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# Measurement of the stray magnetic field of the Faraday isolator

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# 1 Introduction

The Faraday isolator (FI) placed after the mode cleaner has a magnet that generates an intense B (magnetic) field. Part of this B-field leaks out into surrounding areas. Because of the dynamic displacement of FI due to seismic noise, this stray B-field fluctuates with a frequency and amplitude determined by the combination of the seismic noise and the transfer function of the optical table on which FI is placed. This B-field fluctuation can affect the operation of the magnetic actuators used for nearby mirrors. Previously we assessed this effect theoretically by estimating the stray B-field by a solenoid model and calculating the force and torque to the SOS actuators using the estimated stray B-field and its gradients [1]. The result of this assessment indicated that the displacement of the closest SOS mirror due to this effect is smaller than the open-loop displacement of the mirror due to seismic noise, and therefore Compensatable by the actuators. In this test, we measure the stray B-field, calculate the displacement of the closest SOS mirror using the measured B-field and compare the result with the above mentioned previous calculation. Our results indicate that the dynamic fluctuation of the B-field will not disturb the actuator’s operation.

# 2 REQUIREMENTS

The displacement of the SOS caused by the dynamic fluctuation of the B-field must be smaller than the SOS open-loop displacement so that it may be compensated by the actuators. Table1 shows the SOS open-loop displacement.

**Table 1: SOS open-loop displacement**

| *frequency (Hz)* | *SOS open-loop displacement (m/Hz$^{1/2}$)* |
|---|---|
| 0.1 - 1 | $10^{-7.5} * (1/f)^{1.5}$ |
| 1 - 10 | $10^{-5.5} * (1/f)^{6.5}$ |
| >10 | $(1/f)^{12}$ |

# 3 MEASUREMENTS

Fig.1 illustrates the arrangement of the stray B-field measurement. We measured the B-field as a function of r and z where z is the coordinate axis along the optical path and r is that perpendicular to z. We used a Gauss meter (Model 9200, manufactured by F. W. Bell) and measured FRs manufactured by Synoptics, Pasat and Electro-Optics Technology (EOT). (For reasons having to do with vacuum contaminations, we will use EOT isolators in LIGO). At each measuring point, we measured the z and r components of the B-field by orienting the tip of the Gauss meter probe such that its surface becomes normal to the component to be measured.

**Figure 1: Arrangement for B-field measurement**

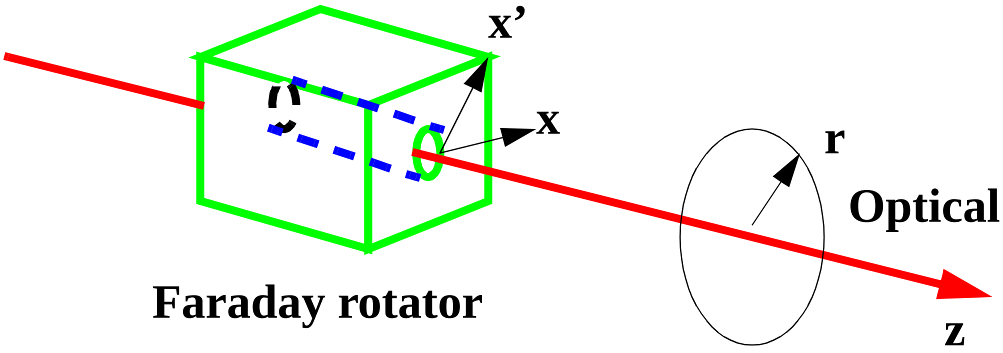


Fig.2 compares the measured z-component of the B-fields among the manufactures and with calculation based on the same solenoid model as ref. [1]. For this calculation we used the dimensions of the EOT FR. All the FRs show very similar $B_z$ for z<4 cm and the Synoptics FR shows an order of magnitude lower value for z>10 cm. The calculated $B_z$ shows good agreement with the value of $B_z$ measured for the EOT FR.

**Figure 2: $B_z$ measured along optical axis**

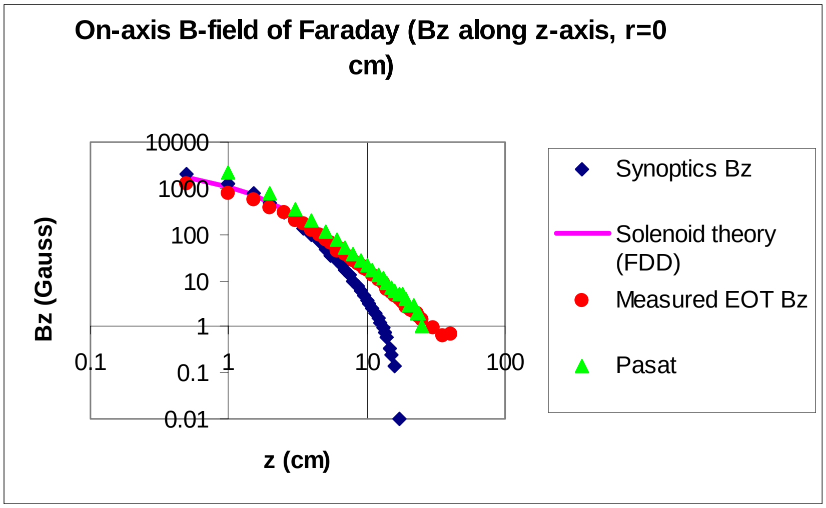


Fig. 3 through Fig. 6 show $B_r$ and $B_z$ measured at z=15 cm and z=25 cm, respectively, for Synoptics and EOT, and compare them with calculation using the same formula used in Fig. 2. Pasat data are not available at these z. Both $B_z$ and $B_r$ measured lower than calculation predicts. *Note that z=25 cm is the realistic location of the closest SOS (4K, HAM1, MC mirror 3)* [2].

**Figure 3: $B_z$ measured at z=15 cm**

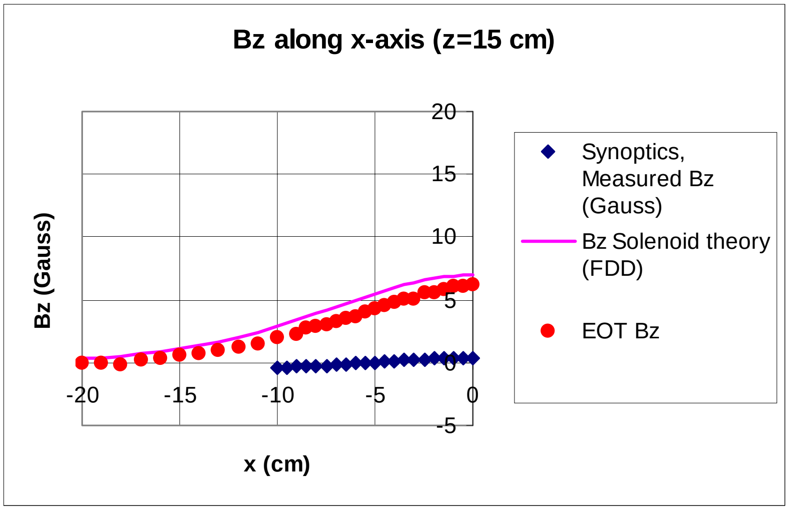


**Figure 4: $B_r$ measured at z=15 cm**

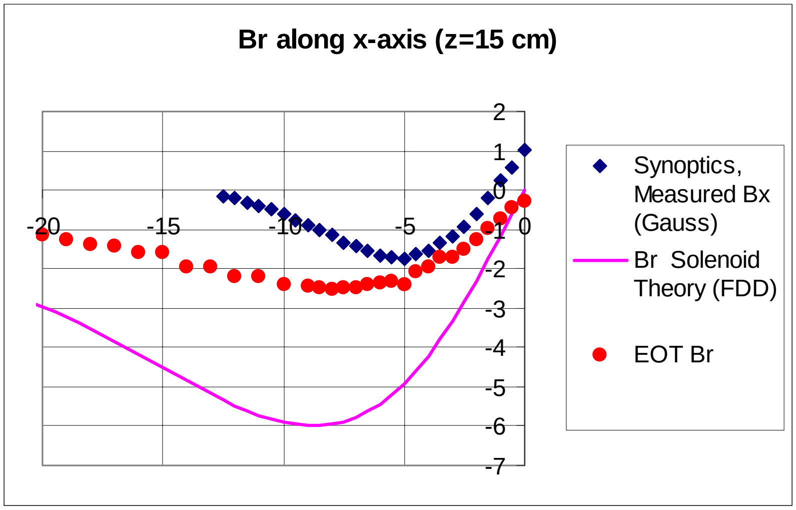


**Figure 5: $B_z$ measured at z=25 cm**

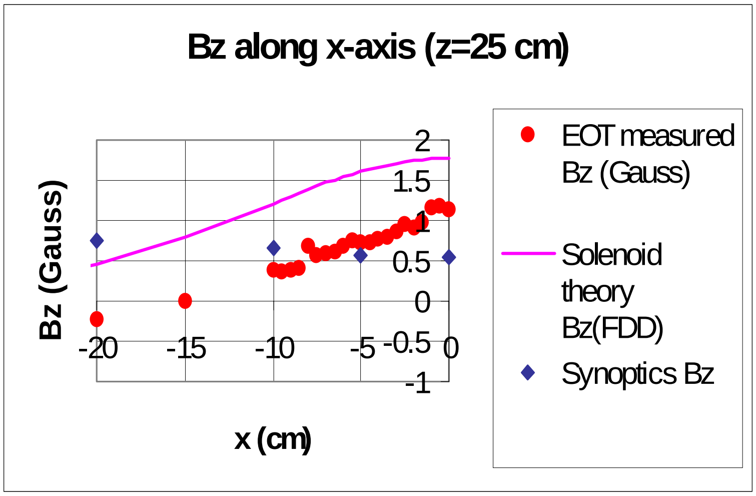
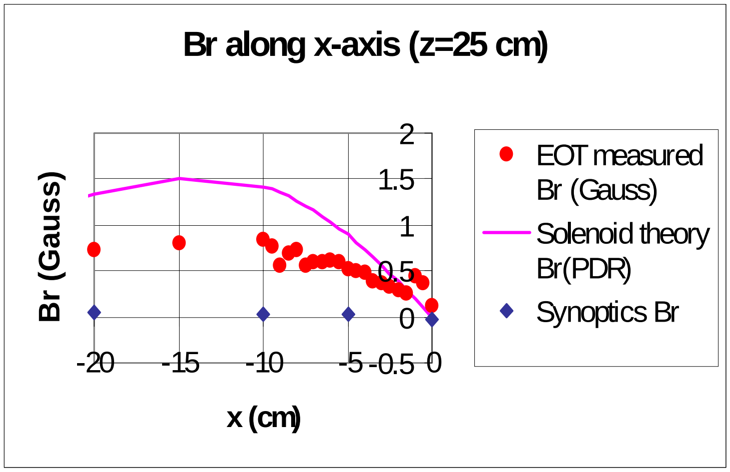


**Figure 6: Br measured at z=25 cm**



We next estimate the actual displacement expected for the mode cleaner mirror at this location. For this estimation, we use the same configuration of the small mirror and the actuator magnets as in [1] except that the distance from the FR to the mirror is now 25 cm (new location). The mode cleaner mirror is tilted to the optical path by 45 deg, and six magnets are used for actuation. For the actuator magnets, we use the nominal dipole moment of 0.0107 $Am^2$ [3]. As is the case of the previous calculation, we consider all the dynamic components of the forces and torques felt by the six magnets to derive translational and rotational equations of motion and solve them for displacements normal and parallel to the mirror surface due to translational motion, and displacements due to horizontal and vertical rotational motion of the mirror. Here the force and torque can be evaluated as follows.

Let B(x, y, z) be the stray B-field. When FI vibrates by seismic noise the stray B-field varies as a function of time as well and in the first order approximation its time derivative can be written as

$$\frac{dB}{dt} = \frac{\partial B}{\partial x} \cdot \frac{dx}{dt} + \frac{\partial B}{\partial y} \cdot \frac{dy}{dt} + \frac{\partial B}{\partial z} \cdot \frac{dz}{dt} \tag{1}$$

where x, y, and z represent the seismic vibration of FI.

$$x(t) = \Delta x(\omega) \exp(i\omega t) \tag{2}$$

$$y(t) = \Delta y(\omega) \exp(i\omega t) \tag{3}$$

$$z(t) = \Delta z(\omega) \exp(i\omega t) \tag{4}$$

Substituting eqs. (2) - (4) into eq.(1), and integrating the result with respect to time, B becomes

$$B = B_0 + \left\{ \frac{\partial}{\partial x} B_0 \cdot \Delta x + \frac{\partial}{\partial y} B_0 \cdot \Delta y + \frac{\partial}{\partial z} B_0 \cdot \Delta z \right\} \exp(i\omega t) \equiv B_0 + B_1 \exp(i\omega t) \tag{5}$$

where $B_0(x, y, z)$ is the static B-field that the actuator magnet would feel if FI was completely stationary.

The force and torque that a magnet having a dipole moment $\mu$ feels can be written as

$$F = grad(\mu \cdot B) = grad(\mu \cdot B_0) + grad(\mu \cdot B_1) \exp(i\omega t) \equiv F_0 + F_1 \exp(i\omega t) \tag{6}$$

and

$$\tau = \mu \times B = \mu \times (B_0 + B_1 \exp(i\omega t)) \equiv \tau_0 + \tau_1 \exp(i\omega t) \quad , \tag{7}$$

respectively.

Thus if B is given as a function of x, y, and z, F and $\tau$ that the actuator magnet feels can be evaluated from eqs. (6) and (7). $F_0$ and $\tau_0$ are the force and torques that the actuator magnet feels as soon as FI is placed, and $F_1$ and $\tau_1$are the amplitude of the temporal fluctuation of the force and torque caused by seimic noise. We call $B_0$, $F_0$ and $\tau_0$ the static component, and $B_1$, $F_1$ and $\tau_1$ the dynamic component.

In this estimation the first and second order spatial derivatives of Bz and Br are needed to calculate the force and torque. Since it is unrealistic to measure these derivatives with a reasonable accuracy, we calculated them using the theoretical formula used in Fig. 2 - 4. Note that in Fig.3 and Fig. 4 at z=25 cm, the calculated values are higher than the measured values both in Bz, Br and their spatial derivatives. This indicates that we overestimate the derivatives. Fig. 7 shows the displacement of the mode cleaner mirror estimated in this way. In the whole frequency range of 0.01 - 100 Hz, all the displacement components are smaller than the open-loop SOS displacement by two to five orders of magnitude.

**Figure 7: SOS displacement estimated by measured B-field**

# 4 CONCLUSION

Table 2 compares the SOS open-loop displacement and the SOS displacement due to the stray B-field.

**Table 2: Comparison of SOS displacements**

| *frequency (Hz)* | *SOS open-loop displacement (m/Hz$^{1/2}$)* | *SOS displacement due to stray B-field* |
|---|---|---|
| 1 | 4 x $10^{-9}$ | 1 x $10^{-12}$ |
| 10 | 1 x $10^{-12}$ | 5 x $10^{-17}$ |
| 100 | 3 x 10-23 | 6 x 10-28 |

Confirming our previous theoretical estimation, this measurement of stray B-field indicates that the dynamic force and torque caused by seismic vibration of the Faraday isolator will not disturb the performance of nearby SOS actuators.

References:

[1] S. Yoshida, R. Adhikari and D. Reitze, Influence of the stray magnetic field generated by the Faraday isolator on SOS mirror actuators, LIGO T970149, 11/2/96

[2] Rana Adhikari, Tom Delker, David Reitze, Qi-Ze Shu, David Tanner and Sanichiro Yoshida, Input Optics Preliminary Design, LIGO-T970144-00D, 8/12/97, Section 7.1.1.2

[3] D. Coyne, Magnetic Dipole Moment Limits, 8/29/96