Statistic for Combination of Results from Multiple Gravitational-Wave Searches

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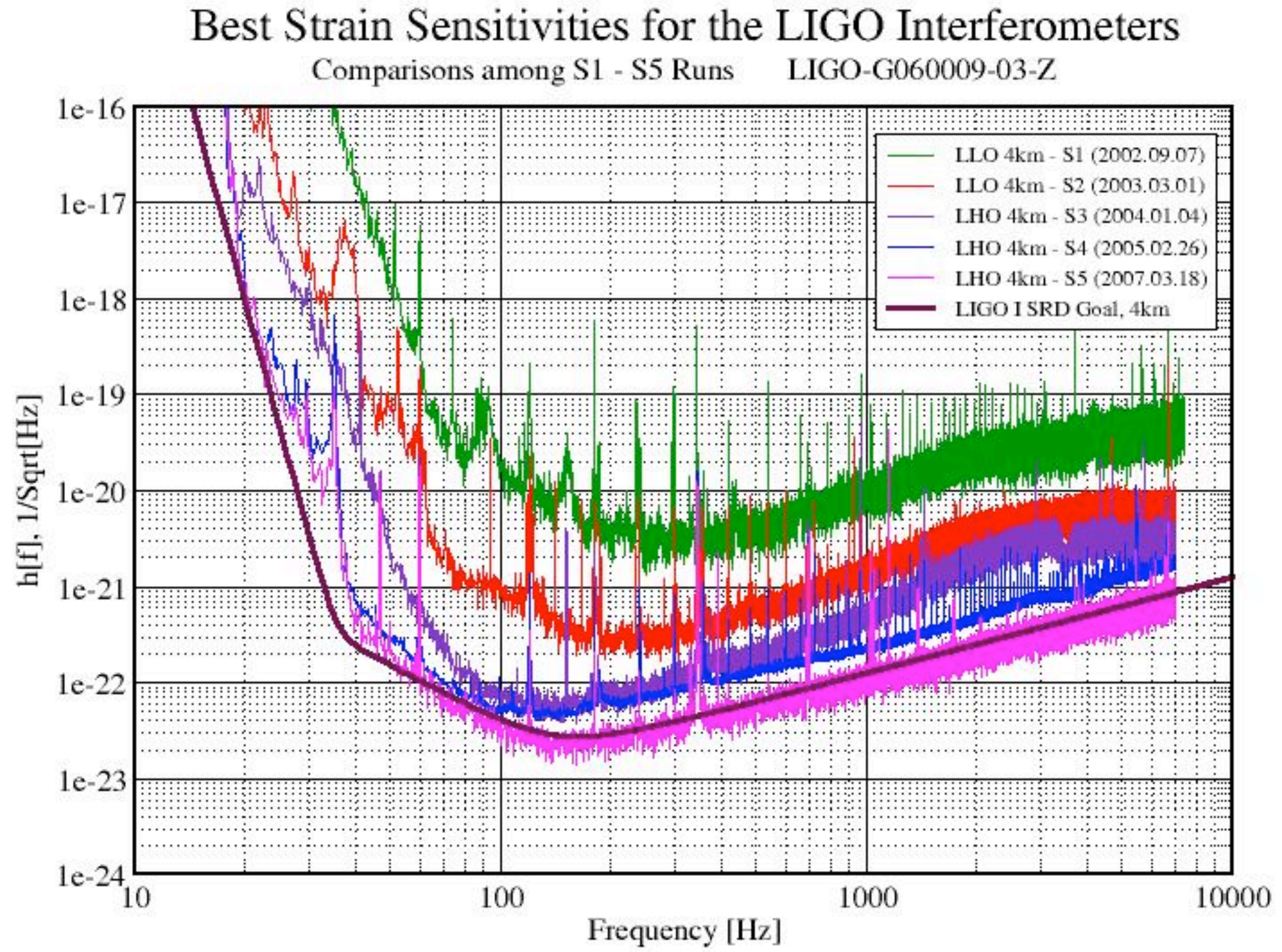
Combining Multiple Observations

- First generation GW interferometers have collected ~1.5 years of observation time spanning several data epochs with different detector configurations, sensitivities, and noise properties
- Putative signals buried deep in the noise: how do we determine the significance of candidate events?
- Expected 1G detection rates are low: given a source, how do we combine results of different searches into a single measurement to optimally utilize the observation time?



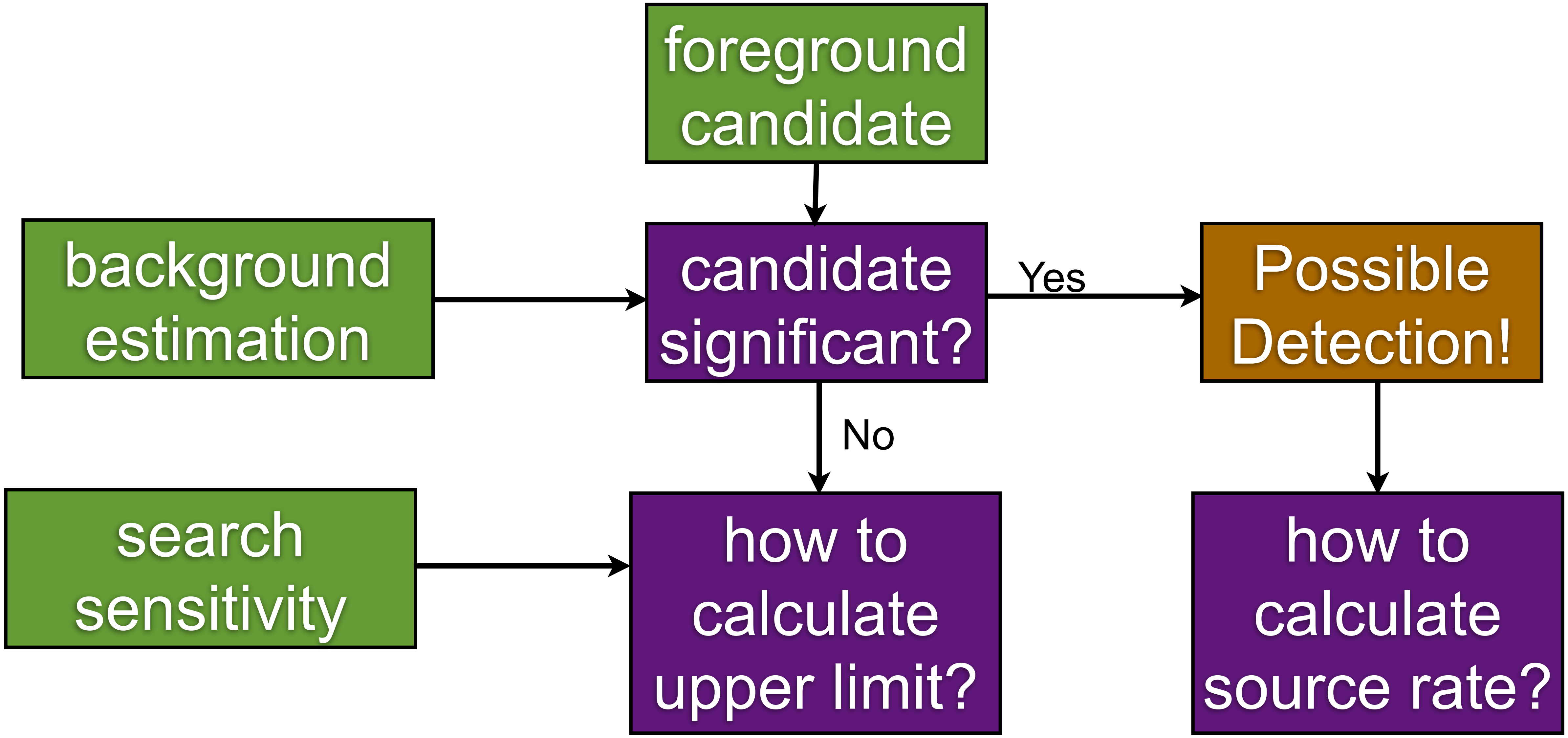
http://www.ligo.caltech.edu/~jzweizig/distribution/LSC_Data/

Definitions

- *Search* — all-sky analysis of GW observation data for a specified **transient** source, e.g.:
  - S4/S5/S6 Burst all-sky search
  - S4/S5/S6 CBC low mass search, S5/S6 high mass search, S5 IMBH search
- *Search Sensitivity Volume* — an effective volume of space in which a search detects candidate events (also called visible volume)
- *False Alarm Rate* (FAR) — the expected rate of triggers from the background
- *False Alarm Probability* (FAP) — the probability that the foreground candidate is produced by the background

Search Decisions

- Any GW search with a candidate event(s) must be able to answer the following questions:



False Alarm Rate Statistic

- How are these issues currently addressed in single searches?

- Small detection rates → Poisson counting experiments

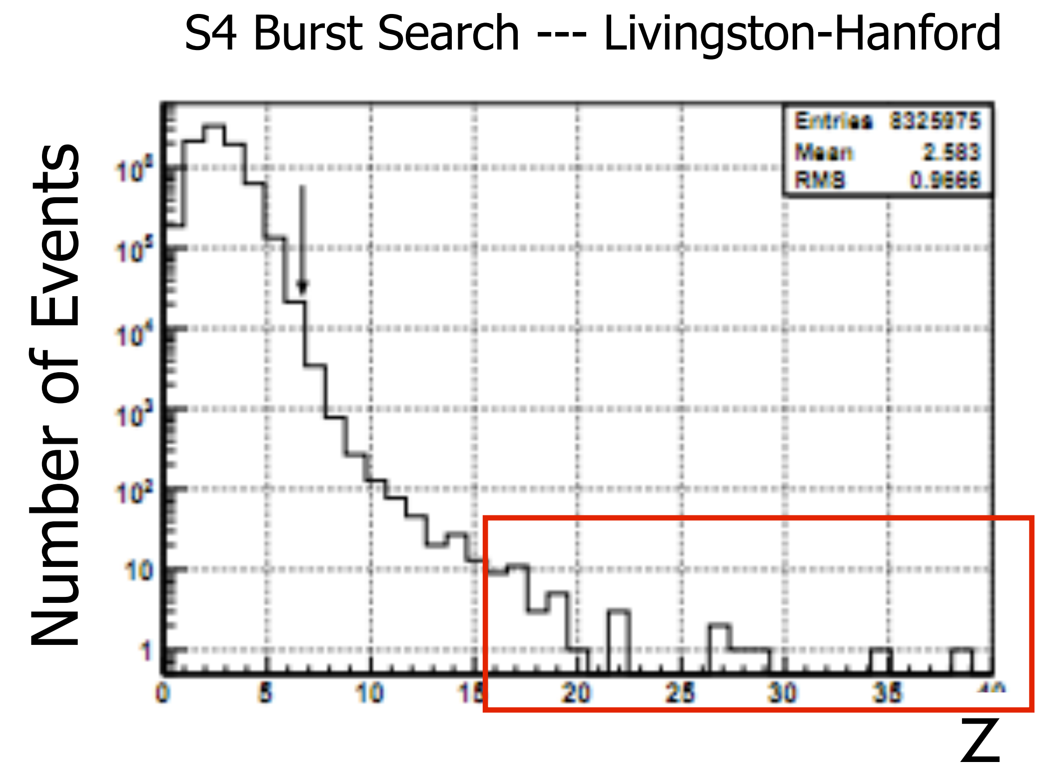
$$\mu(\rho) = \frac{N_{\rm bkg}(\rho) T_{\rm obs}}{T_{\rm bkg}} \qquad \mathrm{FAP}(n|\mu(\rho)) = 1 - \sum_{i=0}^{n-1} \frac{\mu(\rho)^i}{i!} \exp(-\mu(\rho))$$

- False Alarm Probability is determined from the expected background contribution number μ (an implicit function of candidate signal-to-noise ratio ϱ) inserted into Poisson (counting) statistics

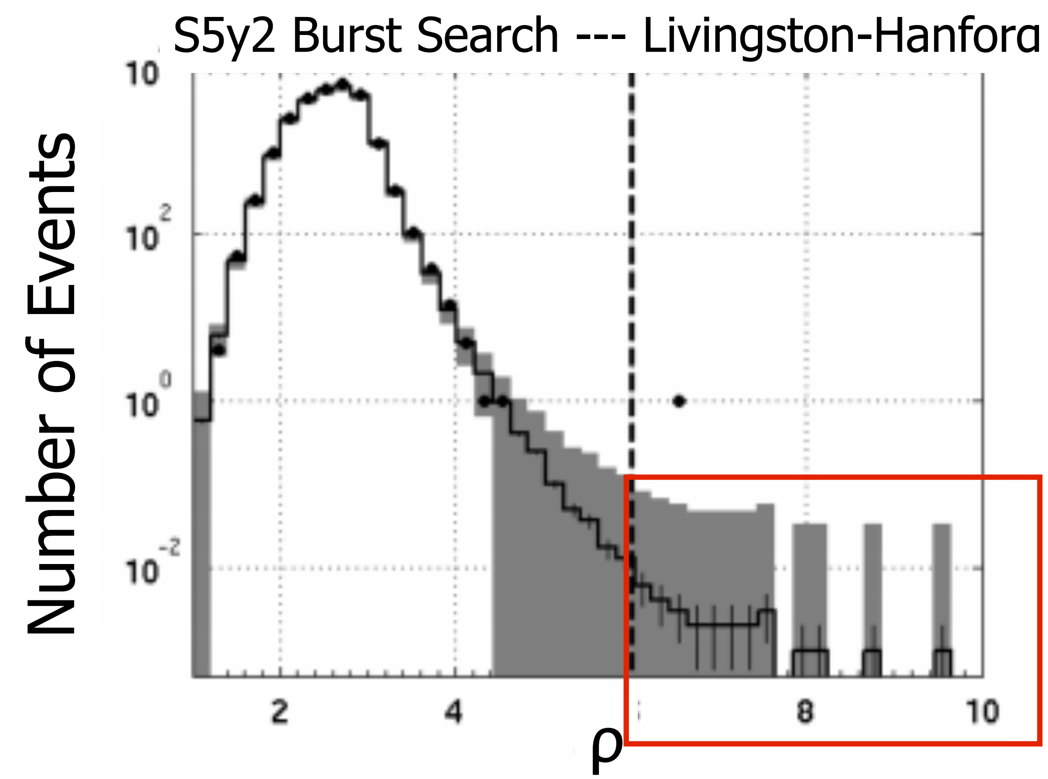
- Interpretation: What is the probability of a background process producing *n* candidates with strength of at least ϱ?

Multiple Searches

- FAR is not always a good measure of candidate significance:



- GW searches can have different false alarm rates and sensitivities
- Different searches for the same source may have different ranking statistics — how can we compare them?



- Measures of significance derived only from background estimation ignores the search sensitivity

Search Sensitivity

- Target a source — search sensitivity is directly related to the astrophysical interpretation of the search
- Searches that target the same source do not always have equivalent sensitivities
  - Different properties such as the detector network and search algorithm affect the sensitivity volume

$$V(\rho) = 3h_0^3 V_0 \int_0^\infty \frac{\epsilon(\rho, h)dh}{h^4}$$

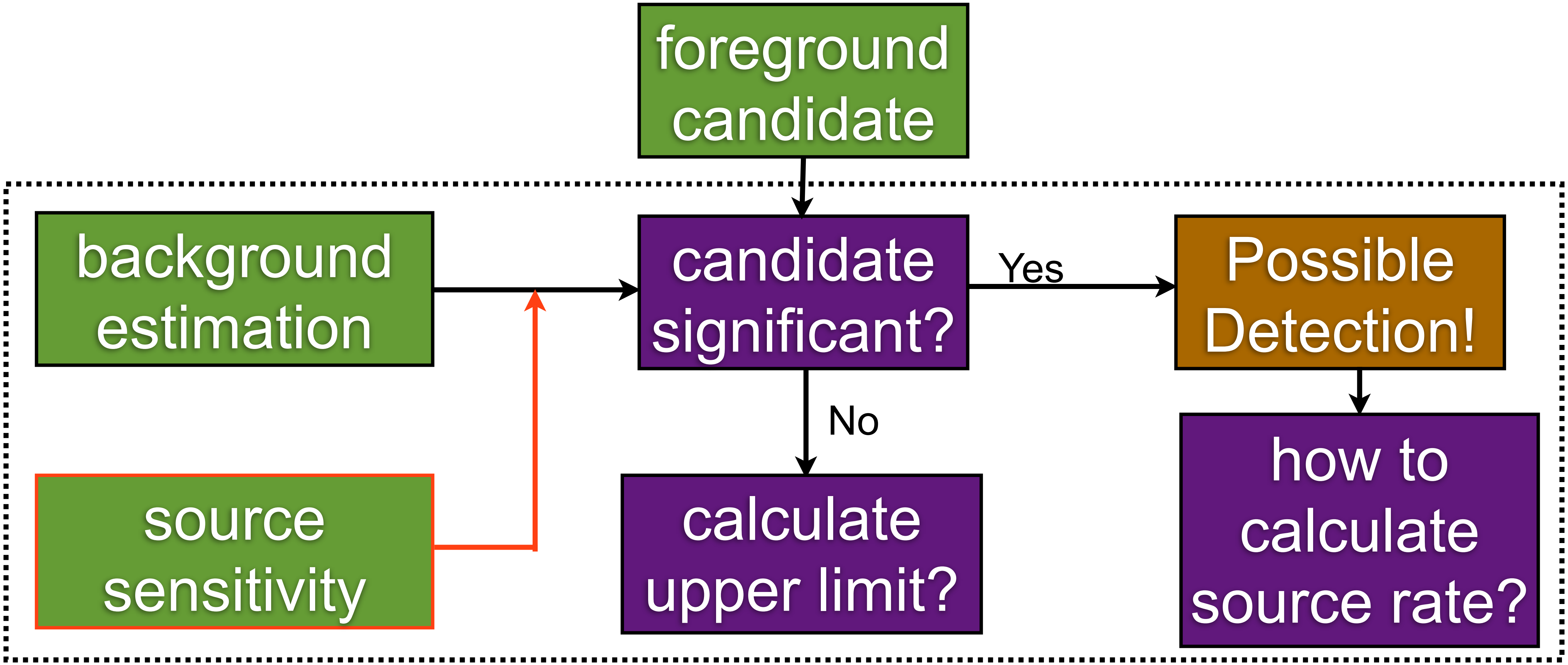
Visible volume within fiducial volume V_0 of uniform isotropic sources with root-square-sum strain amplitude h_0 and detection efficiency ϵ

False Alarm Density

- Combine estimated background with estimated sensitivity:

$$\mathrm{FAD}(\rho) = \frac{1}{T_{\mathrm{bkg}}} \sum_{i}^{\rho_i > \rho} \frac{1}{V(\rho_i)}$$

Estimated rate density of background events in the search volume



Measure of Search Performance

- *Productivity* — time-volume product of all searches targeting a source:

$$\nu(\mathrm{FAD}) = \sum_k V_k(\mathrm{FAD}) T_{obs,k}$$

- Noisy or insensitive searches are weighted out
- Candidate significance is measured against the combined productivity of all searches

$$\mu(\mathrm{FAD}) = \nu(\mathrm{FAD})\mathrm{FAD}$$

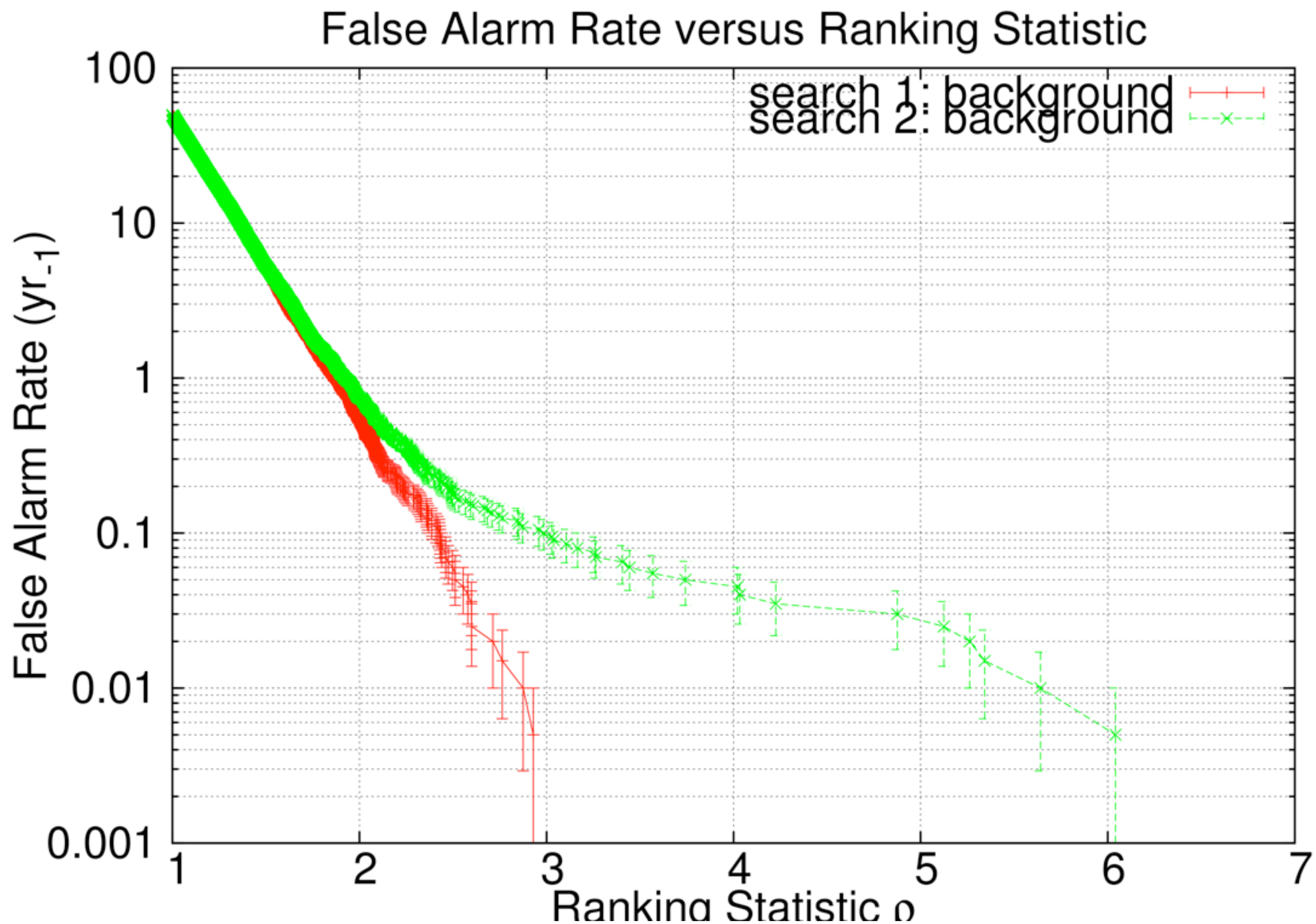
- In the case of no detection, what is the event rate upper limit? We can use the Loudest Event Statistic formulation in the limit of no foreground:

$$R_{90\%} = \frac{2.303}{\nu(\rho)}$$ binary inspirals

$$R_{90\%} = \frac{2.303}{\epsilon(\rho) T_{\mathrm{obs}}}$$ bursts

Examples

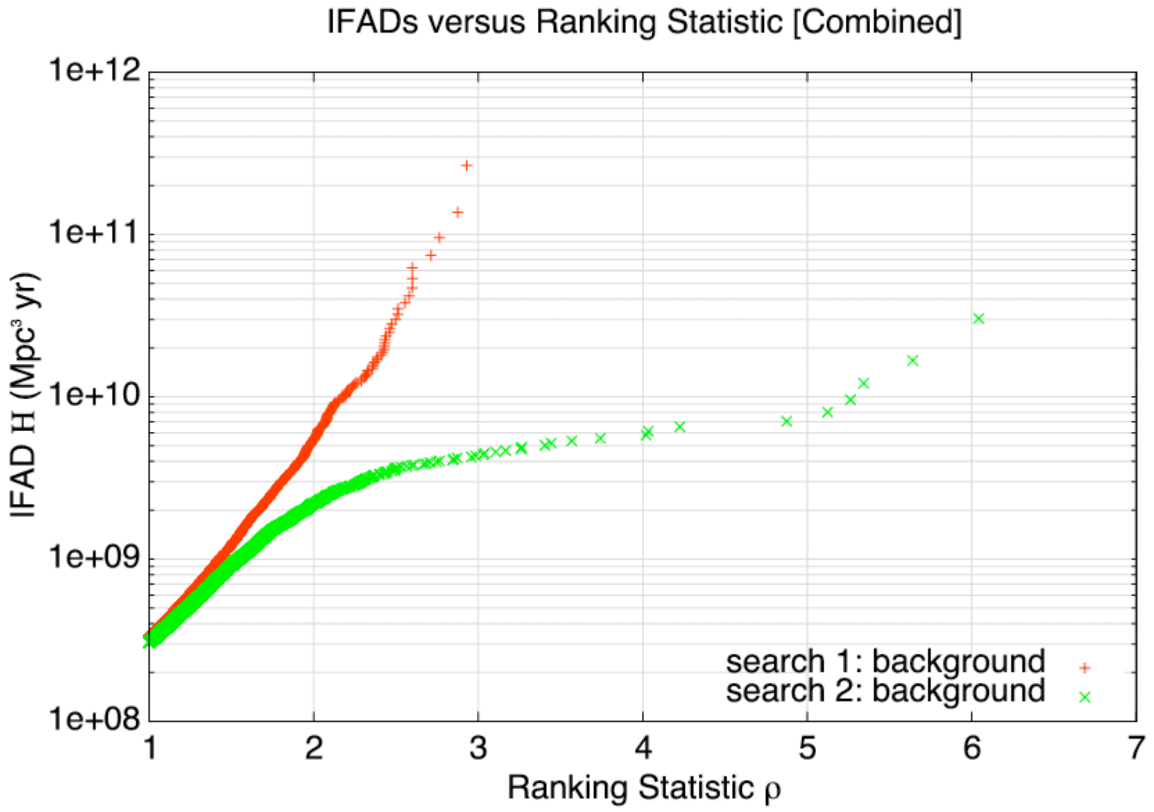
→False Alarm Rate



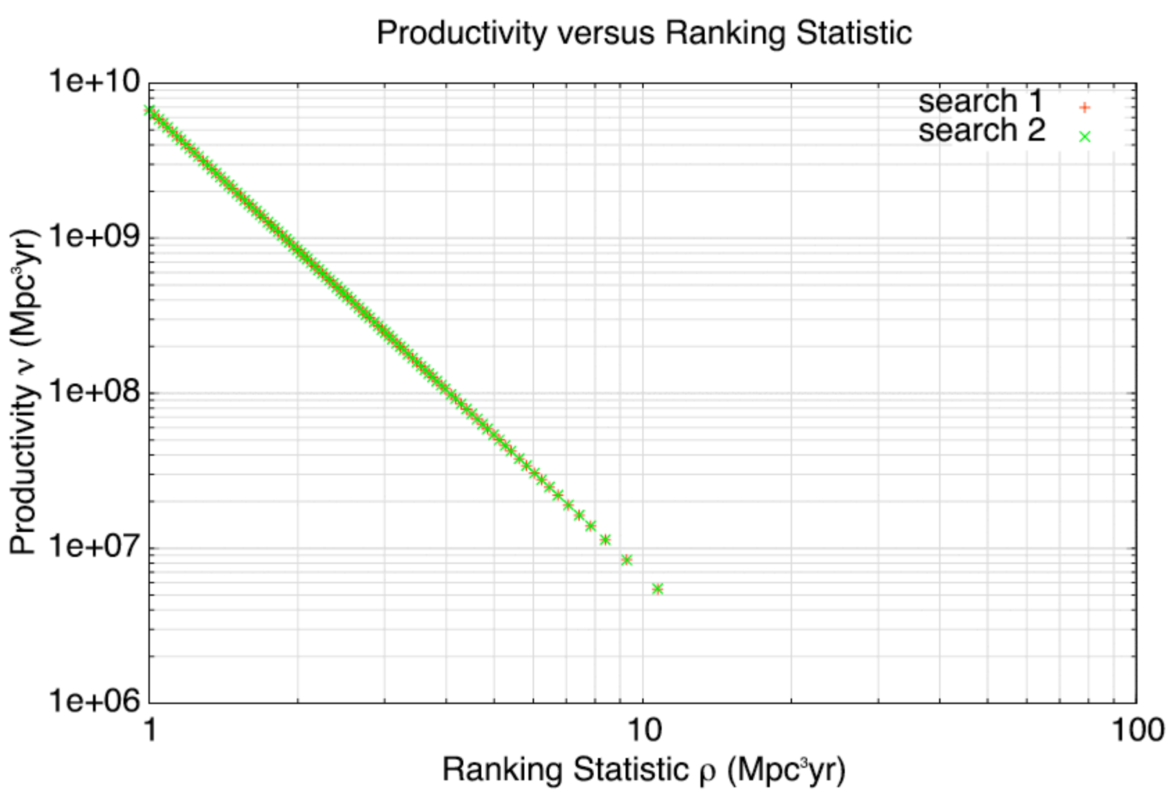
All examples shown will have the same FAR curves as shown above

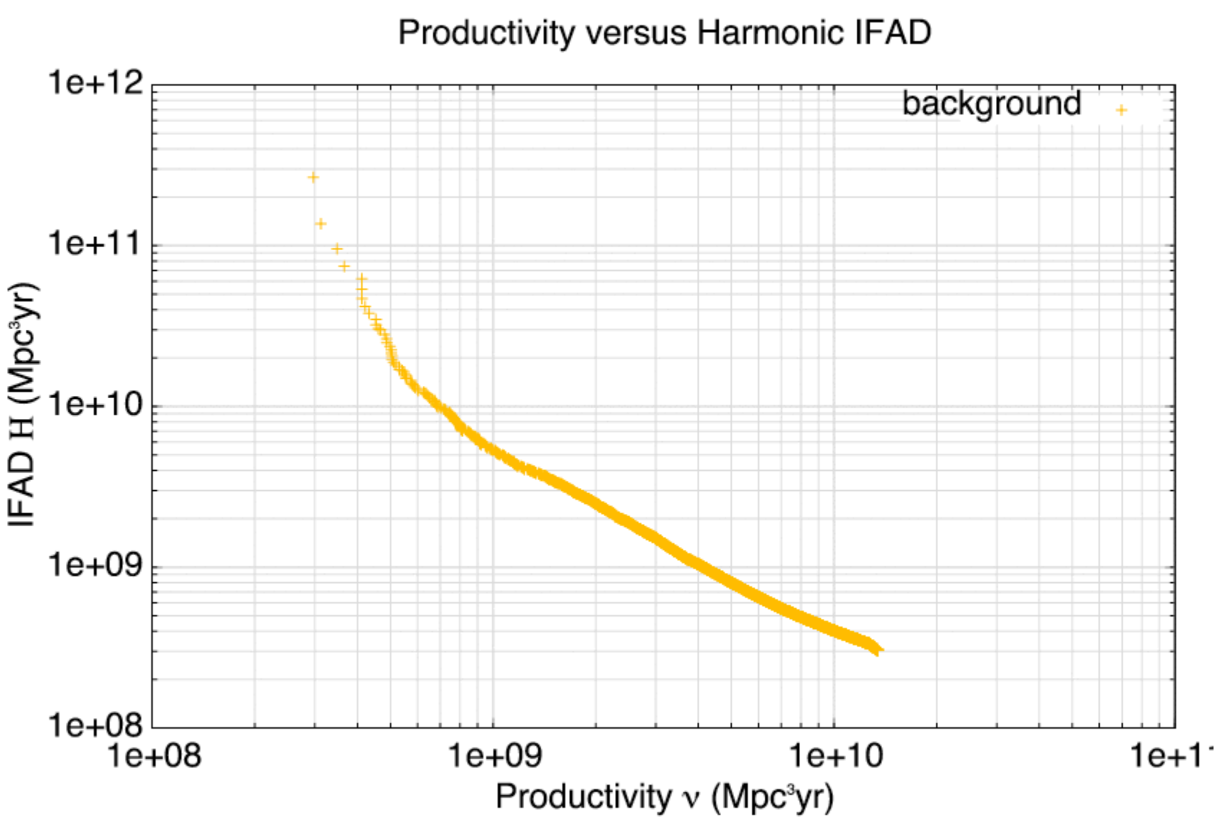
Examples

→Search Combination (1)



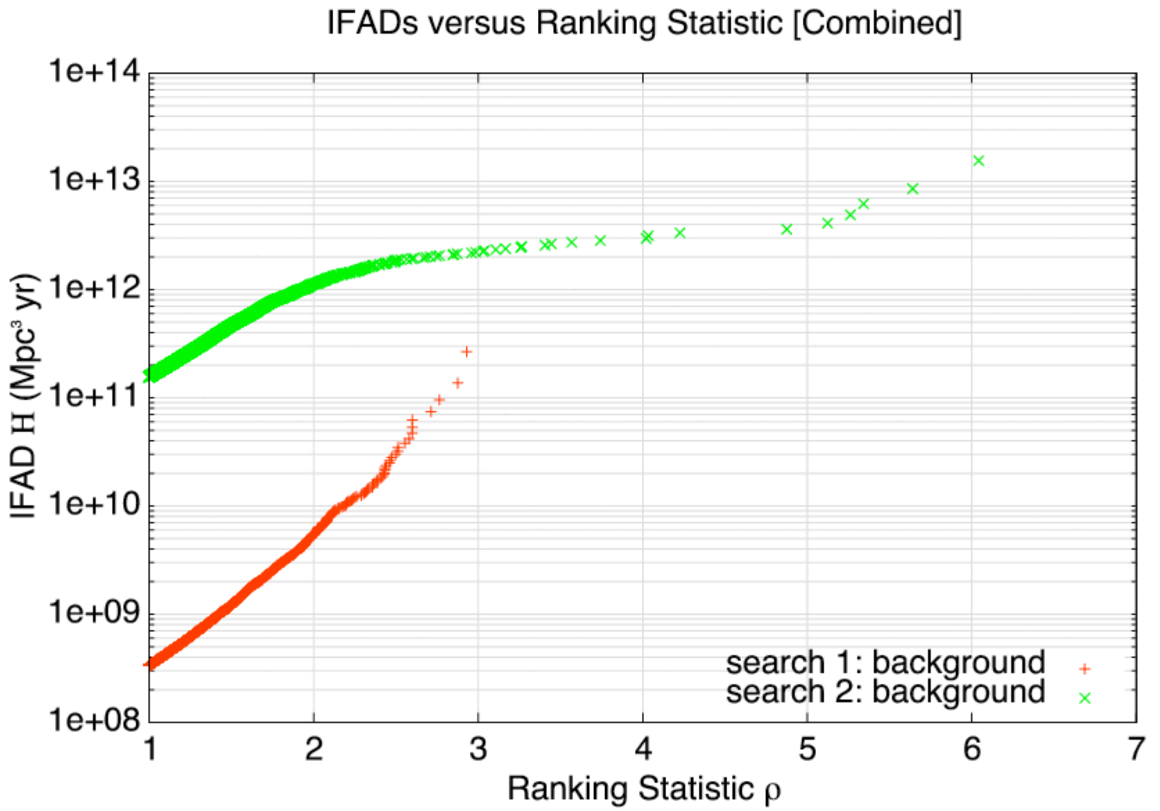
Equal Sensitivity Search 2 More Glitchy		
foreground ρ=2.8	FAP(FAR)	FAP(FAD)
search 1	0.4%	0.2%
search 2	5%	27%



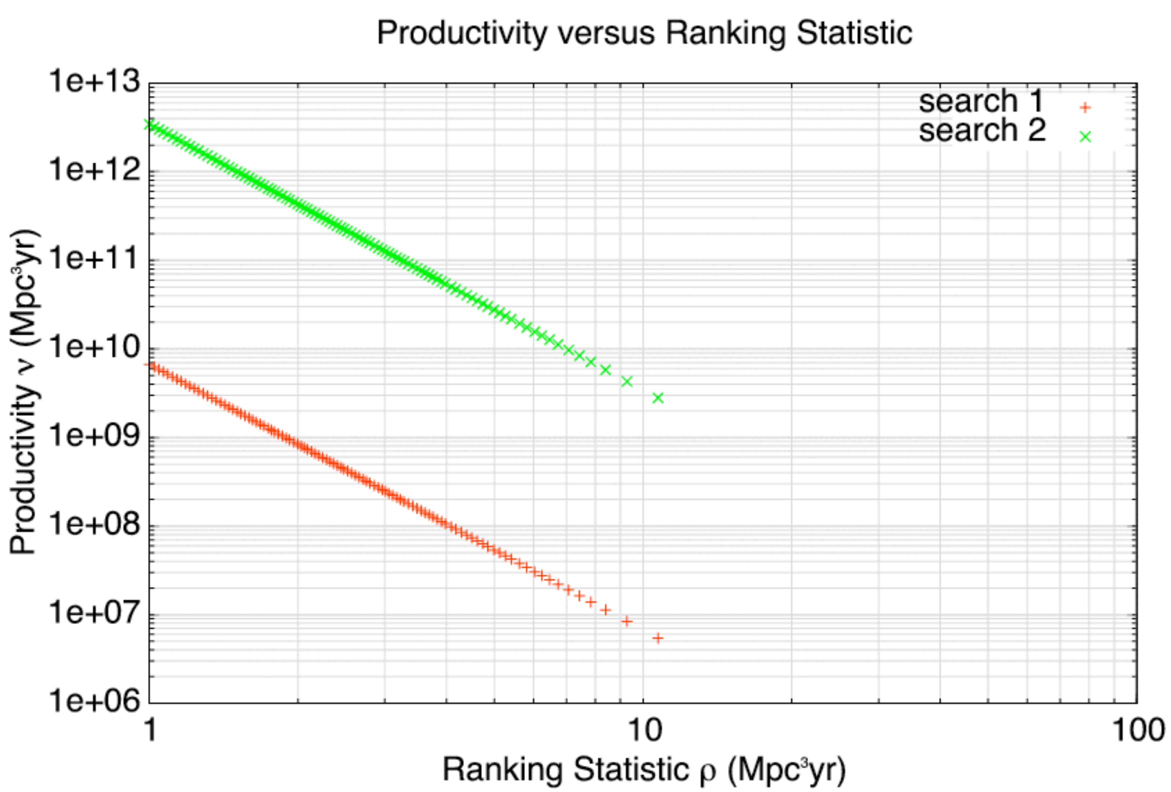


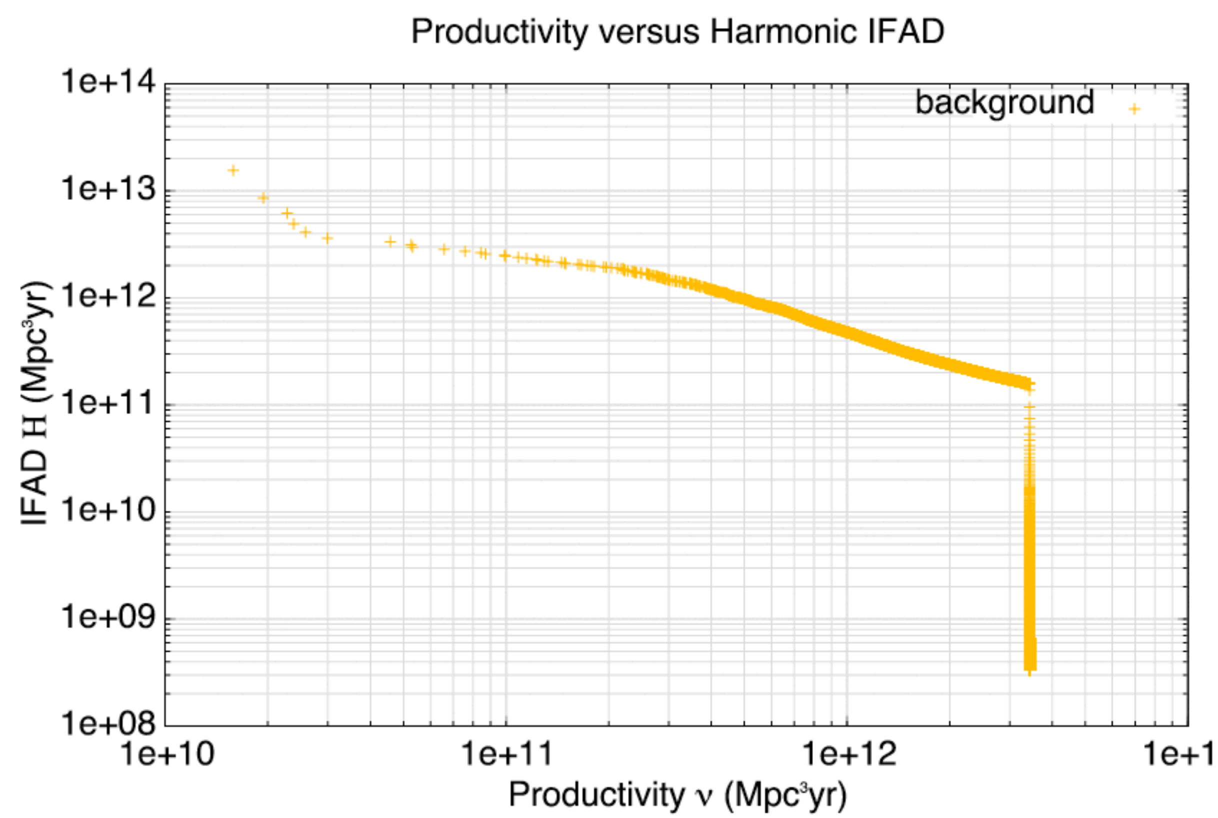
Examples

→Search Combination (2)



Search 1: Less Glitchy		Search 2: More Glitchy, Ten-fold More Sensitive
foreground ρ=2.8	FAP(FAR)	FAP(FAD)
search 1	0.4%	100%
search 2	5%	8%





Conclusions

- Problem of detection is intimately tied to how one defines the significance of a candidate event
- Deriving the significance of an event should be determined by both the sensitivity of a search and its estimated background
- FAD/Productivity allows for combination of multiple searches for the same source, regardless of data epoch and relative sensitivity
- FAD/Productivity addresses both the problem of detection and astrophysical interpretation of upper limits
- Can be extended to searches on same data and folding in trials factors