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Digital Filter Representation of a Pockels Cell

R. Savage and H. Yamamoto

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LIGO Hanford Observatory
P. O. Box 1970; Mail Stop S9-02
Richland, WA 99352
Phone (509) 372-8106
Fax (509) 372-8137
E-mail: info@ligo.caltech.edu

LIGO Livingston Observatory
19100 LIGO Lane
Livingston, LA 70754
Phone (225) 686-3100
Fax (225) 686-7189
E-mail: info@ligo.caltech.edu

California Institute of Technology
LIGO Project - MS 18-34
Pasadena CA 91125
Phone (626) 395-2129
Fax (626) 304-9834
E-mail: info@ligo.caltech.edu

Massachusetts Institute of Technology
LIGO Project - MS 20B-145
Cambridge, MA 01239
Phone (617) 253-4824
Fax (617) 253-7014
E-mail: info@ligo.mit.edu

WWW: http://www.ligo.caltech.edu/

1 ABSTRACT

Pockels cells are often used as frequency actuators for laser frequency stabilization. Because the design of frequency stabilization feedback control loops often utilizes computer modeling tools such as *Matlab* or the *LIGO End-to-End Model*, the digital filter, or s-space, representation of the Pockels cell action is required. The intent of this note is to resolve confusion over the proper representation of the Pockels Cell as a frequency actuator.

2 KEYWORDS

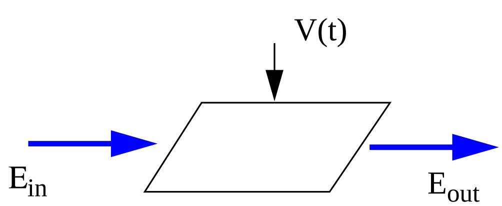
Pockels cell, digital filter, LIGO End-to-End Model, Matlab, Sumulink

3 DEFINITION OF WORDS

Pockels Cell - an optical element that employs the electrooptic effect to change the phase of an optical wave.

Digital filter - Laplace domain or s-space, representation of elements in, for example, feedback control loops.

4 FREQUENCY SHIFT INDUCED BY A VOLTAGE APPLIED TO A POCKELS CELL



When an electromagnetic wave propagates through a Pockels cell with an applied voltage V(t), the phase of the electric field is modified by A·V(t), where A is in radians per volt (0.015 rad/V in the case of a NewFocus PC, for example). Thus,

$$\begin{aligned} E_{out}(t) &= E_{in}(t) \cdot e^{iA \cdot V(t)} \\ &= \overline{E_{in}} \cdot e^{i(2\pi f_0 t + A \cdot V(t))} \end{aligned} \qquad (1)$$

where f_0 is the frequency of the input field and $\overline{E_{in}}$ is the magnitude of the input field.

The instantaneous frequency of the output field at time t is calculated by taking the time derivative of the output field phase and by dividing by 2π to give the frequency in Hz, rather than the angular frequency in rad/sec.

$$\begin{aligned} f(t) &= \frac{1}{2\pi} \cdot \frac{d}{dt}(2\pi f_0 t + A \cdot V(t)) \\ &= f_0 + A \cdot \frac{1}{2\pi} \cdot \frac{d}{dt} V(t) \end{aligned} \tag{2}$$

Thus the frequency shift induced by the applied voltage is given by

$$A \cdot \frac{1}{2\pi} \cdot \frac{d}{dt} V(t) \ . \tag{3}$$

4.1. Example: sinusoidal modulation

Consider a sinusoidally varying applied voltage,

$$V(t) = V_0 \cdot \sin(2\pi f t) \tag{4}$$

The frequency shift induced by this voltage is

$$A \cdot f \cdot \cos(2\pi f t) \tag{5}$$

i.e., an oscillating frequency shift, or, generation of sidebands at frequencies given by f_0 n·fwhere n is an integer.

4.2. Example: linear ramp

Consider a ramp, a linear change of the voltage with time,

$$V(t) = B \cdot t \tag{6}$$

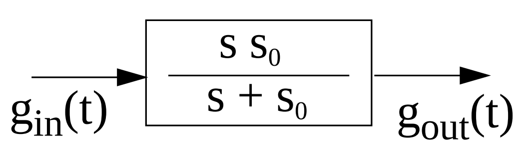
The frequency change caused by this applied voltage is

$$A \cdot \frac{1}{2\pi} \cdot B \tag{7}$$

i.e., a constant frequency shift results from an applied voltage ramp.

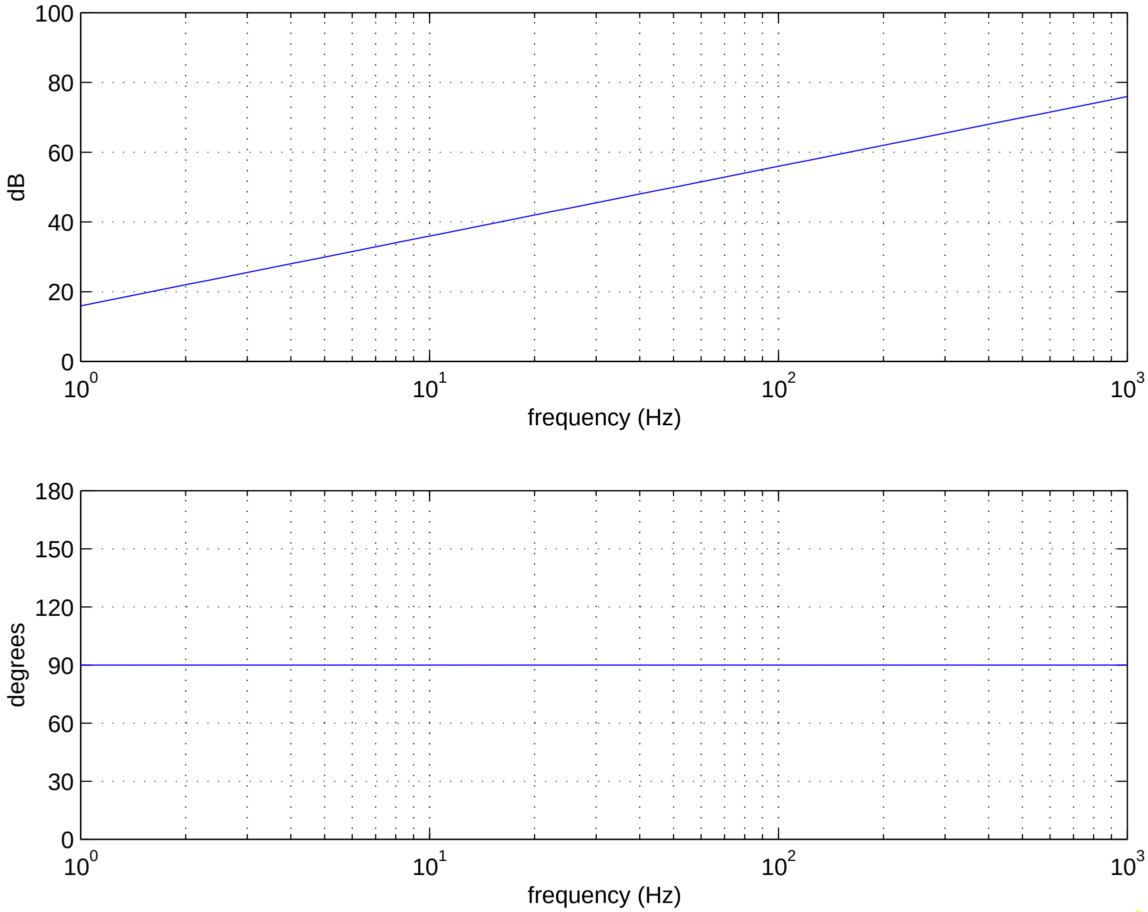
5 DIGITAL FILTER REPRESENTATION OF A POCKELS CELL

A digital filter with a single zero at DC behaves as an differentiator. For stability, a pole at a very high frequency, a regulator, is typically required. Schematically, the zero at DC and the regulator pole are represented as



where the regulator pole is at angular frequency s_0. This s-space representation is equivalent to $g_{out}(t) = d\, g_{in}(t) / dt$.

The following Bode plot, generated by Matlab was generated by a Simulink model of a zero at DC with a regulator pole with s_0=1e8, as shown above. The magnitude is $2\pi f$ and the phase shift is $\pi/2$. This is thus shown to be equivalent to the time derivative operation in the time domain.



Therefore, the digital filter representation of the phase modulation effect of a Pockels cell driven by a time dependent voltage is $A/2\pi \cdot DF[s]$, where DF[s], the digital filter, is a zero at DC and a regulator pole at high frequency, if required.