies

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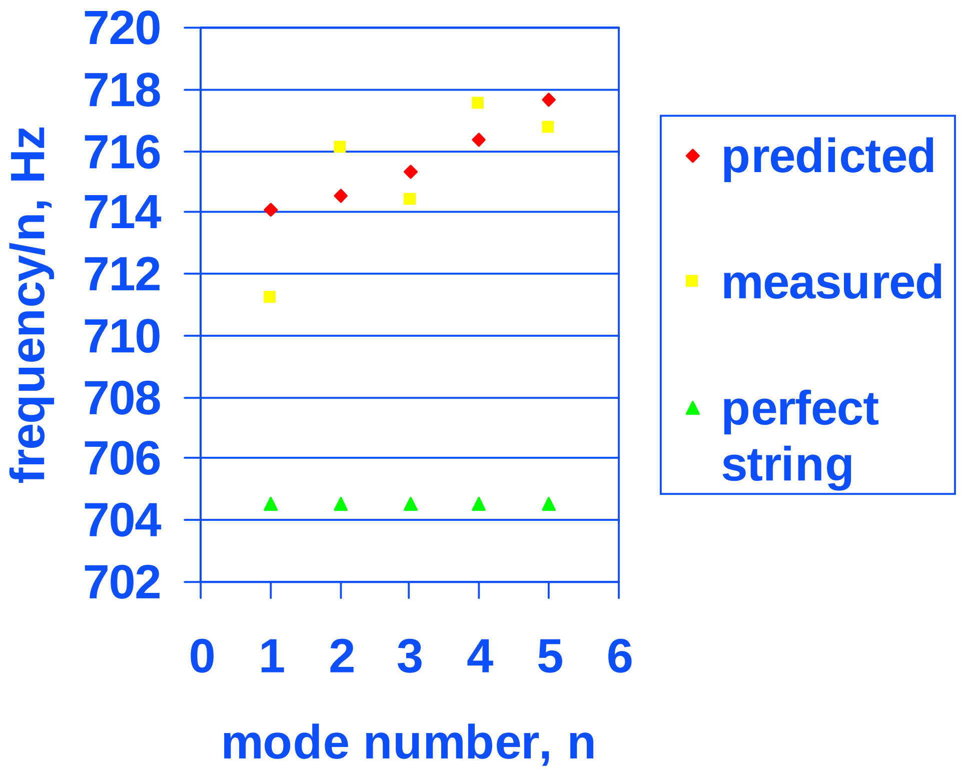
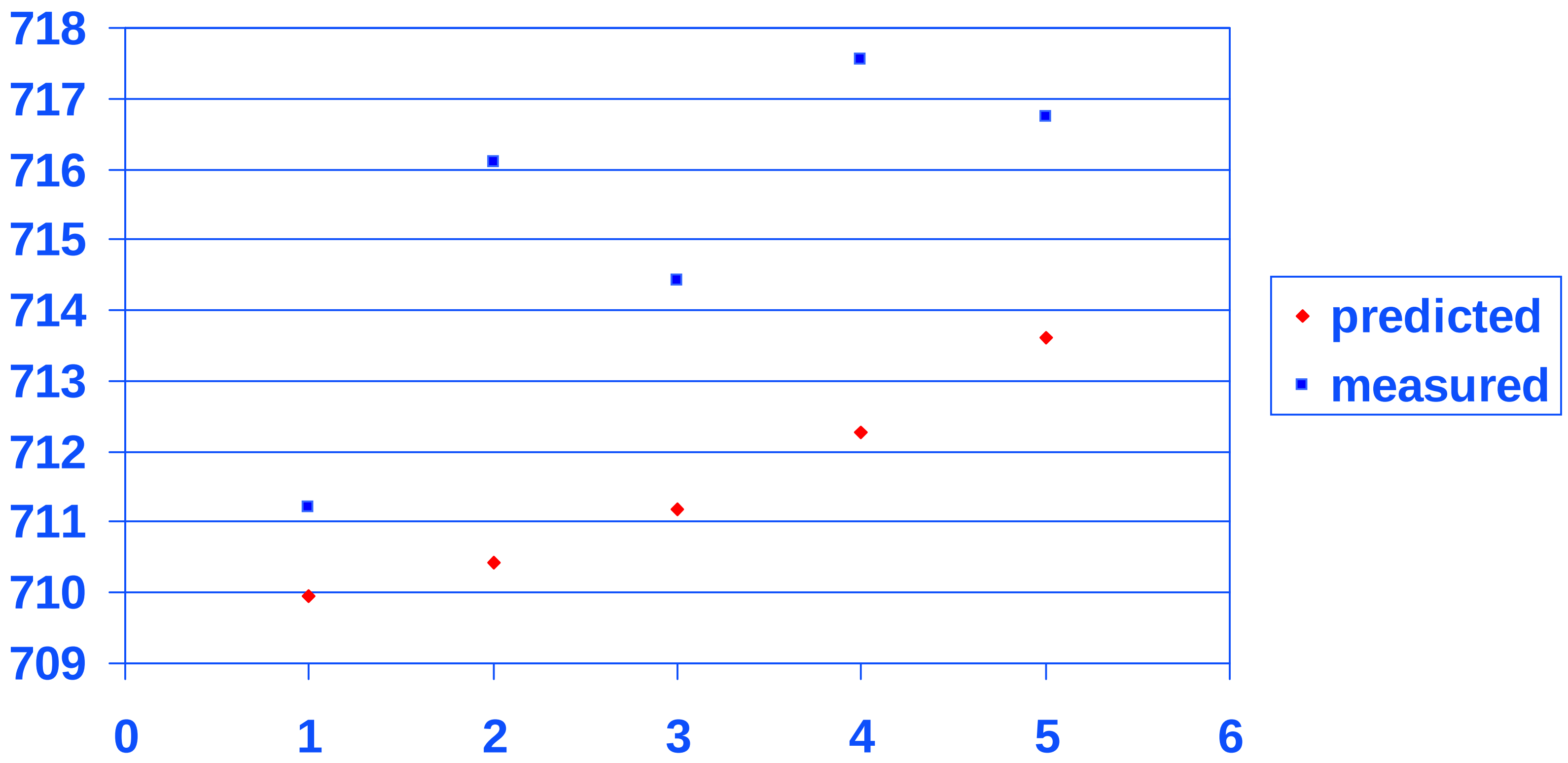
‘Fit’ of measured mode frequencies to model with radius as free parameter gives

$$r_{fiber} = 167.4 \mu m$$

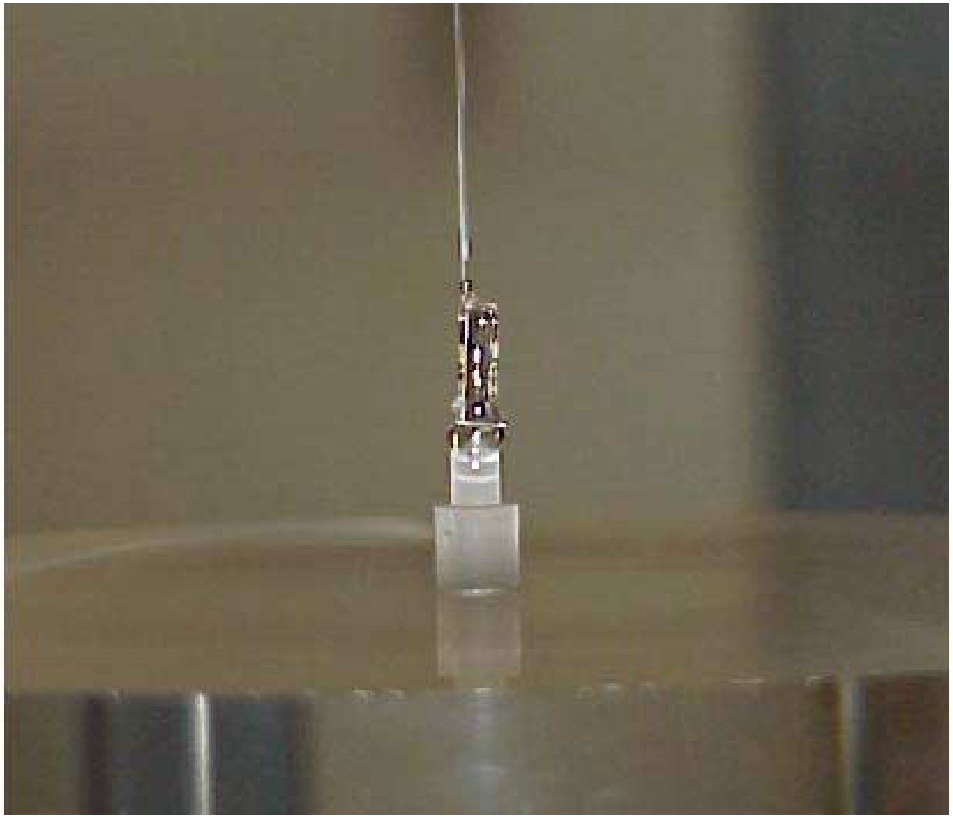
with <.5% error for all modes



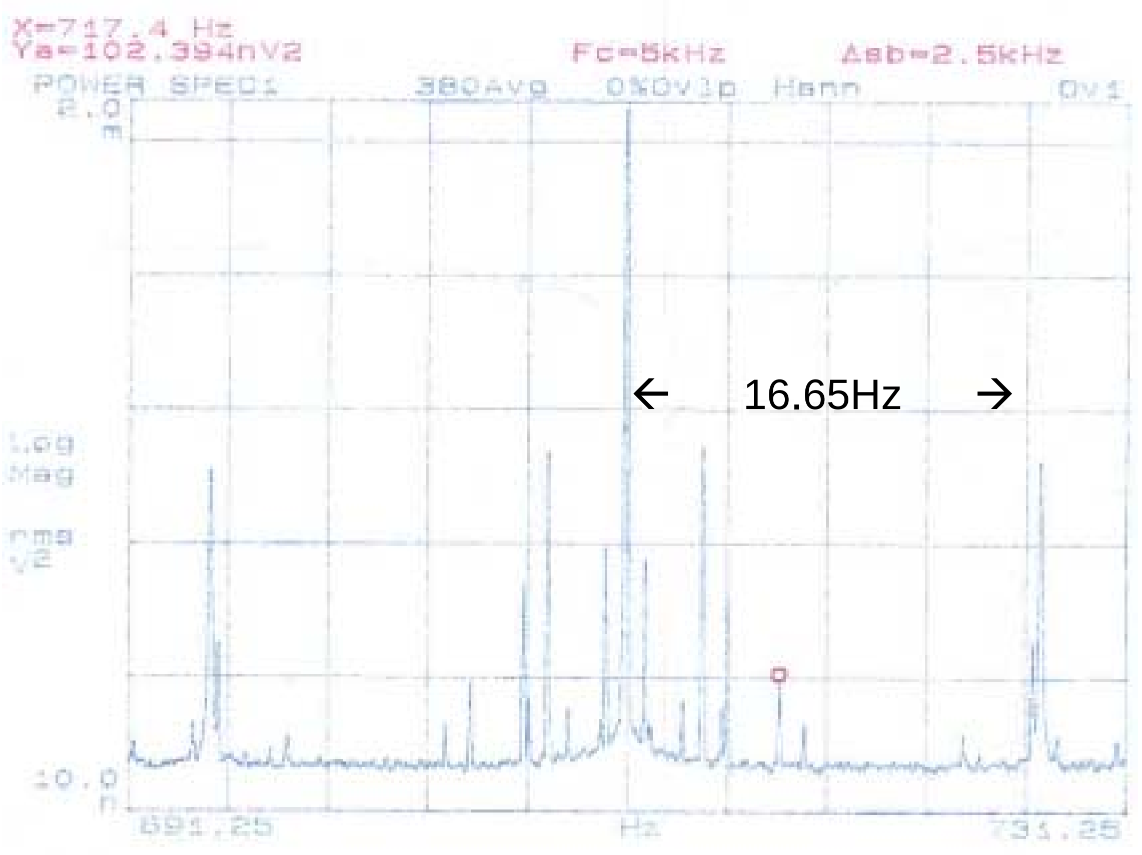
How the frequencies change for 1μm change in fiber radius



Welded Fiber End



Vertical Bounce Modulation



Vertical Bounce and Violin Vibrato

Violin mode amplitude:

$$A(t) \approx A_0 \sin\left(\frac{n\pi}{L}\sqrt{\frac{P}{\rho_L}}t\right) = A_0 \sin(\omega_n t)$$

Strain amplitude:

$$P = P_0 + \delta P \sin(\Omega t)$$

Ω=bounce frequency

$$A(t) \approx A_0 \sin\left(\frac{n\pi}{L}\sqrt{\frac{P_0 + \delta P \sin(\Omega t)}{\rho_L}}t\right) \approx A_0 \sin\left(\omega_n\left[1 + \frac{\delta P}{2P_0}\sin(\Omega t)\right]t\right)$$

$$A(t) \approx A_0\left\{J_0(m)\sin(\omega_n t) + J_1(m)\cos((\omega_n + \Omega)t) + J_1(m)\cos((\omega_n - \Omega)t)\right\}$$

where $m=\delta P/2P_0$ (in this case $\delta P/P_0 \sim .1$)

Nonlinear Thermoelastic Damping

Loss function ϕ:

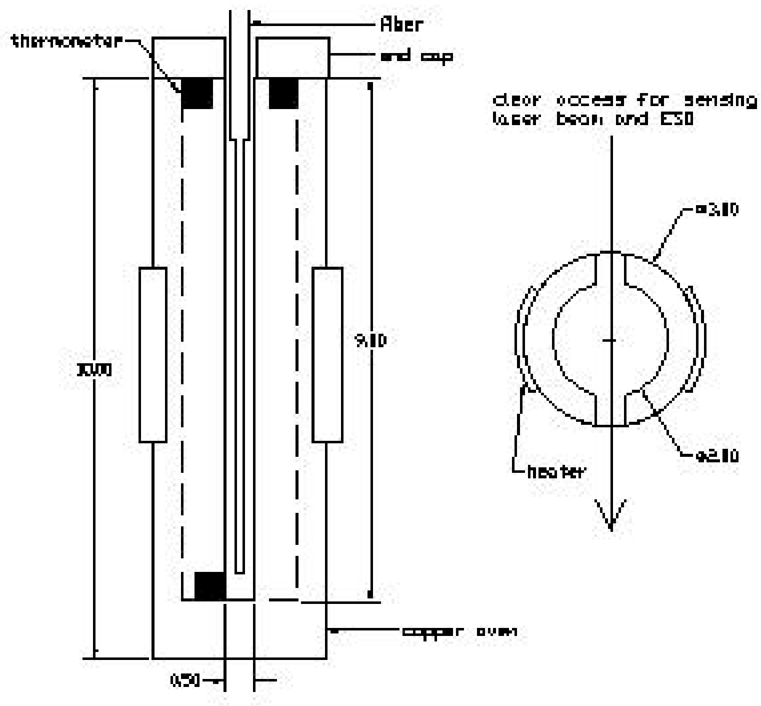
$$\phi_{\mathrm{NTE}} = \frac{\mathrm{TE}}{\rho \mathrm{C_V}} \left(\alpha - \frac{dE}{dT} \frac{u}{E} \right)^2 \frac{\omega \tau}{1 + (\omega \tau)^2}$$

$$\tau_{ribbon} = \frac{t^2}{\pi^2} \frac{\rho C_V}{k} \qquad \tau_{fiber} = \frac{.0737 \rho C_V r^2}{4k}$$

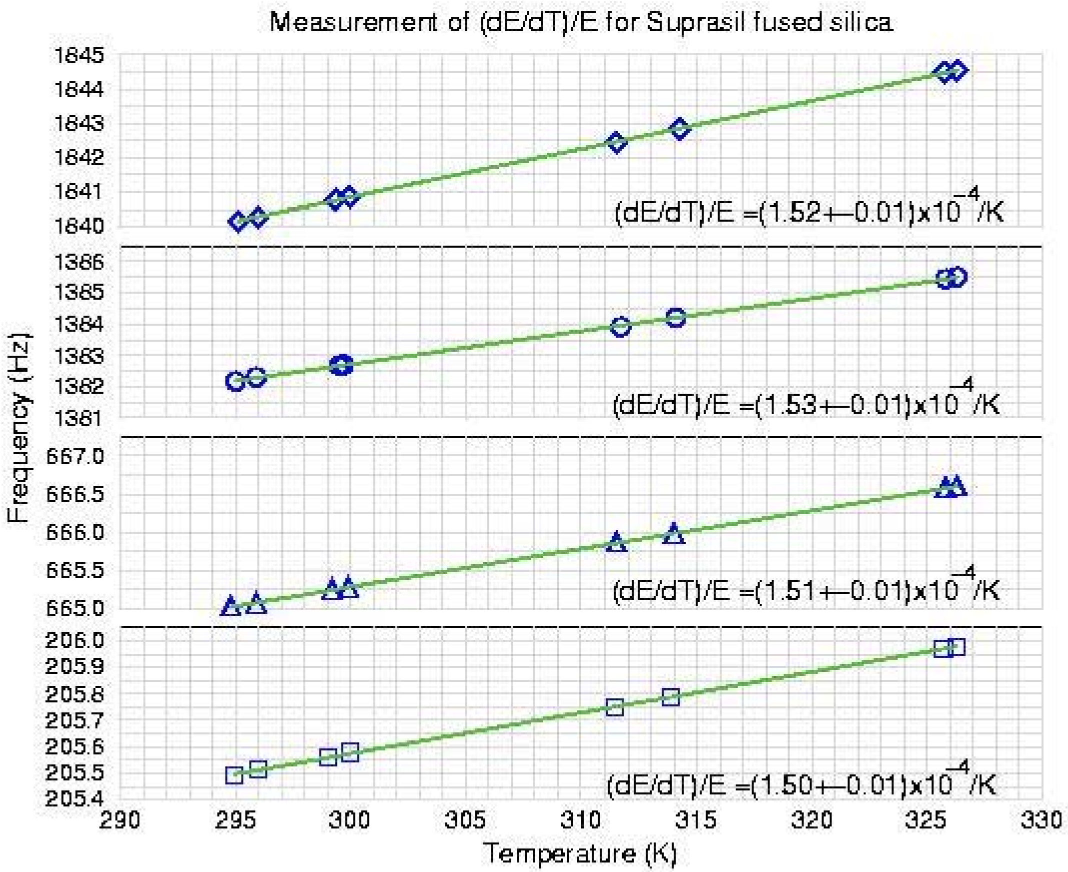
Measurement of $\frac{dE/dT}{E}$

- Of all the parameters that determine ϕ_{NTE}, the temperature dependence of the Young's modulus is most uncertain: published values vary by factor of 3.
  - Measurements made from ~10kHz to GHz frequencies
  - Measurements made using various mechanical & optical techniques
  - Measurements made using many different types of fused silica
- Need exists for measurements on our fused silica (Suprasil 2), in our frequency range, and in our mode of oscillation (bending).

$\frac{dE/dT}{E}$ Measurement Apparatus



$\frac{dE/dT}{E}$ Data



Data consistent with (dE/dT)/E=1.52e-4/K±5%. Main source of error is calibration of oven.

Note: B. Lunin measures 2.2e-4/K for similar glass (Suprasil 300).

Total Intrinsic Loss in Fibers

$$\phi_{total} = \phi_{bulk} + \phi_{surface} + \phi_{NTE}$$

$$\phi_{bulk} = 3\times10^{-8}$$

$$\phi_{surface} = \frac{3\times10^{-11}}{r}$$

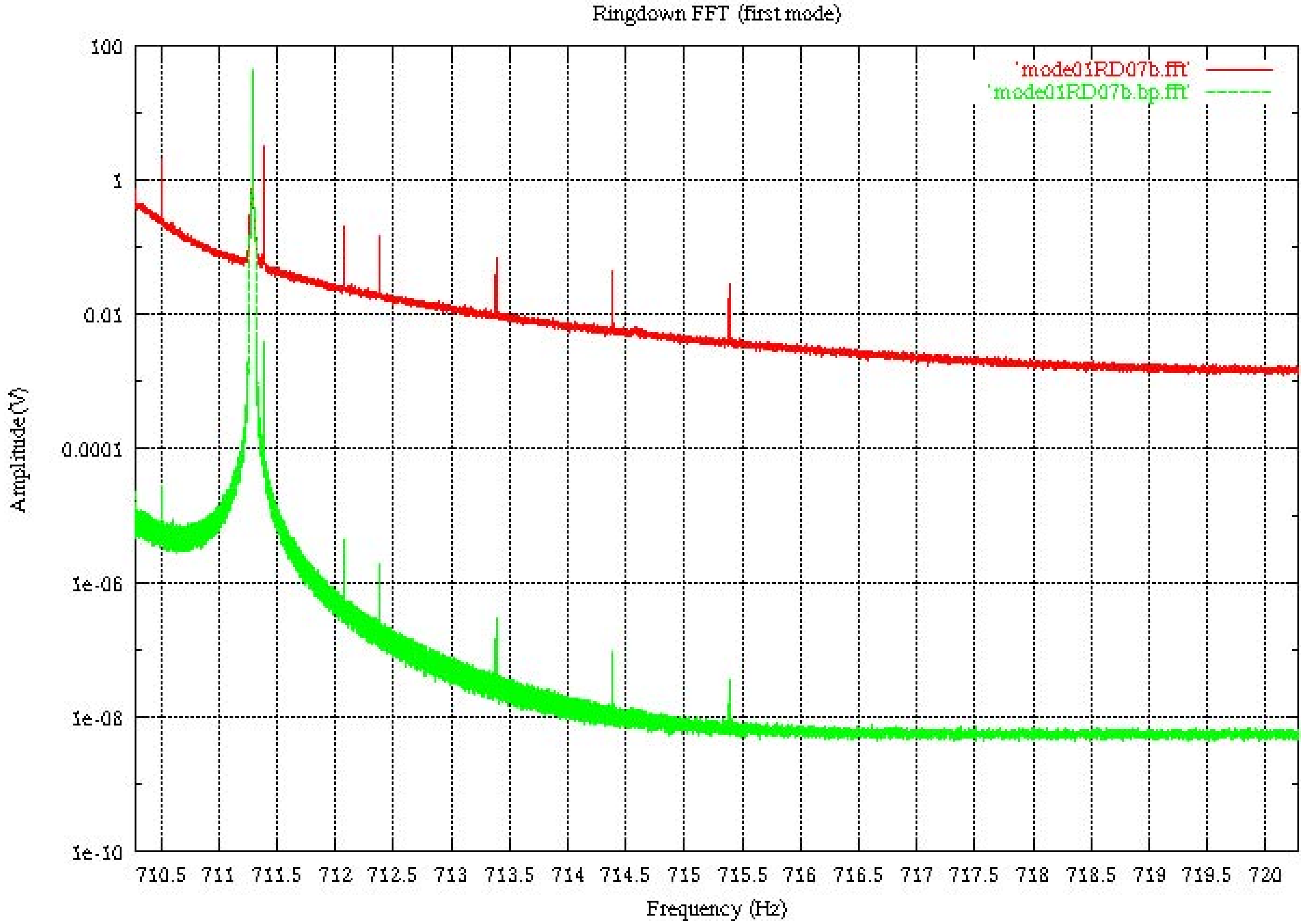
Q of a Violin Mode

$$1/Q_{violin} = \phi_{total} \frac{2}{k_e L}\left[1 + \frac{(n\pi)^2}{2k_e L}\right]$$

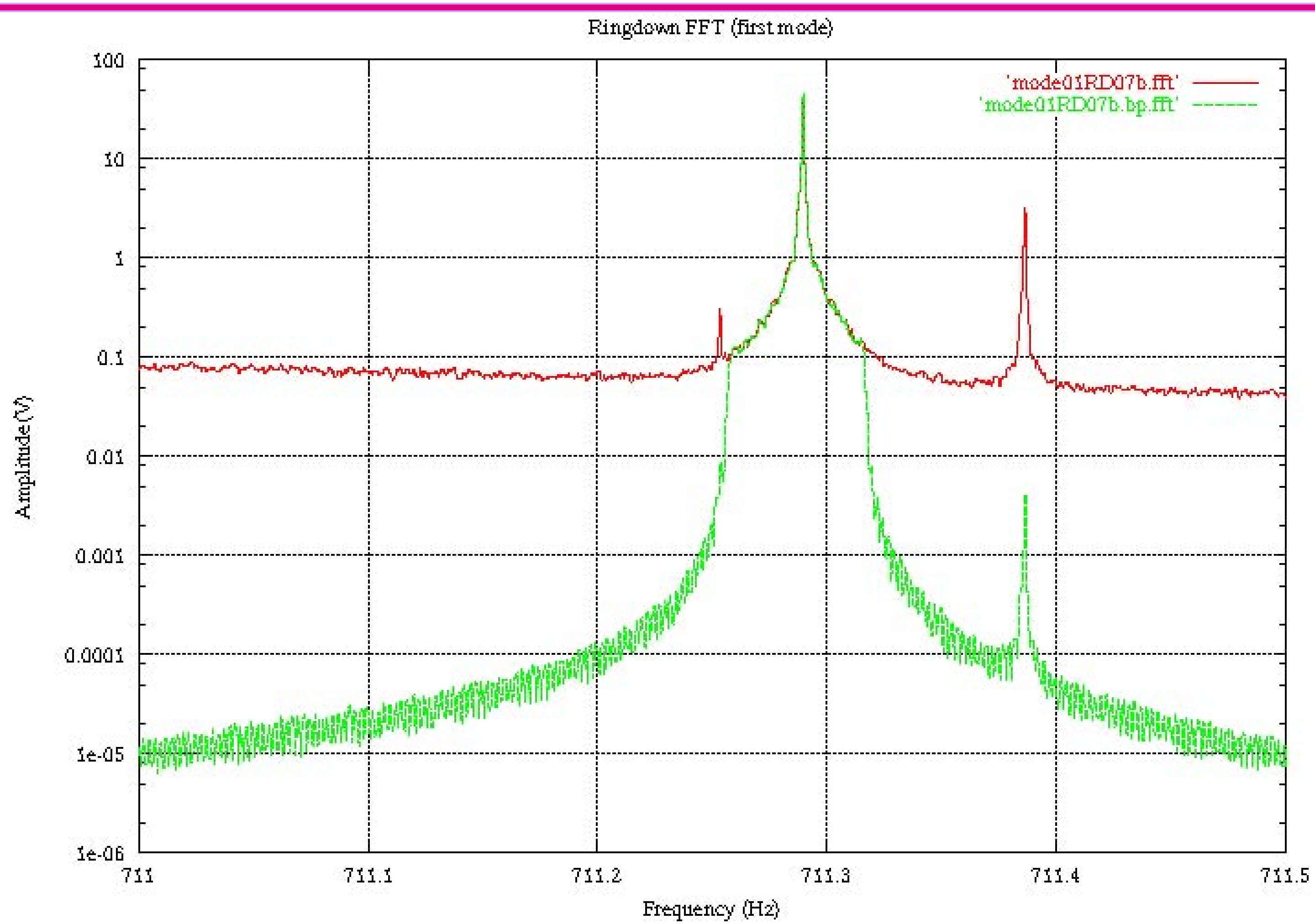
$$n = \text{mode number}$$

$$k_e = \sqrt{\frac{P + \sqrt{P^2 + 4EI\rho_L \omega^2}}{2EI}} = \text{elastic wavenumber}$$

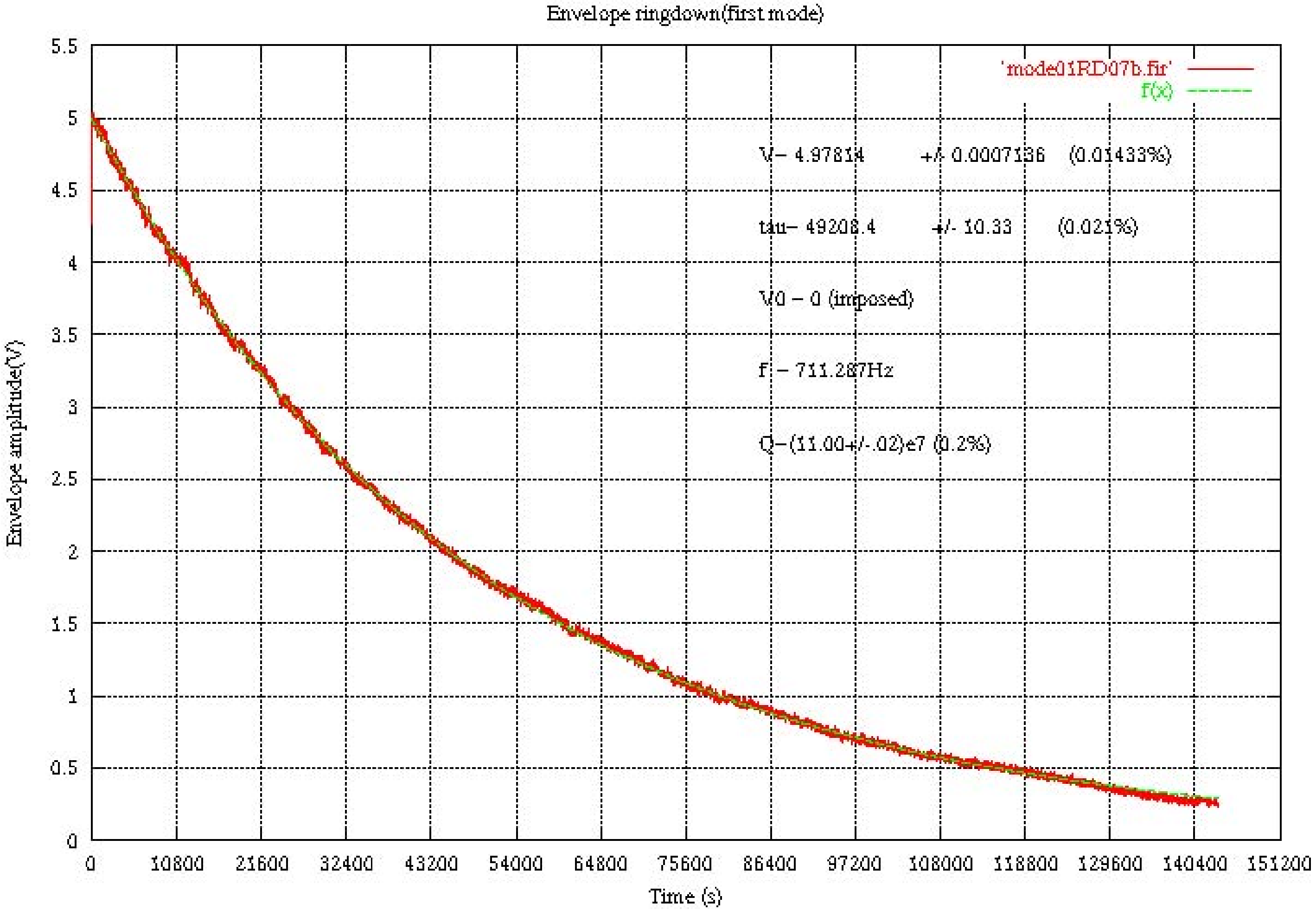
Removal of Sidebands by Digital Filtering



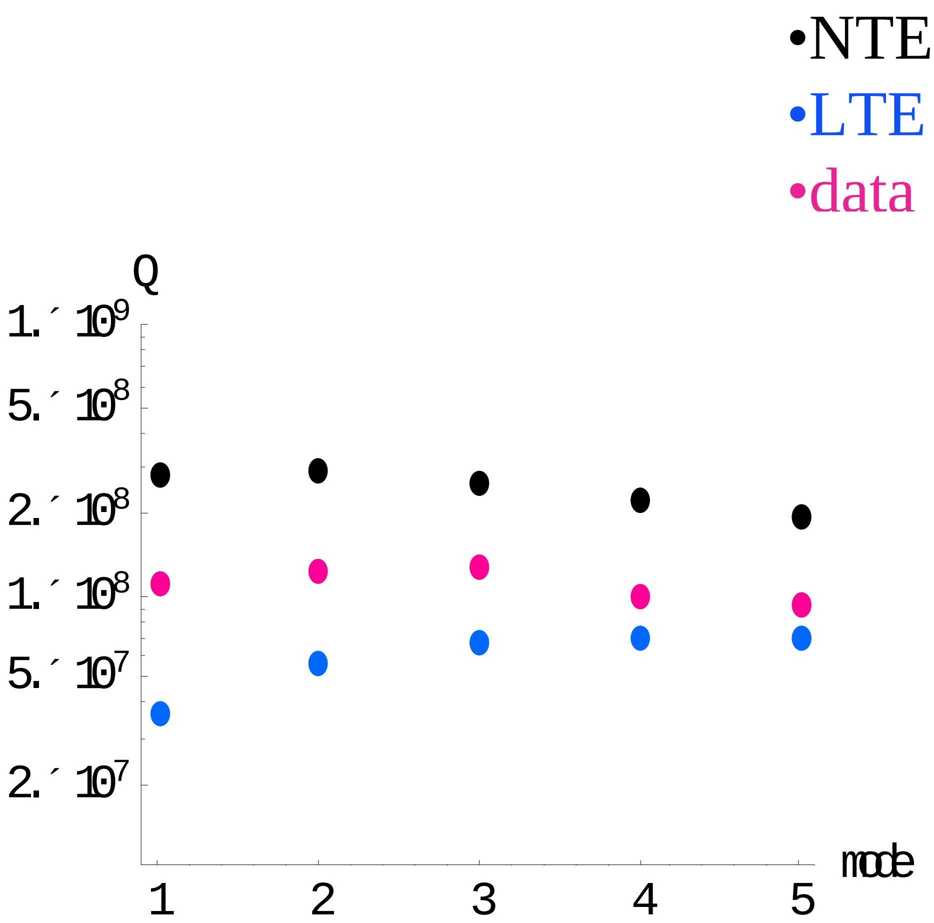
More Digital Filtering



Filtered Ringdown Data



Our Data



- Bear in mind that the predictions of the LTE model are *upper limits only.*
- ‘NTE’ prediction based on published values for ϕ_{bulk}, $\phi_{surface}$.
- We believe this to be the first observation of nonlinear thermoelastic damping.
- Future work:
  - » Measure ϕ_{bulk}, $\phi_{surface}$ in unloaded fiber
  - » Measure Q of vertical bounce mode

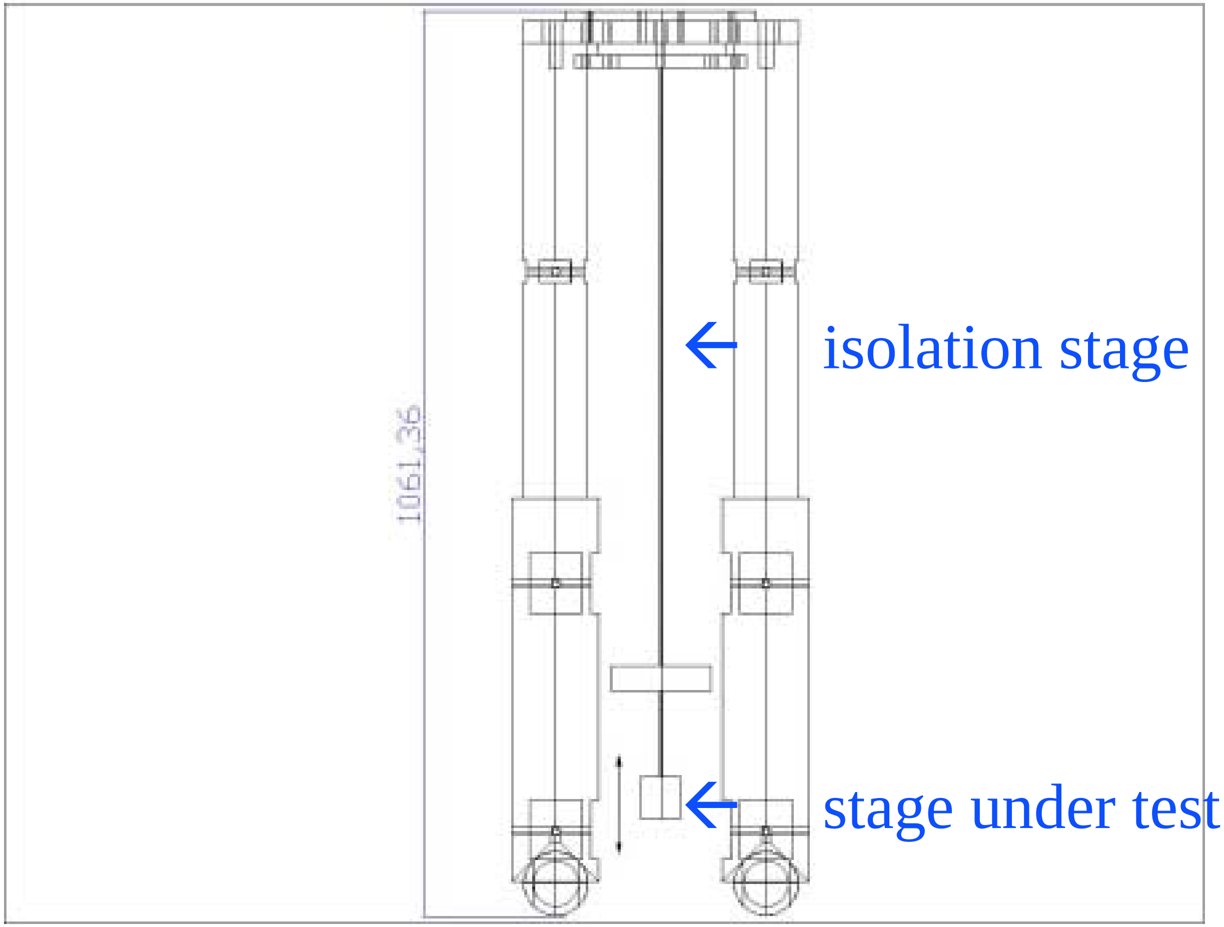
Why Study Vertical Bounce Mode Q's?

- The vertical bounce mode probes the entire fiber uniformly, unlike the pendulum and violin modes which weight the endpoints more heavily.
- The vertical bounce mode has no dilution factor.
- The vertical bounce mode has no thermoelastic damping.
- Thus the vertical bounce mode permits the clean study of the internal friction of fused silica at high strain.

Internal Friction at High Strain in Fused Silica

- So far, no good data exists at strains relevant for Advanced LIGO (u~.005-.01):
  - » Braginsky et al. measure Q consistent with ϕ=1.4e-6 at u=.005
  - » Data above consistent with ϕ=7.8e-7 at u=.003
  - » Rowan et al. measure data consistent with ϕ=7e-7 at u=.0002
  - » Rowan et al. measure data consistent with ϕ=1.5e-6 at u=.002
  - » Advanced LIGO baseline assumes ϕ=2e-7 at u=.01
- Discrepancies could be due to recoil losses or contamination due to welding, but stress-dependent ϕ cannot yet be excluded.
- Fused silica is getting nonlinear at these strains… Young's modulus changes by ~1e-2 at u=.003 (note: ϕ is only 1e-6!).
- It is prudent therefore to check stress-dependence of ϕ

Vertical Bounce Measurement Apparatus- in Preparation



Expected Sensitivity

- Use of monolithic fused silica upper suspension should contribute negligibly to loss from that stage…
- Fundamental recoil limit should be through suspension mounting structure- previous rigidities achieved at Glasgow should allow $1/Q_{recoil}$~1e-7 for 50um fibers holding 200g
- Hopefully new ultraheavy pendulum laboratory will permit larger fibers to be tested.