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Optical Vernier Technique for Measuring the Lengths of LIGO Fabry-Perot Resonators

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Abstract

We propose to apply a method of vernier for measurement of length of long baseline Fabry-Perot cavities. The vernier occurs naturally when the light incident on the cavity has a sideband. By changing length of the cavity over several wavelengths we obtain a set of carrier resonances alternating with sideband resonances. From the measurement of a separation between the carrier and the sideband resonance we determine the length of the cavity. We apply the technique to measure the length of the Fabry-Perot cavity of the Caltech 40m Interferometer and discuss the accuracy of the technique.

1 Introduction

Fabry-Perot cavities serve as measuring devices for the interferometric gravitational wave detectors, which are currently under construction [1]. The length of these Fabry-Perot cavities is made very large to provide sensitivity to gravitational waves. For example, in LIGO (Laser Interferometer Gravitational wave Observatory) the cavities are 4 km long. In the gravitational wave detectors the length of the Fabry-Perot cavities is an important parameter. It determines the sensitivity of the detector and affects detector's performance. Therefore, the length must be known with high precision.

Measurement of very large distances requires special techniques, such as GPS or optical interferometry. The GPS technique, in principle, can provide high accuracy measurement of the cavity length. However, application of the GPS technique to LIGO cavities would be complicated because the mirrors are inside vacuum envelop. The accuracy of such measurements will not be determined by the accuracy of the GPS technique. It will be limited by uncertainty in the position of the GPS receiver relative to the mirror.

The optical interferometry allows for the measurement of the distance between the mirrors coating - to - coating and, therefore, is free from the problems associated with accurate positioning of measuring devices. Several interferometric techniques for measuring distance were proposed in the past. In paper [2] the authors introduced a method to measure distances by shifting wavelength of the laser. A technique based on optical-frequency-scanning was proposed in [3]. Absolute distance measurements using modulated laser are described in [4], [5]. Common to all these techniques are multiple wavelengths of laser. Although these techniques can provide high accuracy in length measurement they are not suited for Fabry-Perot cavities. Application of these techniques to the arm cavities of the gravitational wave detectors would require modification of the interferometer configuration and alignment.

In this paper we propose a technique designed specifically for Fabry-Perot cavities. It is based on the ability of the Fabry-Perot resonator to resolve close spectral lines. The technique does not require special equipment or modification of the interferometer. The only condition is that the input laser for the Fabry-Perot resonator should consist of at least two close frequencies. This condition is easily satisfied in all interferometric gravitational wave detectors which are currently under construction. All these detectors have optical sidebands as an essential part of their signal extraction schemes. Although we developed the technique for measuring the length of the LIGO Fabry-Perot cavities the technique is general and can applied to other Fabry-Perot cavities.

For single frequency input laser Fabry-Perot resonator produces an array of resonances along its optical axis. The resonances are equally spaced and separated by the half-wavelength of the incident light. By moving one of the mirrors over several wavelength and thus changing the cavity length we can observe these resonances. Two slightly different wavelengths give rise to two sets of resonances with slightly different spacings, thus creating a vernier scale along the resonators' optical axis. This vernier can be used as a length measuring tool similar to mechanical verniers.

Mechanical verniers have been extensively used in various precision measurement devices, such as calipers and micrometers. The idea of a vernier is that a greatly enhanced precision is obtained if two slightly different length scales are used simultaneously [6], [7]. The method we present here is nothing but another application of the vernier idea.

Our method is similar to the method developed by Vaziri and Chen [8] for applications to multimode optical fibers. These authors proposed to measure the intermodal beat length of the two-mode optical fiber by measuring a separation between the resonances corresponding to these modes. We developed our method independently of the work by these authors for applications to the long baseline Fabry-Perot cavities of the gravitational wave detectors. Although different in motivation and underlying physics our method resembles the one described by these authors, because of the common vernier idea.

2 Theory of vernier method

Let the input laser for Fabry-Perot cavity consist of a carrier and a sideband with wavelengths λ and λ_1. These wavelengths define the beat length b according to

$$\frac{1}{b} = \frac{1}{\lambda} - \frac{1}{\lambda_1}. \tag{1}$$

For single wavelength laser the resonances of the Fabry-Perot cavity are equally spaced and separated by half-wavelength of the laser. Let z be a coordinate along the optical axis of the resonator with the origin at the surface of the front mirror. As we move the end mirror away from the front mirror we observe a set of resonances corresponding to the carrier and the sideband. The locations of the mirror, where the resonances occur are

$$z = N\frac{\lambda}{2}, \tag{2}$$

$$z_1 = N_1\frac{\lambda_1}{2}, \tag{3}$$

where N and N_1 are integer numbers. These two sets of resonances have slightly different spacings and constitute the vernier scale.

The carrier and the sideband can resonate simultaneously. This happens if the mirror is placed at the beat nodes

$$y = m\frac{b}{2}, \tag{4}$$

where m is the beat number. The beat number is integer and is related to the order of the carrier and sideband resonance at the coincidence (beat node) as follows

$$m = N - N_1. \tag{5}$$

Periodicity of the beats suggests the following similarity relation. The separation between the sideband resonance and the nearest carrier resonance is proportional to the distance between the mirror and the nearest beat node. For example, if the sideband resonance is located exactly in between the two close carrier resonances the mirror must be exactly in between the two beat nodes. This similarity is described by the identity

$$\frac{z_1 - z}{\lambda/2} = \frac{z_1 - y}{b/2}. \tag{6}$$

The identity can be derived as follows

$$z_1 - z = N_1 \frac{\lambda_1}{2} - N\frac{\lambda}{2} \tag{7}$$

$$= N_1 \frac{\lambda_1 - \lambda}{2} - (N - N_1)\frac{\lambda}{2} \tag{8}$$

$$= N_1 \frac{\lambda_1 \lambda}{2b} - m\frac{\lambda}{2}. \tag{9}$$

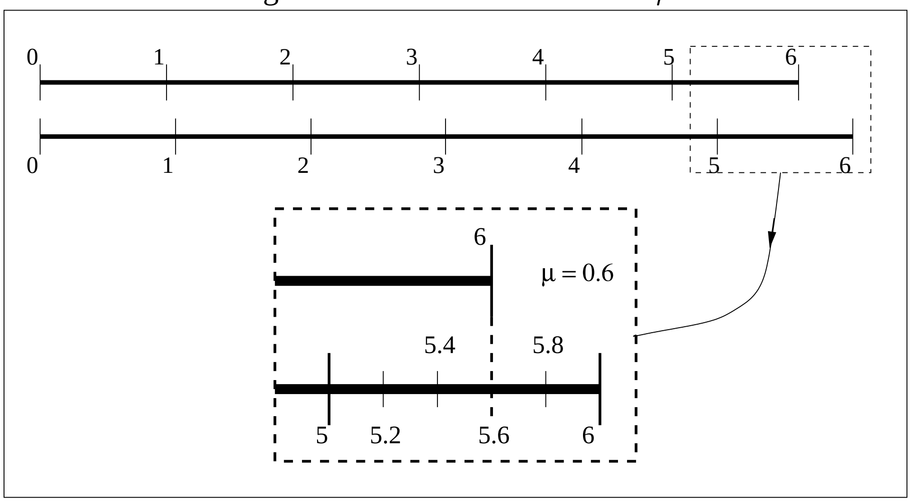
Dividing both sides of this equation by $\lambda/2$ we obtain the identity eq. (6). This identity shows that the sideband-to-carrier distance in units of $\lambda/2$ is the same as the distance between the mirror and the nearest beat node in units of $b/2$.

Let us introduce the ratio

$$\mu = \frac{z_1 - z}{\lambda/2}. \tag{10}$$

Fig. 1 shows several examples of the sideband-carrier separations and the corresponding ratios μ. The horizontal axis represents mirror displacement in units of $\lambda/2$. The ratio, μ, can be found from

Figure 1: Vernier scale and ratio μ



observation of the resonances that occur when the length of the cavity varies over several wavelengths. If we know the ratio μ we can calculate the cavity length using the identity, eq. (6) as follows

$$z_1 = y + \mu\frac{b}{2} \tag{11}$$

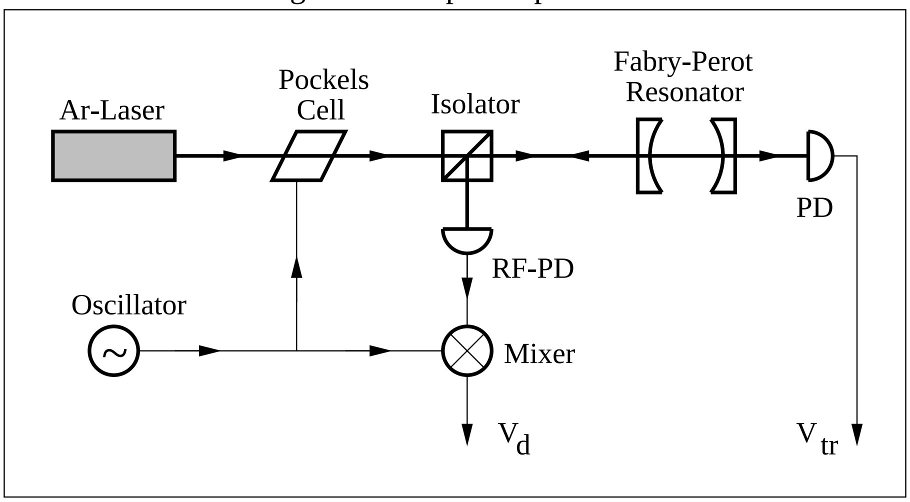
$$= (m + \mu)\frac{b}{2}. \tag{12}$$

Thus to find the length of the cavity we need to know three parameters: the beat length b, the beat number m, and the ratio μ.

3 Measurement results and discussion

We applied the technique to measure the length of the Fabry-Perot resonator of LIGO "40m" interferometer at Caltech. The measurement was a proof of concept and was not aimed at achieving limiting accuracy of the technique. Our experiment utilized the existing setup which was part of Pound-Drever locking scheme. The setup is shown on Fig. 2. Spectra-Physics Ar-laser with wavelength $\lambda = 514.5$

Figure 2: Setup of experiment



nm provided input beam for the Fabry-Perot cavity. The sidebands on the laser were generated by the phase modulation at the Pockels cell. The RF-oscillator provided the reference signal with frequency $f = 32.7$ MHz. Therefore, the beat length between the carrier and the first lower sideband was

$$b = \frac{c}{f} = 9.174 \text{ m}. \tag{13}$$

Two outputs were available to us in the experiment. These were the transmitted light power V_{tr}, measured by the photodiode (PD), and the demodulated output V_d, measured by the RF-tuned photodiode (RF-PD). The demodulated output is the Pound-Drever locking discriminant. Both signals showed sharp resonances corresponding to the carrier and the sideband. Either signals could be used for calculations of the cavity length. However, the demodulated output, V_d, provides a greater precision than the transmitted light power, V_{tr}. For our measurements we used the demodulated output, see Fig. 3.

Approximate length of the resonator, known from previous measurements, is

$$L = 38.5 \pm 0.2 \text{ m}. \tag{14}$$

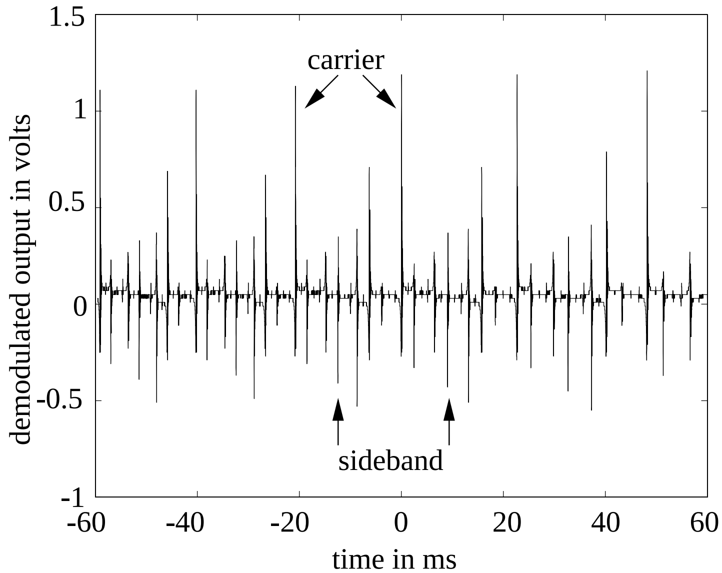
Therefore, the beat number is

$$m = \text{integer}\left[\frac{L}{b}\right] = 8, \tag{15}$$

where "integer" stands for greatest integer less than. The ratio, obtained from the trace of the demodulated output, is

$$\mu = 0.4087 \pm 0.0004. \tag{16}$$

Figure 3: Sideband and carrier resonances



Using the eq. (11) we find the resonator length

$$L = 38.545 \pm 0.004 \text{ m}. \tag{17}$$

The error, approximately 0.01%, is due to uncertainty in determination of the ratio μ from the data.

In the experiment the front mirror was damped by control system and the end mirror was swinging freely through several wavelengths. As the end mirror moved through resonances sharp peaks appeared in time domain traces, taken from the readout channels.

From the data we obtained the times t and t_1 corresponding to the carrier and to the sideband resonances, see Table. 1. The resonances of the carrier are separated by $\lambda/2$. The location of the sideband resonances z_1 can be found from the time series $t_1(p)$ if we know the trajectory of the mirror $x(t)$. We restore the trajectory $x(t)$ by interpolation between the carrier resonances

$$x[t(p+1)] - x[t(p)] = \frac{\lambda}{2}, \tag{18}$$

where $p = 1, 2, \ldots 6$. Then the locations of the sideband resonances are found from the interpolation function $x(t)$. Thus we obtain the ratio as

$$\mu(p) = \frac{x[t_1(p)] - x[t(p)]}{\lambda/2}, \tag{19}$$

The results are collected in the Table 1.

The error in the determination of length comes from the error in the beat length and the error in the ratio μ. Since the beat number is integer there is no error associated with it.

Table 1: Results of experiment

p	carrier peak t (ms)	sideband peak t_1 (ms)	separation μ
1	-59.060	-51.405	0.4072
2	-40.230	-32.390	0.4092
3	-20.765	-12.450	0.4087
4	0.050	09.060	0.4088
5	22.705	32.685	0.4097
6	48.200	- - -	- - -

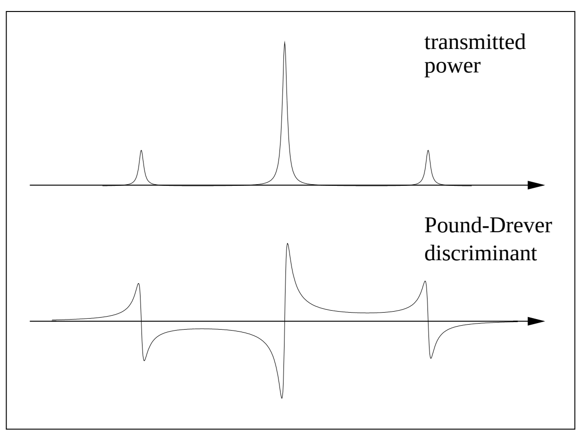
In our experiment the largest error was in the determination of the ratio μ from the data. This error was entirely due to the interpolation. Variation in the values of μ, obtained by the interpolation can be seen from table 1. The interpolation error can be greatly reduced if the change in the cavity length is known with high precision.

The fundamental limit in accuracy of the technique depends on the signal used to obtain the ratio μ. For the transmitted power the limit comes from the finite width of resonance in Fabry-Perot cavity. Separation between the resonance peaks can be measured only upto a width of the resonance. Therefore,

$$\delta L \sim \frac{b}{\text{Finesse}}. \tag{20}$$

This precision limit does not depend on the length of the cavity.

Figure 4: Profiles of output signals



There is no limit due to finite width of resonance if the technique is based on the Pound - Drever locking discriminant. In this case the distance between the resonances is defined by zero - crossings of the demodulated output. These zero - crossings correspond to centers of the resonance peaks and can be found from the date with precision far better than the width of the resonance. For Pound-Drever locking signal the fundamental limit is given by the uncertainty in the beat length

$$\frac{\delta L}{L} \sim \frac{\delta b}{b}, \tag{21}$$

which is defined by stability of the oscillator. To achieve this limit the mirror motion must be slow. Namely, the time it takes for the mirror to move through entire width of resonance must be much less than the cavity storage time. Otherwise the intra-cavity field transients affect the location of the zero - crossings.

By improving stability of the oscillator we can reduce the uncertainty in the beat length. Ultimately, the precision will be limited by the laser line-width

$$\frac{\delta L}{L} \sim \frac{\delta \lambda}{\lambda}. \tag{22}$$

This limit can be comparable to the limit defined by the oscillator stability.

4 Conclusion

We described the method of optical vernier for measuring length of long baseline Fabry-Perot cavities of LIGO interferometers. Using this method we obtained the length of the Fabry-Perot cavity of the LIGO 40m Prototype with the precision of 0.01%. The fundamental limit on the precision of this technique is given by the uncertainty in the beat length.

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