LLO + Allegro: A Unique Opportunity to Experimentally Modulate the Stochastic Gravitational Wave Background

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LIGO Seminar

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Experimental Modulation of the Stochastic Background Correlation

- Work in progress performed in collaboration with L. S. Finn (Penn State University)
- References:
  - P.F. Michelson, *Mon. Not. Roy. Astron. Soc.* **227**, 933 (1987).
  - N. Christensen, *Phys. Rev.* **D46**, 5250 (1992)
  - E. Flanagan, *Phys. Rev.* **D48**, 2389 (1993), astro-ph9305029
  - B. Allen and J. Romano, *Phys. Rev.* **D59**, 102001 (1999), gr-qc9710117
  - M. Maggiore, Trieste, June 2000: Gravitational Waves: A Challenge to Theoretical Astrophysics, gr-qc-0008027

Outline

- Stochastic background detection
  - » Detector cross-correlation
  - » Optimal filter
  - » Modulation by detector rotation
- Application to ALLEGRO + LLO
  - » Numerical results
- Summary of what is known about $\Omega_{GW}(f)$
  - » Previous measurements
  - » Theories

LLO + Allegro:
A Unique Opportunity

- Idea: perform an improved measurement of the stochastic gravitational wave background (SGWB) between a cryogenic resonant bar and one of the LIGO interferometers by introducing a modulation scheme into the measurement - *rotate the bar w.r.t. the interferometer…*
  - + Observation in a regime with little experimental information
  - + Uses a pair of (very nearly) collocated detectors
    - + good geometric overlap
  - + Ability to identify and remove a class of terrestrial backgrounds
  - - Relatively high frequency (920 Hz), narrowband measurement
    - - Less than optimal sensitivity

Stochastic GW Background Detection

- Cross-correlate the output of two (*independent*) detectors with a suitable filter kernel:

$$C(T) = \int_{-T/2}^{T/2} dt \int_{-\tau/2}^{\tau/2} d\tau' \;\; s_1(t) s_2(t-\tau') Q(\tau')$$

- Requires:
  - (i) Two detectors must have overlapping frequency response functions i.e., $s_1(f) s_2(f) \neq 0, \; \{f\} \notin \varnothing$
  - (ii) Detectors sensitive to same polarization state (+, x) of radiation field, h_{GW}.
  - (iii) Baseline separation must be suitably “short”:

$$L < \lambda_{GW}(f) \Rightarrow \frac{fL}{c} < 1$$

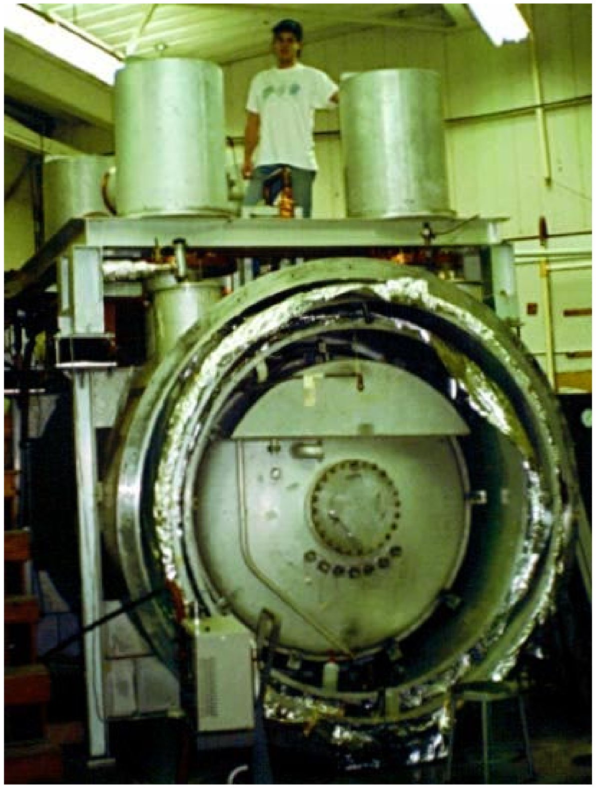
LIGO Livingston Laboratory

To ALLEGRO 42.3 km

LIGO-G000307-01-E

Allegro at LSU

Will be moved to new laboratory



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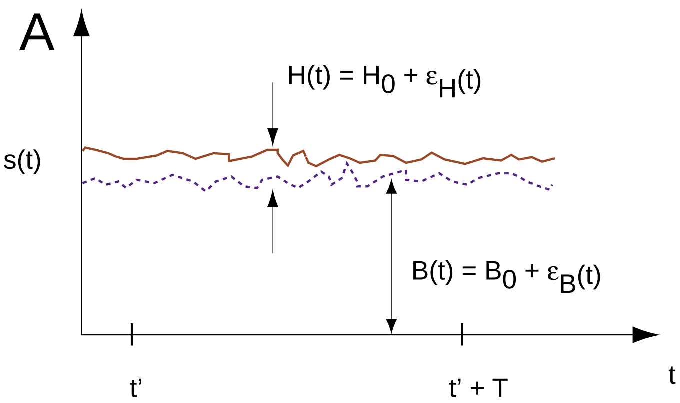
Modulation of the Stochastic Background Correlation

- Ideally, the stochastic background correlation increases with integration time as:

$$SNR \propto \frac{3H_0^2}{10\pi^2}\sqrt{T_{\text{int}}}\left[\frac{\gamma^2(f_0)\Omega_{GW}^2\Delta f}{f^6 S_{1,n}|f|S_{2,n}|f|}\right]^{\frac{1}{2}}$$

  - » Assumes no additional sources of correlated noise
    - – ***cannot discriminate with a single measurement***
  - » Mutual orientation dependence of GW background signal may be exploited to discriminate among possible correlated sources

Case A



$$X = \int_{t'}^{t'+T} s(t)\,dt \quad ; \quad \langle X \rangle = T\left(H_0 + B_0\right)$$

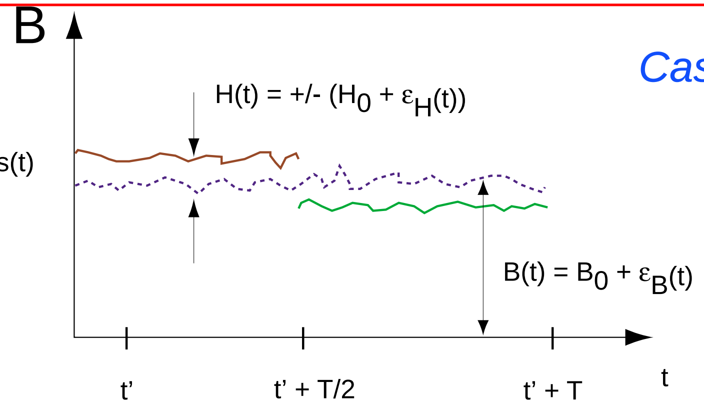
$$\sigma_X^2 = T\left(\sigma_H^2 + \sigma_B^2\right)$$

$$\text{"SNR"} = \frac{\langle X \rangle}{\sigma_X} = \sqrt{\frac{T}{\sigma_H^2 + \sigma_B^2}}\,\left(H_0 + B_0\right)$$

$$X_{\text{measured}} - 1.65\,\sigma_X < \text{"X"}_{\text{obs},90\%} < X_{\text{measured}} + 1.65\,\sigma_X$$

$$\left(H_0 + B_0\right) - 1.65\sqrt{\frac{\sigma_H^2 + \sigma_B^2}{T}} < \text{"H"}_{\text{eff},90\%} < \left(H_0 + B_0\right) + 1.65\sqrt{\frac{\sigma_H^2 + \sigma_B^2}{T}}$$

Case B



$$\tilde{X} = \int_{t'}^{t'+T/2} s(t)\,dt - \int_{t'+T/2}^{t'+T} s(t)\,dt \quad ; \quad \langle \tilde{X} \rangle = T\left(H_0\right)$$

$$\sigma_{\tilde{X}}^2 = \frac{T}{2}\left(\sigma_{X-}^2 + \sigma_{X+}^2\right) = T\left(\sigma_H^2 + \sigma_B^2\right)$$

$$\text{SNR} = \frac{\langle \tilde{X} \rangle}{\sigma_{\tilde{X}}} = \sqrt{\frac{T}{\sigma_H^2 + \sigma_B^2}}\;H_0$$

$$H_0 - 1.65\sqrt{\frac{\sigma_H^2 + \sigma_B^2}{T}} < H_{0,90\%} < H_0 + 1.65\sqrt{\frac{\sigma_H^2 + \sigma_B^2}{T}}$$

$\langle\varepsilon_B(t)\rangle = \langle\varepsilon_H(t)\rangle = 0$

$\langle\varepsilon_B(t)\varepsilon_H(t)\rangle = 0$

$\langle\varepsilon_B(t)^2\rangle = \sigma_B^2$; $\langle\varepsilon_H(t)^2\rangle = \sigma_H^2$

Modulation improves *accuracy*, but not *precision*
- **Effect of background on *mean* removed**
- **Effect of background on *variance* remains**

Optimal filtering in the presence of background correlation

$$C(T) = \int_{-T/2}^{T/2} dt \int_{-T/2}^{T/2} dt' \; s_1(t) s_2(t') Q(t-t') \; ; \; s_i(t) = h_i(t) + n_i(t)$$

h_i=GW signal in detector i

n_i= noise in detector i

$$C(T) = \int_0^{\infty} df \int_0^{\infty} df' \; \delta_T(f-f') \tilde{s}^*_1(f) \tilde{s}_2(f') Q(f') \; ; \; \delta_T(f) \equiv T\left\{\frac{\sin(\pi f T)}{\pi f T}\right\}$$

$$\left\langle \tilde{h}_1^*(f) \tilde{h}_2(f') \right\rangle = \delta(f-f') \frac{3H_0^2}{20\pi^2 |f|^3} \Omega_{GW}(|f|) \gamma\left(|f|, \vec{\Omega}_1, \vec{\Omega}_2\right)$$

$$\left\langle \tilde{n}_i^*(f) \tilde{n}_2(f') \right\rangle = \frac{1}{2} \delta(f-f') S_{ij}(|f|)$$

$$\begin{aligned}\left\langle \tilde{n}_i^*(f) \tilde{n}_2(f') \tilde{n}_i^*(f'') \tilde{n}_2(f''') \right\rangle = \frac{1}{4}\Big(& S_{ii}(|f|) S_{jj}(|f'|) \delta(f+f'') \delta(f'+f''') + \\ & S_{ij}(|f|) S_{ij}(|f''|) \delta(f+f'') \delta(f'+f''') + \\ & S_{ii}(|f|) S_{jj}(|f'|) \delta(f+f'') \delta(f'+f''') \Big)\end{aligned}$$

$\Omega_{GW}(f) = 1/\rho_0 \, d\rho_{GW}/d(\ln[f])$

$\gamma(f, \Omega_1, \Omega_2)$ = geometric overlap reduction factor depends on antenna orientations

Optimal filtering in the presence of background correlation

$$\langle C(T,\vec{\Omega}_1,\vec{\Omega}_2)\rangle = T\int_0^\infty df \left(\pm\frac{3H_0^2}{20\pi^2|f|^3}\Omega_{GW}(|f|)\gamma\left(|f|,\vec{\Omega}_1,\vec{\Omega}_2\right)+S_{12}(|f|)\right)\tilde{Q}(f)\ ;$$

Choose two orientations of one detector { Ω_1, Ω_1' }, for which γ(f, Ω_1, Ω_2) = - γ(f, Ω_1', Ω_2), denote C_+, C_- values of integrated correlation in these two orientations:

$$\langle C(T)\rangle = \langle C_+(T/2) - C_-(T/2)\rangle$$

$$\langle C(T)\rangle = T\int_0^\infty df \left(\frac{3H_0^2}{20\pi^2|f|^3}\Omega_{GW}(|f|)\gamma\left(|f|,\vec{\Omega}_1,\vec{\Omega}_2\right)\right)\tilde{Q}(f)$$

$$\sigma_C^2 = \langle C^2\rangle - \langle C\rangle^2 = 2\sigma_{C+,-}^2$$

$$\sigma_C^2 = \frac{T}{2}\int_0^\infty df\ \left(S_1(|f|)S_2(|f|)+S_{12}^2(|f|)\right)\left[\tilde{Q}(f)\right]^2$$

$$SNR = \frac{\langle C\rangle}{\sigma_C} \overset{max}{\Rightarrow} \frac{\delta[SNR]}{\delta[\tilde{Q}]} = 0 \ \Rightarrow\ \underline{\underline{\tilde{Q}(f) = \left(\frac{\gamma\left(|f|,\vec{\Omega}_1,\vec{\Omega}_2\right)\Omega_{GW,\mathrm{model}}(|f|)}{|f|^3\left(S_1(|f|)S_2(|f|)+S_{12}^2(|f|)\right)}\right)}}$$

Allegro+LLO

Modulation of the Stochastic Background Correlation

- Overlap reduction function, γ, is a function of detector geometries, orientations and detector separations

$$\gamma(f;\Omega_1,\Omega_2) = \frac{5}{8\pi}\int d\hat{\Omega}\ \ e^{2\pi i f \hat{\Omega}\cdot\Delta\vec{x}/c}\left(F_1^{+}F_2^{+} + F_1^{\times}F_2^{\times}\right);$$

$$F_i^{+,\times} = \frac{1}{2}\left(\hat{X}_i^a\hat{X}_i^b - \hat{Y}_i^a\hat{Y}_i^b\right)e_{ab}^{+,\times}\left(\hat{\Omega}\right);$$

$$e_{ab}^{+}\left(\hat{\Omega}\right) = \hat{\phi}_a\hat{\phi}_b - \hat{\theta}_a\hat{\theta}_b$$

$$e_{ab}^{\times}\left(\hat{\Omega}\right) = \hat{\phi}_a\hat{\theta}_b - \hat{\theta}_a\hat{\phi}_b$$

Allegro+LLO

Modulation of the Stochastic Background Correlation

- γ is determined by detector antenna tensors, $\mathbf{d}_{A,L}$, vector separating detectors, $\mathbf{n}_{AL}$ and 3 frequency dependent functions, ρ_i $(\alpha = 2\pi f L/c)$:

$$\gamma(f;\Omega_1,\Omega_2) = \rho_1(\alpha)\ \mathbf{d}_A : \mathbf{d}_L + \rho_2(\alpha)\left(\hat{n}_{AL}\cdot\mathbf{d}_A\right)\cdot\left(\mathbf{d}_L\cdot\hat{n}_{AL}\right) + \rho_3(\alpha)\left(\hat{n}_{AL}\cdot\mathbf{d}_A\cdot\hat{n}_{AL}\right)\left(\hat{n}_{AL}\cdot\mathbf{d}_L\cdot\hat{n}_{AL}\right)$$

Allegro+LLO

Modulation of the Stochastic Background Correlation

$$\begin{pmatrix} \rho_1(\alpha) \\ \rho_2(\alpha) \\ \rho_3(\alpha) \end{pmatrix} = \begin{pmatrix} 5 & \frac{-10}{\alpha} & \frac{5}{\alpha^2} \\ -10 & \frac{40}{\alpha} & \frac{-50}{\alpha^2} \\ \frac{5}{2} & \frac{-25}{\alpha} & \frac{175}{2\alpha^2} \end{pmatrix} \cdot \begin{pmatrix} j_0(\alpha) \\ j_1(\alpha) \\ j_2(\alpha) \end{pmatrix}$$

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Modulation of the Stochastic Background Correlation

$$\mathbf{d}_L\left(\sigma_1\right) = \sin\left(2\sigma_1\right)\left[\frac{\hat{n}_x \otimes \hat{n}_x - \hat{n}_y \otimes \hat{n}_y}{2}\right] - \cos\left(2\sigma_1\right)\left[\frac{\hat{n}_x \otimes \hat{n}_y + \hat{n}_y \otimes \hat{n}_x}{2}\right]$$

$$\mathbf{d}_A\left(\sigma_2\right) = \frac{3\left(\cos(\sigma_2 - \frac{\pi}{4})\,\hat{n}_x + \sin(\sigma_2 - \frac{\pi}{4})\,\hat{n}_y\right) \otimes \left(\cos(\sigma_2 - \frac{\pi}{4})\,\hat{n}_x + \sin(\sigma_2 - \frac{\pi}{4})\,\hat{n}_y\right) - I}{3}$$

Allegro+LLO

Modulation of the Stochastic Background Correlation

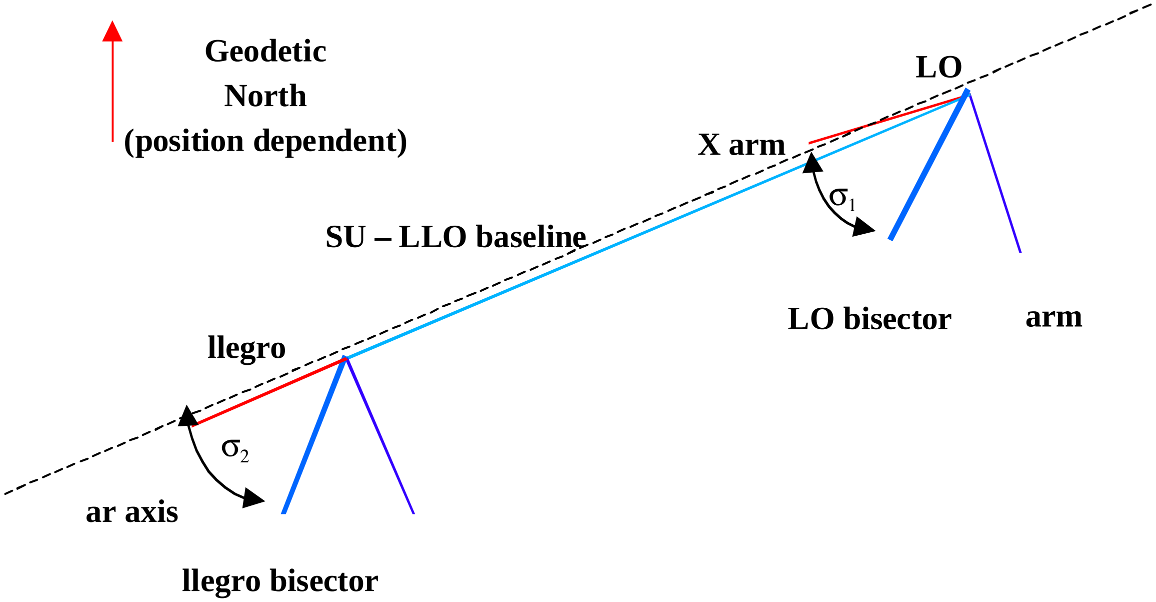
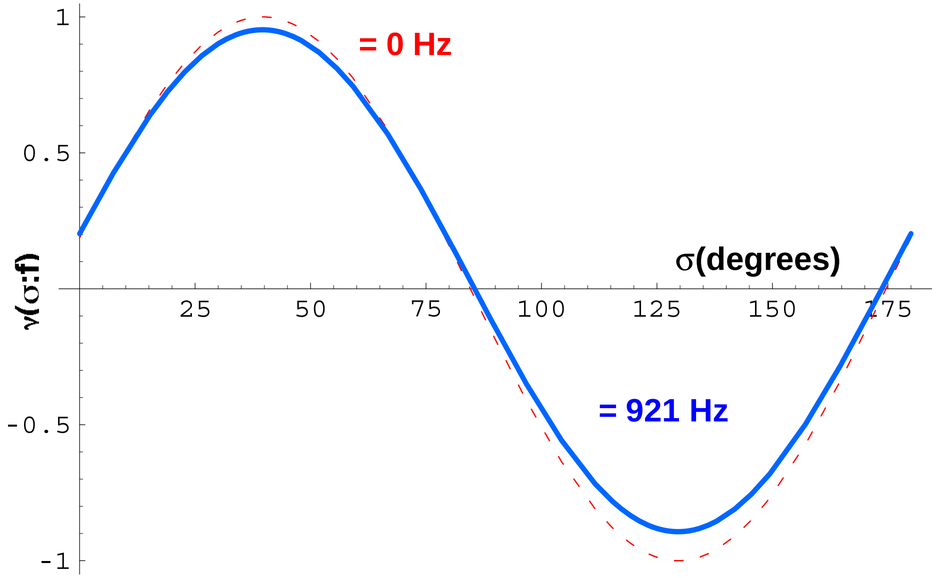


Figure 1: Schematic showing Allegro and LLO orientations with respect to geodetic north and the LLO-LSU baseline.

Allegro+LLO

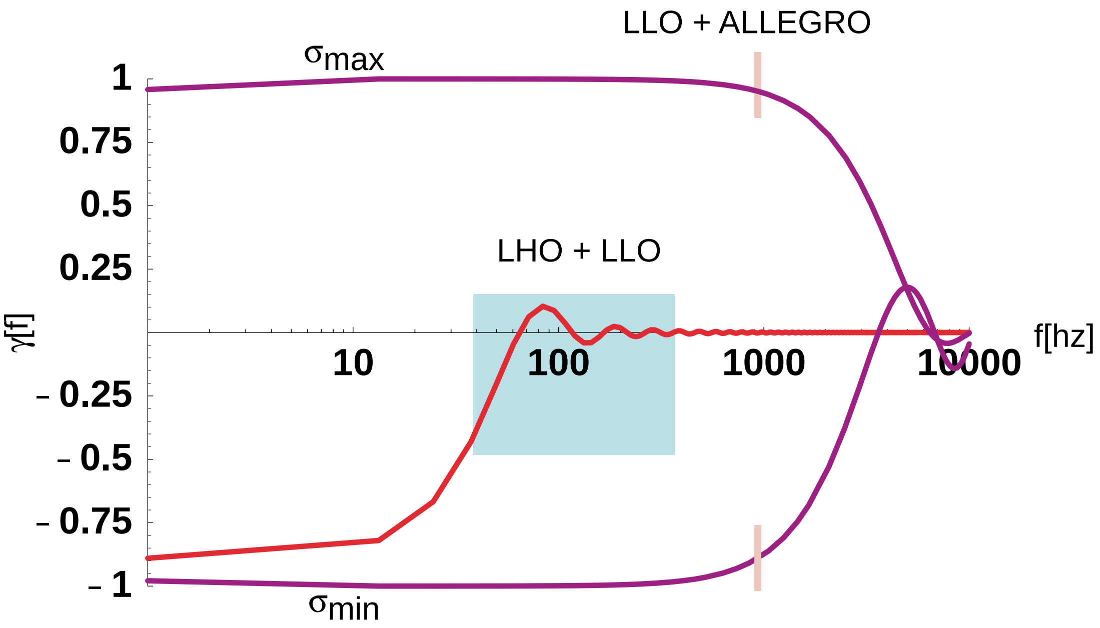
Modulation of the Stochastic Background Correlation



igure 2: Dependence of the Allegro-LLO correlation function on the ngle between the Allegro bar bisector and the LLO-to-LSU baseline refer to Figure 1). Dashed line is for DC and the solid line is for the llegro resonant frequency.

Allegro+LLO

Modulation of the Stochastic Background Correlation



LLO

Geodetic Parameters

Table I: Geographic Data for LIGO Livingston Laboratory (LLO)

Quantity	**Value**	**Units**
LLO Vertex	{-74276.044, -5496283.721, 3224257.018} {-6.568, N30°33'6.871", W90°48'50.229"}	meter* {h(m), θ, λ}
X arm unit vector	{-0.954574,-0.1415805,-0.2621887}	In ECF
Y arm unit vector	{0.2977412,-0.4879104,-0.8205447}	In ECF
Bearing of Allegro at LLO vertex	S66.88°W	Reference is geodetic north (not grid north)
Angle between LLO arm bisector, $(\hat{x}+\hat{y})/\sqrt{2}$,and LLO-LSU baseline	39.59° CCW (Bearing: S27.28°W)	

*Positions are with respect to the Earth Centered Frame [ECF], defined as follows: $\hat{z}$ pierces the Earth at the north pole; $\hat{x}$ pierces the Earth at the intersection of the prime meridian and the equator; $\hat{y} = \hat{z} \times \hat{x}$

Allegro

Geodetic Parameters

Table II: Geographic Data for Allegro Bar Detector at LSU

Quantity	Value	Units
Allegro Vertex	{-113258.848, 5504077.706, 3209892.353} {30°24’ 45.110”, W91°10’43.766”}	meter* $\{\theta,\lambda\}$
Bearing of LLO vertex at Allegro	N66.67°E	Reference is geodetic north (not grid north)
Angle σ, between Allegro coordinate axis bisector, $(\hat{x}+\hat{y})/\sqrt{2}$, and LLO-to-LSU baseline at various values of correlation	Correlation maximum: γ_{max}(921 Hz) = 0.953 σ=39.60°CCW (Bar axis bearing: S72.08°W) Correlation null: γ_{null}(921 Hz) = 0.0 σ=85.52°CCW (Bar axis bearing: S26.15W) Correlation minimum: γ_{min}(921 Hz) = -0.893 σ=129.60°CCW (Bar axis bearing: S17.92E)	
LLO – Allegro Baseline baseline distance	42269.951	meter
Angle subtended by LLO – Allegro baseline at center of Earth	0.358°	

*Positions are with respect to the Earth Centered Frame [ECF], defined as follows: $\hat{z}$ pierces the Earth at the north pole; $\hat{x}$ pierces the Earth at the intersection of the prime meridian and the equator; $\hat{y}=\hat{z}\times\hat{x}$

Allegro+LLO

Modulation of the Stochastic Background Correlation

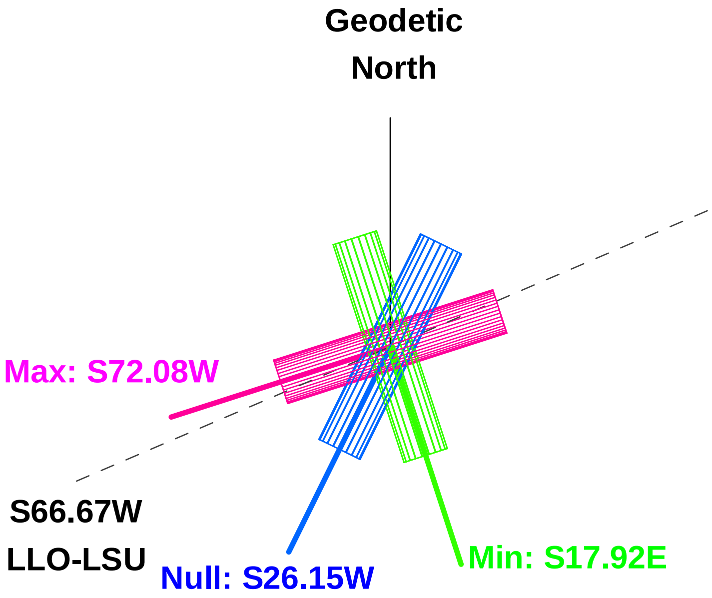


Figure 3: Schematic showing Allegro orientations with respect to geodetic north and the LLO-LSU baseline

$$\boldsymbol{C}_{+}(\sigma_2 = 39.6°) \approx T_{\text{int}}\Delta f\left(\frac{3H_0^2}{20\pi^2}\,\frac{\gamma^2\Omega_{GW}(f_0)\Omega_{GW,\text{model}}(f_0)}{f_0^6\left(S_A(f_0)S_L(f_0)+S_{LA}(f_0)^2\right)}+S_{LA}(f_0)\frac{\gamma\ \ \Omega_{GW,\text{model}}(f_0)}{f_0^3\left(S_A(f_0)S_L(f_0)+S_{LA}(f_0)^2\right)}\right)$$

$$\boldsymbol{C}_{-}(\sigma_2 = 129.6°) \approx T_{\text{int}}\Delta f\left(-\frac{3H_0^2}{20\pi^2}\,\frac{\gamma^2\Omega_{GW}(f_0)\Omega_{GW,\text{model}}(f_0)}{f_0^6\left(S_A(f_0)S_L(f_0)+S_{LA}(f_0)^2\right)}+S_{LA}(f_0)\frac{\gamma\ \ \Omega_{GW,\text{model}}(f_0)}{f_0^3\left(S_A(f_0)S_L(f_0)+S_{LA}(f_0)^2\right)}\right)$$

Allegro+LLO

Modulation of the Stochastic Background Correlation

- After total a observation time T_{int}

$$\boldsymbol{C} = \boldsymbol{C}_{+}(\sigma_2 = 39.6°) - \boldsymbol{C}_{-}(\sigma_2 = 129.6°) = T_{\text{int}}\Delta f\left(\frac{3H_0^2}{20\pi^2}\frac{\gamma^2\Omega_{GW}(f_0)\Omega_{GW,\text{model}}(f_0)}{f_0^6\left(S_A(f_0)S_L(f_0)+S_{LA}^2(f_0)\right)}\right)$$

$$\sigma_C^2 = T_{\text{int}}\Delta f\left(\frac{\gamma^2\Omega_{GW,\text{model}}^2(f_0)}{f_0^6\left(S_A(f_0)S_L(f_0)+S_{LA}^2(f_0)\right)}\right)$$

$$\boldsymbol{C} = \left(\frac{3H_0^2}{20\pi^2}\frac{\Omega_{GW}(f_0)}{\Omega_{GW,\text{model}}(f_0)}\right)\sigma_C^2$$

$$SNR = \frac{\boldsymbol{C}}{\sigma_C} = \sqrt{T_{\text{int}}\Delta f}\left(\frac{3H_0^2}{10\pi^2}\frac{\gamma\ \Omega_{GW}(f_0)}{f_0^3\sqrt{\left(S_A(f_0)S_L(f_0)+S_{LA}^2(f_0)\right)}}\right)$$

ALLEGRO h[f]

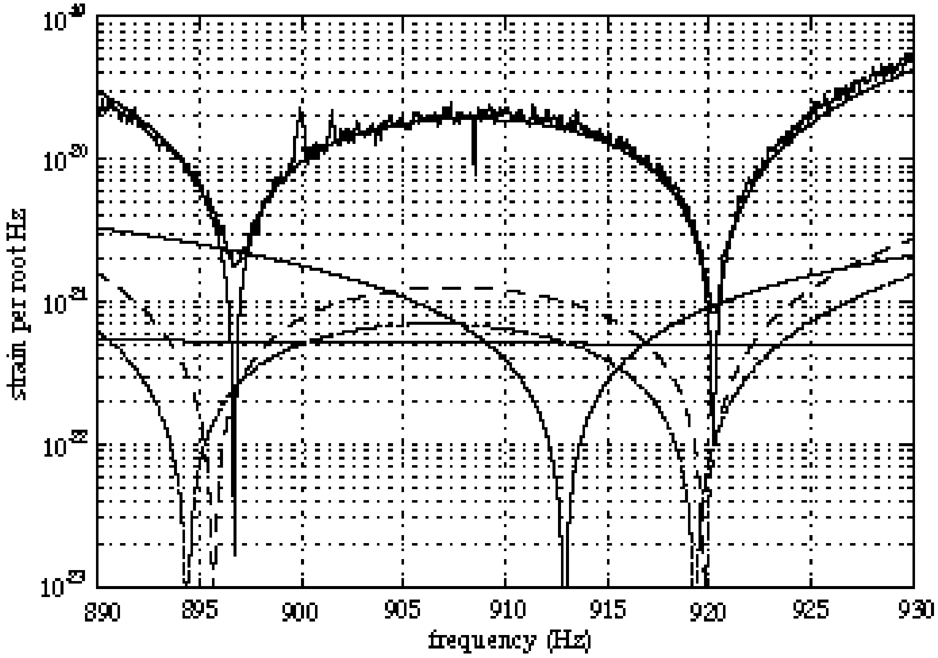
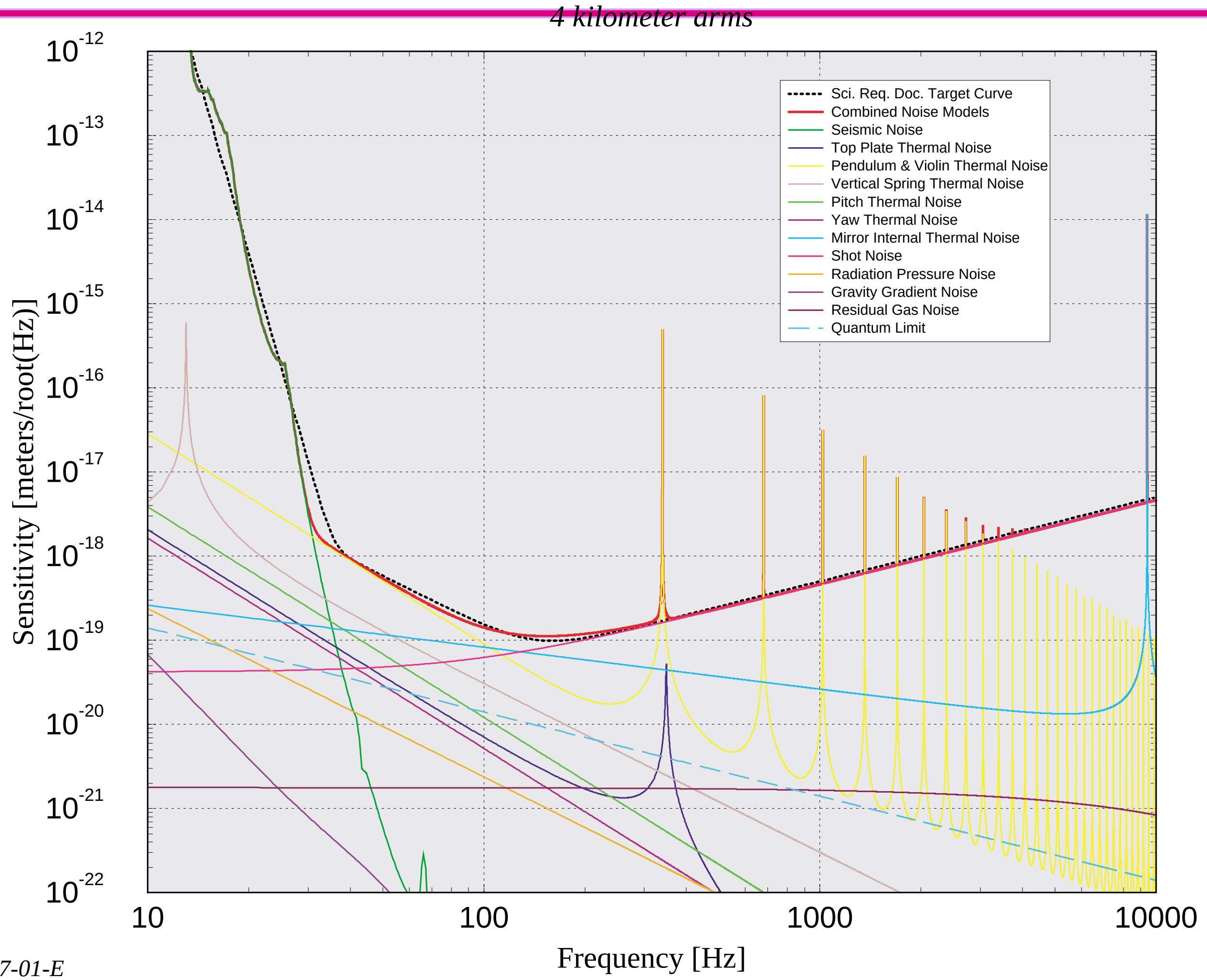


FIGURE 2. The measured strain noise of ALLEGRO, shown as the irregular line. The various noise contributions estimated from the noise model are shown as smooth lines. The noise is dominated by the SQUID's wide band and the transducer's narrow band noise. — Measured total noise, — antenna brownian noise, — transducer brownian, — transducer electrical loss, — SQUID white noise, – – SQUID back action (estimated).

Geng, Hamilton, Johnson, Mauceli, Merkowitz, Morse, Solomonson, Amaldi Conference 1999

LLO h[f]



Blackburn & Shoemaker

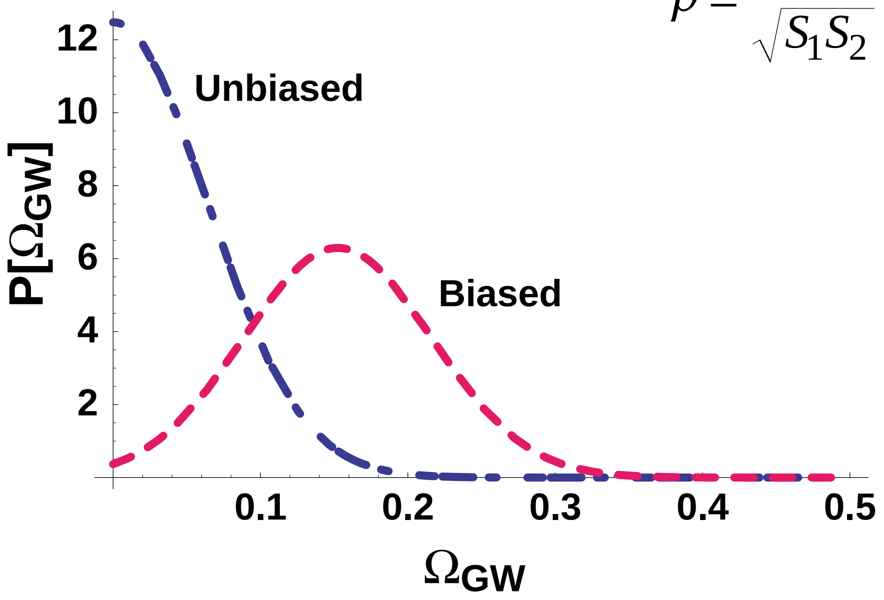
LLO - Allegro Detector Parameters

Quantity	Lower ALLEGRO resonance, $f_<$	Upper ALLEGRO resonance, $f_<$	Units
Frequency	896.8	920.3	Hz
Allegro Bandwidth, Δf	1		Hz
Upgraded Allegro Bandwidth, Δf	50		Hz
Sensitivities	**LIGO**	**Allegro**	**Units**
ALLEGRO, h[f]	1.8×10^{-21}	0.9×10^{-21}	1/√Hz
LLO, h[f]	1×10^{-22}		1/√Hz
LIGO II (narrowband), h[f]	2×10^{-24}		1/√Hz
Ω_{min} for LIGOI + ALLEGRO, T = 1yr	1×10^{-1}		---
Ω_{min} for LIGOII + Upgraded ALLEGRO, T = 1yr	3×10^{-4}		---

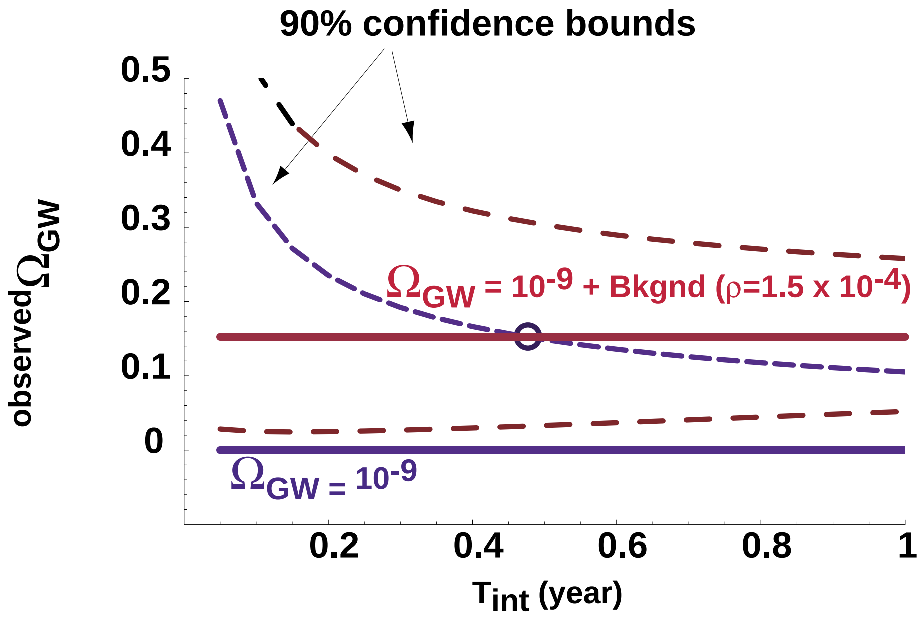
Likelihoods

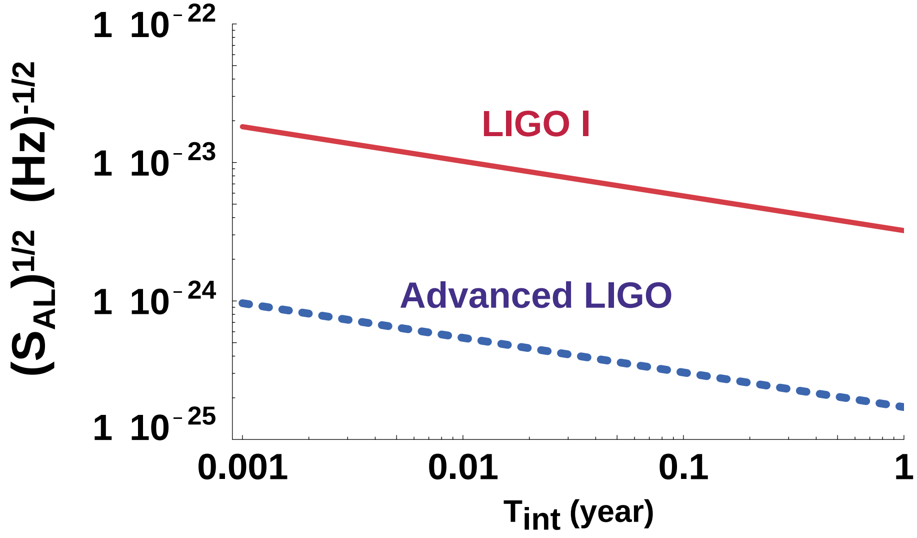
T_{int}=1 yr; ρ=1.5 x 10^{-4}

$$\rho \equiv \frac{S_{12}}{\sqrt{S_1 S_2}}$$



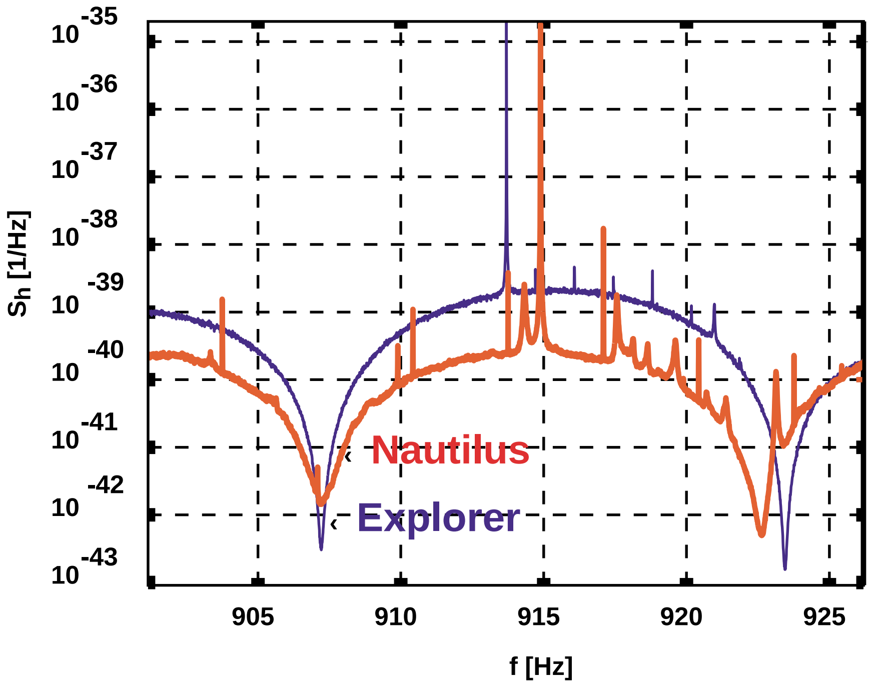
Effect of correlated background on observable upper limits for Ω_{GW}

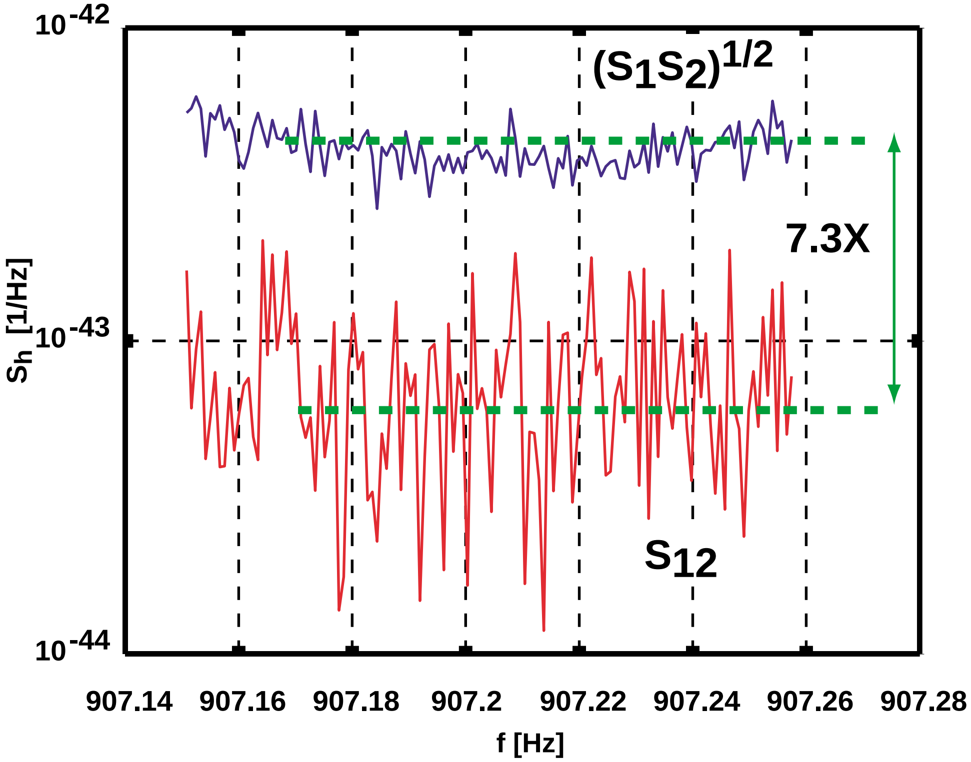




$$\rho \equiv \frac{S_{12}}{\sqrt{S_1 S_2}}$$

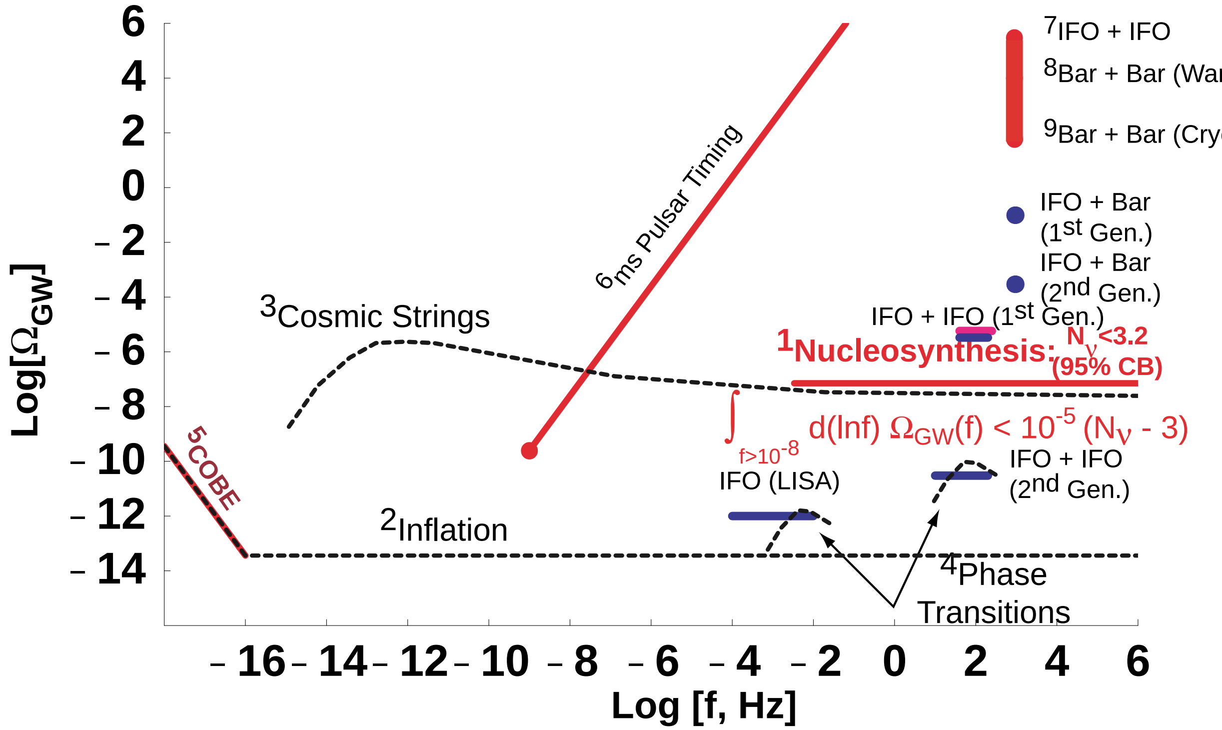
Effect of correlated background on observable upper limits for Ω_{GW}





Astone, et. al., Astr. Astroph. 351 (1999)

Measurements of the Stochastic Background



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[9] Astone, et. al., Astr. Astroph. 351 (1999)

Conclusions

Modulation of the Stochastic Background Correlation

- It is possible to account for correlated detector noise background in deriving the Optimal Wiener filter
- In the presence of correlated detector noise background, the upper limit will be a biased estimate:
  - ! Level of $\rho \sim 10^{-4}$ begins to limit measurement after less than 1 year
  - ! Rome group upper limit of $\Omega_{GW} < 60$ apparently limited by $\rho \sim 0.14$ over baseline L=600 km (Explorer - Nautilus)
- Planned move of ALLEGRO to new quarters is being used to modify bar to allow this measurement
  - » Modulation period >> detector settling time => *dead-time*
  - » Modulation period < total integration time => *multiple orientations*
  - » Choose period of ~ 3 - 5 months (not commensurate with seasonal/annual cycles)