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Optical vernier technique for in-situ measurement of the length of long Fabry-Perot cavities

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Abstract

We propose a method for in-situ measurement of the length of kilometer size Fabry-Perot cavities in laser gravitational wave detectors. The method is based on the vernier, which occurs naturally when the laser incident on the cavity has a sideband. By changing the length of the cavity over several wavelengths we obtain a set of carrier resonances alternating with sideband resonances. From the measurement of the separation between the carrier and a sideband resonance we determine the length of the cavity. We apply the technique to the measurement of the length of a Fabry-Perot cavity in the Caltech 40m Interferometer and discuss the accuracy of the technique.

1 Introduction

Very long Fabry-Perot cavities serve as measuring devices for interferometric gravitational wave detectors, which are currently under construction [1, 2, 3]. Among them is the Laser Interferometer Gravitational wave Observatory (LIGO) which will have 4 km long cavities [2]. The cavity length, defined as the coating-to-coating distance between its mirrors, is an important parameter for these gravitational wave detectors. It determines the detector sensitivity and its overall performance. Therefore, the length must be known with high accuracy, especially if more than one wavelength of laser is required to resonate in the cavity. Since the length of LIGO Fabry-Perot cavities can change by 0.4 mm due to ambient seismic motion of the ground we do not need to measure the length with accuracy better than a millimeter.

Measurement of distances of order a few kilometers with millimeter accuracy requires special techniques, such as GPS or optical interferometry. Application of the GPS technique would be difficult because the mirrors of the gravitational wave detectors are inside a vacuum envelope and the GPS receivers cannot be placed very close to the reflective surfaces of the mirrors. On the other hand optical interferometry provides both convenient and precise way to measure distances [4]. Common techniques are the method of fractional fringe, the method of synthetic wavelength and the method frequency scanning [5]. Variations of these techniques for different applications are discussed in the SPIE Publication [6]. Although these techniques may provide high precision length measurements (100 μm and better) they are not well suited for Fabry-Perot cavities of the gravitational wave detectors. All these techniques require installation of additional optics, modification of the detector configuration and alignment.

In this paper we propose a technique for in-situ measurement of the cavity length which requires no special equipment or modification to the interferometer. The technique is based on the ability of the Fabry-Perot cavity to resolve close spectral lines. The only requirement is that there be at least two close wavelengths in the laser incident on the Fabry-Perot cavity. This requirement will be easily satisfied by all gravitational wave detectors, which are currently under construction, because optical sidebands are an essential part of their signal extraction schemes.

For single wavelength input laser a Fabry-Perot cavity produces an array of resonances along its optical axis. The resonances are equally spaced and separated by the half-wavelength of the laser. By moving one of the mirrors over several wavelengths, and thus changing the cavity length, we can observe these resonances. Two slightly different wavelengths give rise to two sets of resonances with slightly different spacings, thereby forming a vernier scale along the optical axis.

Mechanical verniers have been extensively used in various precision measurement devices, such as calipers and micrometers. The idea of a vernier is that an enhanced precision is obtained if two slightly different length scales are used simultaneously [7], [8]. The technique we propose here is an extension of the vernier idea to the length scales set by the laser wavelengths.

Our method is similar to the method developed by Vaziri and Chen [9] for application to multimode optical fibres. They obtain the intermodal beat length of the two-mode optical fibres by measuring a separation between the resonances corresponding to these modes. We developed our method independently of them for application to the very long Fabry-Perot cavities in gravitational wave detectors. Although different in motivation and underlying physics our method resembles theirs, because of the common vernier idea.

2 Theory of vernier method

A mechanical vernier is a combination of two length scales which usually differ by 10%. The optical vernier, described in this paper, is made out of two laser wavelengths which differ by roughly one part in 10^8. To use the laser wavelengths in exactly the same way the mechanical verniers are used would be impossible. Instead we relate the optical vernier with the beat length, as we describe below.

Let the primary length scale be a and a secondary length scale be a'. Assume that $a' > a$ and consider two overlapping rulers made out of these length scales, which start at the same point. Let z be a coordinate along the rulers with origin at the starting point. The coordinates for the two sets of marks are

$$z = Na, \tag{1}$$
$$z' = N'a', \tag{2}$$

where N and N' are integers. Each mark on the secondary rule is shifted with respect to the corresponding mark on the primary ruler. The shift accumulates as we move along the z-axis. At some locations along z-axis the shift becomes so large that the mark on the secondary ruler passes the nearest mark on the primary ruler. The first passage occurs at $z = b$, where b is the beat length, defined according to the equation

$$\frac{1}{b} = \frac{1}{a} - \frac{1}{a'}. \tag{3}$$

Other passages occur at multiples of the beat length:

$$y = mb, \tag{4}$$

where m is integer. Thus the number of beats within a given length, z', is equal to the integer part of the fraction z'/b. The beat number, m, is related to the order numbers of the two nearest marks on the different rulers:

$$m = N - N'. \tag{5}$$

Periodicity of the beats manifests itself in the similarity relation

$$\frac{z' - z}{a} = \frac{z' - y}{b}. \tag{6}$$

Derivation of this identity is given in the Appendix.

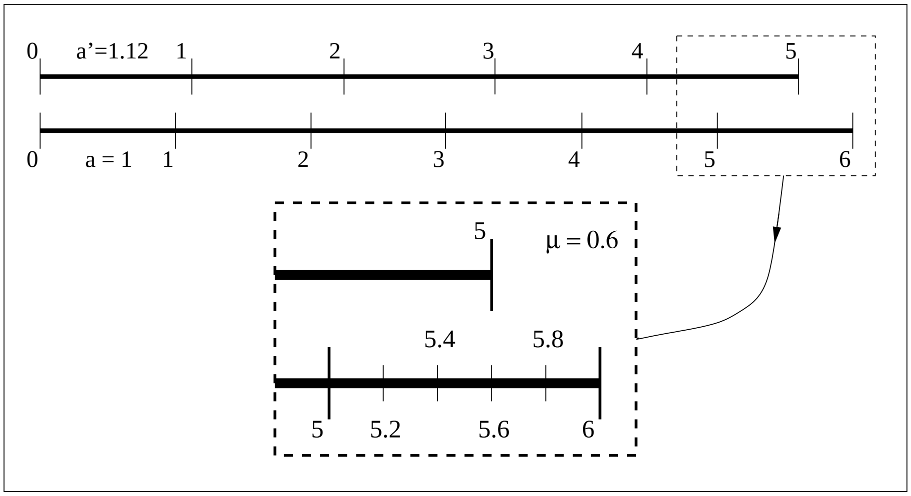
Let us define the shift of the mark at z' on the secondary ruler with respect to the nearest mark at z on the primary ruler as a fraction

$$\mu = \frac{z' - z}{a}. \tag{7}$$

Then the similarity relation, equation (6), allows us to express the length of the secondary ruler in terms of the beat length:

$$z' = y + \mu b \tag{8}$$
$$= (m + \mu)b. \tag{9}$$

Figure 1: An example of vernier. The integers are the order numbers N and N'. The length of the secondary ruler ($z' = 5a'$) is equal to 5.6.



Therefore, if we know the beat number, m, and the shift, μ, we can find the length of the ruler.

We illustrate the method on an example of a vernier with length scales: $a = 1$ and $a' = 1.12$, shown in figure 1. In this case the beat length is $9\frac{1}{3}$. There are no passages within the length shown in the figure, therefore $m = 0$. From the figure we see that the shift is equal to 0.6. Thus we find that the length of the secondary ruler, $z' = \mu b$, is equal to 5.6, which is a correct result as can be seen from the figure.

For a single wavelength laser a Fabry-Perot cavity produces an array of resonances along its optical axis. Two slightly different wavelengths give rise to two overlapping arrays of resonances with slightly different spacings and form a vernier scale.

Let z be a coordinate along the optical axis of the cavity. Assume that the input mirror is placed at $z = 0$ and the end mirror is at $z = L$. In the experiment below different wavelengths are obtained by phase modulation of a single wavelength laser. Let the frequency of the phase modulation be f then the modulation wavelength is $\Lambda = c/f$. Three most prominent components of the phase modulated laser are the carrier with wavelength λ_0 and the first order sidebands with wavelengths $\lambda_{\pm 1}$, which are defined as

$$\frac{1}{\lambda_{\pm 1}} = \frac{1}{\lambda_0} \pm \frac{1}{\Lambda}. \tag{10}$$

Any two wavelengths can be used to form a vernier. For example, the primary scale can be set by the carrier, $a = \frac{1}{2}\lambda_0$, and the secondary scale can be set by either of the sidebands: $a' = \frac{1}{2}\lambda_{\pm 1}$. Then the coordinates for the carrier and the sideband resonances are given by the equations (1)-(2). Correspondingly, the beat length is set by the modulation wavelength:

$$b = \frac{\Lambda}{2}. \tag{11}$$

Thus a vernier occurs in Fabry-Perot cavity when a multiple wavelength laser beam is incident on it. This vernier can be used to find the cavity length. Similar to the length in the equation (9), the cavity length can be can be expressed in terms of the beat length:

$$L = (m + \mu)\frac{\Lambda}{2}. \tag{12}$$

The beat number, m, can be found from the approximate length of the cavity

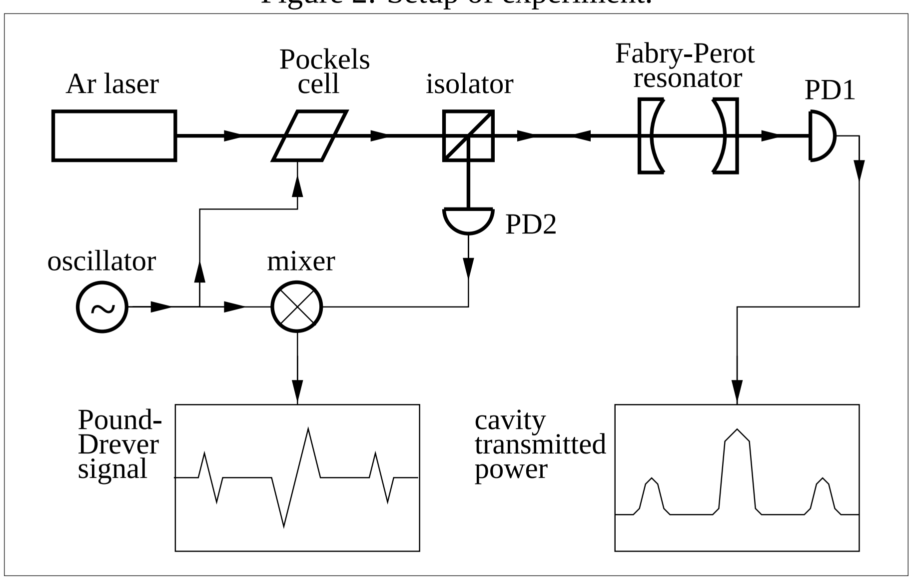
$$m \equiv \text{floor}\left(\frac{L}{\Lambda/2}\right), \tag{13}$$

where "floor" stands for greatest integer less than. As long as the approximate length is known with accuracy better than the beat length the beat number is defined exactly. The shift, μ, can be obtained from observation of the carrier and sideband resonances.

3 Measurement results and discussion

We apply the technique to measure the length of Fabry-Perot cavity of the 40m prototype of LIGO interferometer at Caltech. For our measurement we use one of the arm cavities of the interferometer and the Pound-Drever signal extraction scheme [10]. The setup is shown in figure 2.

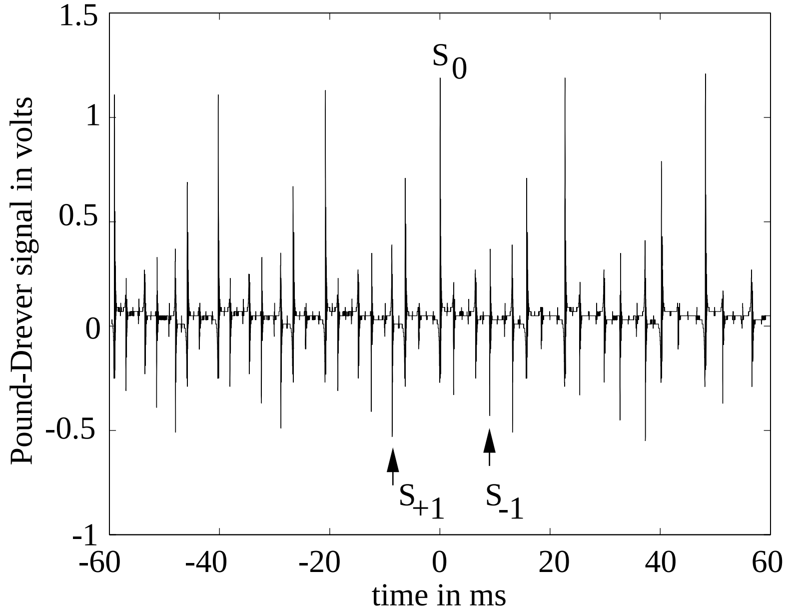
Figure 2: Setup of experiment.



A single wavelength ($\lambda_0 = 514.5$ nm) laser beam is generated by Ar laser. The sidebands on the laser are produced by phase modulation at the Pockels cell, which takes its input from the RF oscillator with modulation frequency of 32.7 MHz. The modulation wavelength corresponding to this frequency is $\Lambda = 9.16795$ m. The resulting multi-wavelength laser beam is incident on the Fabry-Perot cavity, whose approximate length, $L = 38.5 \pm 0.2$ meters, is known from previous measurements. From the approximate length we find the beat number:

$$m = 8. \tag{14}$$

Figure 3: Oscilloscope trace of Pound-Drever signal. The resonances corresponding to the carrier and the sidebands are marked by S_0 and $S_{\pm 1}$. Other resonances result from the higher order modes due to imperfections of the laser and tilts of the mirrors.



Both the input and the end mirror of the cavity are suspended from wires and are free to move along the optical axis of the cavity. The signals are obtained from the photodiodes PD1 and PD2. The signals are: the cavity transmitted power and the Pound-Drever signal, which is the output of the mixer. Although either signal can be used for length measurement we choose the Pound-Drever signal because it provides higher precision than the signal based on the transmitted power.

In the experiment the motion of the front mirror is damped by a local control system and the end mirror is swinging freely through several wavelengths. As the end mirror moves through the resonances sharp peaks appear in the output signals. From the trace on the oscilloscope, figure 3, we obtain the times when the mirror passes through the carrier resonances, $t_0(p)$, and the sideband resonances, $t_{\pm 1}(p)$, where p is integer from 1 to 6. The times are found with a precision of 1 μs, set by the resolution of the oscilloscope.

The carrier resonances are located at

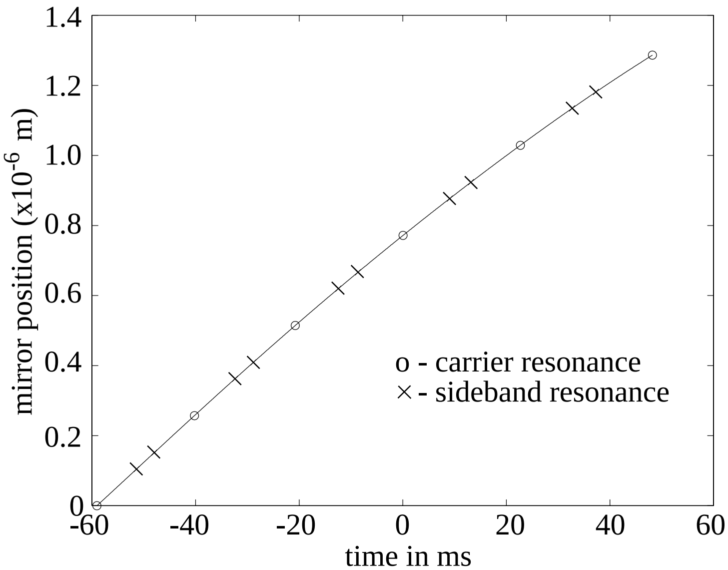
$$z_0(p) = (p-1)\frac{\lambda_0}{2} + u, \tag{15}$$

where u is an unknown constant, which cancels in the calculation. The location of the sideband resonances can be found from the times $t_{\pm 1}(p)$ if the trajectory of the mirror is known. We find the approximate trajectory of the mirror by polynomial interpolation between the carrier resonances. The plot of the interpolated mirror trajectory is shown in figure 4. Let the interpolation polynomial be $F(t)$. Using the polynomial we find the location of the sideband resonances as follows

$$z_{-1}(p) = F(t_{-1}(p)) + u, \tag{16}$$

$$z_{+1}(p) = F(t_{+1}(p)) + u. \tag{17}$$

Figure 4: Interpolated mirror trajectory within the first 6 carrier resonances.



Once the locations of the carrier and the sideband resonances are known we can find the corresponding shifts as

$$\mu(p) = \frac{z_{-1}(p) - z_0(p)}{\lambda_0/2} \tag{18}$$

for the lower sideband, and

$$\mu(p) = \frac{z_0(p) - z_{+1}(p)}{\lambda_0/2} \tag{19}$$

for the upper sideband. The results are shown in the table 1. The average shift and its standard

Table 1: The shifts obtained from the interpolated mirror trajectory. The first and the last fringe contains only one sideband resonance.

p resonance order	μ (-1 sideband)	μ ($+1$ sideband)
1	0.407213	...
2	0.409232	0.410154
3	0.408725	0.408647
4	0.408816	0.409038
5	0.409685	0.409093
6	...	0.408188

deviation is

$$\mu = 0.4089 \pm 0.0008. \tag{20}$$

Using the equation (12) we find the length of the cavity

$$L = 38546 \pm 4 \text{ mm}. \tag{21}$$

The error in the cavity length comes from the error in the beat length and the error in the shift. In our experiment the dominant was the error in the shift, which is mostly the error of the polynomial interpolation. The interpolation error can be greatly reduced if the change in the cavity length is known with high precision. This can be done, for example, by controlling the cavity mirrors at low frequencies.

The limiting precision of the technique, δL, is determined by the signal used to obtain the shift μ. For the transmitted power the limit comes from the finite width of resonances in the Fabry-Perot cavity. A separation between the resonances in the transmitted power can be measured up to a width of a resonance. Therefore,

$$\delta L \sim \frac{\Lambda/2}{\text{Finesse}}, \tag{22}$$

which is roughly 4 mm for our experiment. This precision limit does not depend on the length of the cavity.

There is no limit due to the finite width if the resonances are observed in the Pound-Drever signal. In this case the separation between the resonances are found from zero-crossings or peaks in the Pound-Drever signal and the shifts can be measured with a precision far better than the width of a resonance. For the Pound-Drever signal the limit on the precision is given by the uncertainty in the beat length

$$\frac{\delta L}{L} \sim \frac{\delta \Lambda}{\Lambda}, \tag{23}$$

which is defined by stability of the oscillator. In our case 1 Hz-stability of the oscillator sets the limit of 1 μm to the precision of the technique.

There are two small but noteworthy systematic errors in this method: one is due to the phase change upon reflection off the mirrors, the other is due to the Guoy phase of the Gauss - Hermite modes of Fabry-Perot cavity [11]. If the phase of the reflected laser is not exactly opposite to the phase of the incident laser at the mirror surface the resonances in the cavity become shifted. This effect can be as large as $\lambda/4$ per mirror and is far below the precision of the technique. The Guoy phase also affects the location of the resonances and can be at most $\pi/2$ for the lowest mode of the cavity. Thus the largest contribution due to the Guoy phase is $\lambda/4$ and can also be neglected.

4 Conclusion

The method of optical vernier is a simple and accurate way to measure the cavity length of the laser gravitational wave detectors in-situ. The method requires no special equipment or modification to the detector. We tested the method on the 40m prototype of the LIGO interferometers and attained a precision of 4 mm. The ultimate precision of the method is defined by the uncertainty in the beat length, and is of order a few microns. The method is general and can be used for length measurement of any Fabry-Perot cavity, which allows for small adjustment of its length.

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Appendix

Derivation of the equation (6) is straightforward:

$$\begin{aligned} z' - z &= N'a' - Na && (24) \\ &= N'(a' - a) - (N - N')a && (25) \\ &= N' \frac{aa'}{b} - ma. && (26) \end{aligned}$$

Dividing both sides of this equation by a we obtain the similarity relation, equation (6).

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