A Comparison of Spherical Antennas with Interferometers Using Resonant Sideband Extraction

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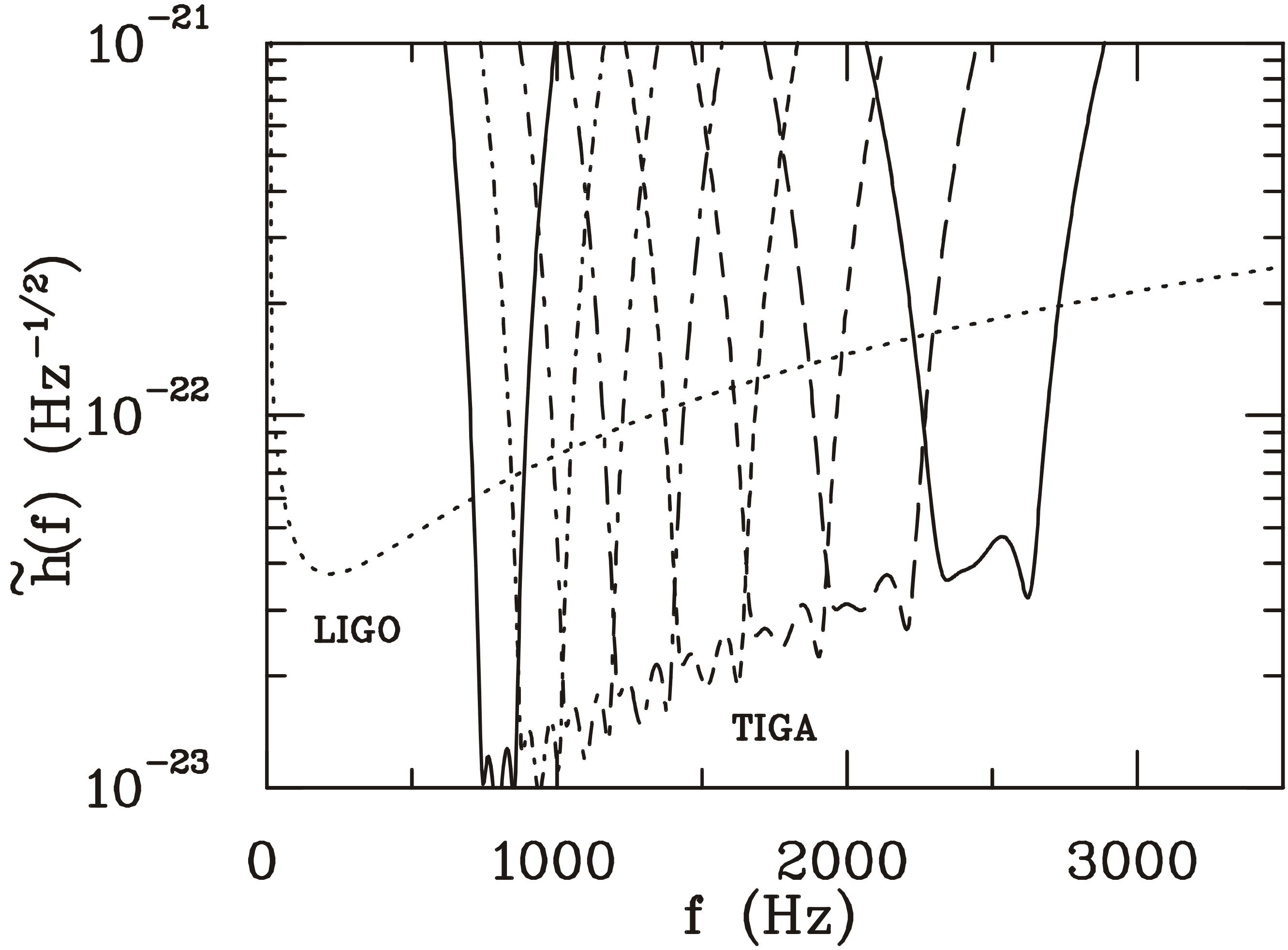
Overview of Presentation

- Review of previous work
- Spherical antennas
- Interferometers with resonant sideband extraction (RSE)
- Sources
- Signal-to-noise ratio calculations
- Conclusion

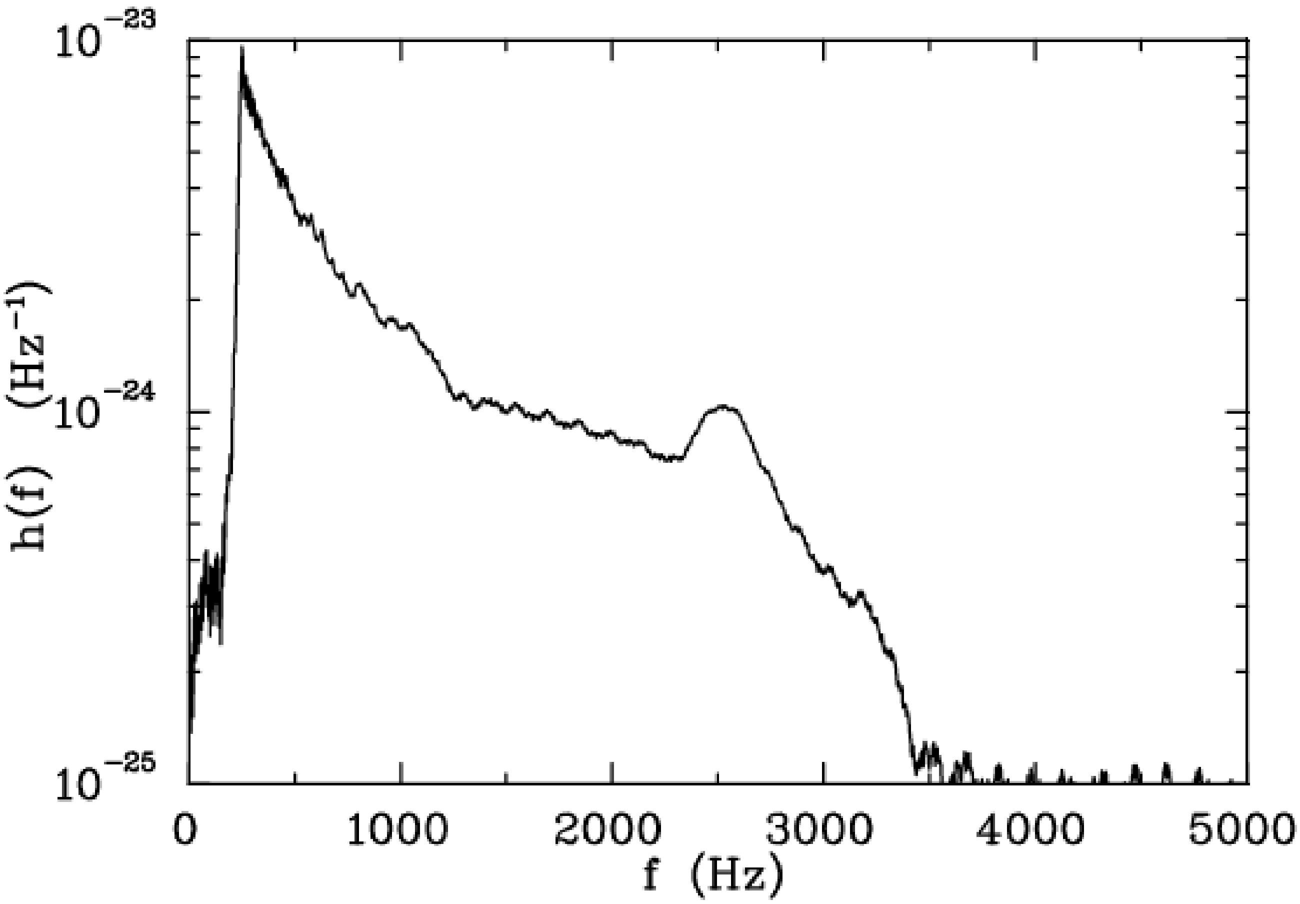
Comparison of Spheres with LIGO I

3 □ spheres (TIGAs) compared with LIGO I
c. 1992



From G M Harry, T R Stevenson, H J Paik, Physical Review D **54**, 2409 (1996).

Binary Neutron Star Inspiral and Coalescence (1994)



From: G M Harry, T R Stevenson, H J Paik, Physical Review D **54**, 2409 (1996); X Zhuge, J M Centrella, S L W McMillan, Physical Review D **50** 6247 (1994).

A Comparison of Spheres and Interferometers

- **Comparison between spheres and a more advanced interferometer now more relevant**
- **More relevant to compare spheres with a narrowband interferometer**

Compare spheres with an interferometer that uses resonant sideband extraction (RSE)

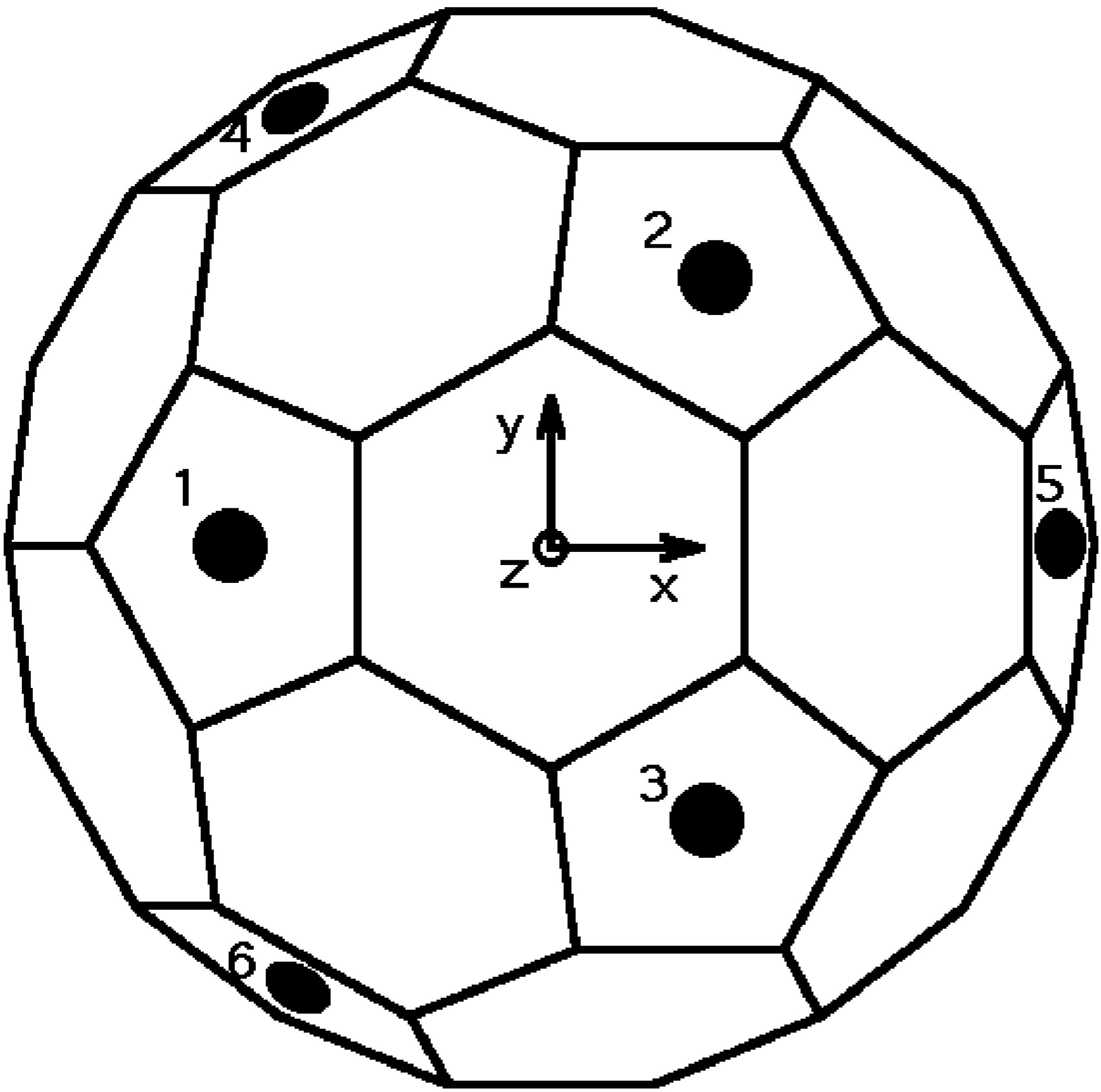
Model Philosophy

- Create strain spectra using experimentally determinable parameters
- Use parameters that have been or will plausibly be demonstrated within the next 5 years

→ Use BENCH v1.5 with LIGO II parameters

Use same sphere model as 1996

Truncated Icosahedral Gravitational-wave Antenna



Sphere Parameters I: Transducer

Type: Inductive, Paik Style

Number of Modes: 3

Transducer Material: Niobium

Transducer Q: 40 X 10^6

Mass Ratio: $m_s / m_1 = m_1 / m_2 = 100$

Relative Bandwidth: 10%

Sphere Parameters II: Thermal Noise

Sphere Material:	**Aluminum**
Sphere Q:	**40 X 10^6**
Intermediate Mass:	**Aluminum**
Intermediate Mass Q:	**40 X 10^6**
Temperature:	**50 mK**

$$S_{sph,thermal} = 2k_B T \,\mathrm{Re}[\,y_{22}(f)]$$

where y (f) is the admittance matrix of the sphere and y_{22} (f) depends on the Q's

Sphere Parameters III: Amplifier Noise

SQUID Noise Number: N_n **= 1 (□)**

Sensing Coil Diameter: d_c **= 9 cm**

Noise Resistance: R_n □□ d_c^2

Velocity Noise:

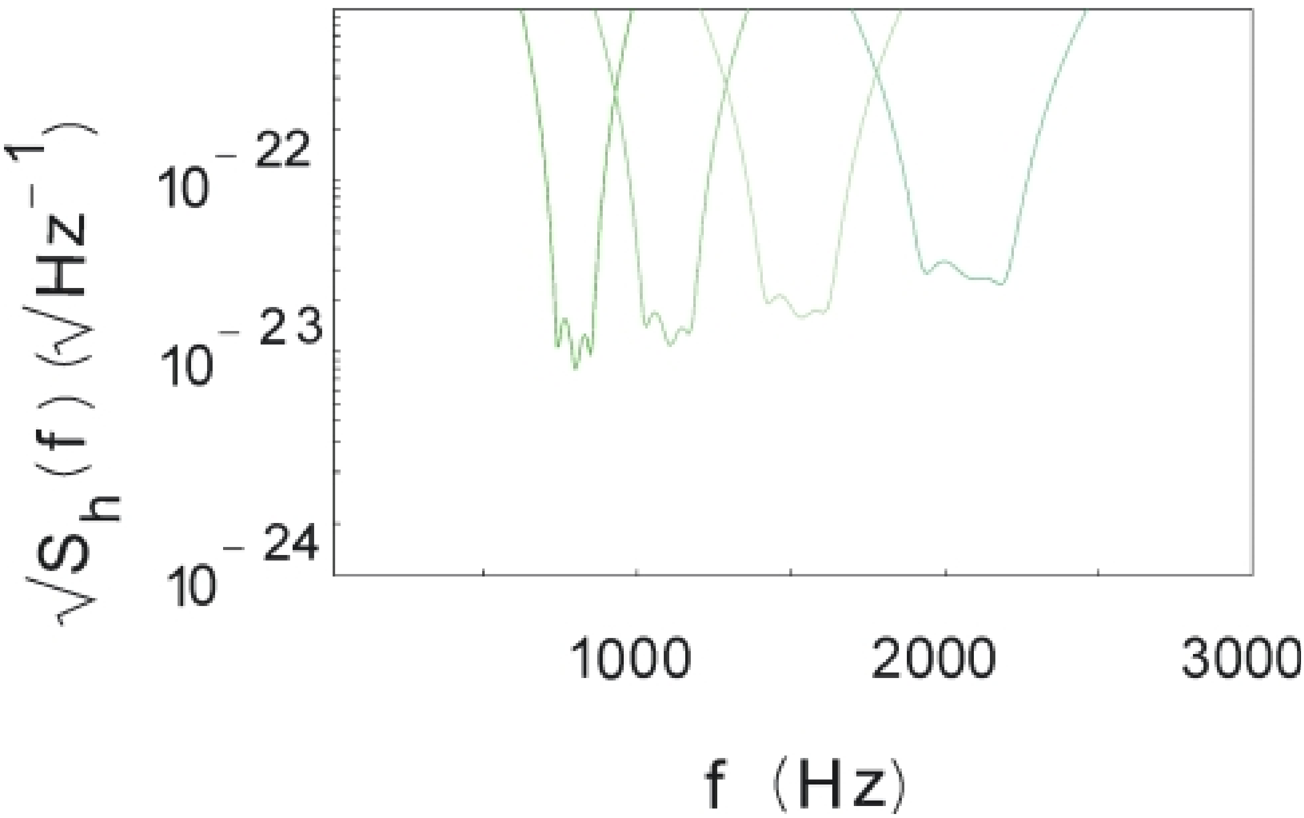
$$S_u(f) = 2\pi \bullet N_n f_0 / R_n$$

Force Noise:

$$S_{f,out}(f) = 2\pi \bullet N_n f_0 R_n \, | y_{22}(f) |^2$$

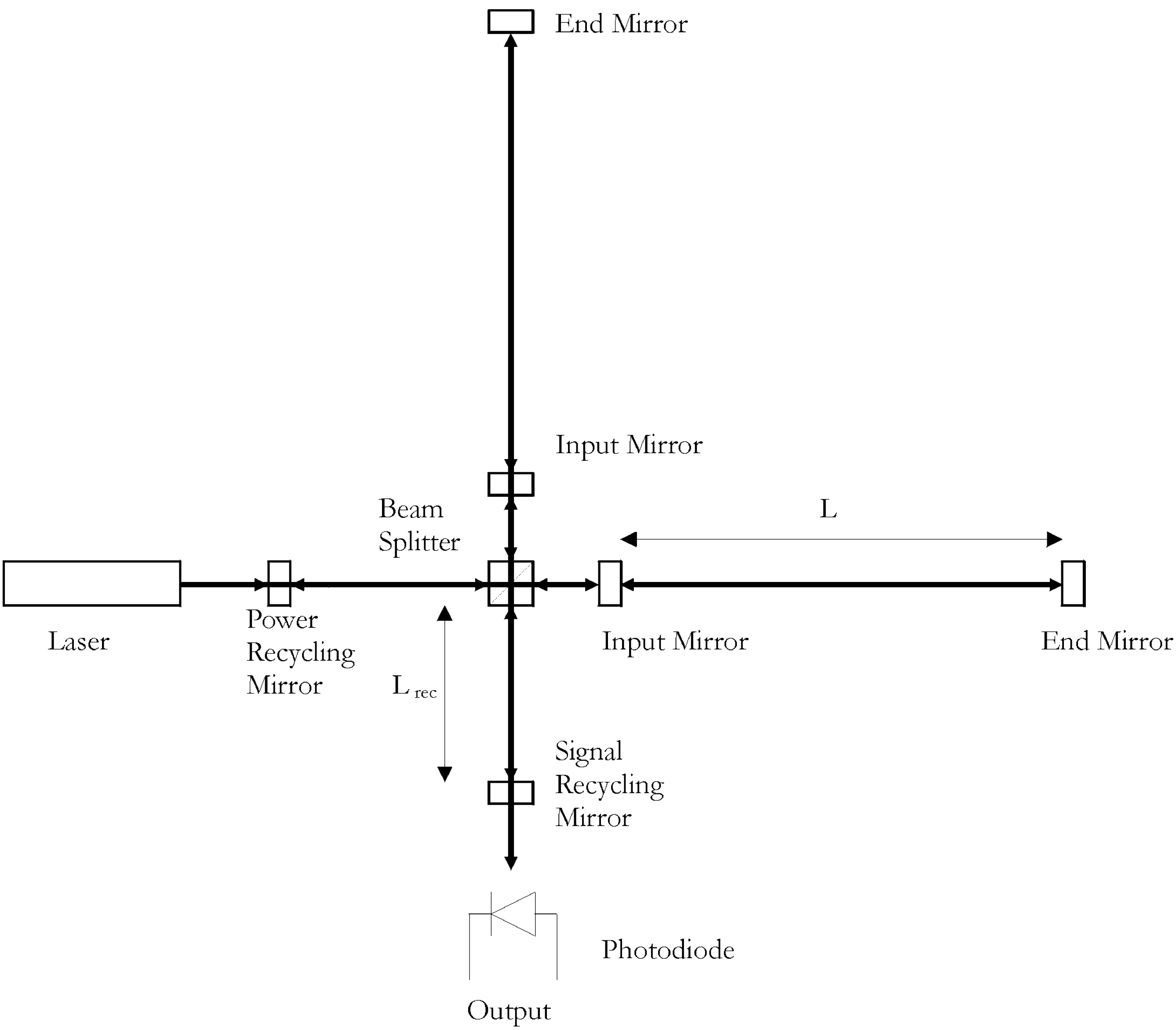
where f_0 is the resonance frequency of the sphere

Sphere Spectrum



Diameter	Mass	Frequency
3.25 m	50 t	795 Hz
2.35 m	19 t	1100 Hz
1.70 m	7 t	1520 Hz
1.25 m	3 t	2067 Hz

Interferometer with Resonant Sideband Extraction



Interferometer Parameters I: Global Values

Arm Length:	**L = 4000 m**
Temperature:	**T = 300 K**
Gaussian Width of Laser:	**w = 6 cm**
Beamsplitter Thickness:	**12 cm**
Mirror Thickness:	**12 cm**
Mirror Radius:	**14 cm**
Laser Power:	**125 W**
Laser Wavelength:	**λ = 1.064 μm**

Interferometer Parameters II: Seismic Noise

Four stages of suspension

Two stages of 6 dof vibration isolation

External hydraulic actuators

Seismic Cutoff Frequency:
$\mathbf{f_{seismic} = 10\ Hz}$

$$S_{seismic} = \infty \;\; \text{if} \;\; f \leq f_{seismic}$$

Interferometer Parameters III: Internal Thermal Noise

Silica Beamsplitter and Sapphire Mirrors

Loss Angle of Sapphire: ┘□□□5.0 X 10^{-9}

$$S_{thermal} = \frac{1}{L^2} \; \frac{4k_B T\phi}{\pi \; f} C + S_{thermo}$$

where C is the overlap integrals between the normal modes of the mirror and the gaussian-profile laser, and S_{thermo} is the noise due to thermoelastic damping

Interferometer Parameters IV: Suspension Thermal Noise

Suspension Length: L_{sus} **= 0.588 m**

Mirror Mass: **m = 30 kg**

Loss Angle of Ribbon: ϕ_{rib} [illegible] $^{-8}$

Ribbon Thickness: **1.7 mm**

Dissipation Depth of Ribbon: **185 μm**

$$S_{susp} = \frac{16\ k_B T \phi_{eff}\, g}{L^2 (L_{sus} 2\pi f\ m(((2\pi f)^2 - \omega_{pen}^2)^2 + \omega_{pen}^4 \phi_{eff}^2))}$$

where ϕ_{eff} is a loss angle that includes the effects of dissipation dilution, thermoelastic damping, and surface loss

Interferometer Parameters V: Radiation Pressure Noise

Treat Each RSE Sideband Separately for Cavity Response Function G_0

Power at the Beamsplitter: $P_{BS} = 9.3$ **kW**

P_{BS} **compared to limits from thermal lensing**

Power Transmittance: $t_1^2 = 3\ \%$

Power Transmittance: $t_2^2 = 3.75 \times 10^{-3}\ \%$

Power Transmittance: $t_3^2 = 0.5\ \%$

$$S_{rad} = \frac{16 P_{BS}\, 2\pi \bullet t_1^4 t_3^2 r_2^2 \left(\frac{1}{|G_{0,1}|} + \frac{1}{|G_{0,2}|}\right)^2}{c\lambda((1 - r_2 r_2)(2\pi f)^2 mL)^2}$$

Interferometer Parameters VI: Shot Noise

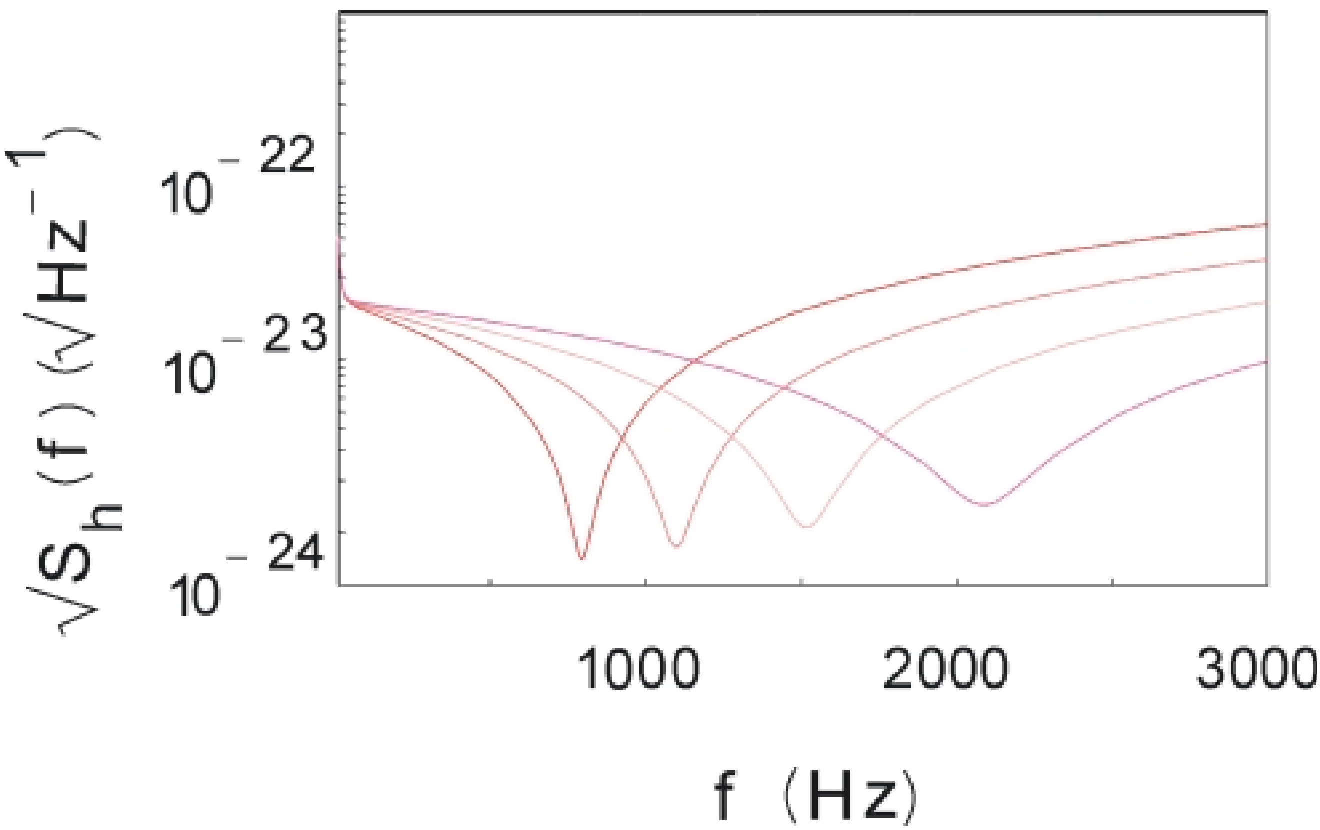
Recycling Cavity Length: $L_{rec} = 10$ **m**

Light Transit Time: $▲_a = 2$ **L / c**

Photodiode Efficiency: • □□ **0.9**

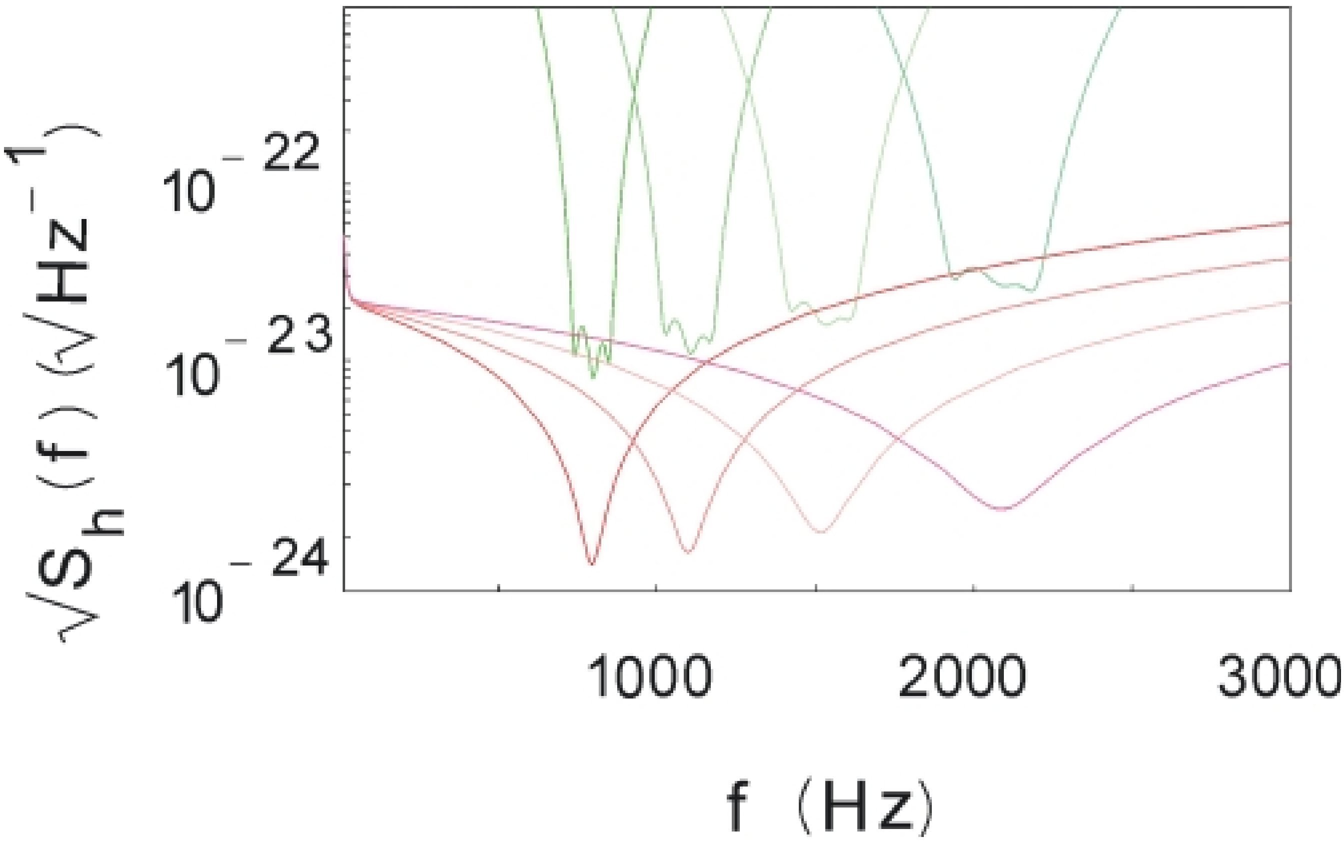
$$S_{shot} = \left(\frac{2\lambda f (1 - r_1 r_2)}{c \,\mathrm{Sin}(\pi f \tau_a)\; t_1^2 r_2 t_3 \left(\frac{1}{|G_{0,1}|} + \frac{1}{|G_{0,2}|} \right)} \right)^2 \frac{2\pi f \;•}{\eta P_{BS}}$$

Interferometer with RSE Spectrum



Accumulated phase δ	Frequency
0.2271	795 Hz
0.1641	1100 Hz
0.1182	1520 Hz
0.08619	2067 Hz

Combined Spectra of Spheres and Interferometers



Accumulated phase δ	Diameter	Frequency
0.2271	3.25 m	795 Hz
0.1641	2.35 m	1100 Hz
0.1182	1.70 m	1520 Hz
0.08619	1.25 m	2067 Hz

Comment on Bandwidths

Sphere's bandwidth depends on impedance matching between the sphere and the SQUID. The maximum available with this three-mode transducer is

BWsphere = 10%

Interferometer's bandwidth depends on input and signal recycling mirrors' reflectivities. The minimum reasonable with our parameters is

BWint = 17%

Since it is unreasonable to match bandwidths, we held t_1 and t_3 fixed and let the interferometer bandwidths vary from 17% to 33%.

Janet’s slides on souces go here.

Signal-to-Noise Ratio Calculations

Each antenna was modeled with each source to find the SNR

$$S / N = \int_{-\infty}^{\infty} \frac{\Sigma}{S_{tot}} \mathrm{d}f$$

where

$$\Sigma_{\text{sphere}} = \frac{\pi E^{5/2} \Pi m_s}{f_0 \rho^{3/2}} f^2 |y_{21} h|^2$$

$$\Sigma_{\text{int}} = |h|^2$$

Conclusions

- With our parameters, interferometers with RSE are more sensitive than spheres
- This sensitivity translates into higher SNRs for interferometers for the two sources we considered
- Sphere do have enough sensitivity to detect sources beyond our galaxy
- LIGO II with RSE can see BNS out to distances where they are "guaranteed"
- Coalescence phase of BNS can be detected at distance where an event is likely in a multiyear run
- Addition of gravitational back reaction to BNS model makes little difference to SNRs

A Niche for Spheres

- Simultaneous detection of a gravitational wave by two separate techniques adds confidence to discovery
- Spheres operating near interferometers can help detect stochastic background gravitational waves
- Symmetry of spheres makes searches for scalar gravitational waves natural. This would allow for exploration of gravity beyond general relativity