Searching for Unmodeled Sources with LIGO

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LIGO Science Seminar

Jan. 25, 2000

LIGO-G000001-00-D

Outline

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   - supernovae
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   - others?
3. Methods for detecting unmodeled sources
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Part II: A New Method

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   - derivation of power filter
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5. Summary and Comments

Classification of Sources

Data analysis classification

Sources of gravitational radiation can be split into four classes for the purposes of detection methods.

Stochastic sources produce long-lasting (years) gravitational radiation which can only be characterized statistically.

- Examples: the big bang, unresolvable populations of compact sources.
- Detection Schemes: statistical correlation between multiple detectors, "maximum likelihood" statistical schemes.

Periodic sources produce long-lasting (days - years) gravitational radiation at a single frequency, which is Doppler modulated by the motion of the detector (e.g. Earth's orbital motion) or source.

- Examples: rotating neutron stars or other compact objects.
- Detection Schemes: Fourier transforms demodulated by sliding stacks or Hough transforms.

Modeled burst sources produce bursts (msec - minutes) of gravitational radiation in the detectable frequency band by a process for which there is an trusted theoretical model.

- Examples: inspiral of a compact binary, "ringing" black hole.
- Detection Schemes: "matched filtering" with banks of template waveforms.

Unmodeled burst sources produce bursts (msec - minutes) of gravitational radiation in the detectable frequency band by a process for which there is no trusted theoretical model.

- Examples: core collapse supernovae, black hole mergers.
- Detection Schemes: under development.

Unmodeled Sources

Definition

Unmodeled sources have the following properties:

- they emit gravitational radiation in the frequency band in which our detectors are sensitive.
- they are "burst" sources, i.e. the time for which the radiation is emitted in the detector band is small (sampling period $\lesssim$ duration $\lesssim$ minutes).
- no trusted theoretical waveforms exist for the radiation.

Supernovae (K. Thorne, gr-qc/9706079)

Core collapse of stars can lead to various mechanisms which generate gravitational radiation, including:

1. axisymmetric collapse (unmodeled)
   Centrifugally flattened core collapses to nuclear density, bounces back, and oscillates axisymmetrically, producing weak gravitational radiation.
   - signal duration $\sim$ 0.1 sec.
   - signal frequency $\sim$ 200 Hz - 1000 Hz.
   - abundance $\sim$ (100 years)$^{-1}$ with LIGO I.
2. collapse induced convection (unmodeled)
   Energy of collapse heats core and causes convective currents which produce gravitational radiation.
   - duration $\sim$ 1 sec.
   - frequency $\sim$ 100 Hz.
   - abundance $\sim$ (100 years)$^{-1}$ with LIGO I.
3. bar instabilities (unmodeled)
   Inherent instabilities in the nuclear fluid causes core to distort into a rotating bar which emits gravitational radiation.
   - duration $\lesssim$ 1000 sec.
   - frequency $\sim$ 10 Hz - 100 Hz.
   - abundance $\lesssim$ few/year with LIGO I?

Black Hole Mergers (Flanagan & Hughes, PRD**57**, 4535, (1998))

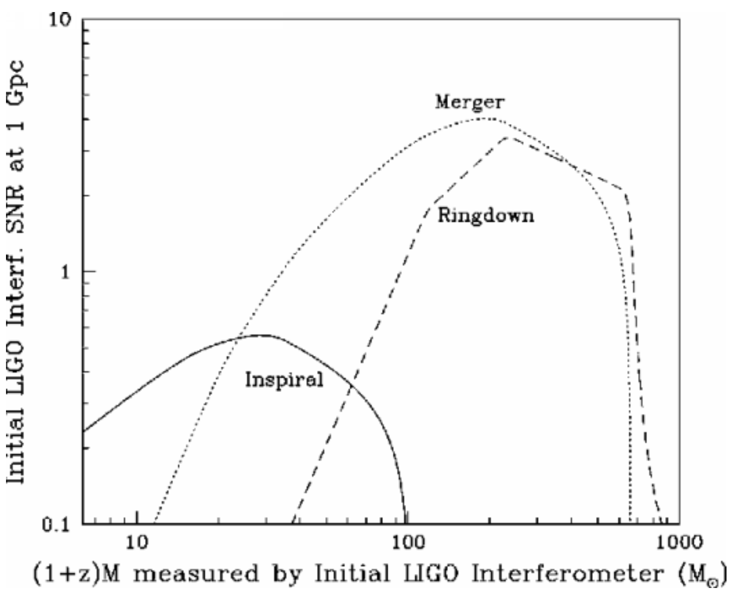
Coalescing black hole binaries undergo three phases of evolution:

1. inspiral (modeled)
   Two black holes slowly spiral inward as they orbit each other, producing gravitational radiation. Accurate waveforms are calculated theoretically using post-Newtonian expansions of the equations of motion.

2. merger (unmodeled)
   The orbit becomes unstable, the black holes plunge toward each other, and the event horizons merge, producing gravitational radiation. No waveform has been calculated. If M is the total mass of the system:

   - duration $\sim 2.5 \times 10^{-4} M/M_{\odot}$ sec.
   - frequency $\sim 4.1 \times 10^{3} M_{\odot}/M$ Hz – $2.6 \times 10^{4} M_{\odot}/M$ Hz.
   - abundance $\lesssim$ 5/year with LIGO I?

3. ringdown (modeled)
   Enveloped by a single event horizon, the newly merged black hole vibrates as it approaches its equilibrium configuration, producing gravitational radiation. Accurate waveforms are calculated using black hole perturbation theory.



Signal strengths for three phases of a binary black hole system
- Flanagan and Hughes.

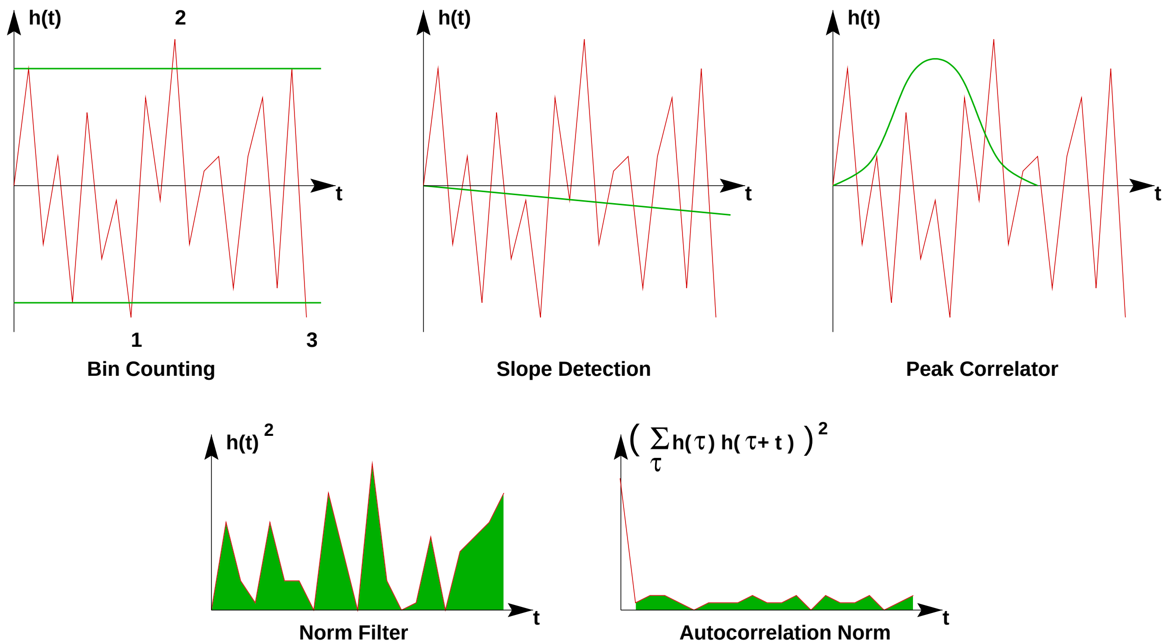
Other unmodeled sources?

" . . . when gravitational waves are finally seen, they will come predominantly from sources we have not thought of or we have underestimated." (Thorne, *300 Years of Gravitation*, 1987)

Methods for Detecting Unmodeled Sources

Ideas from Orsay (Arnaud et al, gr-qc/9903035)

The VIRGO group in Orsay is comparing several methods.

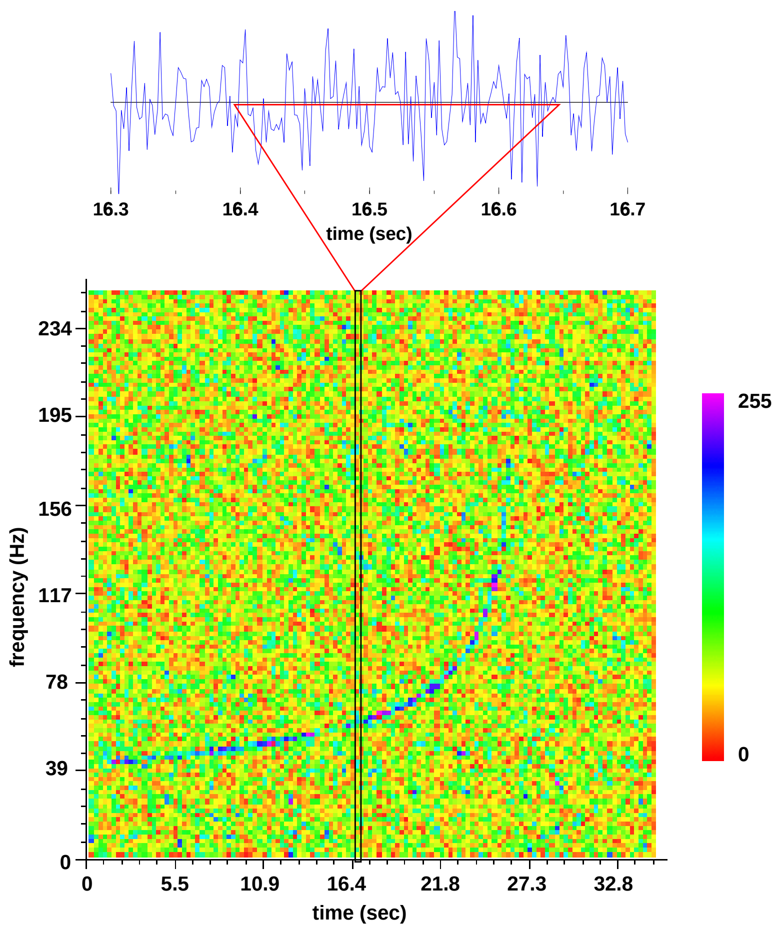


Student's t-test method (Mohanty, gr-qc/9910027)

- looks for non-stationarity in data by comparing mean power at each frequency in sections of data separated by a fixed time.
- if means are significantly different (Student's t-test) then flag sections.
- proposed for robust detector characterization.

Time-frequency method (WGA & Balasubramanian, PRD60, 102001, (1999))

1. Produce a time-frequency representation of data, e.g. spectrogram
   (a) divide data segment into subsegments $(0,\delta t)$, $(\delta t,2\delta t)$, $\ldots$
   (b) Fourier transform each subsegment
   (c) calculate power in each frequency bin of each subsegment
   (d) plot graph of power as a function of time subsegment and frequency bin.
2. Find curves using methods developed for image analysis, e.g. for finding roads in satellite images.
3. Use curve vetoing and an appropriate statistical threshold to sort signal curves from noise.



$1.4M_\odot - 1.4M_\odot$ binary inspiral signal at SNR 30 in white Gaussian noise.

The Excess Power Method

Theory of Optimal Filters

Because interferometer noise is stochastic, detecting signals is a statistical process. Start with some useful definitions:

$\mathbf{h} = \mathbf{n} + \mathbf{s}$: a vector of N consecutive interferometer data samples.

$\mathbf{n}$: a detector noise vector (usually stochastic).

$\mathbf{s}$: a signal vector (possibly $\mathbf{0}$).

A Filter: an algorithm whose input is detector data and whose output is a list of signal candidates.

False Alarm Probability: probability that the filter detects a signal when no signal is present. We want this to be low.

False Dismissal Probability: probability that a filter does not detect a signal when one is present. We want this to be low also.

An Optimal Filter: filter that has the lowest false dismissal probability for any given false alarm probability. Depends on the set of signals.

The Likelihood Ratio:

$$\Lambda[\mathbf{h}] \equiv \int \mathcal{D}[\mathbf{s}] \, \frac{p[\mathbf{h}|\mathbf{s}]}{p[\mathbf{h}|\mathbf{0}]}$$

$p[\mathbf{h}|\mathbf{s}]$: probability of obtaining $\mathbf{h}$ if signal $\mathbf{s}$ is present.
$p[\mathbf{h}|\mathbf{0}]$: probability of obtaining $\mathbf{h}$ if no signal is present.
$\mathcal{D}[\mathbf{s}]$: measure over space of signals.

Question: can one determine the optimal filter for a given set of signals?

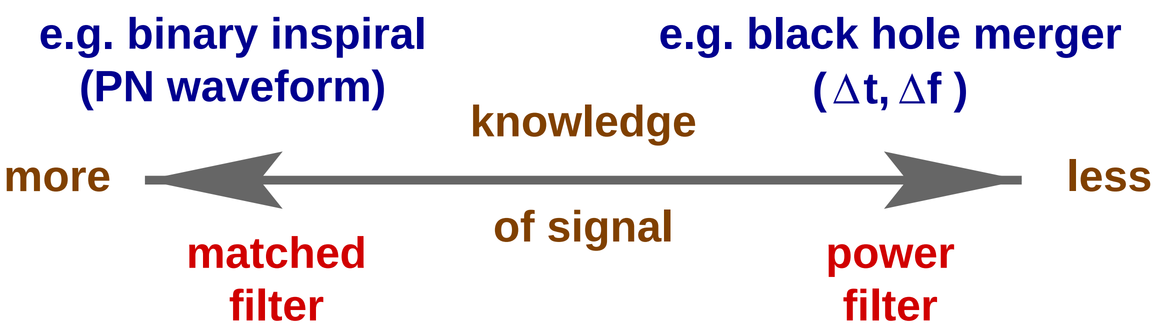
Answer (Neyman-Pearson Lemma): the optimal filter is a threshold decision rule for the likelihood function, i.e.

1. calculate the likelihood ratio $\Lambda[\mathbf{h}]$ for the detector output $\mathbf{h}$.
2. compare to a threshold value determined by setting the false alarm probability.
3. if $\Lambda[\mathbf{h}]$ exceeds the threshold value, signal is detected.

The Power Filter (WGA, P. Brady, J. Creighton, É. Flanagan)

Choosing a signal

choice of signal ⟶ degree of prior knowledge about the signal ⟶ what the optimal filter for that signal is.

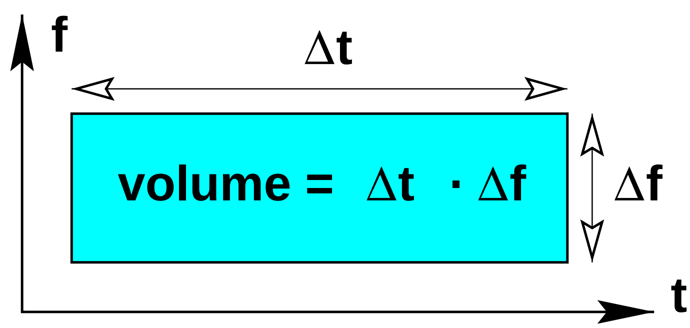


Black hole mergers

for black hole mergers, Flanagan and Hughes estimate duration Δt and frequency band Δf of signal.

Question

What is the optimal filter when the prior knowledge is only the time duration and frequency band of the signal?



Answer

Construct the likelihood ratio.

Integrand: assume noise ($\mathbf{n}$) is Gaussian and stationary:

$$p[\mathbf{n}] = C \exp\left[-\frac{\mathbf{n}\cdot\mathbf{n}}{2}\right]$$

where

C is a constant

$\mathbf{n}\cdot\mathbf{n}$ is an inner product

both of which depend on the autocorrelation matrix of the noise.

Integrand (cont.)

If no signal is present

$$\mathbf{n} = \mathbf{h} \longrightarrow p[\mathbf{h} \mid \mathbf{0}] = C \exp\left[-\frac{\mathbf{h}\cdot\mathbf{h}}{2}\right].$$

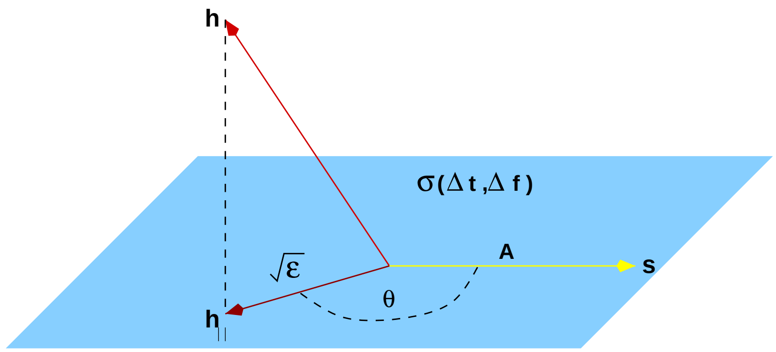
If a signal $\mathbf{s}$ is present,

$$\mathbf{n} = \mathbf{h} - \mathbf{s} \longrightarrow p[\mathbf{h} \mid \mathbf{s}] = C \exp\left[-\frac{(\mathbf{h}-\mathbf{s})\cdot(\mathbf{h}-\mathbf{s})}{2}\right].$$

Thus the likelihood ratio is

$$\Lambda[\mathbf{h}] = \int \mathcal{D}[\mathbf{s}]\ \exp\left[\mathbf{h}\cdot\mathbf{s} - \frac{\mathbf{s}\cdot\mathbf{s}}{2}\right].$$

Measure: reflects our prior knowledge of the signal. Signal has duration Δt and frequency band Δf.



This restricts $\mathbf{h}\cdot\mathbf{s}$ to the subspace $\sigma(\Delta t, \Delta f)$ of vectors with this duration and frequency band.

Every $\mathbf{s}$ in σ is equally likely, so the measure over θ is uniform over an $(2V-1)$-sphere, where $2V = 2\Delta t \times \Delta f$ is the dimensionality of σ. Now the likelihood ratio is

$$\Lambda[\mathbf{h}] = \int \mathcal{D}[A]\ e^{-A^2/2} \int_0^{\pi} d\theta\ \sin^{2V-2}\theta\ \exp\left[A\sqrt{\mathcal{E}}\cos\theta\right].$$

Shortcut: $\Lambda[\mathbf{h}]$ increases monotonically with $\mathcal{E}$ which implies that for every $\Lambda^\star$ there is a $\mathcal{E}^\star$ such that $\Lambda[\mathbf{h}] > \Lambda^\star \longrightarrow \mathcal{E} > \mathcal{E}^\star$.

Optimal Filter: threshold on

$$\boxed{\mathcal{E} = \mathbf{h}_{\|}\cdot\mathbf{h}_{\|} = \sum_{\Delta f} |\tilde{h}_k|^2/S_k}$$

$\tilde{h}_k$ frequency domain representation data vector.

S_k is the interferometer noise spectrum.

The optimal filter is a threshold on the total power of the interferometer data for the duration and in the frequency band of the signal.

Operating Characteristics

Use known statistical properties of Gaussian variables to get false alarm and false dismissal probabilities:

False Alarm: if there is no signal, then $\mathcal{E}$ is a sum of V random variables. It therefore has a χ^2 distribution with V degrees of freedom. False alarm probability for threshold $\mathcal{E}^\star$ is

$$P(\mathcal{E} > \mathcal{E}^\star \mid A = 0) = \frac{\Gamma(V, \mathcal{E}^\star/2)}{\Gamma(V)}$$

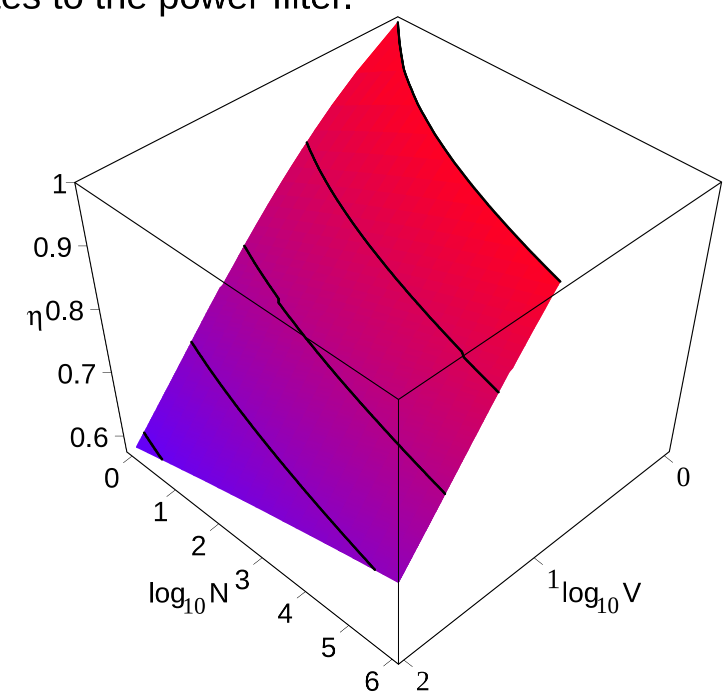
where $\Gamma(a, x)$ is incomplete Gamma function.

False Dismissal: if a signal of amplitude A is present, then $\mathcal{E}$ is distributed as a non-central χ^2 distribution with V degrees of freedom. False dismissal probabilities can be easily calculated numerically.

Comparison to Matched Filters

The matched filter is the optimal filter for a signal where the prior knowledge is the waveform. It is instructive to compare the effectiveness of the power filter. For a given time duration and frequency band we consider a bank of templates to search for N signals of that duration and band. We compare the performance of that bank of templates to the power filter.

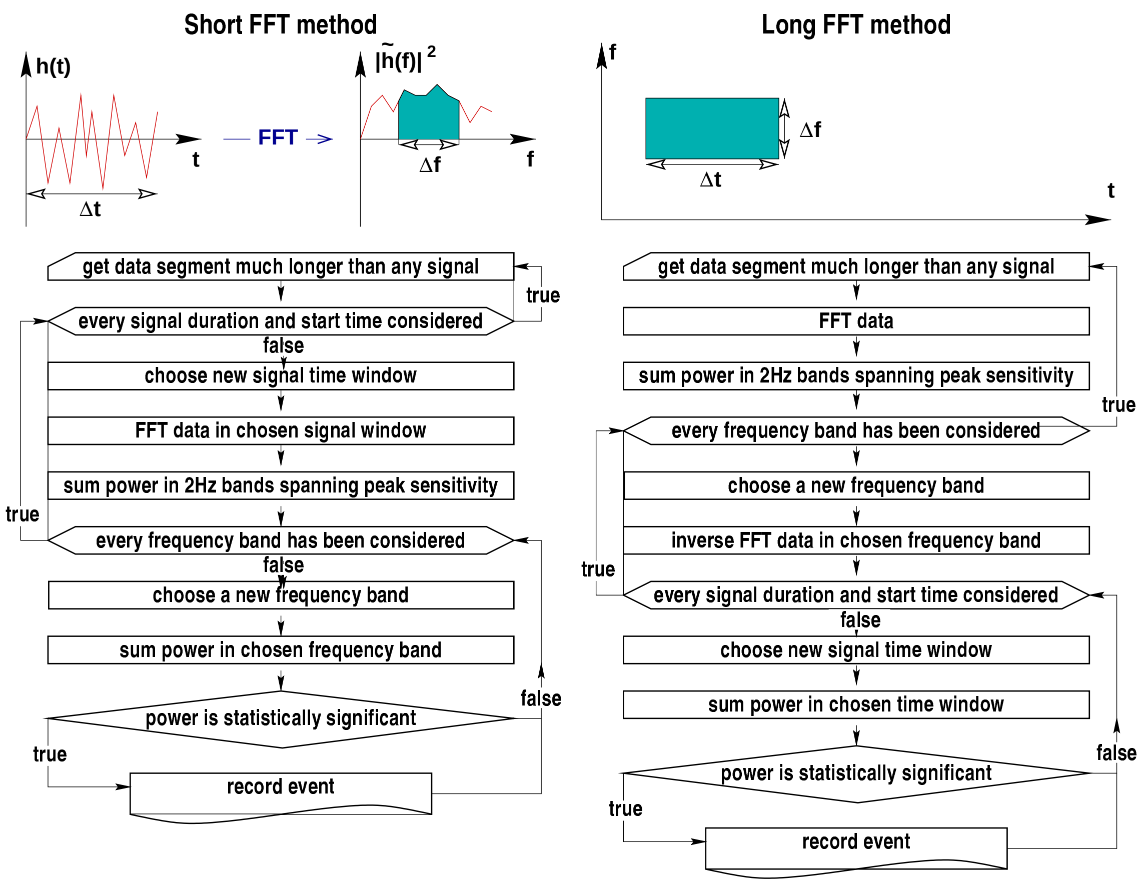
For fixed false alarm and false dismissal probabiliies, we obtain the required signal amplitude A for the power filter, and likewise for a bank of matched filters. η is the ratio of these amplitudes.



If the number of filters in the bank is large and the time-frequency volume is small, the power filter is almost as effective as a bank of matched filters.

Algorithms and Implementation

We have investigated two implementations:



For frequency bands of $2Hz \leq \Delta f \leq 200Hz$ and signal durations of $0.005s \leq \Delta t \leq 0.5s$ and sampling rate of $1kHz$ second method is ~ 10 faster.

Multiple Detectors

An optimal multi-detector statistic can be derived in the same way.

$$\mathcal{E}_2(\theta, \psi, \phi) = \sum_{\Delta f} \frac{|A_1 \tilde{h}_k^{(1)}(t_1) + A_2 \tilde{h}_k^{(2)}(t_2)|^2}{A_1 S_k^{(1)} + A_2 S_k^{(2)}}$$

- $(\theta, \phi, \psi) \rightarrow$ sky position of source.
- $A_n \rightarrow$ detector weightings [functions of (θ, ϕ, ψ)].
- $t_n \rightarrow$ arrival times [functions of (θ, ϕ)].

Note that this statistic contains both cross and autocorellations.

Summary

- unmodeled sources require new methods to detect.
- a number of methods for unmodeled sources are under investigation.
- a new method for detecting black hole mergers, the power filter, will be available soon.
- the power filter can perform almost as well as a matched filter.
- efficient implementations of the power filter are available.
- multiple detector versions of the power filter are available.

Comments

- while this derivation of the power filter used Gaussian noise, it can be shown that this filter is optimal for weak signals in other types of noise.
- generalized power filters have recently been "attracting considerable interest" in the signal processing community (Streit and Willis. IEEE Trans. Signal Processing **47**, 1823 (1999)).
- Brady has written preliminary code to implement the long FFT algorithm. Flanagan will lead the effort to produce a LAL implementation.

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Note 1, Linda Turner, 01/28/00 10:19:55 AM
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Note 1, Linda Turner, 01/28/00 10:20:42 AM
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