Q factor measurements on prototype fused quartz pendulum suspensions for use in gravitational wave detectors

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Abstract

This paper describes measurements of the pendulum mode Q factors of prototype fused quartz test mass suspensions for use in interferometric gravitational wave detectors. Pendulums were constructed using bobs of masses ranging from approximately 12 g to 1.9 kg suspended by cylindrical fibres of standard fused quartz and Q factors greater than 10^7 obtained.

1 Introduction

Of the many noise sources which may degrade the sensitivity of ground based laser interferometric gravitational wave detectors, thermal noise in the mirror test masses and their suspensions is likely to be one of the most significant in the frequency range of interest. The design sensitivity of the GEO 600 detector [1], less than $h \simeq 2 \times 10^{-22}/\sqrt{(\text{Hz})}$ between 50 Hz and 150 Hz, is based on the premise that in this frequency range the dominant noise source results from losses associated with the internal modes of the fused silica test masses. This requires the thermally induced motion at 50 Hz from the pendulum mode of each test mass to be significantly less than the internal mode contribution.

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A target of approximately $2 \times 10^{-20}\text{m}/\sqrt{(\text{Hz})}$ for the pendulum motion, a tenth in power terms of the internal mode contribution, is chosen. It can be shown that a loss factor associated with the pendulum mode of each suspension of less than 4×10^{-8} at 50 Hz is thus required [2].

Experiments [3,4], suggest fused silica (quartz) may be a promising choice of fibre suspension material. We wished in particular to investigate the suitability of standard commercial quality fused quartz for this purpose.

For a mass suspended on four fibres or two fibre loops the loss factor of the pendulum mode of the suspension, $\phi_{pend}(\omega)$, and the loss factor of the suspension fibre material, $\phi_{mat}(\omega)$, are related by [4,5]

$$\frac{1}{\phi_{pend}(\omega)} = \frac{1}{\phi_{mat}(\omega)} \frac{mgl}{4\sqrt{TEI}} \qquad (1)$$

where m is the mass of the pendulum, l is the length of pendulum, T is the tension in each wire, E is the Young's modulus of the material of the suspension wires and I is the moment of inertia of each fibre ($I = \pi r^4/4$ for a cylindrical fibre). The above equation holds when there are no excess losses at the jointing points where the suspension fibres leave the pendulum bob. The 16 kg test masses of GEO 600 thus require suspension fibres with a material loss factor of less than approximately 6×10^{-6} at 50 Hz. Braginsky and colleagues [3] have demonstrated lower loss factors in very carefully prepared fused silica fibres and we have demonstrated that, with standard normal grade fused quartz ribbon fibres, loss factors of the order of 10^{-6} may be achieved [4].

The loss factor of a pendulum, at a frequency away from its resonance, is very difficult to measure experimentally. However it can be deduced from ringdown, or Q, [2] measurements of the pendulum resonance if the frequency dependence of the loss factor of the material of the suspension fibres is known. The simplest case is for the material loss to be structural in form, then the loss factor is independent of frequency. Experiments suggest this is a reasonable assumption when considering commercially available fused quartz [4] even when the effect of thermo-elastic damping [6] is taken into account, and this will be discussed in a later section.

Measurements of the Q factors of all-welded pendulums constructed from very high purity fused silica have yielded Q factors of up to 10^8 for the pendulum and bifilar torsional modes [7,8].

In this paper we report the results of experiments in which the Q factors of the pendulum mode of a selection of pendulums of various masses were measured with, in each case, the pendulum bobs being suspended for simplicity by two cylindrical fibres of commercial grade fused quartz. The Q factor of the pendulum mode of each suspension was obtained by measuring the decay of the amplitude of the pendulum motion using a shadow sensor system as shown

[2] note that the Quality Factor, Q, of a system resonant at angular frequency ω_0 is related to the loss factor $\phi(\omega)$ by $Q = 1/\phi(\omega_0)$

in figure 1, with the resulting signals being recorded using a computer based data aquisition system and also using a chart recorder to allow cross checking of results. These experiments show the effect of various fibre attachment techniques on the measured pendulum mode Q factor and demonstrate that Q factors of greater than 10^7 may be achieved using commercial grade fused quartz suspension fibres. The highest Q values were limited by a combination of the material loss in the suspension fibres and the recoil loss of the structure from which the pendulums were suspended.

2 Measurements of the Q factors of the pendulum modes of masses suspended by cylindrical fused quartz fibres

2.1 Support Structure for the Pendulums

When a pendulum swings there is a tendency for its support structure to recoil and thus for energy to be lost into the surroundings. This can be a very important experimental limit to the measurement of pendulum Q factors with the limiting value of Q being given by [9]

$$Q_{limit} = \frac{1}{m\omega_0^2}\frac{k}{\delta} \tag{2}$$

where k and δ are the spring constant and loss angle of the structure at the pendulum frequency. For all the pendulum Q factor measurements described in this paper the pendulums were suspended as shown in figure 2, from a rigid aluminium support box structure itself rigidly attached to an aluminium plate of thickness 1.6 cm. The plate is mounted on four aluminium rods of 5 cm diameter inside a vacuum tank of 45 cm diameter, the rods being pushed out by screwed braces to fit tightly against the tank walls, and the plate being firmly jammed in place by means of aluminium wedges hammered in between its corners and the walls of the tank.

The mounting of the box structure to the top plate is stiffened by the addition of aluminium fillets. The vacuum tank sits on 3 small stainless steel pads on a stainless steel plate of thickness 1 cm which is cemented to a concrete block of mass 1080 kg. This concrete block sits on two smaller concrete blocks which connect to a concrete floor. The vacuum tank is connected to a heavy ion pump by means of a short length of rigid pipe of diameter 13 cm and the ion pump sits on a separate concrete block of mass 678 kg. In order to determine the rigidity of the structure a lead pendulum of mass 10 kg, hung on steel wires, was attached to an aluminium plate clamped to the rigid support box, and when the pendulum was swung with a measured amplitude the acceleration of the mounting point was measured using a home-built inverted pendulum feedback accelerometer. A lock-in amplifier was used to determine the magnitude and phase of the acceleration, the reference signal being provided by a shadow sensor monitoring the pendulum swing. Considerable care was taken to subtract out or compensate for phase delays in the measurement system. The force exerted on the mounting point was calculated from the amplitude of swing of the pendulum, the mass of its bob, and the lengths of the suspending wires. Using the value obtained for this force, and the measured acceleration, the values of the spring constant and loss angle were found to be $k = (5.5 \pm 0.7) \times 10^6\,\mathrm{Nm}^{-1}$ and $\delta = (-1.61 \pm 0.05)^\circ$ The limiting Q factor for a pendulum of mass m may be obtained using equation 2.

2.2 *Experimental measurements*

Cylindrical fused quartz fibres were drawn from fused quartz rods using a radio frequency (r.f.) oven with a graphite susceptor. The initial diameter of the quartz rod was 3 mm and fibres with a diameter of approximately 290 μm at the point of bending were produced, with the ends of the fibres still having thicker portions of rod attached. In initial experiments the rod ends of each of two fibres were attached to glass or ceramic bobs, of mass approximately 200 g, and to aluminium top clamps, by a mixture of clamping and glueing as shown in figure 3. The fibres were splayed out slightly at their tops at an angle of approximately 2° to the vertical to clearly separate the main pendulum frequency from the frequency of the orthogonal mode. This essentially eliminates coupling between the main pendulum mode and the orthogonal mode induced by the Coriolis effect. The resulting pendulums, of length approximately 26.4 cm, were rigidly clamped inside the vacuum tank described above and the tank was then evacuated to a pressure of approximately 5×10^{-7} mbar. The pendulum mode, in each case, was excited using a mechanical pusher operated by a coil and magnet drive as shown in figure 1.

Experiments showed that the amplitude of the pendulum motion did not decay exponentially as expected, and over the course of the decay the Q factor was observed to decrease from a value of a few times 10^6 to a few times 10^4. A series of experiments showed that electrostatic charging of the pendulum bob due to UV radiation produced by the ion pump on the system was significantly degrading the pendulum mode Q factor [2]. This problem was circumvented by careful shielding of the pendulum from the UV. However even then the pendulum Q factor was still of the order of 3×10^6, a factor of approximately 7.5 lower than predicted from a knowledge of the suspension material loss and the loss due to recoil of the supporting structure. A value for the material loss factor for the quartz suspension fibres of $(7 \pm 2) \times 10^{-7}$ was used (equivalent to a material Q factor of $(1.5 \pm 0.5) \times 10^6$) [4]. Some excess loss factor was thus present.

In order to investigate the source of this discrepancy a series of experiments on pendulums of different mass, with different dimensions of suspending fibres and with different methods of clamping the suspending fibres, was performed. The results of these experiments are summarised in table 1. The measured pendulum loss, $1/Q_{meas}$ can be expressed as the sum of the expected losses of the system $(1/Q_{theory} + 1/Q_{recoil})$ and an excess loss term, $(1/Q_{excess})$ such that

$$\frac{1}{Q_{meas}} = \left(\frac{1}{Q_{theory}} + \frac{1}{Q_{recoil}}\right) + \frac{1}{Q_{excess}} = \frac{1}{Q_{expected}} + \frac{1}{Q_{excess}} \qquad (3)$$

$1/Q_{expected}$ and $1/Q_{excess}$ are included in table 1.

2.3 Results

One possible source of the excess loss measured was the presence of some stress dependent loss in the fused quartz suspension fibres such that $1/Q_{mat} \propto m/r^2$. However results 4 and 7 in table 1 are for two pendulums with suspension fibres of the same radius but with bobs of different mass. If the excess loss was caused by a stress dependence in the suspension material as described above it can be shown that $1/Q_{excess}$ should increase as $\sqrt{m}$ as the pendulum mass is increased. The excess loss can however be seen to have decreased. Results 3 and 7 are for two pendulums of the same mass hung from fibres of different diameter. In this case it can be shown that $1/Q_{excess}$ should be independent of fibre radius; but this is clearly not observed in practice.
Another possible source of $1/Q_{excess}$ is excess loss associated with the clamping of the fibres. Although clamping is carried out on the thick rod ends of the fibres in order to minimise the stick slip effects described by Quinn et al [10] there may still be some such residual loss remaining. Results 1 to 3 in table 1 were for pendulums where a combination of glueing and clamping, as discussed earlier, was used for attaching the fibre rods at the top. Results 4 to 7 were for pendulums where precision engineered pin vices were used to clamp the fibre rods at the top, while the rods at the bottom were welded to a fused quartz bob. A review of the results 2 to 7 suggests that the loss $1/Q_{excess}$ could be reduced to an insignificant value by using the precision clamps; and it is interesting to note how the loss was reduced between results 5 and 6,7 by tightening of the precision clamps.

2.3.1 Seismic noise

When attempting to measure the Q factor of the pendulum mode of a suspension, care must be taken to ensure that seismic excitation of the pendulum mode does not result in an artificially high value being obtained for the Q

factor. Assuming that the driving seismic noise spectrum is approximately white, a reasonable assumption for frequencies around 1 Hz, it can be shown that the root mean square motion of a pendulum bob, x_1, induced by seismic motion of amplitude spectral density, x_0, can be expressed in the form

$$\frac{x_1}{x_0} = \sqrt{\frac{\pi f_0 Q}{2}} \tag{4}$$

where f_0 is the resonant frequency of the pendulum and Q is the true Q factor of the pendulum mode.

Measurements of the magnitude of horizontal seismic motion of the floor in the direction of pendulum swing close to where our pendulums were suspended suggested the ground motion was such that $x_0 \simeq 10^{-8}\,\mathrm{m}/\sqrt{\mathrm{Hz}}$ at 1 Hz. Assuming a value for the pendulum Q factor of 3×10^7 suggests that the maximum r.m.s. induced motion of the pendulum bob is $\simeq 68\mu$m, or equivalently $\simeq 192\mu$m peak to peak motion.

In our measurements the initial peak to peak amplitude of pendulum swing was approximately 3 to 4 mm, a factor of 15 to 20 times larger than any induced seismic excitation, suggesting that our measurements of pendulum Q factor should not be significantly affected by seismic noise.

2.3.2 Additional experiments

The experiments described above highlighted the need for care to be taken when choosing a method for attaching the fused quartz fibres to a test mass. In view of this an all-welded fused quartz pendulum was constructed with the rod ends of the two suspension fibres welded to a 96g fused quartz bob at the bottom and to a fused quartz plate, 0.5 cm thick, 7.5 cm wide and 12 cm long at the top. The plate was rigidly clamped to the structure inside the vacuum tank. The pendulum mode Q factor for this was measured and the logarithmic fit of a typical ringdown for this pendulum is shown in figure 4. The Q factor was found to be $(3.3 \pm 0.3) \times 10^7$, equivalent to a loss factor of $(3.0 \pm 0.3) \times 10^{-8}$, confirming our conclusion that for result 7 in table 1 the use of carefully tightened precision pin vices allowed excess loss in the fibre clamps to be reduced to an insignificant level.

We have also measured the pendulum Q factor of a 1.9 kg brass pendulum suspended on two of our fused quartz fibres from a rigid structure at the University of Perugia and achieved a Q factor of approximately 1.4×10^7, limited we believe by a combination of losses in the clamps used to attach the fibres to the pendulum bob and damping of the conducting bob moving in the Earths magnetic field.

2.3.3 *Thermo-elastic damping and its significance for the measurements reported here*

When a thin fibre is flexed there is a loss mechanism associated with the periodic transfer of thermal energy from one side of the fibre to the other known as thermo-elastic damping. For a fibre of diameter d the loss factor due to this mechanism is given by the equation [6]

$$\phi_{th}(\omega) = \Delta \frac{\omega \tau}{1 + \omega^2 \tau^2} \tag{5}$$

where Δ is the relaxation strength and τ^{-1} is the characteristic angular frequency of the loss mechanism given by

$$\Delta = \frac{E\alpha^2 \mathcal{T}}{\rho c} \tag{6}$$

$$\tau = 7.37 \times 10^{-2} \frac{\rho c d^2}{K} \tag{7}$$

with α the coefficient of linear expansion, c the specific heat capacity at constant tension, K the thermal conductivity, ρ the density, and $\mathcal{T}$ the temperature of the fibre.

For fused quartz fibres at room temperature $\phi_{th}(\omega)$ has a maximum value of $\Delta/2 = 1.6 \times 10^{-6}$. The characteristic frequency, f_{char}, at which this maximum loss occurs is a function of fibre diameter and for fibres such as those used in constructing pendulums described earlier, of diameter 290μm, maximum damping occurs at $f_{char} = 21$ Hz.

At the pendulum resonant frequency of 1 Hz the thermo-elastic loss imposed is 1.5×10^{-7} which is in fact smaller than the mean experimental value for the material loss of the fused quartz fibres of 7×10^{-7} used in section 2.2. (This implies that other material loss mechanisms are dominant away from the thermoelastic peak.)

At 50 Hz the loss factor set by thermo-elastic damping is 1.1×10^{-6} which is slightly larger than the values used in section 2.2. Thus for a pendulum hung on fibres of 290μm diameter the off resonance pendulum loss factor at 50 Hz will be greater than that predicted by the measured Q at 1 Hz by the ratio of 1.1×10^{-6} to 7×10^{-7} - a relatively small effect.

It should be noted that for pendulums of heavier mass supported on correspondingly thicker fibres the frequency at which thermoelastic damping is largest will be different, but in all cases the peak value of the damping factor for fused silica is smaller than that required for the GEO 600 pendulums and thus thermo-elastic damping should present no problems.

With other materials of very low intrinsic loss but higher coefficients of expansion, such as sapphire [11], consideration of the equations 5, 6 and 7 suggests that thermoelastic damping may contribute significantly to the loss factor of a pendulum at frequencies of interest for gravitational wave detectors.

3 Conclusions

In this paper we have reported experiments which demonstrate that measured Q factors of the order of 3×10^7 may be achieved for pendulums using standard fused quartz as the material for the suspension fibres. These experiments emphasise the importance of choosing a very low loss method of attaching the fused quartz fibres both to the pendulum bob and also to the upper suspension stage. Subtracting the measured loss due to recoil of the support structure for the 96 g all-welded pendulum from the measured loss of the pendulum, gives a corrected value for the pendulum loss of $(1.1 \pm 0.3) \times 10^{-8}$ (equivalent to a pendulum Q factor of $(9 \pm 2) \times 10^7$) which is limited by the material loss of the fused quartz suspension fibres. The results presented here suggest that the use of standard commercial quality fused quartz as a suspension material should allow pendulum Q factors of the magnitude necessary for use in gravitational wave detectors.

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Fig. 1. Shadow sensing system used to measure the decay of the pendulum swing. Also shown is the coil and magnet arrangement used to excite the pendulum mode.

Fig. 2. Schematic diagram of the rigid support structure from which pendulums were suspended inside a vacuum system.

Fig. 3. Initial method of attachment of the rod ends of the fused quartz fibres to the pendulum bob and aluminium plate from which the pendulums were suspended.

Fig. 4. Logarithmic fit to a typical ringdown for an all-welded fused quartz pendulum of mass 96 g.

		Pendulum mass (g)	Fibre diameter (μm)	$1/Q_{pend}$ (measured)	$1/Q_{expected}$	$1/Q_{excess}$
Original clamps	1	200	290	$(3.5 \pm 0.4) \times 10^{-7}$	$(4.6 \pm 0.6) \times 10^{-8}$	$(3.0 \pm 0.4) \times 10^{-7}$
	2	32	60	$(7.2 \pm 0.7) \times 10^{-8}$	$(6.8 \pm 0.9) \times 10^{-9}$	$(6.5 \pm 1.0) \times 10^{-8}$
	3	96	60	$(2.2 \pm 0.1) \times 10^{-7}$	$(1.9 \pm 0.2) \times 10^{-8}$	$(2.0 \pm 0.1) \times 10^{-7}$
New clamps	4	12	290	$(6.9 \pm 0.5) \times 10^{-8}$	$(3 \pm 1) \times 10^{-8}$	$(3.9 \pm 1.0) \times 10^{-8}$
	5	28	290	$(6.0 \pm 0.6) \times 10^{-8}$	$(2.3 \pm 0.7) \times 10^{-8}$	$(3.7 \pm 0.9) \times 10^{-8}$
Clamps tightened	6	28	290	$(3.0 \pm 0.4) \times 10^{-8}$	$(2.3 \pm 0.7) \times 10^{-8}$	$(0.7 \pm 0.8) \times 10^{-8}$
	7	96	290	$(2.8 \pm 0.7) \times 10^{-8}$	$(2.8 \pm 0.4) \times 10^{-8}$	$(0 \pm 0.8) \times 10^{-8}$

Table 1

A summary of the results of measurements of pendulum mode Q factors for pendulums suspended on fused quartz fibres

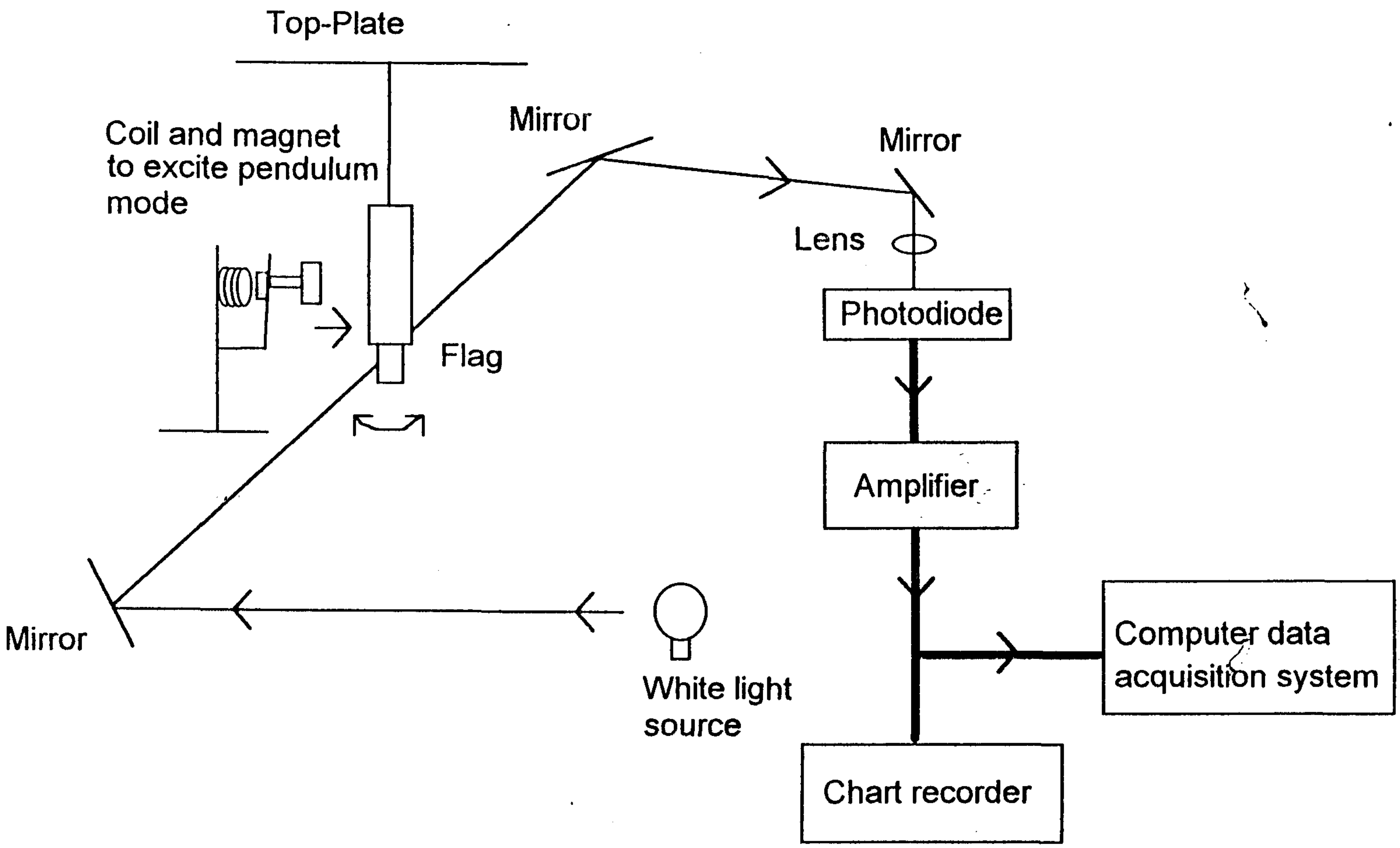
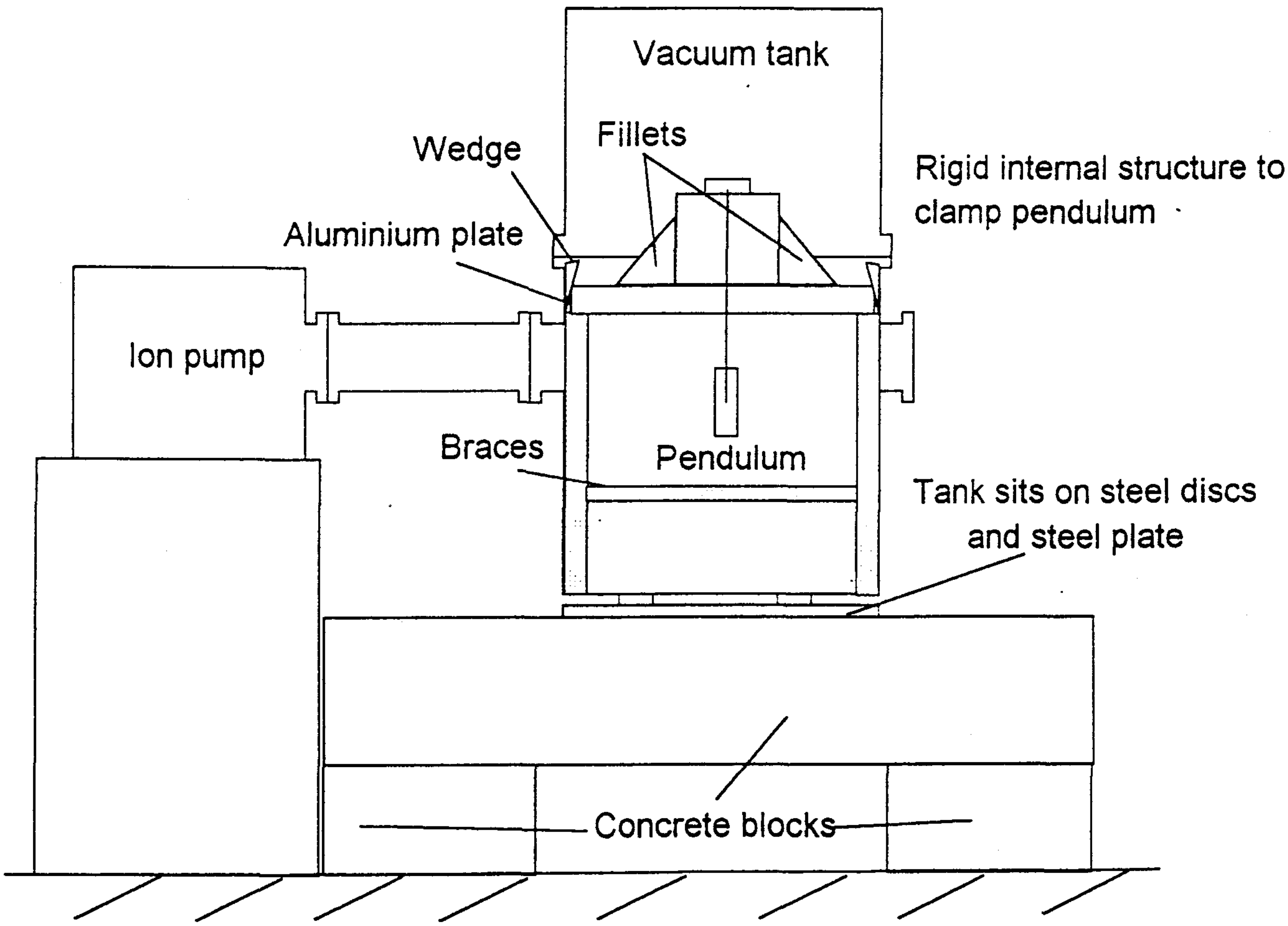

Top-Plate
Coil and magnet to excite pendulum mode
Mirror
Mirror
Lens
Photodiode
Flag
Amplifier
Mirror
Computer data acquisition system
White light source
Chart recorder

FIG 7

FIG 2


Vacuum tank
Wedge
Fillets
Rigid internal structure to clamp pendulum
Aluminium plate
Ion pump
Braces
Pendulum
Tank sits on steel discs and steel plate
Concrete blocks

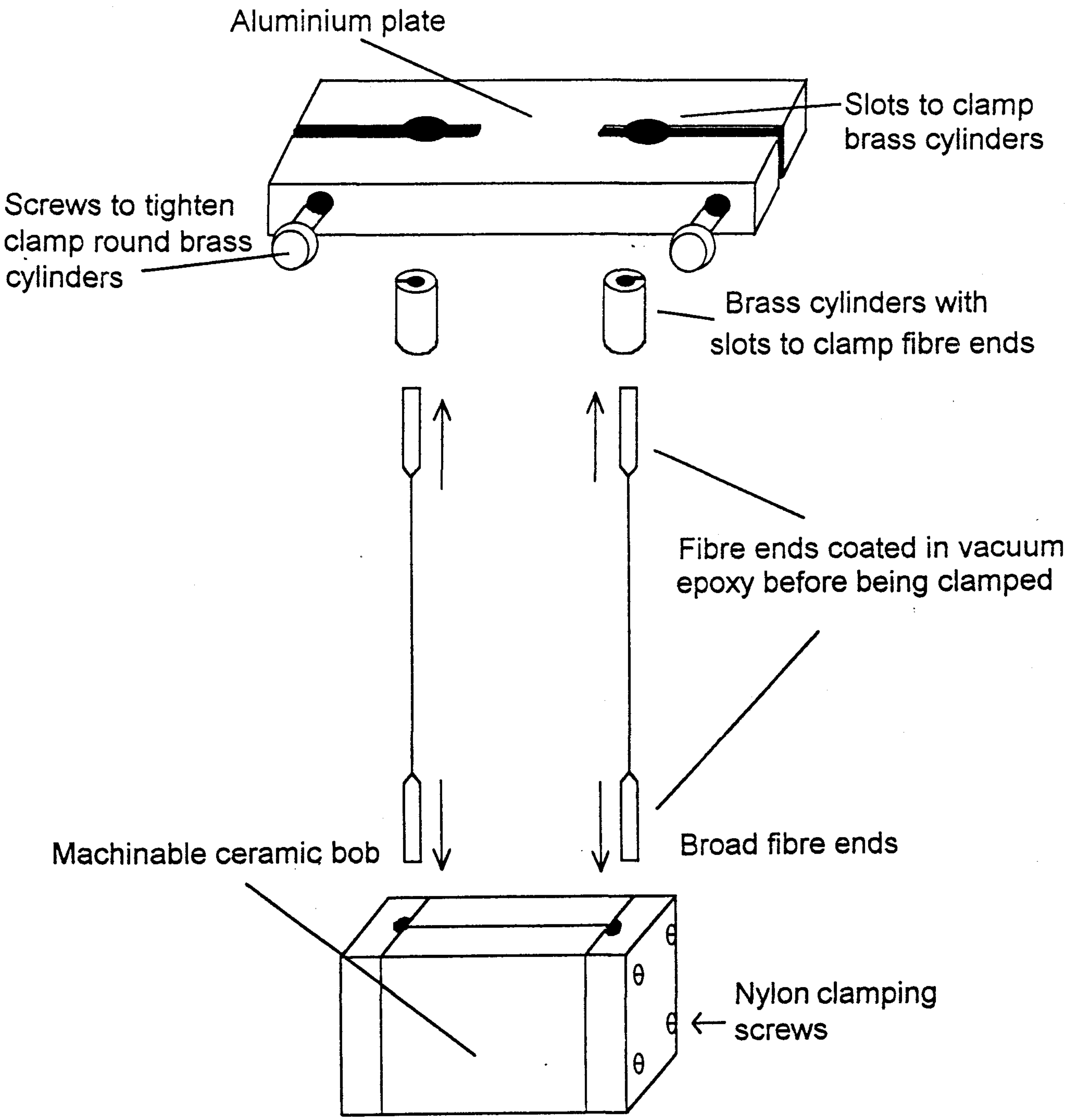
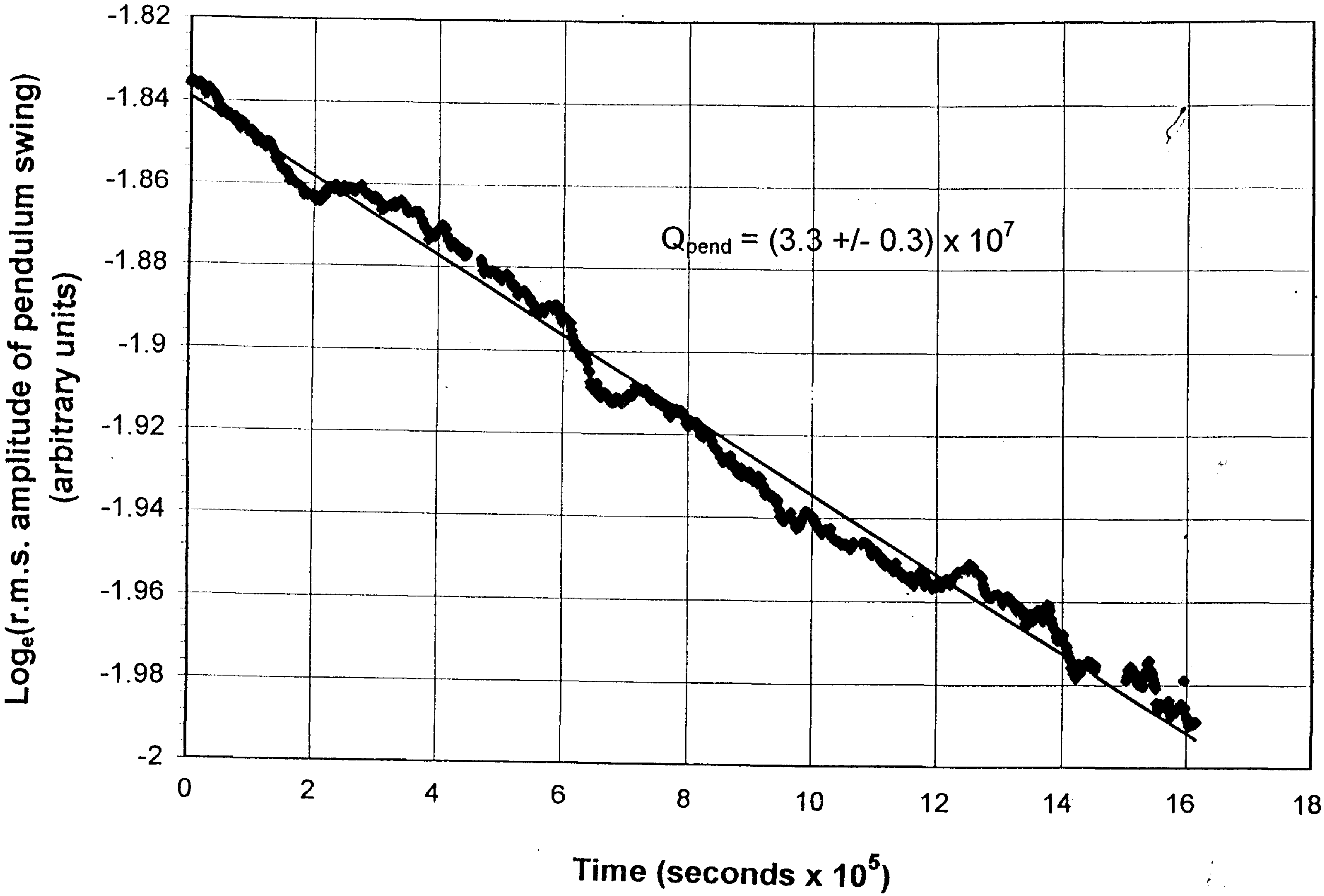

Aluminium plate
Slots to clamp brass cylinders
Screws to tighten clamp round brass cylinders
Brass cylinders with slots to clamp fibre ends
Fibre ends coated in vacuum epoxy before being clamped
Broad fibre ends
Machinable ceramic bob
Nylon clamping screws

FIG. 4



$Q_{pend} = (3.3 +/- 0.3) \times 10^7$
Log$_e$(r.m.s. amplitude of pendulum swing) (arbitrary units)
-1.82
-1.84
-1.86
-1.88
-1.9
-1.92
-1.94
-1.96
-1.98
-2
0
2
4
6
8
10
12
14
16
18
Time (seconds x 10^5)