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HAM and BSC Dynamic Models Users Manual

Eric Ponslet
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Abstract

This note provides the necessary information to use the numerical models of the BSC and HAM isolation systems. The models are provided as a set of MATLAB data files and functions that can be used to perform simulations of those systems with frequency dependent properties for the isolation springs.

DRAFT

Design Engineering
Advanced Composite Applications
Ultra-Stable Platforms

110 Eastgate Dr., Ste. 100
Los Alamos, NM 87544

Phone 505 661 3000
Fax 505 662 5179
www.hytecinc.com

LIGO PROJECT

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1. General Notes about Modeling Techniques and Assumptions

All dynamic models use lumped parameter simplifications: the systems are discretized into a small number of mass elements (point masses with rotational inertias) and a number of 3D spring elements, with complex stiffnesses in 3 mutually orthogonal directions. Note that this lumped modeling is very realistic for the stacks themselves (very stiff mass element and very soft springs), but much less realistic for the support systems (continuous elastic structures).

The coil spring stiffnesses and damping are frequency dependent quantities Note that the effect of this dependence on the overall system dynamics is small: models generated with spring properties at a single, fixed frequency near the stack resonances are reasonable approximations of the real systems.

The mass and stiffness matrices provided define the system with 6 degrees of freedom per mass element, aligned along the local BSC or HAM reference frame and defined as follows:

- u : translation of the center of mass along U, the beam direction.
- v : translation of the center of mass along V, orthogonal to the beam direction in the local horizontal plane.
- w : translation of the center of mass along W, the local vertical, positive upwards.
- α : rotation around U, right hand rule.
- β : rotation around V, right hand rule
- γ : rotation around W, right hand rule

The d.o.f.'s are ordered as shown above from mass element 1 up. The "ground" is always mass element number 1; it is constrained in 6 d.o.f. This implies that the $(u,v,w,\alpha,\beta,\gamma)$ d.o.f.'s of mass element number i are contained in rows and columns number $6(i\text{-}2)+(1,2,3,4,5,6)$.

All models are expressed in the SI system (kg, m, sec, N). All LIGO models use the complex stiffness approach for modeling damping. There is no viscous term in the dynamic equations, and therefore no viscous damping matrix. The damping is implicit as the imaginary part of the stiffness matrix. The system dynamic equation can be written as:

$$M\ddot{x} + K(f)x = F + K_f(f)x_f, \qquad (1)$$

where M and K are the mass and stiffness matrix for the free d.o.f.'s, x is the vector of free d.o.f.'s, F is a vector of input forces applied on the free d.o.f.'s, x_f is a vector of forced displacements of the fixed degrees of freedom (the 6 d.o.f.'s of the ground or the support platform in the LIGO SEI and SIS models, respectively), and K_f is the stiffness matrix for those fixed degrees of freedom. Both K and K_f are frequency dependent matrices (see section 2).

Finally, note that the cylinders used to represent the mass elements in the figures below are only visual tools; their dimensions and shape are not taken into account in the modeling. The green dots in the figures represent spring elements.

2. Specifics of LIGO models

To allow the user to perform simulations with frequency dependent properties for the isolations springs, the stiffness matrices have been split into fixed terms K_0, and $K_{f,0}$ (support structures) and direction cosine matrices K_{sh}, $K_{f,sh}$ and K_{ax}, $K_{f,sh}$ for the isolation springs, in the shear and axial directions, respectively. The full, frequency dependent system stiffness matrices $K(f)$ and $K_f(f)$ can be rebuilt for any given frequency f as

$$K(f) = K_0 + k_{sh}(f)K_{sh} + k_{ax}(f)K_{ax}\,, \tag{2}$$

$$K_f(f) = K_{f,0} + k_{sh}(f)K_{f,sh} + k_{ax}(f)K_{f,ax}\,, \tag{3}$$

where $k_{sh}(f)$ and $k_{ax}(f)$ are the complex stiffnesses of an isolation spring, in the shear and axial directions, respectively, at a frequency f (Hz).

Note that for the SIS models, which only include the isolation stacks, the fixed portions of the stiffness matrices, K_0 and $K_{f,0}$ contains only zeros; they are included in the files anyway to avoid confusion and allow the use of standard processing routines.

A separate routine is provided that evaluates the shear and axial stiffnesses of the LIGO coil spring at any given frequency(ies). The routine interpolates data points that are a mix of experimental results and analytical estimates.

The mass matrix M is not a function of frequency; it incorporates a nominal payload of 500 lb, uniformly spread on the optics surface. The LIGO specifications call for dead masses to be used in the payload to maintain it to 500 lb (which is necessary to obtain full isolation performance from the stack) and keep the payload center of mass at the geometric center of the optics platform. Small changes in the moments of inertia of the payload are possible due to changes in the exact locations of the various elements. Such changes can be modeled by adding a correction term to the mass matrix as

$$M(f) = M_0 + \overline{M}_p^T\left(m_p - m_{0,p}\right)\overline{M}_p\,, \tag{4}$$

where M_0 is the default mass matrix, including a uniformly distributed payload of 500 lb, $m_{0,p}$ is the 6×6 tensor of mass moments of inertia of the default payload at the center of the optics mounting surface, m_p is the 6×6 inertia tensor of the actual payload at the same point, and $\overline{M}_p$ is a 6×*ndof* projection matrix. Note that, unless the payload is know to substantially deviate from essentially equivalent to a uniform distribution of mass across the optics platform, the effect of this correction is small and can usually be neglected.

3. Identification and Content of the Files Provided

Six (6) files are provided:

- `Model Description.pdf`: this manual, in Adobe Acrobat format.
- `BSC_SIS.mat`: Matlab data file, BSC isolation system, stack only.
- `BSC_SEI.mat`: Matlab data file, BSC isolation system, floor-up, includes support systems.
- `HAM_SIS.mat`: Matlab data file, HAM isolation system, stack only.
- `HAM_SEI.mat`: Matlab data file, HAM isolation system, floor-up, includes support systems.

- `coil.m`: Matlab data file, Matlab function, LIGO coil spring properties.

Each of the four ".MAT" files provided contains the following data:

- `n_dof` : number of degrees of freedom, system size.
- `M_o` : square matrix [ndof×ndof] of real numbers, default mass matrix for a uniform payload.
- `M_p` : projection matrix for payload inertia tensor.
- `K_o` : square matrix [ndof×ndof] of complex numbers, Stiffness matrix, fixed portion (support structures).
- `K_sh` : square matrix [ndof×ndof] of real numbers, direction cosines for shear stiffness of isolation springs.
- `K_ax` : square matrix [ndof×ndof] of real numbers, direction cosines for axial stiffness of isolation springs.
- `mass_dof` : array of integers; this array has one line for each mass element; each line contains 6 integers for the d.o.f. numbers for $u,v,w,\alpha,\beta,\gamma$ for that mass element. A zero indicates a fixed degree of freedom (i.e. not included on the left hand side of the dynamic equation (x) but on the right hand side (x_f).
- `Kf_o` : rectangular matrix of complex numbers, stiffness matrix for fixed d.o.f.s, fixed portion
- `Kf_sh` : rectangular matrix of real numbers, direction cosines for shear stiffness of isolation springs.
- `Kf_ax` : rectangular matrix of real numbers, direction cosines for axial stiffness of isolation springs.
- `readme` : information about model, creation date, and version.

The file "coil.m" is a Matlab function that returns shear and axial properties of a LIGO coil spring at a given frequency; type '`help coil`' at the Matlab prompt for more information.

4. BSC Models

4.1 BSC_SIS: Stack Only

This model includes the stack only (leg elements and downtube). The stack is sitting on element #1 (the "floor"), representing the top surface of the support platform. The stack elements are numbered from bottom to top within each leg, starting with the [U+, V+] leg and going around in a right hand rule rotation. The last element in the model is the downtube (#14). The optics table surface is at W= - 0.4811 meter from the element 14 (the center of mass of the downtube).

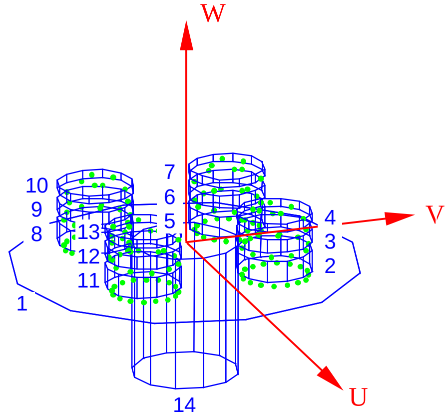

Figure 1: BSC_SIS model configuration showing mass element numbers.

4.2 BSC_SEI: Stack and Support System

This model includes the stack and a 2-body (12 d.o.f) *approximation* of the support system (piers, actuators, cross beams, support beams, and support platform). The floor is element #1; element #2 loosely represents the piers and cross beams, and element #3 loosely represents the support beams and support platform (note that there is really no exact physical equivalent of the separation between elements 2 and 3 since their definition is based on effective mass principles for the lower modes of the support system).

The stack is now sitting on element #3. The stack elements are numbered as before from bottom to top within each leg, starting with the [U+, V+] leg and going around in a right hand rule rotation. The last element in the model is the downtube (#14). The optics table surface is at W= - 0.4811 meter from the element 16 (the center of mass of the downtube).

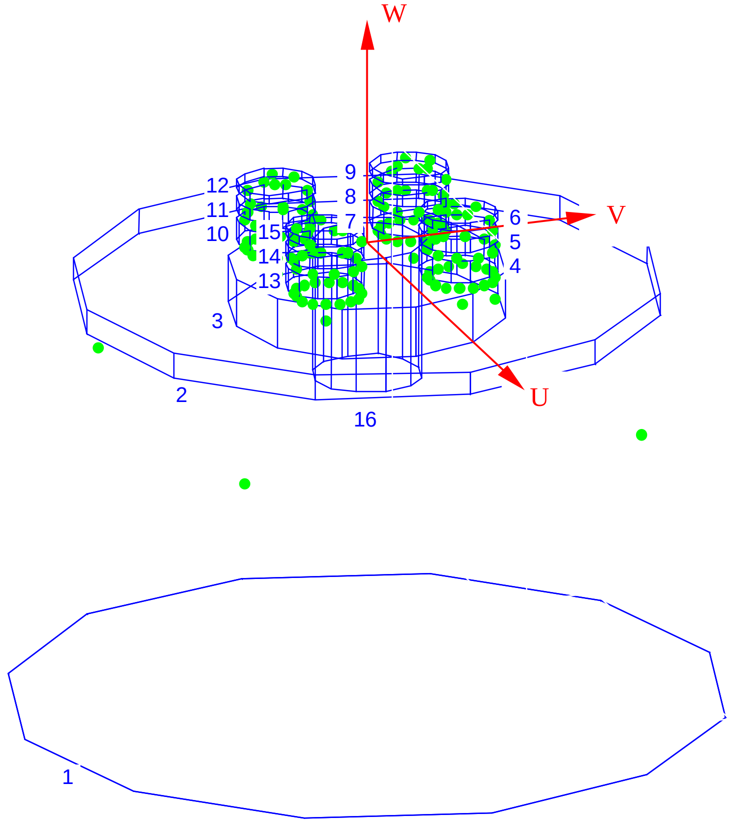

Figure 2: BSC_SEI model configuration showing mass element numbers.

5. HAM Models

5.1 HAM_SIS: Stack Only

This model includes the stack only (leg elements and optics table). The stack is sitting on element #1 (the "floor"), representing the top surface of the support platform. The stack elements are numbered from bottom to top within each leg, starting with the [U+, V+] leg and going around in a right hand rule rotation. The last element in the model is the optics table (#10). The optics table surface is at W= + 0.062 meter from the element 10 (the center of mass of the downtube).

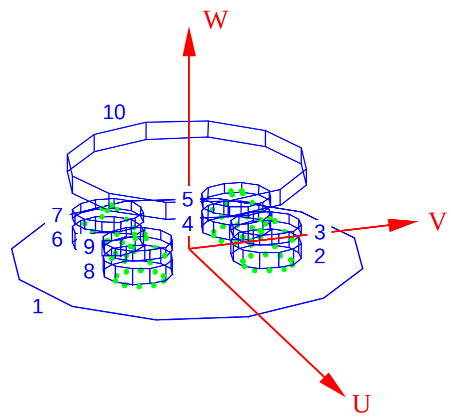

Figure 3: HAM_SIS model configuration showing mass element numbers

5.2 HAM_SEI: Stack and Support System

This model includes the stack and a 2-body (12 d.o.f) *approximation* of the support system (piers, actuators, cross beams, support beams, and support platform). The floor is element #1; element #2 loosely represents the piers and cross beams, and element #3 loosely represents the support beams and support platform (note that there is really no exact physical equivalent of the separation between elements 2 and 3 since their definition is based on effective mass principles for the lower modes of the support system).

The stack is now sitting on element #3. The stack elements are numbered as before from bottom to top within each leg, starting with the [U+, V+] leg and going around in a right hand rule rotation. The last element in the model is the optics table (#12). The optics table surface is at W= + 0.062 meter from the element 12 (the center of mass of the downtube).

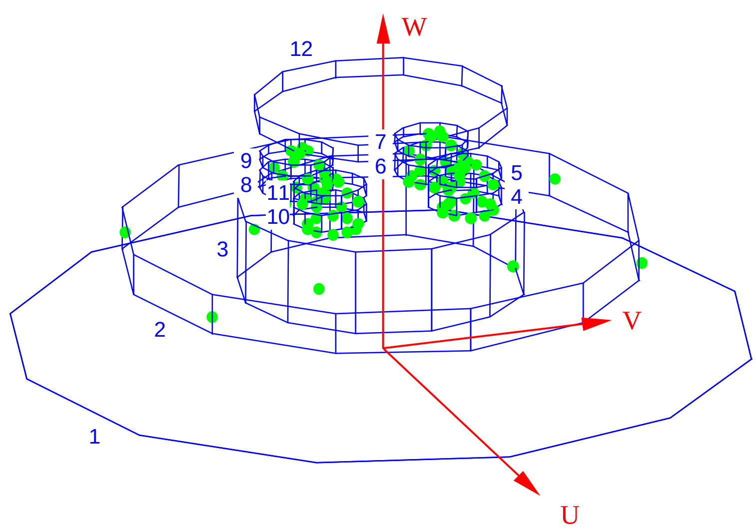

Figure 4: HAM_SEI model configuration showing mass element numbers.